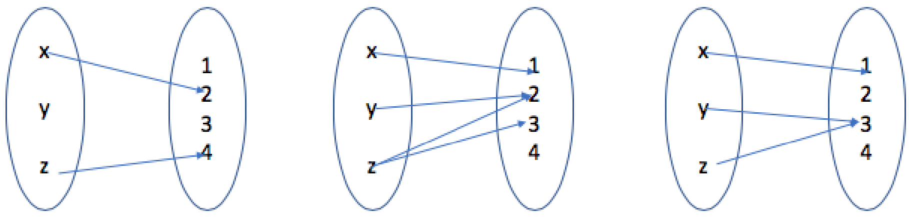
Discrete Mathematics

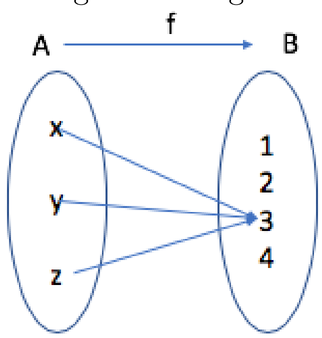
Tutorial sheet | Funtions

Question 1.
Let A and B be two sets with $A = \{x, y, z\}$ and $B = \{1, 2, 3, 4\}$. Which of the following arrow diagrams define functions from A to B?



(i) (ii) (iii)

Question 2.
Let A and B be two sets with $A = \{x, y, z\}$ and $B = \{1, 2, 3, 4\}$. Let f from A to B defined by the following arrow diagram:



1. Write the domain, the co-domain and the range of f.
2. Find $f(x)$ and $f(y)$.
3. Write down the set of pre-images of 3 and the set of pre-images of 1 .
4. represent f as a set of ordered pairs.

Question 3.

The Hamming distance function is very important in coding theory. It gives a measure of the difference between two strings of 0's and 1's that have the same length. Let S_n be the set of all strings of 0's and 1's of length n. The Humming function H is defined as follows:

$H: \;\; S_n \times S_n \to \mathbb{N} \cup \{0\}$

$\quad (s,t) \to H(s,t) =$ The number of positions in which s and t have different values.

For $n = 5$, Find $H(11111, 00000)$, $H(11000, 00000)$, $H(00101, 01110)$ and $H(10001, 01111)$.

Question 4.

Digital messages consist of a finite sequence of 0's and 1's. When they are communicated across a transmission channel, they are frequently coded in special ways to reduce the chance that they will be garbled by interfering noise in the transmission lines. A simple way to encode a message of 0's and 1's is to write each bit three times, for example: the message 0010111 would be encoded as 000 000 111 000 111 111 111.

Let A be the set of all strings of 0's and 1's and let E and D be the encoding and the decoding function on the set A defined for each string, s, in A as follows:

$E(s) =$ The string obtained from s by replacing each bit of s with the same bit written three times.

$D(s) =$ The string obtained from s by replacing each consecutive triple of three identical bits of s by a single copy of that bit.

Find $E(0110), E(0101), D(000111000111000111111)$ and $D(111111000111000111000000)$

Question 5.

Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{a, b, c, d\}$ and $C = \{w, x, y, z\}$ be three sets.

Let f and g be two functions defined as follows:

$f : A \to B$ is defined by the following table.

x	1	2	3	4	5	6
$f(x)$	a	b	a	c	d	d

$g : B \to C$ is defined by the following table.

x	a	b	c	d
$g(x)$	w	x	y	z

1. Draw arrow diagrams to represent the function f and g.
2. List the domain; the co-domain and the range of f and g.
3. Find $f(1)$, the ancestor (pre-image) of d. and (g o f)(3)
4. Show that f is not a one to one function.
5. Show that f is an onto function.

6. Show that g is both one to one and onto.

Question 6.
Suppose you read that a function $f : \mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{Q}$ is defined by the formula $f(m, n) = \frac{m}{n}$ for all $(m, n) \in \mathbb{Z} \times \mathbb{Z}^+$.

1. Is f a one to one function?

2. Is f an onto function?

Question 7.

Given a function f definted by $f(x) = \lfloor x \rfloor$. where $f : \mathbb{R} \to \mathbb{Z}$,

1. Plot the graph of the function f(x) for $x \in [-3, 3]$.

2. Use this graph to find $\lfloor \pi \rfloor$, $\lfloor -2.5 \rfloor$, $\lfloor -1 \rfloor$.

3. Use the graph in (1) to show that f is not a one to one (not injective) function.

4. Is f onto (surjective)? Justify your answer.

Question 8.
Let S denote the set of all 3 bit binary strings and $B = (0, 1, 2, 3)$. The function $f : S \to B$ is defined by the rule

f(x)= the number of zeros in x for each $x \in S$.

Find the following.

1. The domain of f.

2. $f(001)$ and $f(101)$.

3. The set of ancestors of 2.

4. The range of f.

5. Say whether or not f is one to one, giving a reason for your answer.

6. Say whether or not f is onto, giving a reason for your answer.

Question 9.
Let $f(x) = x \bmod 3$, where $f(x)$ is the remainder when x is divided by 3, and $f : \mathbb{Z}^+ \to \{0, 1, 2\}$.

1. Find $f(7)$ and $f(12)$.
2. Find the ancestors of 2.
3. Say whether or not $f(x)$ is one to one, justifying your answer.
4. Say whether or not $f(x)$ is onto, justifying your answer.

Question 10.

Given the following function $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = 4x - 1$, for any real number x.

1. Is f a one to one function? Prove or give a counter-example.
2. Is f an onto function? Prove or give a counter-example.
3. Is f invertible? and why? if the answer yes define f^{-1}.

Question 11.
Given the following function $f : \mathbb{Z} \to \mathbb{Z}$ with $g(x) = 4x - 1$, for any real number x.

1. Is g a one to one function? Prove or give a counterexample.
2. Is g an onto function? Prove or give a counterexample.
3. Is g invertible? and why? if the answer yes define g^{-1}.

Question 12.

Given the following function $h : \mathbb{R} \to \mathbb{R}$ with $h(x) = x^2 - 1$, for any real number x.

1. What is co-domain and the range of h
2. Is h a one to one function? Prove or give a counterexample.
3. Is h an onto function? Prove or give a counterexample.
4. Is h invertible? and why? if the answer yes define h^{-1}.

Question 13.
Given the following function $h : [0, +\infty[\to [-1, +\infty[$with $h(x) = x^2 - 1$, for any real number x.

1. What is co-domain and the range of h

2. Is h a one to one function? Prove or give a counterexample.

3. Is h an onto function? Prove or give a counterexample.

4. Is h invertible? and why? if the answer yes define h^{-1}.

5. On the same graph, plot the curve of h and that of h^{-1} if it exists.

Question 14.
Consider the following function $f : \mathbb{R} \to \mathbb{R}^+$ with $f(x) = 2^{x+3}$.

1. Show that f is a bijective function.

2. Find the inverse function f^{-1}.

3. Plot the both curves of f and of f^{-1} on the same graph.

Question 15.
Consider the following function $f : \mathbb{R} - \{-1\} \to \mathbb{R}$ with $f(x) = \frac{2x}{x+1}$.

1. Show that f is a one to one function.

2. Show that f is not an onto funcion.

Question 16.

Find the inverse of the following functions:

1. $f(x) = e^{x^2-5}$

2. $g(x) = e^x + 5$

Question 17.
Find the inverse of the following functions:

1. $f(x) = \ln(x+2) + 2$

2. $g(x) = \log_2(x-5) + 3$

Question 18.
Let A, B and C be three sets/and $f : A \to B$ and $g : B \to C$ be two functions. Prove that if gof is an onto function then g must be onto.
Question 19.
Let A, B and C be three sets and $f : A \to B$ and $g : B \to C$ be two functions. Prove that if $g\o f$ is a one to one function then f must be one to one.
Question 20.
Let $f : \mathbb{Q} \times \mathbb{Q} \to \mathbb{R}$ with $f(x, y) = x + \sqrt{2}y$ for all $x, y \in \mathbb{Q}$
Is f a one to one function? Prove or give a counter-example.

End of questions