Formula Sheet 4 CXC

Area and Perimeter Formula

Perimeter = distance around the outside (add all sides).

$Area\ \text{triangle} = \frac{1}{2}bh$

$Area\ \text{parallelogram} = bh$

$Area\ \text{rectangle} = bh \text{ or } lw$

$Area\ \text{square} = bh \text{ or } s^2$

$Area\ \text{trapezoid} = \frac{1}{2}h(b_1 + b_2)$

$Area\ \text{circle} = \pi r^2$

$Circumference\ \text{of circle} = 2\pi r = \pi d$

Volume and Surface Area

$V(\text{rectangular solid}) = lwh$

$SA(\text{rect. solid}) = 2lh + 2hw + 2lw$

$V(\text{cylinder}) = \pi r^2 h$

$SA(\text{cylinder}) = 2\pi rh + 2\pi r^2$

Equation of a line

Slope-Intercept Method:

$y = mx + c$

Point-Gradient Method:

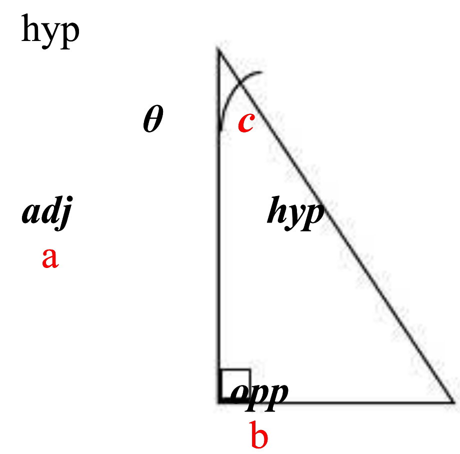
$y - y_1 = m(x - x_1)$

Trigonometric Formula

Opposite – side opposite to angle

Adjacent –side beside (adjacent) to angle

Hypotenuse- longest side



$\sin\theta = \frac{opposite}{hypotenus}$

$\cos\theta = \frac{adjacent}{hypotenus}$

$\tan\theta = \frac{opposite}{adjacent}$

Remember: works only on right angle triangles

Pythagorean Theorem

$$c^2 = a^2 + b^2$$

Triples: 3,4,5 5,12,13 8,15,17

C is the hypotenuse, a and b are the other sides.

Remember: works only on right angle triangles

Coordinate Geometry Formula

Distance Formula:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Midpoint Formula:

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Gradient Formula:

$$m = \frac{y_1 - y_2}{x_1 - x_2} \qquad m = \frac{rise}{run}$$

Parallel lines have equal slope.
Perpendicular lines have negative reciprocal gradients.

Angle Information

Complementary angles - two angles whose sum is 90.
Supplementary angles - two angles whose sum is 180.

General Triangle Information

Sum of angles of triangle = 180.

Measure of exterior angle of triangle = the sum of the two non-adjacent interior angles.

The sum of any two sides of a triangle is greater than the third side.

Polygons

Sum of Interior Angles:
$180(n-2)$

Sum of Exterior Angles:
360

Each Interior Angle (regular poly):
$\frac{180(n-2)}{n}$

Each Exterior Angle (regular poly):
$\frac{360}{n}$

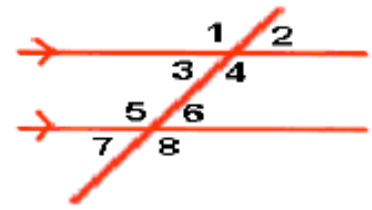
Quadratic Formula

If $ax^2 + bx + c = 0$

then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

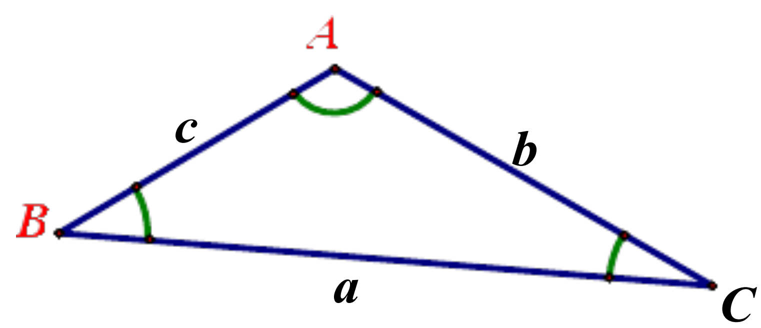
Circle Facts

Parallel lines



Corresponding angles are equal. 1= 5, 2= 6, 3= 7, 4= 8
Alternate Interior angles are equal. 3= 6, 4= 5
Alternate Exterior angles are equal.
1= 8, 2= 7
Same side interior angles are supplementary.
m 3+m 5=180, m 4+m 6=180

Solving triangles



Sine rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Used when any two sides and their corresponding angles are involved to find one missing side or angle.

Cosine rule

$a^2 = b^2 + c^2 - 2bc \times \cos A$

$b^2 = a^2 + c^2 - 2ac \times \cos B$

$c^2 = a^2 + b^2 - 2ab \times \cos C$

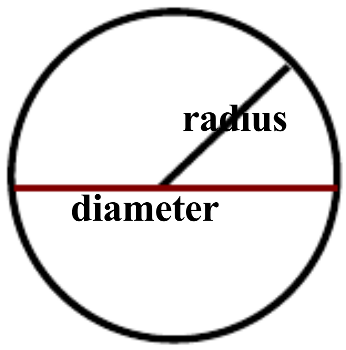
Used when three sides and an angle between them are given to find the other side

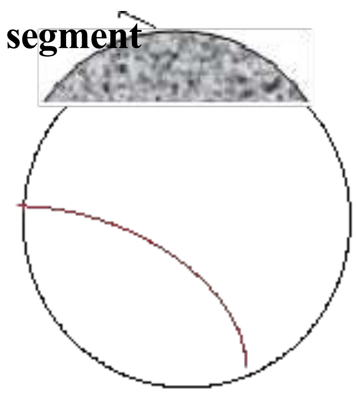
Heron's Formula
Area of a triangle given only the length of the sides

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$where\ s = \frac{a+b+c}{2}$

Capital letters represent Angles
Common letters represent sides





Sector

Diameter = 2× radius

Area of circle = πr^2

Circumference of circle = $2\pi r$ or πd

Length of arc = $2\pi r \times \frac{\theta}{360}$

Area of sector = $\pi r^2 \times \frac{\theta}{360}$

$$Area\ of\ Segment = Area\ of\ sector - Area\ of\ triangle$$

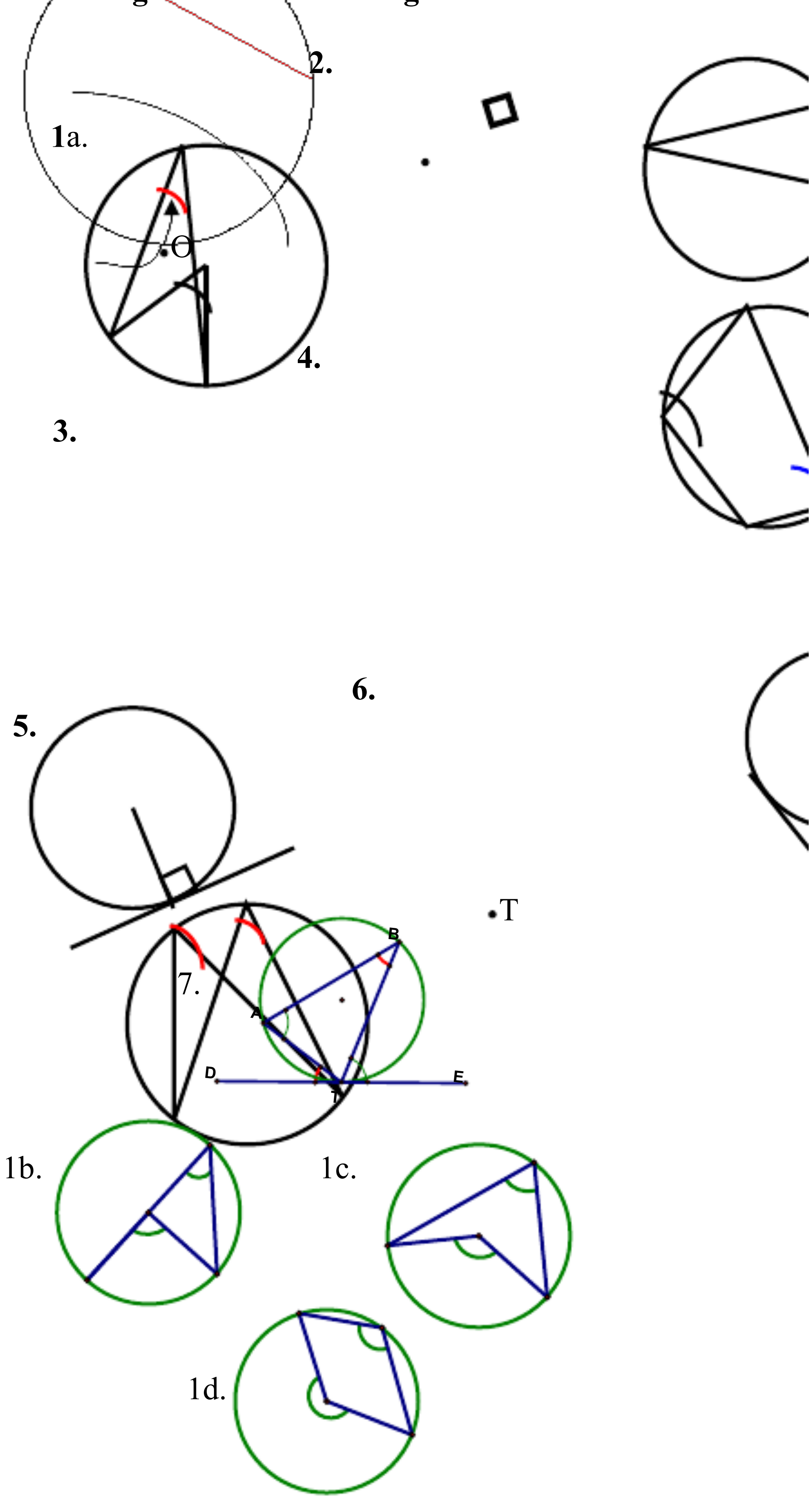
$$= \left(\pi r^2 \times \frac{\theta}{360}\right) - \left(\frac{1}{2} r^2 \sin\theta\right)$$

Angles in circles

1. **a,b,c,d Angle at the center in twice angle at the circumference.**
2. **Angle formed on the diameter in 90°**
3. **Angles in the same segment are equal**
4. **opposite angles in a cyclic Quadrilateral are supplimentary(add up to 180°)**

Tangent

5. **Radius to tangent is 90° at point of contact.**
6. **The tangents to a circle from an external point T are equal in length.**
7. **Angle between tangent to circle and chord at the point of contact is eqaual to the to the angle in the alternet segment**



Matrices

Transformational Matrices

Adding or subtracting matrices

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \pm \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a \pm e & b \pm f \\ c \pm g & d \pm h \end{pmatrix}$$

Multiplying Matrices

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}$$

Determinant of 2×2 Matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

If $|A| = ad - cb$

A singular matrices has a determinant of 0

Adjoint of 2×2 matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A \quad adjo\,\text{int} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Inverse of 2×2 matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times A \ \text{ adjoint}$$

or

$$A^{-1} = \frac{1}{ad - bc} \times \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Reverse opportions

$+$	$-$	$\sin\theta$	$\sin^{-1}\theta$
$-$	$+$	$\cos\theta$	$\cos^{-1}\theta$
$\times$	$\div$	$\tan\theta$	$\tan^{-1}\theta$
$\div$	$\times$	$f(x)$	$f^{-1}(x)$
$\sqrt[x]{a}$	a^x		

REFLECTION

Multiply matrices by each point to get reflection in

x- axis	y- axis	$y=x$	$y=-x$
$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

TRANSLATION

Movement of x in x direction and y in ydirection add matrix

$\begin{pmatrix} x \\ y \end{pmatrix}$ *to eack point to get its image.*

ROTATION

Multiply by matrices to get rotation of Θ-degrees clockwise about origin (0,0)

$$R_\theta \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad R_{90} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$R_{180} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \qquad R_{270} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Enlargment

Multiply each point by scale factor K to get the image of the point for an enlargment from the origin.

Sets

ξ = universal set

$\in$= is a member of

$\notin$= is not a member of

$\cup$ = union

$\cap$ = intersect

$\varnothing$= null set

A' = Elements not in set A

Factoring Formulas

$$x^2 - a^2 = (x+a)(x-a)$$

$$x^2 + 2ax + a^2 = (x+a)^2$$

$$x^2 - 2ax + a^2 = (x-a)^2$$

$$x^2 + (a+b)x + ab = (x+a)(x+b)$$

Arithmetic Operations

$$ab + ac = a(b + c) \qquad a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc} \qquad \frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \qquad \frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

$$\frac{a - b}{c - d} = \frac{b - a}{d - c} \qquad \frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{ab + ac}{a} = b + c, \ a \neq 0 \qquad \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$$

Exponent Properties

$$a^n a^m = a^{n+m} \qquad \frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$$

$$\left(a^n\right)^m = a^{nm} \qquad a^0 = 1, \ a \neq 0$$

$$(ab)^n = a^n b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{-n} = \frac{1}{a^n} \qquad \frac{1}{a^{-n}} = a^n$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n} \qquad a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = \left(a^n\right)^{\frac{1}{m}}$$

Completing the Square

Solve $2x^2 - 6x - 10 = 0$

(1) Divide by the coefficient of the x^2

$$x^2 - 3x - 5 = 0$$

(2) Move the constant to the other side.

$$x^2 - 3x = 5$$

(3) Take half the coefficient of x, square it and add it to both sides

$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2 = 5 + \frac{9}{4} = \frac{29}{4}$$

(4) Factor the left side

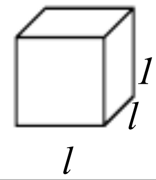
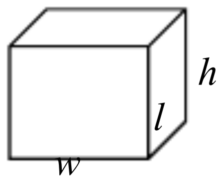
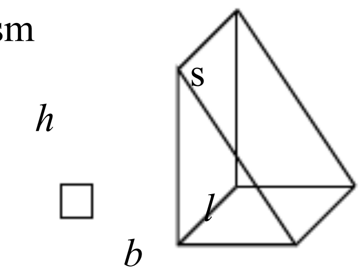
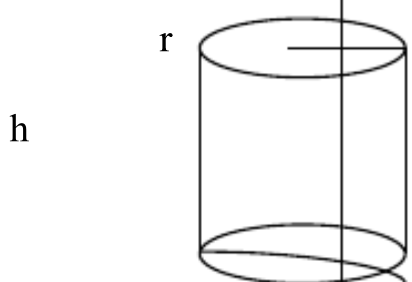
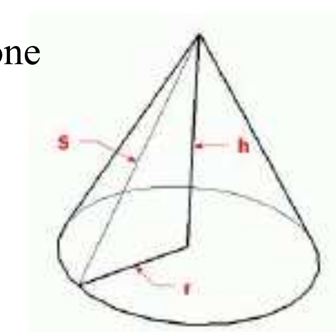
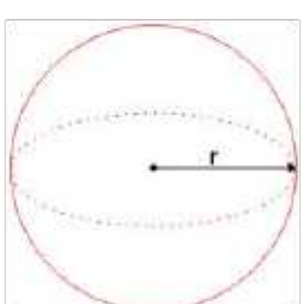
$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

(5) Use Square Root Property

$$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{2}$$

(6) Solve for x

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

Shape	Volume	Surface Area
Cube 	$l \times l \times l = l^3$	$6l^2$
Cuboid 	lwh	$2lw+2hw+2lh$
Prism 	$\frac{1}{2}bhl$	$bh+lb+sl+hl$
Cylinder 	$\pi r^2 h$	$2\pi r^2 + 2\pi rh$ or $2\pi r(r+h)$
Cone 	$\frac{1}{3}\pi r^2 h$	$\pi r^2 + \pi rs$
Sphere 	$\frac{4}{3}\pi r^3$	$4\pi r^2$