量子力学での測定 と波動関数の収縮. (坂田量子本 chap 9.)

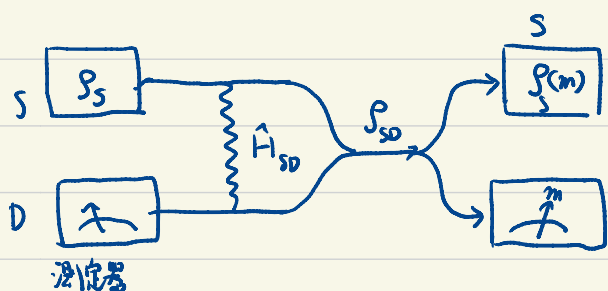S系の 物理量 $\hat{O}$ を測定したい. $\hat{O} = \sum_n |n\rangle\langle n| O_n.$ $\hat{P}_O(n) = |n\rangle\langle n|$

$\hat{M} = \sum_m |u_m\rangle\langle u_m| m.$ $\hat{P}_M(m) = |u_m\rangle\langle u_m|$

ρ_S, ρ_D が 測定前の 状態.

測定器

- ρ_{SD} についての Born則のみ用いて $\hat{M} = m$ を得たときの $\hat{\rho}(m)$ を求める.

Born則 ⑨ $\boxed{\hat{p}(\hat{O}=O_n ; \hat{M}=m) = \hat{p}(\hat{O}=O_n|m)\,\hat{p}(\hat{M}=m)}$

$$\boxed{\hat{p}(\hat{O}=O_n ; \hat{M}=m)} = \mathop{Tr}_{SD}\left\{\rho_{SD}\left(\hat{P}_O(n)\otimes\hat{I}\right)\left(\hat{I}\otimes\hat{P}_M(m)\right)\right\}$$

$\parallel$ なぜ

$$= \mathop{Tr}_{S}\left[\mathop{Tr}_{D}\left(\rho_{SD}\left(\hat{I}\otimes\hat{P}_M(m)\right)\right)\hat{p}_O(n)\right]$$

$$\boxed{\hat{p}(\hat{O}=O_n|m)\times\hat{p}(\hat{M}=m)} = \left\{\mathop{Tr}_{S}\left(\rho_S(m)\,\hat{P}_O(n)\right)\right\}\left\{\mathop{Tr}_{SD}\rho_{SD}\left(\hat{I}\otimes\hat{P}_M(m)\right)\right\}$$

$$= \mathop{Tr}_{S}\left[\hat{P}_S(m)\left[\mathop{Tr}_{SD}\rho_{SD}\left(\hat{I}\otimes\hat{P}_M(m)\right)\right]\hat{p}_O(n)\right]$$

$$\boxed{\hat{\rho}_S(m)\left(\mathop{Tr}_{SD}\hat{\rho}_{SD}\left(\hat{I}\otimes\hat{P}_M(m)\right)\right) = \mathop{Tr}_{D}\hat{\rho}_{SD}\left(\hat{I}\otimes\hat{P}_M(m)\right)}$$

$$\boxed{\rho_S(m) = \frac{\mathop{Tr}_{D}\left(\hat{I}\otimes\hat{P}_M(m)\right)\hat{\rho}_{SD}\left(\hat{I}\otimes\hat{P}_M(m)\right)}{\mathop{Tr}_{SD}\hat{\rho}_{SD}\left(\hat{I}\otimes\hat{P}_M(m)\right)}}$$

波動関数の収縮