Question. 9-02

다음과 같이 Sigmoid function과 Binary Cross Entropy function이 주어졌을 때, $\frac{\partial L}{\partial z}$ 를 구하라.

$$\hat{y} = \sigma(z) = \frac{1}{1+e^{-z}}$$

$$L = BCE = -[y log(\hat{y}) + (1-y)\log(1-\hat{y})]$$

1) $\frac{\partial L}{\partial z} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z}$ 이므로

$$\frac{\partial L}{\partial \hat{y}} = \frac{\partial}{\partial \hat{y}}\left(-[y\log(\hat{y}) + (1-y)\log(1-\hat{y})]\right)$$

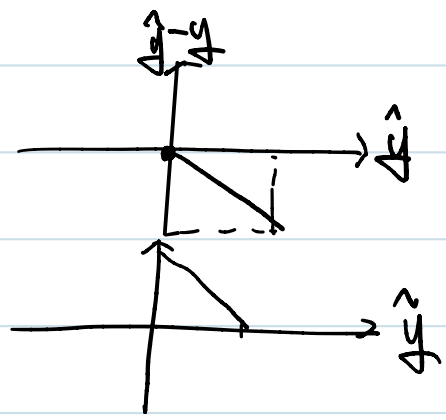
$$= -1\cdot\left(\frac{\partial}{\partial \hat{y}}(y\log(\hat{y})) + \frac{\partial}{\partial \hat{y}}(1-y)(\log(1-\hat{y}))\right)$$

$$= -\frac{\partial}{\partial \hat{y}}\left((y\log(\hat{y})) + (1-y)\log(1-\hat{y})\right)$$

$$= -\left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) = \frac{\hat{y}-y}{\hat{y}(1-\hat{y})}$$

$$\therefore \frac{\partial L}{\partial z} = \frac{\hat{y}-y}{\hat{y}(1-\hat{y})} \cdot \hat{y}(1-\hat{y}) = \hat{y}-y$$

2) $\frac{\partial L}{\partial z} = \hat{y}-y$ 는 $y=0$ 일 때,



의 그래프가 된다.

$y=1$ 일 때,

따라서, $\frac{\partial L}{\partial z}$ 는 prediction과 label의 차이를 잘 나타내는 형태가 된다.