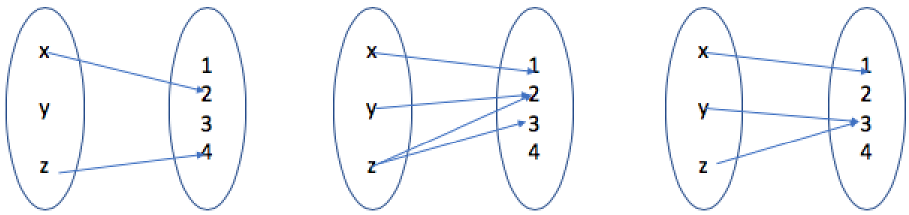
# Discrete Mathematics

Tutorial sheet

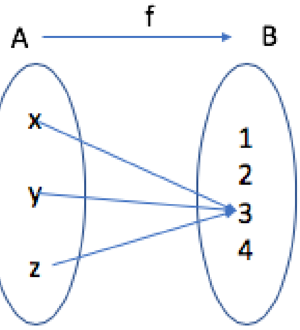
Funtions

**Question 1.**
Let $A$ and $B$ be two sets with $A = \{x, y, z\}$ and $B = \{1, 2, 3, 4\}$. Which of the following arrow diagrams define functions from $A$ to $B$?




(i) (ii) (iii)

**Question 2.**
Let $A$ and $B$ be two sets with $A = \{x, y, z\}$ and $B = \{1, 2, 3, 4\}$. Let $f$ from $A$ to $B$ defined by the following arrow diagram:




1. Write the domain, the co-domain and the range of $f$.
2. Find $f(x)$ and $f(y)$ .
3. Write down the set of pre-images of 3 and the set of pre-images of 1 .
4. represent $f$ as a set of ordered pairs.

**Question 3.**

The Hamming distance function is very important in coding theory. It gives a measure of the difference between two strings of 0's and 1's that have the same length. Let $S_n$ be the set of all strings of 0's and 1's of length $n$. The Humming function $H$ is defined as follows:

$H: \ S_n \times S_n \to \mathbb{N} \cup \{0\}$

$(s,t) \to H(s,t) =$ The number of positions in which s and t have different values.

For $n = 5$, Find $H(11111, 00000)$, $H(11000, 00000)$ , $H(00101, 01110)$ and $H(10001, 01111)$ .

**Question 4.**

Digital messages consist of a finite sequence of 0's and 1's. When they are communicated across a transmission channel, they are frequently coded in special ways to reduce the chance that they will be garbled by interfering noise in the transmission lines. A simple way to encode a message of 0's and 1's is to write each bit three times, for example: the message 0010111 would be encoded as 000 000 111 000 111 111 111.

Let $A$ be the set of all strings of 0's and 1's and let $E$ and $D$ be the encoding and the decoding function on the set $A$ defined for each string, $s$, in $A$ as follows:

$E(s) =$ The string obtained from $s$ by replacing each bit of $s$ with the same bit written three times.

$D(s) =$ The string obtained from $s$ by replacing each consecutive triple of three identical bits of $s$ by a single copy of that bit.

Find $E(0110), E(0101), D(000111000111000111111)$ and $D(111111000111000111000000)$

**Question 5.**

Let $A = \{1,2,3,4,5,6\}$, $B = \{a,b,c,d\}$ and $C = \{w,x,y,z\}$ be three sets.

Let f and g be two functions defined as follows:

$f : A \to B$ is defined by the following table.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(x)$ | $a$ | $b$ | $a$ | $c$ | $d$ | $d$ |

$g : B \to C$ is defined by the following table.

| $x$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $g(x)$ | $w$ | $x$ | $y$ | $z$ |

1. Draw arrow diagrams to represent the function f and g.
2. List the domain; the co-domain and the range of $f$ and $g$.
3. Find $f(1)$, the ancestor (pre-image) of $d$. and (g o f)(3)
4. Show that $f$ is not a one to one function.
5. Show that $f$ is an onto function.

6. Show that g is both one to one and onto.

**Question 6.**
Suppose you read that a function $f : \mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{Q}$ is defined by the formula $f(m, n) = \frac{m}{n}$ for all $(m, n) \in \mathbb{Z} \times \mathbb{Z}^+$.

1. Is $f$ a one to one function?
2. Is $f$ an onto function?

**Question 7.**

Given a function $f$ definted by $f(x) = \lfloor x \rfloor$. where $f : \mathbb{R} \to \mathbb{Z}$,

1. Plot the graph of the function f(x) for $x \in [-3, 3]$.
2. Use this graph to find $\lfloor \pi \rfloor$, $\lfloor -2.5 \rfloor$, $\lfloor -1 \rfloor$.
3. Use the graph in (1) to show that $f$ is not a one to one (not injective) function.
4. Is $f$ onto (surjective)? Justify your answer.

**Question 8.**
Let $S$ denote the set of all 3 bit binary strings and $B = (0, 1, 2, 3)$. The function $f : S \to B$ is defined by the rule

f(x)= the number of zeros in $x$ for each $x \in S$.

Find the following.

1. The domain of $f$.
2. $f(001)$ and $f(101)$.
3. The set of ancestors of 2.
4. The range of $f$.
5. Say whether or not $f$ is one to one, giving a reason for your answer.
6. Say whether or not $f$ is onto, giving a reason for your answer.

**Question 9.**
Let $f(x) = x \bmod 3$, where $f(x)$ is the remainder when $x$ is divided by 3, and $f : \mathbb{Z}^+ \to \{0, 1, 2\}$.

1. Find $f(7)$ and $f(12)$.
2. Find the ancestors of 2.
3. Say whether or not $f(x)$ is one to one, justifying your answer.
4. Say whether or not $f(x)$ is onto, justifying your answer.

**Question 10.**

Given the following function $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = 4x - 1$, for any real number x.

1. Is $f$ a one to one function? Prove or give a counter-example.
2. Is $f$ an onto function? Prove or give a counter-example.
3. Is $f$ invertible? and why? if the answer yes define $f^{-1}$.

**Question 11.**
Given the following function $f : \mathbb{Z} \to \mathbb{Z}$ with $g(x) = 4x - 1$, for any real number x.

1. Is $g$ a one to one function? Prove or give a counterexample.
2. Is $g$ an onto function? Prove or give a counterexample.
3. Is $g$ invertible? and why? if the answer yes define $g^{-1}$.

**Question 12.**

Given the following function $h : \mathbb{R} \to \mathbb{R}$ with $h(x) = x^2 - 1$, for any real number x.

1. What is co-domain and the range of $h$
2. Is $h$ a one to one function? Prove or give a counterexample.
3. Is $h$ an onto function? Prove or give a counterexample.
4. Is $h$ invertible? and why? if the answer yes define $h^{-1}$.

**Question 13.**
Given the following function $h : [0, +\infty[ \to [-1, +\infty[$with $h(x) = x^2 - 1$, for any real number x.

1. What is co-domain and the range of $h$

2. Is $h$ a one to one function? Prove or give a counterexample.
3. Is $h$ an onto function? Prove or give a counterexample.
4. Is $h$ invertible? and why? if the answer yes define $h^{-1}$.
5. On the same graph, plot the curve of $h$ and that of $h^{-1}$ if it exists.

**Question 14.**
Consider the following function $f : \mathbb{R} \to \mathbb{R}^+$ with $f(x) = 2^{x+3}$.

1. Show that $f$ is a bijective function.
2. Find the inverse function $f^{-1}$.
3. Plot the both curves of $f$ and of $f^{-1}$ on the same graph.

**Question 15.**
Consider the following function $f : \mathbb{R} - \{-1\} \to \mathbb{R}$ with $f(x) = \frac{2x}{x+1}$.

1. Show that $f$ is a one to one function.
2. Show that $f$ is not an onto funcion.

**Question 16.**

Find the inverse of the following functions:

1. $f(x) = e^{x^2-5}$
2. $g(x) = e^x + 5$

**Question 17.**
Find the inverse of the following functions:

1. $f(x) = \ln(x+2) + 2$
2. $g(x) = \log_2(x-5) + 3$

**Question 18.**
Let $A, B$ and $C$ be three sets/and $f : A \to B$ and $g : B \to C$ be two functions. Prove that if $gof$ is an onto function then g must be onto.
**Question 19.**
Let $A, B$ and $C$ be three sets and $f : A \to B$ and $g : B \to C$ be two functions. Prove that if $g\o f$ is a one to one function then f must be one to one.
**Question 20.**
Let $f : \mathbb{Q} \times \mathbb{Q} \to \mathbb{R}$ with $f(x, y) = x + \sqrt{2}y$ for all $x, y \in \mathbb{Q}$
Is $f$ a one to one function? Prove or give a counter-example.

End of questions