Discrete Mathematics

Set Theory Practice Exercises

These exercises are designed to help reinforce the concepts we have covered in topic 1 and provide you with an opportunity to apply what you have learned. By working through these problems, you will enhance your understanding and improve your problem-solving skills.

Question 1.

Describe the following sets by the listing method:

1. $\{n : n \in \mathbb{Z} \text{ and } 5 \leq n < 8\}$
2. $\{3n : n \in \mathbb{Z} \text{ and } 5 \leq n < 8\}$
3. $\{2^n : n \in \mathbb{Z} \text{ and } 5 \leq n < 8\}$

Question 2.

Let $\sum = \{x, y\}$ be an alphabet. List the element of the set L_1 and L_2

1. L_1 is the language consisting of all strings over $\sum$ of length **less or equal to 4** that are **palindromes**.
2. L_2 is the language consisting of all strings over $\sum$ of length **less or equal to 3** in which all the x's appear to the left of all the y's.

Question 3.

Describe the following sets by giving a suitable universal set and rules of inclusion:

1. $\{4, 8, 12, 16, 20\}$
2. $\{0, 2, -2, 4, -4, \cdots\}$
3. $\{2, 4, 8, 16, 32\}$
4. $\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}\}$

Question 4.

Let $X = \{f, g, h, i, k\}$ and $Y = \{d, g, h, k\}$ be subsets of a universal set $U = \{d, e, f, g, h, j, k, l\}$. Find each of the following:

1. $\overline{X}$
2. $X \cap Y$
3. $X \cup \overline{Y}$
4. $X - Y$
5. $X \oplus Y$
6. $\overline{(X \cap Y)}$

Question 5.
Let $A = \{2, \frac{1}{2}, \sqrt{2}\}$ and $B = \{x \in \mathbb{Q} : x \notin \mathbb{Z}\}$ be two sets. List the following sets:

$$A \cap B, \quad A - B, \quad A \cap \mathbb{R}, \quad A \cap \mathbb{Z},$$

Question 6.
Let X and Y be two sets with $X = \{f, g, h, j, k\}$ and $Y = \{f, g\}$.

1. What is cardinality of X?
2. What is the total number of subsets of X?
3. Put the correct sign $\in \quad \notin \quad \subset \quad \subseteq$ between the following pairs:

$$f \quad X, \qquad Y \quad X, \qquad X \quad X, \qquad \emptyset \quad X, \qquad \text{and} \qquad h \quad Y$$

Question 7.
Let X and Y be two sets of the universal set U.

1. Use Venn diagram to show to show that $\overline{X \cap Y} = \overline{X} \cup \overline{Y}$.
2. Use membership tables to prove that $\overline{X \cap Y} = \overline{X} \cup \overline{Y}$.

Question 8.

Let A and B and C be subsets of a universal set $\mathcal{U}$.

1. Draw a three binary digit labelled Venn diagram depicting A, B, C in such a way that they divide $\mathcal{U}$ into 8 disjoint regions.

2. The subset $X \subseteq \mathcal{U}$ is defined by the following membership table:

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

   Identify the region X on your diagram. Describe the region you have identified in set notation as simply as you can.

3. let Y be the set represented by the region 000, 011, 101, 110, and 111. Describe the set Y using the set notation.

Question 9.

Given three sets A, B and C, subsets of the universal set U. For each of each of the following Venn diagram write, in terms of A, B and C, the set representing the area coloured in yellow:

1. First Venn diagram given in Figure 1 below

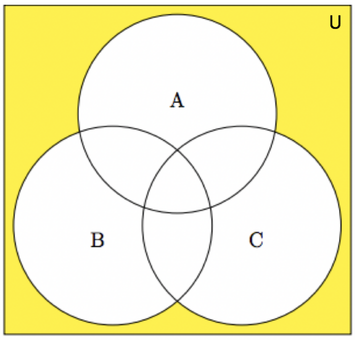

Figure 1: The diagram consists of three overlapping circles labeled A, B, and C, and a surrounding yellow rectangle labeled U, which represents the universal set.

2. Second Venn diagram given in Figure 2 below

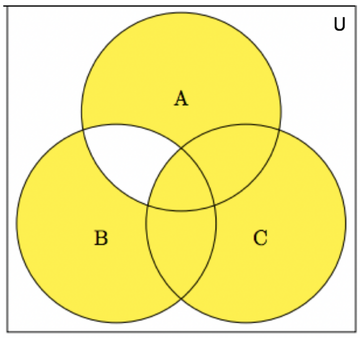

Figure 2: This Venn diagram consists of three overlapping circles labeled A, B, and C, all placed within a square that represents the universal set labeled U. The outside of the three circles is coloured white, the circles are coloured yellow a part the area overlapping A and B excluding C, which is coloured in white.

Question 10.
Let $A = \{t, u, v, w\}$ and let S_1 be the set of all subsets of A that do not contain w and S_2 be the set of all subsets of A that contain w.

1. Find S_1 and S_2.
2. Are S_1 and S_2 disjoint?
3. Find $S_1 \cup S_2$.
4. What is the relation between $S_1 \cup S_2$ and $\mathcal{P}(A)$?

Question 11.

Let $A = \{1, 2\}$ and let $B = \{2, 3\}$. Find each of the following:

1. $\mathcal{P}(A \cap B)$
2. $\mathcal{P}(A \cup B)$
3. $\mathcal{P}(A \times B)$

Question 12.

Given three sets A, B and C. Prove that the expression $\overline{(A \cup B) \cap \overline{C} \cup \overline{B}}$ is equivalent to $B \cap C$ by re-writing the expression using algebraic laws, state the name of each law used.

Question 13.
Given three sets A, B and C. Using set identities, prove that:

$$(A \cup B) - (C - A) = A \cup (B - C).$$

Question 14.
Given two sets A and B. Simplify $\overline{(\overline{A} \cup \overline{B}) - A}$.

Question 15.
Show that for all sets A and B, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

Question 16.
Let A, B and C be three sets. Prove that if $C \subseteq (B - A)$ then $A \cap C = \emptyset$.