

MAE 8 - Winter 2023 Project

Simulation of Magnus Effect in a Soccer Game

I, Introduction

In this project, you are to simulate the motion of a soccer ball during a free kick. The ball is kicked from an initial position in the field at an initial speed along a given direction. In the field, there are four defenders and a goalkeeper who attempt to block the ball from going into the goal. In flight, the motion of the ball is influenced by gravitational, frictional drag and Magnus forces. The Magnus force which results from the spinning motion causes the ball trajectories to bend. You will analyze the trajectories of the ball for different sets of forcing and initial conditions.

The trajectory of the soccer ball can be described by the following equations:

$$\begin{aligned}\frac{\partial U}{\partial t} &= -C_d \frac{\rho A}{2m} U \sqrt{U^2 + V^2 + W^2} + C_m \frac{\rho A r}{2m} (\omega_y W - \omega_z V), \\ \frac{\partial V}{\partial t} &= -C_d \frac{\rho A}{2m} V \sqrt{U^2 + V^2 + W^2} + C_m \frac{\rho A r}{2m} (\omega_z U - \omega_x W), \\ \frac{\partial W}{\partial t} &= -C_d \frac{\rho A}{2m} W \sqrt{U^2 + V^2 + W^2} + C_m \frac{\rho A r}{2m} (\omega_x V - \omega_y U) - g, \\ \frac{\partial X}{\partial t} &= U, \\ \frac{\partial Y}{\partial t} &= V, \\ \frac{\partial Z}{\partial t} &= W.\end{aligned}\tag{1}$$

The parameters are defined as follows:

$m = 0.4 \text{ (kg)}$: mass of the ball

$r = 0.11 \text{ (m)}$: radius of the ball

$A = \pi r^2 \text{ (m}^2\text{)}$: cross-sectional area of the ball

$\rho = 1.2 \text{ (kg/m}^3\text{)}$: density of air

$g = 9.81 \text{ (m/s}^2\text{)}$: gravity

$C_d = 0.3$: drag coefficient

$C_m = 0.6$: Magnus coefficient

$t \text{ (s)}$: time

$X, Y, Z \text{ (m)}$: position of the ball in rectilinear coordinate

$U, V, W \text{ (m/s)}$: velocity components of the ball in the x, y, and z directions, respectively

ω_x, ω_y and $\omega_z \text{ (rad/s)}$: angular velocity around the x, y and z axis, respectively

$U_{mag,0} \text{ (m/s)}$: initial speed of the ball

θ and $\phi \text{ (}^\circ\text{)}$: kick angles with respect to the positive x-axis and z-axis, respectively

During the analysis, the following parameters are to be computed:

$V_{mag} = \sqrt{U^2 + V^2 + W^2}$ (m/s): speed of the ball

$Acc = dV_{mag}/dt$ (m/s²): acceleration of the ball

$KE = 0.5mV_{mag}^2$ (J): kinetic energy of the ball

$PE = mgZ$ (J): potential energy of the ball

max_height_location: location where the absolute value of vertical velocity W is smallest

final_location: final position of the trajectory

You are to simulate numerous free kicks. The initial position of the ball ($X0, Y0, Z0$), the initial speed ($U_{mag,0}$), the direction angles (θ, ϕ), and the three components of the angular velocities ($\omega_x, \omega_y, \omega_z$) for 100 simulations are given in the file **input_parameter.txt**. The file will be sent to your UCSD email.

The geometries of the field and the goal are given **field.mat** and **goal.mat**, respectively. File **field.mat** contains a structure **field** with 3 fields (X, Y, Z) which give the coordinates of the points used to render the field. File **goal.mat** contains a structure **goal** with 3 fields of the goal post ($Xpost, Ypost, Zpost$) and 3 fields of the net ($Xnet, Ynet, Znet$) which give the coordinates of the points used to render the goal. The surface render of the four defenders and the goalkeeper (i.e., the fifth defender) are produced by the function **defender.m**. Note that the surfaces vary in time as the defenders and the goalkeeper try to block the ball. The file **plot_layout.m** provides an example of how to plot the layout of the soccer field. Download these four files from CANVAS.

II, Approach

Using Euler-Cromer method, equations 1 can be transformed into the following algebraic form:

$$\begin{aligned}
 U_{n+1} &= U_n - \left[C_d \frac{\rho A}{2m} U_n \sqrt{U_n^2 + V_n^2 + W_n^2} - C_m \frac{\rho A r}{2m} (\omega_y W_n - \omega_z V_n) \right] \Delta t, \\
 V_{n+1} &= V_n - \left[C_d \frac{\rho A}{2m} V_n \sqrt{U_n^2 + V_n^2 + W_n^2} - C_m \frac{\rho A r}{2m} (\omega_z U_n - \omega_x W_n) \right] \Delta t, \\
 W_{n+1} &= W_n - \left[C_d \frac{\rho A}{2m} W_n \sqrt{U_n^2 + V_n^2 + W_n^2} - C_m \frac{\rho A r}{2m} (\omega_x V_n - \omega_y U_n) + g \right] \Delta t, \\
 X_{n+1} &= X_n + U_{n+1} \Delta t, \\
 Y_{n+1} &= Y_n + V_{n+1} \Delta t, \\
 Z_{n+1} &= Z_n + W_{n+1} \Delta t.
 \end{aligned} \tag{2}$$

Implement the equations 2 above with $\Delta t = 1/100$ s to obtain the trajectories of the ball. Use the following conditions to end the trajectories:

- The ball needs to be above the ground at $z = 0$ m at all times.

- The ball needs to be inside the perimeter of the field at all times except when it is inside the goal.
- The ball can not be in contact with the defenders or the goalkeeper at any time.

You are to write three MATLAB files: **soccer.m**, **read_input.m** and **project.m**. The descriptions of these files are given below.

- File **soccer.m**: This is the function which solves the equations 2 for the trajectories of the ball for a given set of parameters. The function should have the following header: **function [T, X, Y, Z, U, V, W] = soccer(X0, Y0, Z0, Umag0, theta, phi, omgX, omgY, omgZ)** where the inputs and outputs are defined above. All inputs are scalars while all outputs are vectors.
- File **read_input.m**: This is a function that reads the parameters stored in the text file **input_parameter.txt** into MATLAB. The function should have the following declaration: **function [X0, Y0, Z0, Umag0, theta, phi, omgX, omgY, omgZ] = read_input(input_filename, kick_id)** where **input_filename** is a string denoting the name of the file to be read and **kick_id** is an integer indicating the kick ID number. The outputs are the initial position ($X0, Y0, Z0$), the initial speed (**Umag0**), the direction angles (**theta, phi**), and the components of the angular velocity (**omgX, omgY, omgZ**). When the input **kick_id** is not available in the file, the function should set all outputs to **NaN** and display a warning to screen.
- File **project.m** is a script that includes the results of the tasks (and subtasks) described below. Make sure that the two figures are plotted and a text file **report.txt** is generated when your **project.m** is executed.

III, Tasks to perform

To simulate the trajectories, call function **read_input** to get the necessary parameters and then call function **soccer** to obtain the trajectories. For each simulation, the outputs should include time (T), three components of position (X, Y, Z) and three components of velocity (U, V, W). All of these variables should be vectors with the same length. The project will be graded based on the following 3 tasks.

Task 1: Simulate the kick ID # 1 through 7, and create **figure 1** to show how the seven trajectories vary with different rate of rotation ω_z . The figure should include the following items: the seven trajectories and final locations of the ball, field, goal, defenders and goalkeeper. The surface of the defenders and the goalkeeper should be rendered using the longest time among the seven trajectories. Use function **plot3** and **surf** as demonstrated in **plot_layout.m**.

Task 2: Simulate the kick ID # 8 through 13.

2A. Create **figure 2** to compare the six different trajectories. The figure should include the six trajectories and final locations of the ball, field, goal, defenders and goalkeeper. The surface of the defenders and the goalkeeper should be rendered using the final time of the simulation in which the ball the blocked by the goalkeeper.

2B. Create **figure 3** which includes 6 panels, three panels on the top row and three panels on the bottom row. Use function **subplot** to create the panels. In each of the panel, plot the kinetic energy and the potential energy of the ball versus time. Put time on the horizontal axis.

2C. Create a 6-element data structure named **sim_res** with the following fields:

- **kick_ID**: to include the kick ID number.
- **final_time**: to include the total travel time of the ball.
- **max_height_location**: to include a 3-element vector indicating the location of the ball when it is at the maximum height.
- **final_location**: to include a 3-element vector indicating the final location of the ball.
- **travel_distance**: to include the total travel distance that the ball makes along the trajectories

2D. Use function **fprintf** to create a text file named **report.txt**. The text file should include the following lines:

- Your name on the first line.
- Your PID on the second line.
- A string '**kick_ID, final_time (s), travel_distance (m)**' on the third line.
- Corresponding values of kick ID number, travel time, and travel distance for each of the six trajectories (kick ID: 8 through 13) from the fourth to the ninth lines. Use single digit for the launch number and format %15.9e for others.

Task 3: Between kick ID # 14 through 100, there is one kick that would score a goal. Identify the kick ID and create **figure 4** to show how the goal is made. The figure should include the trajectory and final location of the ball, field, goal, defenders and goalkeeper. Be sure to include the kick ID in the figure title.

IV, Submission instructions:

At the end of your **project.m** script, set the following:

```
p1a = evalc('help read_input');
p1b = evalc('help soccer');
p1c = 'See figure 1';
p2a = 'See figure 2';
p2b = 'See figure 3';
p2c = sim_res(1);
p2d = sim_res(2);
p2e = sim_res(3);
p2f = sim_res(4);
p2g = sim_res(5);
p2h = sim_res(6);
p2i = evalc('type report.txt');
p3a = 'See figure 4';
```

Follow the homework solution template. Remember to **clear all**, **close all**, **clc**, and fill in your name and PID. Set **hw_num = 'project'**. Create a zip archive named **project.zip**. The zip archive should include the following files: **project.m**, **soccer.m**, **read_input.m**, **defender.m**, **input_parameter.txt**, **field.mat**, **goal.mat** and any other scripts / functions that you have written for the project. Make sure that you include all necessary files so that your **project.m** will execute properly. Submit **project.zip** through CANVAS before 10 PM on 03/19/2023. Use double precision unless otherwise stated.