Homework 2

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Part 1:

Part 2:

1. " Calculate analytically the closed-loop equations of the system."

The following network control system is given.

$$\dot{x} = Ax + Bu \tag{1}$$

Where the matrices A, B are set to be 0, I since the system is a simple integrator. The input u is set to be.

$$u = -Kx$$

In the network two delays are introduced. One before and one after the controller. This results in that the input of the system is delayed $\tau_{sc} + \tau_{ea}$ seconds. Since $\tau_{sc} + \tau_{ea} < h$ the input signal can be written.

$$u = \begin{cases} u_0 = -Kx(kh - h) & kh < t < kh + \tau_{sc} + \tau_{ea} \\ u_1 = -Kx(kh) & kh + \tau_{sc} + \tau_{ea} < t < kh + h \end{cases} \tag{2}$$

With this we can formulate the closed loop equation.

$$x(kh + h) = \mathrm{e}^{Ah} + \int_{kh}^{kh+h} \mathrm{e}^{AS}\, \mathrm{d}SBu$$

$$= \mathrm{e}^{Ah}x(kh) + \int_{kh}^{kh+\tau_{sc}+\tau_{ea}} \mathrm{e}^{AS}\, \mathrm{d}SBu_0 + \int_{kh+\tau_{sc}+\tau_{ea}}^{kh+h} \mathrm{e}^{AS}\, \mathrm{d}SBu_1 \tag{3}$$

Setting the time delay $\tau = \tau_{sc} + \tau_{ea}$ and combining the equations 2 with 3 we result in the following closed loop equation.

$$x(kh + h) = \begin{bmatrix} 1 + K(\tau - h) & 0 \\ 0 & -K\tau \end{bmatrix} \begin{bmatrix} x(kh) \\ x(kh - 1) \end{bmatrix}$$

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Since the continuous plant is a simple integrator $G(s) = \frac{1}{s}$ we get that the zero-order hold of the continuous plant will have the following pulse-transfer function.

$$H(z) = \frac{h}{z - 1}$$

With the controller as $C(z) = -K$ we will have a stable system if.

$$|\frac{G(z)C(z)}{1 + G(z)C(z)}| < \frac{1}{\tau z}, z \in R$$

This can be written as.

$$hK(1 - z\tau) < z - 1, z \in R$$