Discrete Mathematics

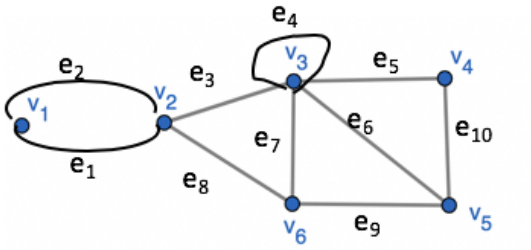
Tutorial sheet | Topic-7 | Graph Theory

Question 1.
Given the following graph $G := (V, E)$

$$\begin{aligned} V &= \{v_1, v_2, v_3, v_4, v_5, v_6\} \\ E &= \{\{v_1v_2\}, \{v_2v_5\}, \{v_5v_3\}, \{v_3v_4\}, \{v_2v_4\}\{v_6v_6\}\}. \end{aligned}$$

1. Draw the graph G.
2. List the set of vertices adjacent to v_2.
3. List the set of edges incident with v_3.
4. Give an example of a path of length 3 starting at the vertex v_2 and ending at the vertex v_5.
5. Give an example of a cycle of length 4.

Question 2.
Given the following graph:

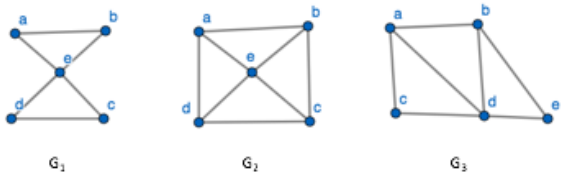


Determine which of the following walks are trails, paths or circuits.

1. $v_1e_1v_2e_3v_3e_4v_3e_5v_4$
2. $e_1e_3e_5e_5e_6$
3. $v_2v_3v_4v_5v_3v_6v_2$
4. $v_2v_3v_4v_5v_6v_2$
5. $v_1e_1v_2e_1v_1$

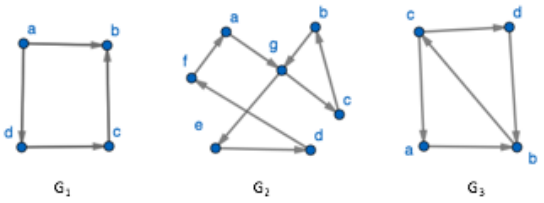
Question 3.

ScreenCast
Which of the following undirected graphs have an Euler circuit? Which of those that do not an Euler circuit have an Euler path.



Question 4.

Which of the following directed graphs have an Euler circuit? Which of those that do not an Euler circuit have an Euler path.



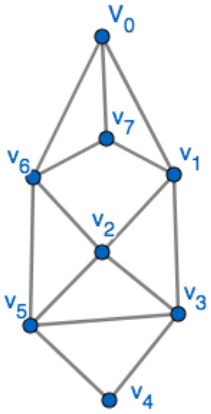
Question 5.
In each of the following either construct a graph with the specified properties or say why it is not possible to do so.

1. A graph with degree sequence 4,3,3,1.
2. A simple graph with degree sequence 4,3,3,2,2.
3. A simple 3 regular graph with 6 vertices.

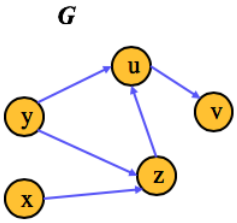
Question 6.
In a group of 25 people, is it possible to each shake hands with exactly 3 other people? Explain your your answer.

Question 7.

Find a Hamiltonian circuit in the following graph:



Question 8.
Given he following directed graph:



Find the transitive closure, G^*, of the graph G.
Question 9.

Suppose that 7 sites are connected in a network. The number of other sites to which each site has a direct connection is given by the following sequence

$$1, 2, 2, 3, 3, , 4, 7$$

1. Describe how a communications network such as this may be modeled by a graph, saying what the vertices and edges represent and what it means when two vertices are adjacent.

2. Say how many vertices it has.

3. Find how many connections there are between pairs of sites, giving a brief explanation of your method.

4. Say why it is impossible to construct a *simple* graph with this degree sequence.

5. Say why it is impossible to construct a network with 9 sites, in which each site has a direct connection to exactly 5 of the other sites.

Question 10.

1. What is meant by a **complete** graph?
2. What is the degree of each vertex of the complete graph K_8? Calculate the number of edges in K_8. Draw K_8.
3. What is the degree of each vertex of the complete graph K_n? How many edges will it have?

Question 11.
Construct 3 non isomorphic graphs with 5 vertices and 5 edges. Give one property for each graph that neither of the others has, which makes it non-isomorphic.

Question 12.
Draw the two graphs with adjacency lists

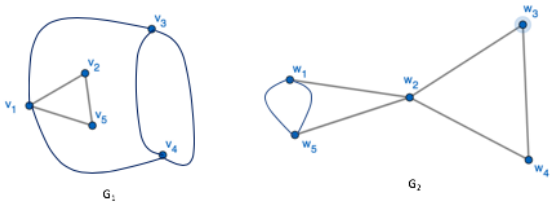
- $v_1 : v_2, v_5$
- $v_2 : v_1, v_3, v_4, v_5$
- $v_3 : v_2, v_4, v_5$
- $v_4 : v_2, v_3, v_5$
- $v_5 : v_1, v_2, v_3, v_4$

and

- $v_1 : v_2, v_3, v_4, v_5$
- $v_2 : v_1, v_5$
- $v_3 : v_1, v_4, v_5$
- $v_4 : v_1, v_3, v_5$
- $v_5 : v_1, v_2, v_3, v_4$

Are these graphs isomorphic? If so, show the correspondence between them.

Question 13.
Show that the following graphs are isomorphic:



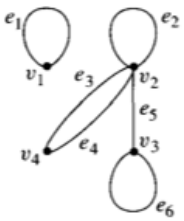
Question 14.

1. A simple connected graph has 7 vertices, all having the same degree d. Give the possible values of d and for each value of d give the number of edges of the graph.

2. Another simple connected graph has 8 vertices, all of the same degree sequence d. Draw this graph when $d = 4$ and give the other possible values of d.

Question 15.
Find adjacency matrices for the following undirected graphs

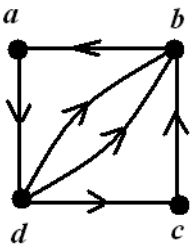
1. G_1



2. K_4, the complete graph with 4 vertices.

3. $K_{2,3}$, the complete bipartite graph on $(2, 3)$ vertices.

Question 16.
Given the following digraph G



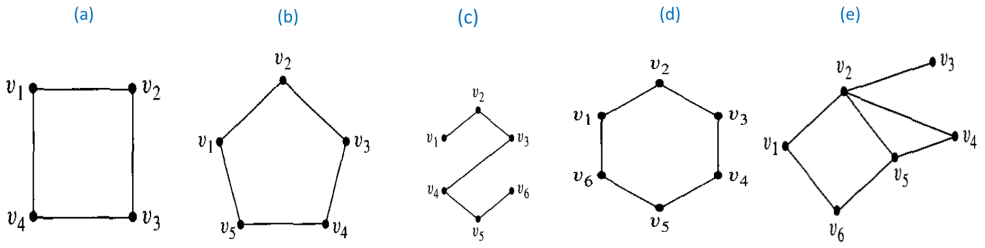
1. Write down the adjacency matrix M of G
2. Compute M^2
3. Find the number of paths of length 2 starting from the vertex a and ending in b.
4. What information does M^2 contain?
5. What information does M^3 contain?

Question 17.
Given the following adjacency matrix of a graph G:

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 2 & 1 \\ 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \end{array}$$

1. Find A^2 and A^3.
2. How many walks of length 2 are there from v_2 to v_3.
3. How many walks of length 2 are there from v_3 to v_4.
4. How many walks of length 3 are there from v_1 to v_4.
5. How many walks of length 3 are there from v_2 to v_3.

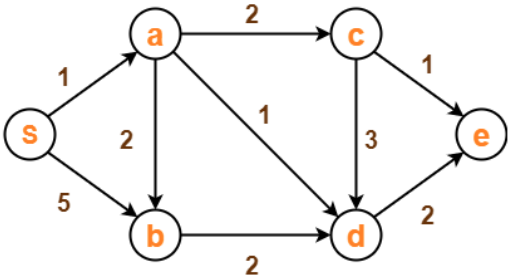
Question 18.
creenCast:
Find which of the following graphs are bipartite, redraw the bipartite graphs so that their bipartite nature is evident.



Question 19.

ScreenCast

Using Dijkstra's Algorithm, find the shortest distance from source vertex ÔSÕ to remaining vertices in the following graph:



Question 20.

Use Dijkstra's algorithm to find the shortest distance from A to H in the following network: