Performing t-tests

HYPOTHESIS TESTING IN PYTHON

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Two-sample problems

- Compare sample statistics across groups of a variable
- `converted_comp` is a numerical variable
- `age_first_code_cut` is a categorical variable with levels (`"child"` and `"adult"`)
- Are users who first programmed as a child compensated higher than those that started as adults?

Hypotheses

H_0: The mean compensation (in USD) is **the same** for those that coded first as a child and those that coded first as an adult.

H_0: $\mu_{child} = \mu_{adult}$

H_0: $\mu_{child} - \mu_{adult} = 0$

H_A: The mean compensation (in USD) is **greater** for those that coded first as a child compared to those that coded first as an adult.

H_A: $\mu_{child} > \mu_{adult}$

H_A: $\mu_{child} - \mu_{adult} > 0$

Calculating groupwise summary statistics

```
stack_overflow.groupby('age_first_code_cut')['converted_comp'].mean()
```

```
age_first_code_cut
adult    111313.311047
child    132419.570621
Name: converted_comp, dtype: float64
```

Test statistics

- Sample mean estimates the population mean
- $\bar{x}$ - a sample mean
- $\bar{x}_{child}$ - sample mean compensation for coding first as a child
- $\bar{x}_{adult}$ - sample mean compensation for coding first as an adult
- $\bar{x}_{child} - \bar{x}_{adult}$ - a *test statistic*
- z-score - a (standardized) test statistic

Standardizing the test statistic

$$z = \frac{\text{sample stat} - \text{population parameter}}{\text{standard error}}$$

$$t = \frac{\text{difference in sample stats} - \text{difference in population parameters}}{\text{standard error}}$$

$$t = \frac{(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}}) - (\mu_{\text{child}} - \mu_{\text{adult}})}{SE(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}})}$$

Standard error

$$SE(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}}) \approx \sqrt{\frac{s_{\text{child}}^2}{n_{\text{child}}} + \frac{s_{\text{adult}}^2}{n_{\text{adult}}}}$$

s is the standard deviation of the variable

n is the sample size (number of observations/rows in sample)

Assuming the null hypothesis is true

$$t = \frac{(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}}) - (\mu_{\text{child}} - \mu_{\text{adult}})}{SE(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}})}$$

$$H_0\colon \mu_{\text{child}} - \mu_{\text{adult}} = 0 \quad \rightarrow \quad t = \frac{(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}})}{SE(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}})}$$

$$t = \frac{(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}})}{\sqrt{\frac{s_{\text{child}}^2}{n_{\text{child}}} + \frac{s_{\text{adult}}^2}{n_{\text{adult}}}}}$$

Calculations assuming the null hypothesis is true

```
xbar = stack_overflow.groupby('age_first_code_cut')['converted_comp'].mean()
```

```
adult    111313.311047
child    132419.570621
Name: converted_comp, dtype: float64 age_first_code_cut
```

```
s = stack_overflow.groupby('age_first_code_cut')['converted_comp'].std()
```

```
adult    271546.521729
child    255585.240115
Name: converted_comp, dtype: float64 age_first_code_cut
```

```
n = stack_overflow.groupby('age_first_code_cut')['converted_comp'].count()
```

```
adult    1376
child     885
Name: converted_comp, dtype: int64
```

Calculating the test statistic

$$t = \frac{(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}})}{\sqrt{\frac{s_{\text{child}}^2}{n_{\text{child}}} + \frac{s_{\text{adult}}^2}{n_{\text{adult}}}}}$$

```python
import numpy as np
numerator = xbar_child - xbar_adult
denominator = np.sqrt(s_child ** 2 / n_child + s_adult ** 2 / n_adult)
t_stat = numerator / denominator
```

```
1.8699313316221844
```

Let's practice!

HYPOTHESIS TESTING IN PYTHON

Calculating p-values from t-statistics

HYPOTHESIS TESTING IN PYTHON

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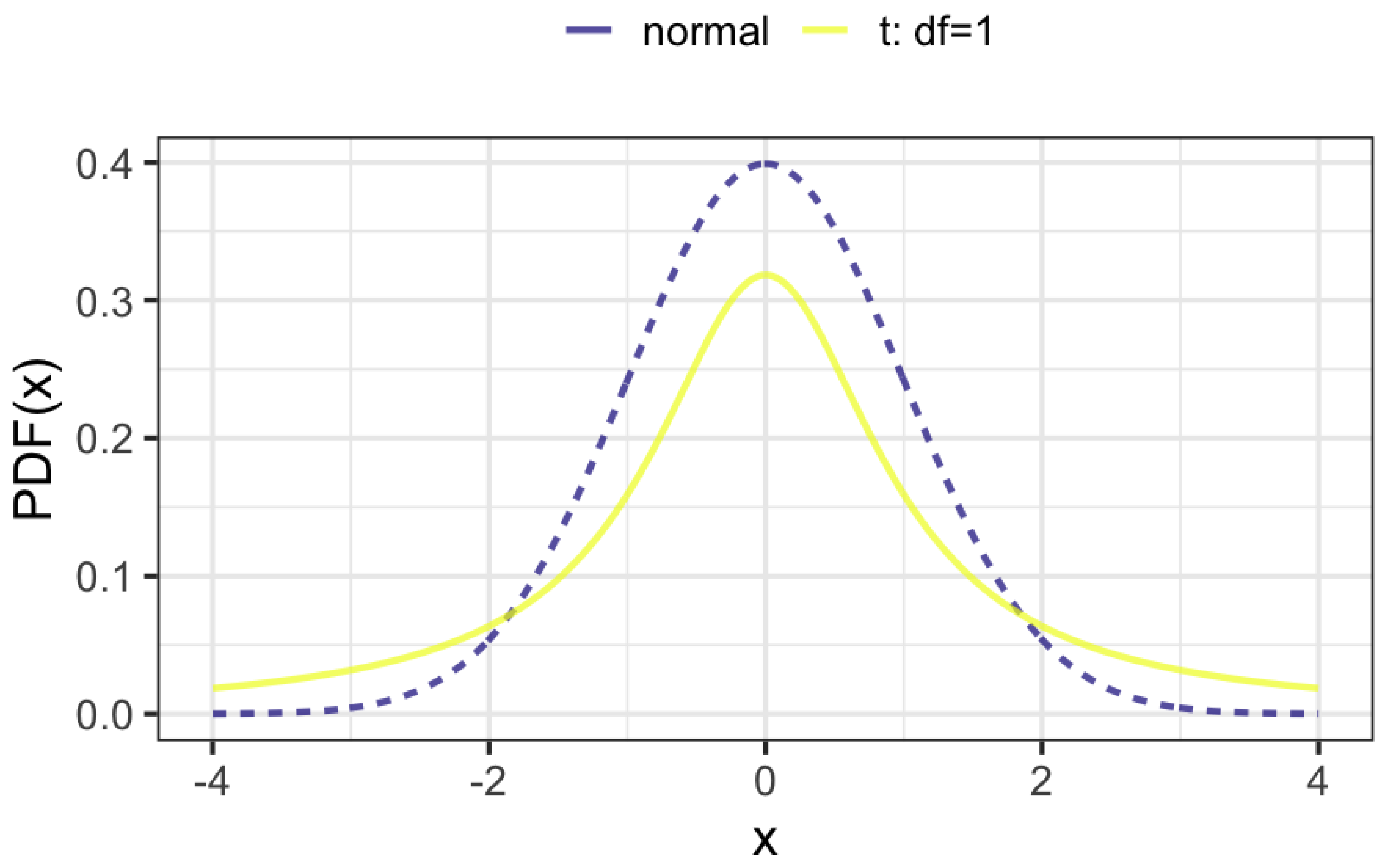
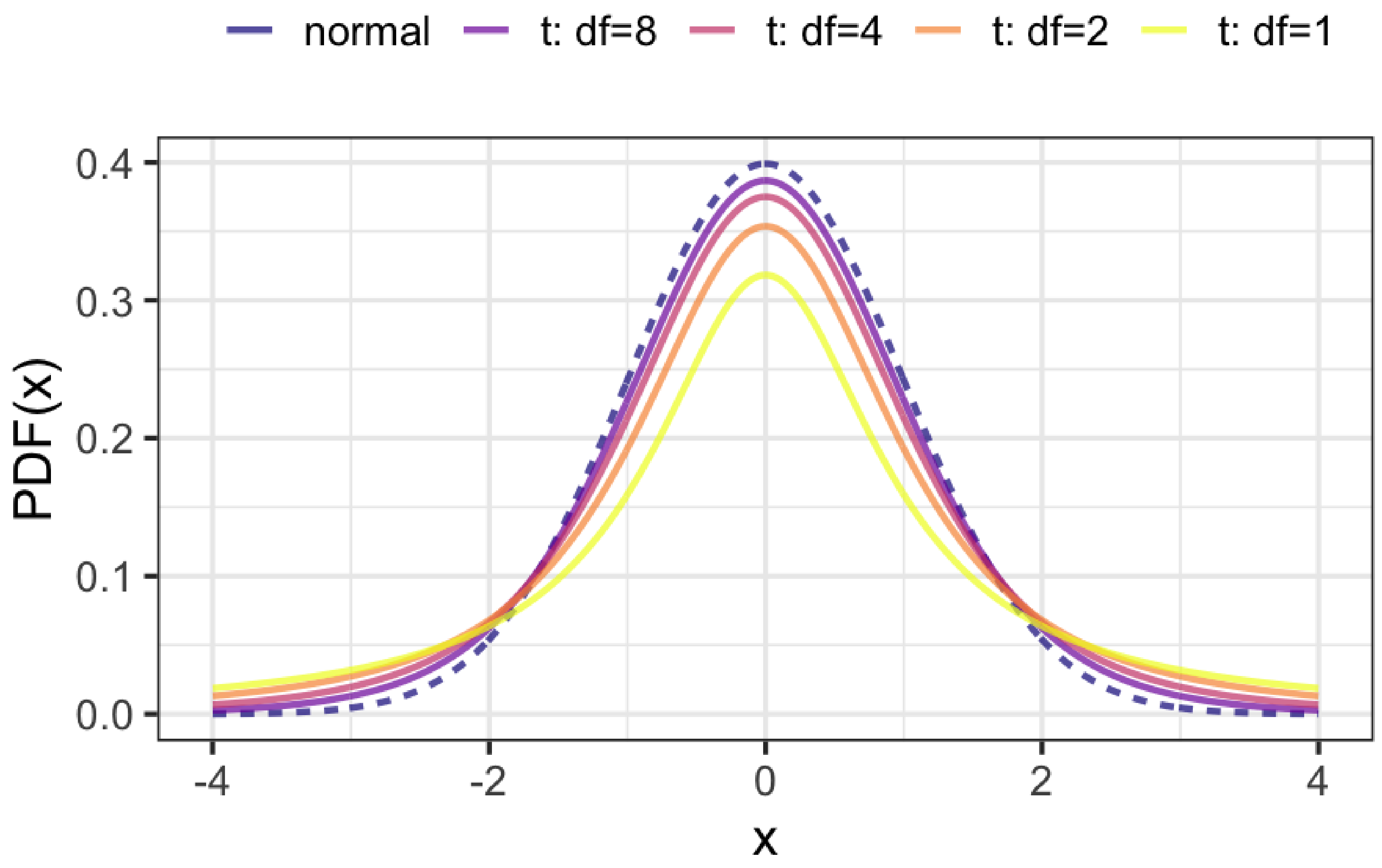
t-distributions

- t statistic follows a t-distribution
- Have a parameter named *degrees of freedom*, or *df*
- Look like normal distributions, with fatter tails



Degrees of freedom

- Larger degrees of freedom $\rightarrow$ t-distribution gets closer to the normal distribution
- Normal distribution $\rightarrow$ t-distribution with infinite df
- Degrees of freedom: maximum number of logically independent values in the data sample



Calculating degrees of freedom

- Dataset has 5 independent observations
- Four of the values are 2, 6, 8, and 5
- The sample mean is 5
- The last value must be 4
- Here, there are 4 degrees of freedom

- $df = n_{child} + n_{adult} - 2$

Hypotheses

H_0: The mean compensation (in USD) is **the same** for those that coded first as a child and those that coded first as an adult

H_A: The mean compensation (in USD) is **greater** for those that coded first as a child compared to those that coded first as an adult

Use a **right-tailed test**

Significance level

$\alpha = 0.1$

If $p \leq \alpha$ then reject H_0.

Calculating p-values: one proportion vs. a value

```
from scipy.stats import norm
1 - norm.cdf(z_score)
```

$$SE(\bar{x}_{\text{child}} - \bar{x}_{\text{adult}}) \approx \sqrt{\frac{s_{\text{child}}^2}{n_{\text{child}}} + \frac{s_{\text{adult}}^2}{n_{\text{adult}}}}$$

- z-statistic: needed when using one sample statistic to estimate a population parameter
- t-statistic: needed when using multiple sample statistics to estimate a population parameter

Calculating p-values: two means from different groups

```
numerator = xbar_child - xbar_adult
denominator = np.sqrt(s_child ** 2 / n_child + s_adult ** 2 / n_adult)
t_stat = numerator / denominator
```

```
1.8699313316221844
```

```
degrees_of_freedom = n_child + n_adult - 2
```

```
2259
```

Calculating p-values: two means from different groups

- Use t-distribution CDF not normal CDF

```
from scipy.stats import t
1 - t.cdf(t_stat, df=degrees_of_freedom)
```

```
0.030811302165157595
```

- Evidence that Stack Overflow data scientists who started coding as a child earn more.

Let's practice!

HYPOTHESIS TESTING IN PYTHON

Paired t-tests

HYPOTHESIS TESTING IN PYTHON

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US Republican presidents dataset

```
         state       county  repub_percent_08  repub_percent_12
0      Alabama         Hale         38.957877         37.139882
1     Arkansas       Nevada         56.726272         58.983452
2   California         Lake         38.896719         39.331367
3   California      Ventura         42.923190         45.250693
..         ...          ...               ...               ...
96   Wisconsin   La Crosse          37.490904         40.577038
97   Wisconsin    Lafayette         38.104967         41.675050
98     Wyoming       Weston         76.684241         83.983328
99      Alaska  District 34         77.063259         40.789626

[100 rows x 4 columns]
```

100 rows; each row represents county-level votes in a presidential election.

[1] https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/VOQCHQ

Hypotheses

Question: Was the percentage of Republican candidate votes lower in 2008 than 2012?

H_0: $\mu_{2008} - \mu_{2012} = 0$

H_A: $\mu_{2008} - \mu_{2012} < 0$

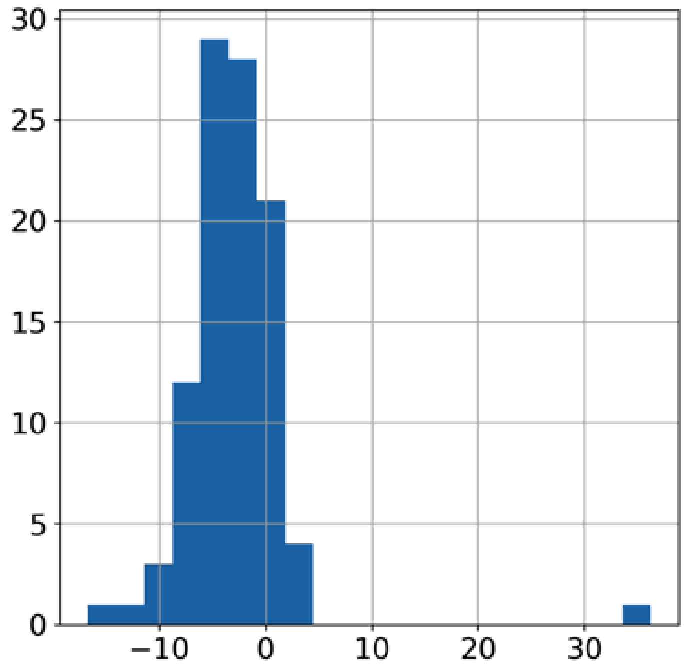
Set $\alpha = 0.05$ significance level.

- Data is **paired** → each voter percentage refers to the same county
  - Want to capture voting patterns in model

From two samples to one

```
sample_data = repub_votes_potus_08_12
sample_data['diff'] = sample_data['repub_percent_08'] - sample_data['repub_percent_12']
```

```
import matplotlib.pyplot as plt
sample_data['diff'].hist(bins=20)
```



Calculate sample statistics of the difference

```
xbar_diff = sample_data['diff'].mean()
```

```
-2.877109041242944
```

Revised hypotheses

Old hypotheses:

H_0: $\mu_{2008} - \mu_{2012} = 0$

H_A: $\mu_{2008} - \mu_{2012} < 0$

$$t = \frac{\bar{x}_{\text{diff}} - \mu_{\text{diff}}}{\sqrt{\frac{s_{diff}^2}{n_{\text{diff}}}}}$$

$$df = n_{diff} - 1$$

New hypotheses:

H_0: $\mu_{\text{diff}} = 0$

H_A: $\mu_{\text{diff}} < 0$

Calculating the p-value

```
n_diff = len(sample_data)
```

```
100
```

```
s_diff = sample_data['diff'].std()
```

```
t_stat = (xbar_diff-0) / np.sqrt(s_diff**2/n_diff)
```

```
-5.601043121928489
```

```
degrees_of_freedom = n_diff - 1
```

```
99
```

$$t = \frac{\bar{x}_{\text{diff}} - \mu_{\text{diff}}}{\sqrt{\frac{s_{\text{diff}}^2}{n_{\text{diff}}}}}$$

$$df = n_{\text{diff}} - 1$$

```
from scipy.stats import t
p_value = t.cdf(t_stat, df=n_diff-1)
```

```
9.572537285272411e-08
```

Testing differences between two means using ttest()

```
import pingouin
pingouin.ttest(x=sample_data['diff'],
               y=0,
               alternative="less")
```

```
               T  dof alternative         p-val          CI95%   cohen-d  \
T-test -5.601043   99        less  9.572537e-08  [-inf, -2.02]  0.560104


             BF10  power
T-test  1.323e+05    1.0
```

[1] Details on Returns from pingouin.ttest() are available in the API docs for pingouin at https://pingouin-stats.org/generated/pingouin.ttest.html#pingouin.ttest.

ttest() with paired=True

```
pingouin.ttest(x=sample_data['repub_percent_08'],
               y=sample_data['repub_percent_12'],
               paired=True,
               alternative="less")
```

```
               T  dof alternative         p-val          CI95%   cohen-d  \
T-test -5.601043   99        less  9.572537e-08  [-inf, -2.02]  0.217364


             BF10     power
T-test  1.323e+05  0.696338
```

Unpaired ttest()

```
pingouin.ttest(x=sample_data['repub_percent_08'],
               y=sample_data['repub_percent_12'],
               paired=False, # The default
               alternative="less")
```

```
              T  dof alternative     p-val         CI95%   cohen-d   BF10  \
T-test -1.536997  198        less  0.062945  [-inf, 0.22]  0.217364  0.927


           power
T-test  0.454972
```

- Unpaired t-tests on paired data increases the chances of false negative errors

Let's practice!

ANOVA tests

HYPOTHESIS TESTING IN PYTHON

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Job satisfaction: 5 categories

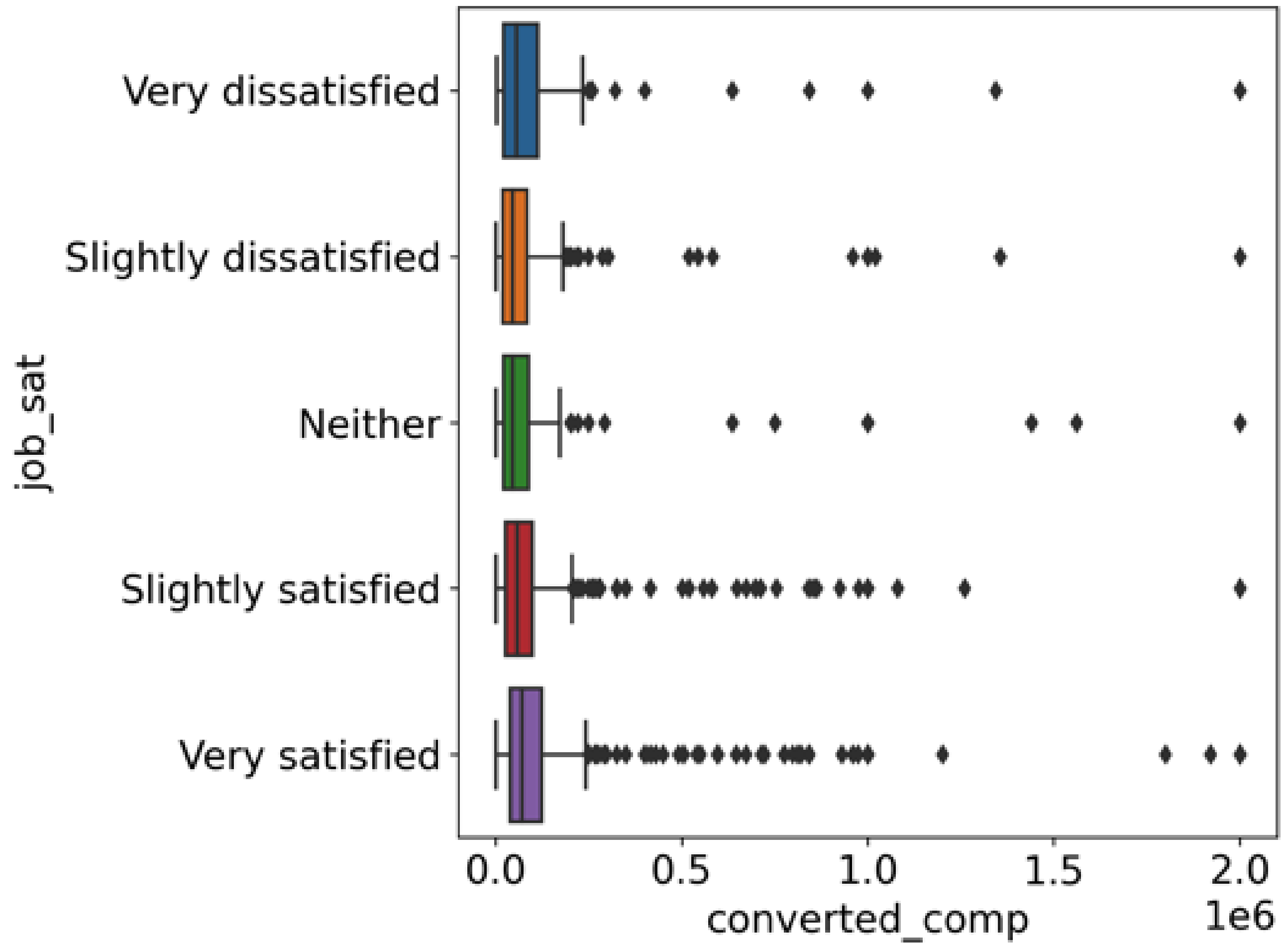
```
stack_overflow['job_sat'].value_counts()
```

```
Very satisfied           879
Slightly satisfied       680
Slightly dissatisfied    342
Neither                  201
Very dissatisfied        159
Name: job_sat, dtype: int64
```

Visualizing multiple distributions

Is mean annual compensation different for different levels of job satisfaction?

```
import seaborn as sns
import matplotlib.pyplot as plt
sns.boxplot(x="converted_comp",
            y="job_sat",
            data=stack_overflow)
plt.show()
```



Analysis of variance (ANOVA)

- A test for differences *between* groups

```
alpha = 0.2

pingouin.anova(data=stack_overflow,
               dv="converted_comp",
               between="job_sat")
```

```
    Source  ddof1  ddof2         F     p-unc       np2
0  job_sat      4   2256  4.480485  0.001315  0.007882
```

- `0.001315` $< \alpha$
- At least two categories have *significantly different* compensation

Pairwise tests

- $\mu_{\text{very dissatisfied}} \neq \mu_{\text{slightly dissatisfied}}$
- $\mu_{\text{very dissatisfied}} \neq \mu_{\text{neither}}$
- $\mu_{\text{very dissatisfied}} \neq \mu_{\text{slightly satisfied}}$
- $\mu_{\text{very dissatisfied}} \neq \mu_{\text{very satisfied}}$
- $\mu_{\text{slightly dissatisfied}} \neq \mu_{\text{neither}}$
- $\mu_{\text{slightly dissatisfied}} \neq \mu_{\text{slightly satisfied}}$
- $\mu_{\text{slightly dissatisfied}} \neq \mu_{\text{very satisfied}}$
- $\mu_{\text{neither}} \neq \mu_{\text{slightly satisfied}}$
- $\mu_{\text{neither}} \neq \mu_{\text{very satisfied}}$
- $\mu_{\text{slightly satisfied}} \neq \mu_{\text{very satisfied}}$

Set significance level to $\alpha = 0.2$.

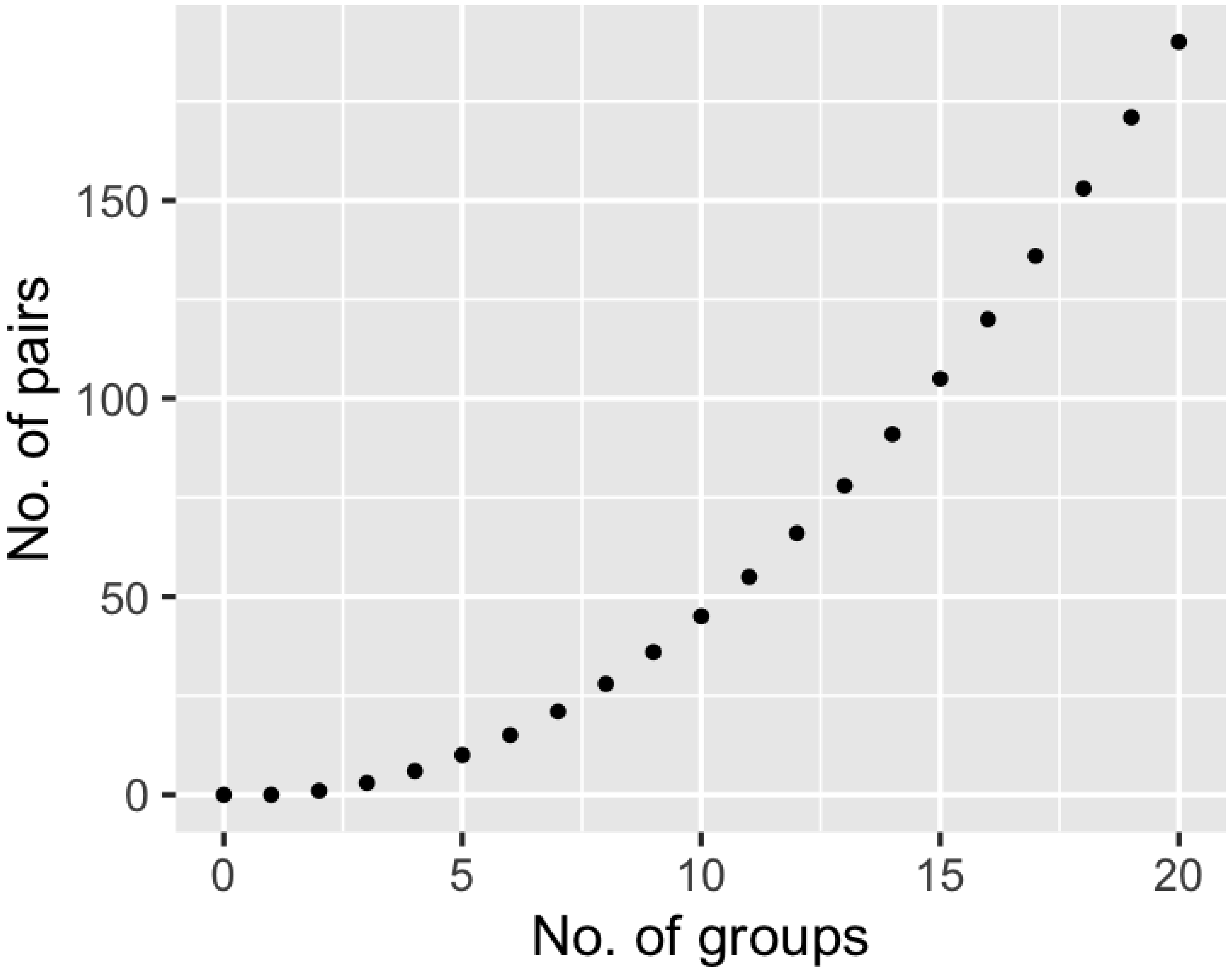
pairwise_tests()

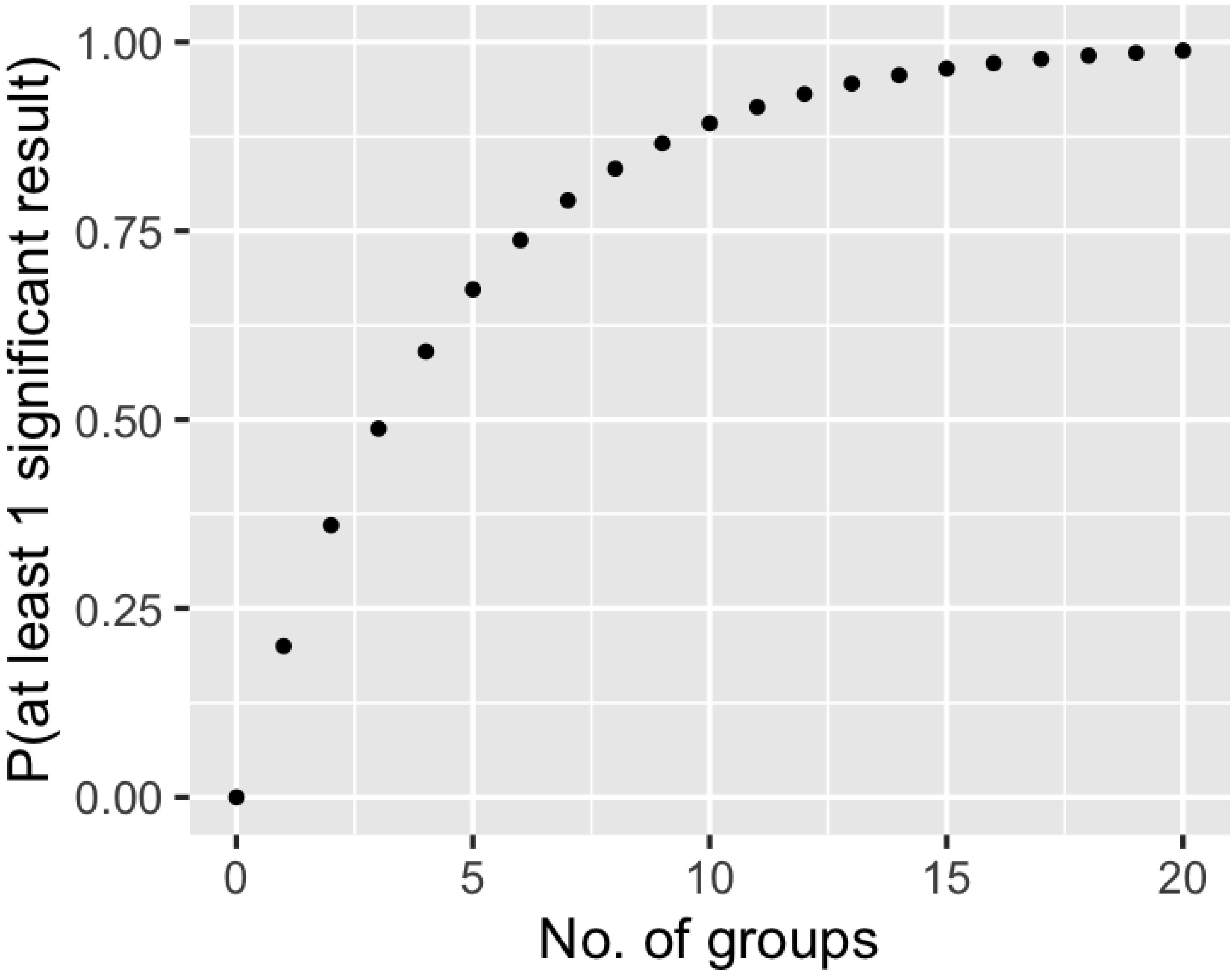
```
pingouin.pairwise_tests(data=stack_overflow,
                        dv="converted_comp",
                        between="job_sat",
                        padjust="none")
```

```
  Contrast                    A                     B  Paired  Parametric  ...          dof  alternative     p-unc     BF10    hedges
0  job_sat  Slightly satisfied        Very satisfied   False        True  ...  1478.622799    two-sided  0.000064  158.564 -0.192931
1  job_sat  Slightly satisfied               Neither   False        True  ...   258.204546    two-sided  0.484088    0.114 -0.068513
2  job_sat  Slightly satisfied     Very dissatisfied   False        True  ...   187.153329    two-sided  0.215179    0.208 -0.145624
3  job_sat  Slightly satisfied  Slightly dissatisfied   False        True  ...   569.926329    two-sided  0.969491    0.074 -0.002719
4  job_sat      Very satisfied               Neither   False        True  ...   328.326639    two-sided  0.097286    0.337  0.120115
5  job_sat      Very satisfied     Very dissatisfied   False        True  ...   221.666205    two-sided  0.455627    0.126  0.063479
6  job_sat      Very satisfied  Slightly dissatisfied   False        True  ...   821.303063    two-sided  0.002166     7.43  0.173247
7  job_sat             Neither     Very dissatisfied   False        True  ...   321.165726    two-sided  0.585481    0.135 -0.058537
8  job_sat             Neither  Slightly dissatisfied   False        True  ...   367.730081    two-sided  0.547406    0.118  0.055707
9  job_sat   Very dissatisfied  Slightly dissatisfied   False        True  ...   247.570187    two-sided  0.259590    0.197  0.119131

[10 rows x 11 columns]
```

As the number of groups increases...





Bonferroni correction

```python
pingouin.pairwise_tests(data=stack_overflow,
                        dv="converted_comp",
                        between="job_sat",
                        padjust="bonf")
```

```
  Contrast                    A                      B  ...     p-unc    p-corr p-adjust       BF10    hedges
0  job_sat  Slightly satisfied         Very satisfied  ...  0.000064  0.000638     bonf    158.564 -0.192931
1  job_sat  Slightly satisfied                Neither  ...  0.484088  1.000000     bonf      0.114 -0.068513
2  job_sat  Slightly satisfied      Very dissatisfied  ...  0.215179  1.000000     bonf      0.208 -0.145624
3  job_sat  Slightly satisfied  Slightly dissatisfied  ...  0.969491  1.000000     bonf      0.074 -0.002719
4  job_sat      Very satisfied                Neither  ...  0.097286  0.972864     bonf      0.337  0.120115
5  job_sat      Very satisfied      Very dissatisfied  ...  0.455627  1.000000     bonf      0.126  0.063479
6  job_sat      Very satisfied  Slightly dissatisfied  ...  0.002166  0.021659     bonf       7.43  0.173247
7  job_sat             Neither      Very dissatisfied  ...  0.585481  1.000000     bonf      0.135 -0.058537
8  job_sat             Neither  Slightly dissatisfied  ...  0.547406  1.000000     bonf      0.118  0.055707
9  job_sat   Very dissatisfied  Slightly dissatisfied  ...  0.259590  1.000000     bonf      0.197  0.119131

[10 rows x 11 columns]
```

More methods

padjust : *string*

Method used for testing and adjustment of pvalues.

- `'none'` : no correction [default]
- `'bonf'` : one-step Bonferroni correction
- `'sidak'` : one-step Sidak correction
- `'holm'` : step-down method using Bonferroni adjustments
- `'fdr_bh'` : Benjamini/Hochberg FDR correction
- `'fdr_by'` : Benjamini/Yekutieli FDR correction

Let's practice!

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