UNIVERSITY OF LONDON

BSc COMPUTER SCIENCE

CM1025 Fundamentals of Computer Science

Midterm coursework assignment

This coursework assignment consists of **FIVE** questions. You should answer **ALL** questions.

There are 50 marks available in this coursework assignment. The marks for each question are indicated at the end of the part in [.] brackets. **Full marks will be awarded for complete answers to a total of FIVE questions. Make sure to show your work clearly in each of the questions.**

The aim of this assessment is to give you the opportunity to consolidate your learning and to assess your understanding of the topics.

Submission requirements

Please submit **one** PDF document for this coursework assignment.

IMPORTANT: you must submit work that is properly formatted using the maths mode of your word processor. Any handwritten submissions will be subject to penalty of being capped at 40 marks.

Question 1

Show your work. Full marks will only be awarded if the workings are shown.

Answer the following questions. Explain your reasoning:

(a) Without using the truth table, determine whether the following two expressions are logically equivalent:

$(P \vee Q) \rightarrow R$ and $(P \rightarrow R) \wedge (Q \rightarrow R)$

Hint: Try reasoning through each direction of the equivalence.

[3 marks]

(b) Which of the following is a tautology?

A. $(P \vee \neg P)$
B. $(P \wedge \neg P)$
C. $(P \leftrightarrow P)$
D. $(P \rightarrow P)$
E. All of the above

[3 marks]

(c) If $P \rightarrow (Q \rightarrow R)$ is logically equivalent to $(P \wedge Q) \rightarrow R$, prove it using logical equivalences only, no truth tables, no examples.

[2 marks]

(d) Translate this into formal logic and determine if the structure is valid:
"Everyone who owns a cat loves animals. Some people own a cat. Therefore, some people love animals."

[2 marks]

Question 2

Show your work. Full marks will only be awarded if the workings are shown.

(a) If a function $f: R \to R$ is continuous and $f(x) \neq 0$ for all $x \in R$, then $1/f(x)$ is also continuous.

(Hint: Think logically – prove the contrapositive: If $1/f(x)$ is not continuous, then $f(x) = 0$ at some point.)

[2 marks]

(b) Prove that if $\sqrt{2} \,.\sqrt{3}$ is rational, then both $\sqrt{2}$ and $\sqrt{3}$ are rational. Try assuming the product is rational and at least one factor is irrational.

[2 marks]

(c) How many non-negative integer solutions are there to:

$$x + y + z = 12$$

such that $x \leq 5$, $y \leq 6$, and $z \leq 7$?

[4 marks]

(d) How many ways can six people sit around a circular table if two particular people must not sit next to each other?

[2 marks]

Question 3

Show your work. Full marks will only be awarded if the workings are shown.

(a) How many people must be in a room to guarantee that at least two people share a birthday, assuming 365 possible birthdays (ignoring leap years)? Use the Pigeonhole Principle to answer this question.

[3 marks]

(b) How many 8-character passwords can be made using only lowercase English letters, such that:

i. Each password contains at least one vowel.
ii. No letter repeats.

[4 marks]

(c) How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 20$$

such that $x_i > 1$, and no x_i equals 2 or 3?

[3 marks]

Question 4

Show your work. Full marks will only be awarded if the workings are shown.

(a)

i. Design a deterministic finite automaton (DFA) that accepts all binary strings that contain the substring "101".

[1 mark]

ii. Find a string that causes the automaton to enter a loop and then exit that loop to reach an accepting state. What is the minimal (shortest length) such string?

[2 marks]

iii. The automaton accepts strings that contain exactly two 1s. Modify it so that it accepts strings with an even number of 1s. Give an accepted string under the new rules but not under the old.

[3 marks]

(b) Given a DFA with 5 states over $\sum = \{0, 1\}$, it accepts all strings that:

i. Do not contain “11”.
ii. End in “0”.

Write a regular expression for this language. Then, describe the language in plain English.

[4 marks]

Question 5

Show your work. Full marks will only be awarded if the workings are shown.

(a) Write a regular expression for passwords with:

- At least one digit
- Only letters and digits
- No spaces or special characters.

[3 marks]

(b) Describe a regular language where all strings contain “101” but never contain “111”.

[2 marks]

(c) Prove using the Pumping Lemma that the language

$$L = \{ a^p \mid p \text{ is a prime number} \} \text{ is not regular.}$$

[2 marks]

(d) Provide an example of a string that belongs to the language generated by the context-free grammar G1, but not to the language generated by the context-free grammar G2, where G1 and G2 differ in their production rules for recursive structures.

[3 marks]

[END OF COURSEWORK ASSIGNMENT]