Trees - Problem sheet

Question 1.
Define the following terms:

1. Tree
2. Spanning Tree
3. Degree of a vertex in a tree
4. Leaf node
5. Minimum Spanning Tree

Question 2.
Let G be an unweighted undirected graph with five vertices labelled A, B, C, D and E. This graph is defined by it's adjacency matrix A_G given below. List at least four spanning trees of G.

$$\mathbf{A}_G = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Question 3.
Use Kruskal's algorithm to find the Minimum Spanning Tree (MST) of the following undirected weighted graph G with five vertices labelled A, B, C, D and E, represented by its Adjacency matrix, A_G given below. List the edges included in the MST and the total weight.

$$\mathbf{A}_G = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 4 & 3 & 0 \\ 1 & 0 & 4 & 2 & 0 \\ 4 & 4 & 0 & 5 & 6 \\ 3 & 2 & 5 & 0 & 7 \\ 0 & 0 & 6 & 7 & 0 \end{bmatrix} \end{array}$$

Question 4.
Given an undirected weighted graph G with five vertices labelled A, B, C, C

and E, represented by its corresponding adjacency matrix, A_G, given below. Find the Minimum Spanning Tree (MST) using Prim's algorithm. Clearly show each step, including the selection of edges and the updating of the MST.

$$\mathbf{A}_G = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \left[\begin{array}{ccccc} 0 & 2 & 0 & 6 & 0 \\ 2 & 0 & 3 & 8 & 5 \\ 0 & 3 & 0 & 0 & 7 \\ 6 & 8 & 0 & 0 & 9 \\ 0 & 5 & 7 & 9 & 0 \end{array}\right] \end{array}$$

Question 5.
Define the following:

1. A binary tree.
2. A balanced binary tree.
3. A full binary tree.
4. A binary search tree.

Question 6.
Draw a binary search tree the store the records 1 to 15. What is height of this binary search tree?