Constrained Multi-variable – Nonlinear Optimization

The problem is stated as follow:

Find X yeild to :

$$X = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

According to Kuhn – Tucker, find minimum of f(X) function:

$$f(X) = z = 8x_1^2 + 10x_2^2 + 12x_1x_2 + 50x_1 - 80x_2$$

Consider:

$$\begin{cases} h_1(X) = x_1 + x_2 - 1 \le 0 \\ h_2(X) = x_1 - \frac{1}{2} \le 0 \\ x_1, x_2 \ge 0 \to \begin{cases} h_3(X) = -x_1 \le 0 \\ h_4(X) = -x_2 \le 0 \end{cases} \end{cases}$$

We replace the constrained problem according to the Kuhn – Tucker condition with the unconstrained optimization problem as follows:

$$K(X, w, v) = f(X) + \sum_{j=1}^{n_j} w_j h_j(X) + \sum_{k=1}^{n_k} v_k l_k(X) = 0$$

$$\to K(X, w, v) = (8x_1^2 + 10x_2^2 + 12x_1x_2 + 50x_1 - 80x_2) + v_1(x_1 + x_2 - 1) + v_2\left(x_1 - \frac{1}{2}\right) - v_3x_1 - v_3x_2 = 0$$

Taking partial derivatives, we have equations

$$\begin{cases} \frac{\partial K}{\partial x_1} = 16x_1 + 12x_2 + v_1 + v_2 - v_3 = -50 \\ \frac{\partial K}{\partial x_2} = 12x_1 + 20x_2 + v_1 - v_3 = 80 \\ \frac{\partial K}{\partial v_1} = x_1 + x_2 = 1 \\ \frac{\partial K}{\partial v_2} = x_1 = \frac{1}{2} \\ \frac{\partial K}{\partial v_3} = -x_1 - x_2 = 0 \end{cases} \to \begin{cases} x_1 = \frac{1}{2} \\ x_2 = \frac{1}{2} \\ v_1 - v_3 = 64 \\ v_2 = -128 \\ v_1 - v_3 = 64 \end{cases}$$

Thus, the function is optimal only when

$$\begin{cases} x_1 = \frac{1}{2} \\ x_2 = \frac{1}{2} \end{cases}$$