# Homework 3 - 131/231

### Due on Friday March 1, 2019 at 11:59 pm

For this homework you will need use the following packages.

```
library(tidyverse)
library(ROCR)
library(tree)
library(maptree)
library(class)
library(ggridges)
library(superheat)
library(kableExtra)
```

### Analyzing drug use

The first half of this homework involves the analysis of drug use. The data set includes a total of 1885 observations on 32 variables. A detailed description of the data set can be found here. For each observation, 12 attributes are known:

- ID: number of record in original database. Used for reference only.
- Age: Age of the participant
- Gender: Gender of the participant (M/F)
- Education: Level of education of the participant
- Country: Country of current residence of the participant
- Ethnicity: Ethnicity of the participant

Many of the covariates have been transformed: some ordinal or categorical variables have been given numeric codes. Part of this problem will involve appropriately re-transforming these variables. The data also contains the following personality measurements:

- Nscore: NEO- FFI- R Neuroticism (Ranging from 12 to 60)
- Escore: NEO- FFI- R Extraversion (Ranging from 16 to 59)
- Oscore: NEO- FFI- R Openness (Ranging from 24 to 60)
- Ascore: NEO- FFI- R Agreeableness (Ranging from 12 to 60)
- Cscore: NEO- FFI- R Conscientiousness (Ranging from 17 to 59)
- Impulsive: Impulsiveness measured by BIS- 11
- SS: Sensation Seeking measured by ImpSS

Finally, participants were questioned concerning their use of 18 legal and illegal drugs (alcohol, amphetamines, amyl nitrite, benzodiazepine, cannabis, chocolate, cocaine, caffeine, crack, ecstasy, heroin, ketamine, legal highs, LSD, methadone, mushrooms, nicotine and volatile substance abuse) and one fictitious drug (Semeron) which was introduced to identify over-claimers. All of the drugs use the class system of CL0-CL6: CL0 = "Never Used", CL1 = "Used over a decade ago", CL2 = "Used in last decade", CL3 = "Used in last year", CL4 = "Used in last month", CL5 = "Used in last week", CL6 = "Used in last day".

### 1. Logistic regression for drug use prediction

This problem has 3 parts for 131 students and 4 parts for 231 students. As mentioned, the data uses some strange encodings for variables. For instance, you may notice that the gender variable has type double. Here the value -0.48246 means male and 0.48246 means female. Age was recorded at a set of categories but rescaled to a mean 0 numeric variable (we will leave that variable as is). Similarly education is a scaled numeric quantity (we will also leave this variable as is). We will however, start by transforming gender, ethnicity, and country to factors, and the drug response variables as ordered factors:

(a). Define a new factor response variable recent\_cannabis\_use which is "Yes" if a person has used cannabis within a year, and "No" otherwise. This can be done by checking if the Cannabis variable is greater than or equal to CL3. Hint: use mutate with the ifelse command. When creating the new factor set levels argument to levels=c("No", "Yes") (in that order).

(b). We will create a new tibble that includes a subset of the original variables. We will focus on all variables between age and SS as well as the new factor related to recent cannabis use. Create drug\_use\_subset with the command:

```
drug_use_subset <- drug_use %>% select(Age:SS, recent_cannabis_use)
```

Split drug\_use\_subset into a training data set and a test data set called drug\_use\_train and drug\_use\_test. The training data should include 1500 randomly sampled observation and the test data should include the remaining observations in drug\_use\_subset. Verify that the data sets are of the right size by printing dim(drug\_use\_train) and dim(drug\_use\_test).

```
train_index <- sample(nrow(drug_use_subset), 1500)
drug_use_train <- drug_use_subset[train_index,]
drug_use_test <- drug_use_subset[-train_index,]</pre>
```

(c). Fit a logistic regression to model recent\_cannabis\_use as a function of all other predictors in drug\_use\_train. Fit this regression using the training data only. Display the results by calling the summary function on the logistic regression object.

```
logistic.cannabis =glm(recent_cannabis_use~., data=drug_use_train, family =binomial)
summary(logistic.cannabis)
```

```
##
## Call:
   glm(formula = recent_cannabis_use ~ ., family = binomial, data = drug_use_train)
##
## Deviance Residuals:
                                    ЗQ
##
                      Median
       Min
                 10
                                            Max
  -2.7781
           -0.5911
                      0.1419
                                0.5535
                                         2.7083
##
##
  Coefficients:
                                Estimate Std. Error z value Pr(>|z|)
##
## (Intercept)
                                0.71923
                                            0.69696
                                                      1.032 0.302095
                                                     -8.894 < 2e-16 ***
## Age
                                -0.81136
                                            0.09123
## GenderFemale
                                -0.62917
                                            0.15744
                                                     -3.996 6.43e-05 ***
## Education
                                -0.33506
                                            0.08080
                                                     -4.147 3.37e-05 ***
## CountryCanada
                                                     -0.333 0.739300
                                -0.41293
                                            1.24086
## CountryNew Zealand
                                -0.86878
                                            0.32959
                                                     -2.636 0.008390 **
## CountryOther
                                0.16978
                                            0.51503
                                                      0.330 0.741670
## CountryIreland
                                -0.80256
                                            0.74884
                                                     -1.072 0.283833
## CountryUK
                                -0.65229
                                            0.38072
                                                     -1.713 0.086658 .
## CountryUSA
                                -1.79573
                                                     -9.490 < 2e-16 ***
                                            0.18923
## EthnicityAsian
                                -0.38434
                                            1.01590
                                                     -0.378 0.705190
## EthnicityWhite
                                 1.00331
                                            0.68672
                                                      1.461 0.144008
## EthnicityMixed:White/Black
                                0.06578
                                            1.18594
                                                      0.055 0.955769
## EthnicityOther
                                 0.95539
                                            0.80870
                                                      1.181 0.237450
## EthnicityMixed:White/Asian
                                1.12184
                                            1.07767
                                                      1.041 0.297884
## EthnicityMixed:Black/Asian
                                12.82035
                                          343.25331
                                                      0.037 0.970206
## Nscore
                                                     -0.991 0.321800
                                -0.09007
                                            0.09091
## Escore
                                -0.13750
                                            0.09469
                                                     -1.452 0.146492
## Oscore
                                 0.64074
                                            0.09007
                                                      7.114 1.13e-12 ***
## Ascore
                                 0.05903
                                            0.08075
                                                      0.731 0.464723
## Cscore
                                -0.34979
                                            0.09150
                                                     -3.823 0.000132 ***
                                -0.05757
                                            0.10073
                                                     -0.572 0.567640
## Impulsive
## SS
                                 0.64607
                                            0.11057
                                                      5.843 5.12e-09 ***
##
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
   (Dispersion parameter for binomial family taken to be 1)
##
##
##
       Null deviance: 2077.5
                               on 1499
                                        degrees of freedom
## Residual deviance: 1195.0 on 1477
                                        degrees of freedom
## AIC: 1241
## Number of Fisher Scoring iterations: 12
```

(d). (231 only). Generalized linear models for binary data involve a link function, g which relates a linear function of the predictors to a function of the probability p:  $g(p) = \beta_0 + \beta_1 X_1 + ... + \beta_p X_p$ . g is a function which

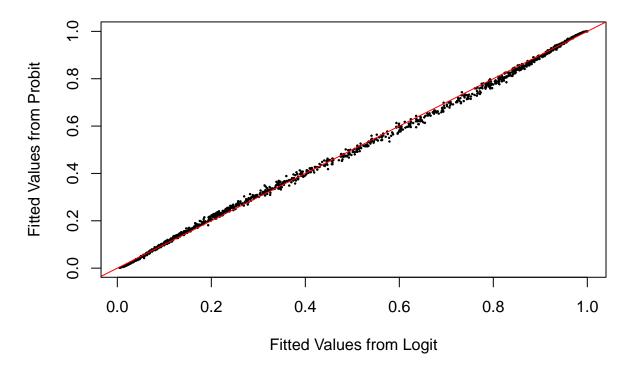
maps  $p \in [0, 1]$  to  $\mathbb{R}$ . Logistic regression is based on the logit link function,  $g(p) = \log(p/(1-p))$ . In class we mentioned another link function, called the probit:  $g(p) = \Phi^{-1}(p)$  where  $\Phi$  is the cumulative density function of the normal distribution. Another often used link function is the "c-log-log" link:  $g(p) = \log(-\log(1-p))$ .

Plot the fitted values for logistic regression fit of the training data on the x-axis and the fitted values for the probit regression on y-axis. In the plot command (assuming you use the base plotting package, not ggplot) set pch=19 and cex=0.2 (this makes the points smaller and more legible). Include the line y=x with the command abline(a=0, b=1, col="red"). Make another identical plot, this time replacing the y-axis with the predicted values from a cloglog

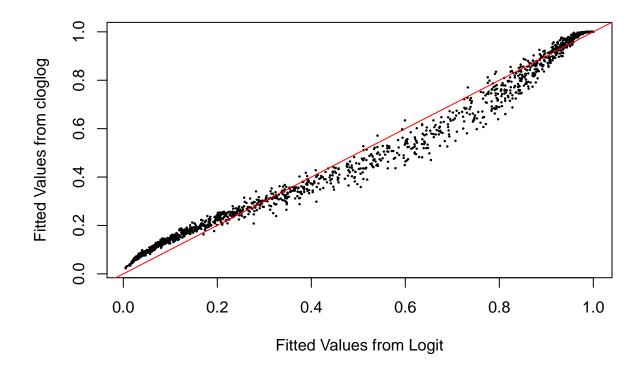
```
probit.cannabis = glm(recent_cannabis_use~., data=drug_use_train, family = binomial(link = "probit"))
cloglog.cannabis = glm(recent_cannabis_use~., data=drug_use_train, family = binomial(link = "cloglog"))
```

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

plot(logistic.cannabis[["fitted.values"]], probit.cannabis[["fitted.values"]], pch = 19, cex = 0.2, xla
abline(a=0, b=1, col="red")



plot(logistic.cannabis[["fitted.values"]], cloglog.cannabis[["fitted.values"]], pch = 19, cex = 0.2, xl
abline(a=0, b=1, col="red")



Comment on the differences between the estimated probabilities in each plot. Things you should comment on include: 1) which link function (probit or cloglog) leads to predictions that are most similar to the logistic regression predictions? 2) for what range of probabilities are the probit and cloglog predictions values more or less extreme than the logit values? 3) Does either probit or cloglog regression seem to estimate systematically smaller or larger probabilities than the logistic regression for a certain range of probabilities?

The probit function leads to predictions that are more similar to the logistic predictions because the fitted values most closely follow the line y = x. The cloglog fitted vales are more extreme than the logit function for probabilities of roughly 0.3 and below and they are less extreme than the logit function for probabilities of about 0.5 to 0.9. As a result, the cloglog seems to systematically estimate smaller or larger probabilities over these certain probability ranges. The probit function tends to have slightly higher values than the logit function for the range of probabilities from 0.3 until close to 0.5 and tends to estimate lower values than the logit function for the range from 0.6 until close to 0.9. These differences between the probit and logit are much less pronounced than in the cloglog vs logit comparison. Due to this minimal difference it feels a little strange to call these differences between probit and logit "systematic" but maybe this description is still fair given the consistently higher or lower points.

Hint: in logistic regression we set family=binomial(link="logit""). To fit probit and cloglog regressions change the value of the link argument appropriately.

#### 2. Decision tree models of drug use

This problem has 3 parts for all students.

Construct a decision tree to predict recent\_cannabis\_use using all other predictors in drug\_use\_train. Set the value of the argument control = tree\_parameters where tree\_parameters are:

```
tree_parameters = tree.control(nobs=nrow(drug_use_train), minsize=10, mindev=1e-3)
```

This sets the smallest number of allowed observations in each leaf node to 10 and requires a deviance of at least 1e-3 to split a node.

```
cannabis.tree = tree(recent_cannabis_use~.,data = drug_use_train, control = tree_parameters)
summary(cannabis.tree)

##
## Classification tree:
## tree(formula = recent_cannabis_use ~ ., data = drug_use_train,
## control = tree_parameters)
## Number of terminal nodes: 129
## Residual mean deviance: 0.4202 = 576.1 / 1371
## Misclassification error rate: 0.098 = 147 / 1500
```

(a). Use 10-fold CV to select the a tree which minimizes the cross-validation misclassification rate. Use the function cv.tree, and set the argument FUN=prune.misclass. Note: you do not need to use a do.chunk function since the tree package will do cross validation for you. Find the size of the tree which minimizes the cross validation error. If multiple trees have the same minimum cross validated misclassification rate, set best\_size to the smallest tree size with that minimum rate.

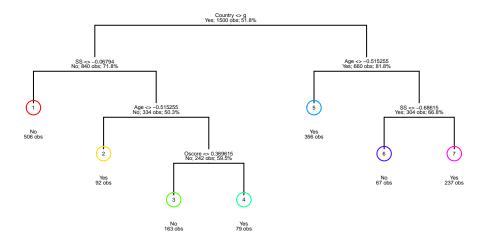
```
nfold = 10
set.seed(1)
folds = seq.int(nrow(drug_use_train)) %>%
                                          ## sequential obs ids
   cut(breaks = nfold, labels=FALSE) %>%
                                      ## sequential fold ids
                                      ## random fold ids
   sample
cannabis.tree.cv = cv.tree(cannabis.tree, FUN = prune.misclass, K = 10, rand = folds)
best.cv = min(cannabis.tree.cv\size[which(cannabis.tree.cv\dev=min(cannabis.tree.cv\detadev, na.rm = TRUE)
cannabis.tree.cv$size
## [1] 129
                  79
                     74
                         68 65 51 45 30 25
                                             19
                                                 15
                                                    14 10
                                                             8
                                                                7
## [18]
cannabis.tree.cv$dev
## [18] 322 318 356 723
best.cv
```

## [1] 7

The tree that minimizes the cross validation error has 7 nodes.

(b). Prune the tree to the size found in the previous part and plot the tree using the draw.tree function from the maptree package. Set nodeinfo=TRUE. Which variable is split first in this decision tree?

```
pruned.cannabis.tree = prune.tree(tree = cannabis.tree, best = best.cv)
draw.tree(tree = pruned.cannabis.tree, size = 2, cex = .3, nodeinfo=TRUE)
```



(c). Compute and print the confusion matrix for the *test* data using the function table(truth, predictions) where truth and predictions are the true classes and the predicted classes from the tree model respectively. Note: when generated the predicted classes for the test data, set type="class" in the predict function. Calculate the true positive rate (TPR) and false positive rate (FPR) for the confusion matrix. Show how you arrived at your answer.

```
pruned.predict.test = predict(pruned.cannabis.tree, drug_use_test,
                              type = "class")
testtable = table(pruned.predict.test, drug_use_test$recent_cannabis_use)
testtable
## pruned.predict.test No Yes
##
                      128
                            46
                   No
##
                   Yes 35 176
tpr = testtable[2,2]/(testtable[2,2] + testtable[1,2])
fpr = testtable[2,1]/(testtable[2,1] + testtable[1,1])
tpr
## [1] 0.7927928
fpr
```

## [1] 0.2147239

The true positive rate is 0.729 and the false positive rate is 0.15. TPR = TP/(TP + FN) and FPR = FP/(FP + TN)

### 3. Model Comparison

This problem has 2 parts for all students.

(a). Plot the ROC curves for both the logistic regression fit and the decision tree on the same plot. Use drug\_use\_test to compute the ROC curves for both the logistic regression model and the best pruned tree model.

```
pruned.predict.test = predict(pruned.cannabis.tree, drug_use_test, type = "vector")
pred.tree <- prediction(pruned.predict.test[,2], drug_use_test$recent_cannabis_use)
perf.tree <- performance(pred.tree, "tpr", "fpr")

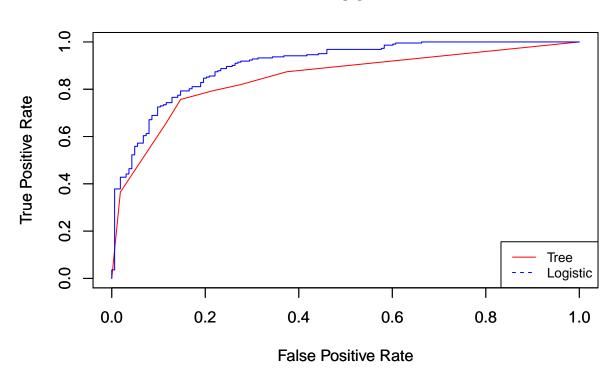
logistic.probs.test = predict(logistic.cannabis, drug_use_test, type="response")
pred.logistic = prediction(logistic.probs.test, drug_use_test$recent_cannabis_use)
perf.logistic <- performance(pred.logistic, "tpr", "fpr")

perf.tree@x.name = perf.logistic@x.name= "False Positive Rate"
perf.tree@y.name = perf.logistic@y.name= "True Positive Rate"

# plot the two ROC curves on the same plot
plot(perf.tree, type = "l", col = "red")
par(new=TRUE)
plot(perf.logistic, type="l", col = "blue")

legend("bottomright", legend=c("Tree", "Logistic"), col=c("red", "blue"), lty=1:2, cex=0.8)
title("ROC", cex = 0.8)</pre>
```

### **ROC**



(b). Compute the AUC for both models and print them. Which model has larger AUC?

## [1] 0.8464185

```
# Calculate AUC
auc.tree = performance(pred.tree, "auc")@y.values
auc.logistic = performance(pred.logistic, "auc")@y.values
auc.tree
## [[1]]
```

Table 1: Number of Patients with Each Sub-Type of Leukemia

Var1	Freq
BCR-ABL	15
E2A-PBX1	27
Hyperdip50	64
MLL	20
OTHERS	79
T-ALL	43
TEL-AML1	79

```
auc.logistic
```

```
## [[1]]
## [1] 0.9038026
```

The AUC for the tree is 0.85 which is smaller than the AUC from the logistic model which is 0.87.

### 4. Clustering and dimension reduction for gene expression data

This problem involves the analysis of gene expression data from 327 subjects from Yeoh et al (2002). The data set includes abundance levels for 3141 genes and a class label indicating one of 7 leukemia subtypes the patient was diagnosed with. The paper describing their analysis of this data can be found here. Read in the csv data in leukemia\_data.csv. It is posted on Piazza in the resources tab with the homework:

```
leukemia_data <- read_csv("leukemia_data.csv")</pre>
```

(a). The class of the first column of leukemia\_data, Type, is set to character by default. Convert the Type column to a factor using the mutate function. Use the table command to print the number of patients with each leukemia subtype. Which leukemia subtype occurs the least in this data?

```
leukemia_data_factor <- leukemia_data %>%
  mutate(Type = factor(Type))

type_table = table(leukemia_data_factor$Type)
```

The BCR-ABL sub-type of leukemia has the fewest patients in this data set at 15 individuals.

(b). Run PCA on the leukemia data using prcomp function with scale=TRUE and center=TRUE (this scales each gene to have mean 0 and variance 1). Make sure you exclude the Type column when you run the PCA function (we are only interested in reducing the dimension of the gene expression values and PCA doesn't work with categorical data anyway). Plot the proportion of variance explained by each principal component (PVE) and the cumulative PVE side-by-side.

```
# Standardize the variables by subtracting mean and divided by standard deviation
scale.leukemia = scale(leukemia_data_factor[,-c(1)], center=TRUE, scale=TRUE)

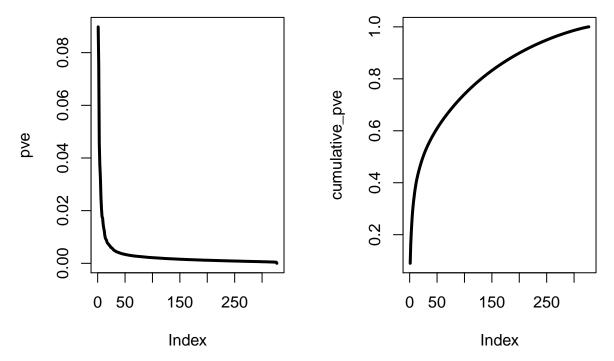
pr.out = prcomp(scale.leukemia, scale =TRUE, center=TRUE)

pr.var = pr.out$sdev^2 # variance is the square of the standard deviation of the PCA output

pve <- pr.var/sum(pr.var)
cumulative_pve <- cumsum(pve)

## This will put the next two plots side by side
par(mfrow=c(1, 2))

## Plot proportion of variance explained
plot(pve, type="l", lwd=3)
plot(cumulative_pve, type="l", lwd=3)</pre>
```



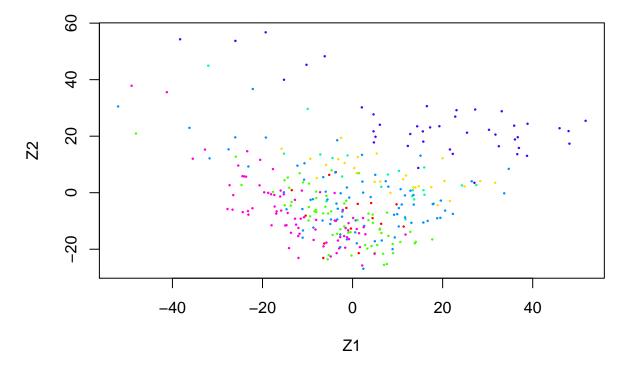
(c). Use the results of PCA to project the data into the first two principal component dimensions. prcomp returns this dimension reduced data in the first columns of x. Plot the data as a scatter plot using plot function with col=plot\_colors where plot\_colors is defined

```
rainbow_colors <- rainbow(7)
plot_colors <- rainbow_colors[leukemia_data_factor$Type]</pre>
```

This will color the points according to the leukemia subtype. Add the leukemia type labels to the plot using text with labels argument set to the leukemia type and the col to plot\_colors (it may help legibility to make the points on the plot very small by setting cex to a small number). Which group is most clearly separated from the others along the PC1 axis? Which genes have the highest absolute loadings for PC1 (the genes that have the largest weights in the weighted average used to create the new variable PC1)? You can find these by taking the absolute values of the first principal component loadings and sorting them. Print the first 6 genes in this sorted vector using the head function.

```
par(mfrow=c(1, 1))

PC1 = pr.out$x[,1]
PC2 = pr.out$x[,2]
plot(PC1, PC2, col = plot_colors, pch =19, xlab ="Z1",ylab="Z2", cex = 0.2)
```

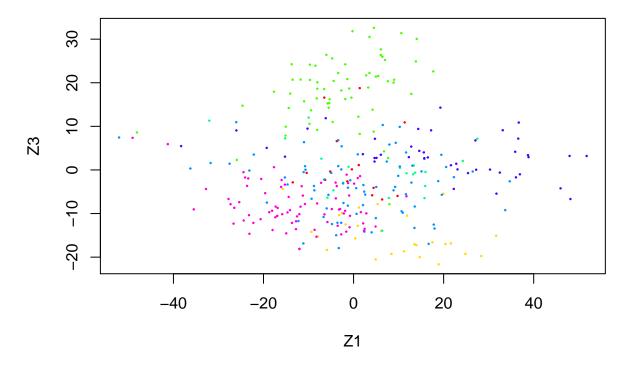


#text("bottomright", labels=paste(leukemia\_data\_factor\$Type))

(d). (231 Only) PCA orders the principal components according to the amount of total variation in the data that they explain. This does not mean, however, that the principal components are sorted in terms of how useful they are at capturing variation between the leukemia groups. For example, if gene expression varied significantly with age and gender (independent of leukemia status), the first principal components could reflect genetic variation due to age and gender, but not to leukemia. The first scatter plot shows that the second PC is not a good discriminator of leukemia type. See if the 3rd PC is better at discriminating between leukemia types by plotting the data projected onto the first and third principal components (not the second).

```
par(mfrow=c(1, 1))

PC3 = pr.out$x[,3]
plot(PC1, PC3, col = plot_colors, pch =19, xlab ="Z1",ylab="Z3", cex = 0.2)
```



#### # text(labels(leukemia\_data\_factor\$Type))

(e.) (231 Only) For this part we will be using the ggridges library. Create a new tibble where the first column (call it z1) is the projection of the data onto the first principal component and the second column is the leukemia subtype (Type). Use ggplot with geom\_density\_ridges to create multiple stacked density plots of the projected gene expression data. Set the ggplot aesthetics to aes(x = z1, y = Type, fill = Type). Make another identical plot, except replace z1 with z3, the projection of the data onto the third principal component. Identify two leukemia subtypes that are nearly indistinguishable when the gene expression data is projected onto the first PC direction, but easily distinguishable when projected onto the third PC direction.

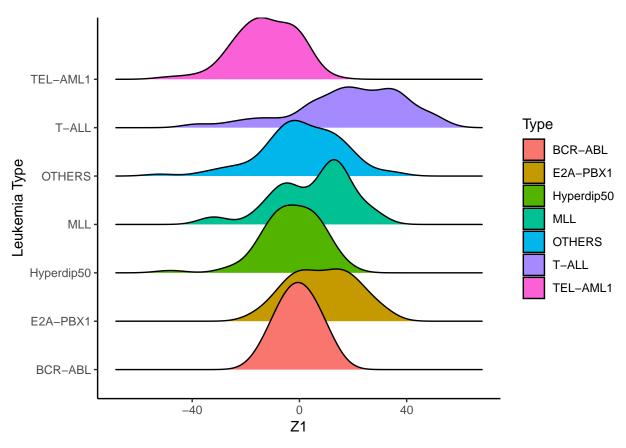
```
pca_df = data.frame("z1" = PC1, "Type" = leukemia_data_factor$Type)
pca_tibble = as.tibble(pca_df)

plot.new()
ggridge_plot_z1 = ggplot(pca_tibble, aes(x=z1, y = Type, fill = Type)) +
    geom_density_ridges() +
    title("Projection of Leukemia Type Data on PC1 from PCA") +
    labs(x = "Z1", y = "Leukemia Type") +
    theme_classic()
```

# Projection of Leukemia Type Data on PC1 from PCA

```
ggridge_plot_z1
```

## Picking joint bandwidth of 5.46



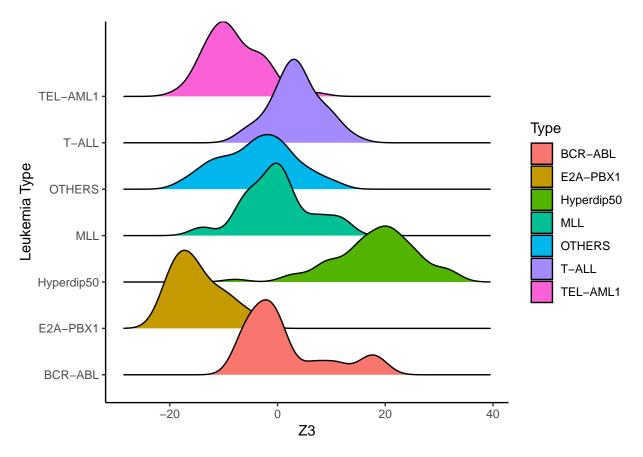
```
plot.new()
pca_df_z3 = data.frame("z3" = PC3, "Type" = leukemia_data_factor$Type)
pca_tibble_z3 = as.tibble(pca_df_z3)

ggridge_plot_z3 = ggplot(pca_tibble_z3, aes(x=z3, y = Type, fill = Type)) +
    geom_density_ridges() +
    title("Projection of Leukemia Type Data on PC3 from PCA") +
    labs(x = "Z3", y = "Leukemia Type") +
    theme_classic()
```

## Projection of Leukemia Type Data on PC3 from PCA

```
ggridge_plot_z3
```

## Picking joint bandwidth of 2.29

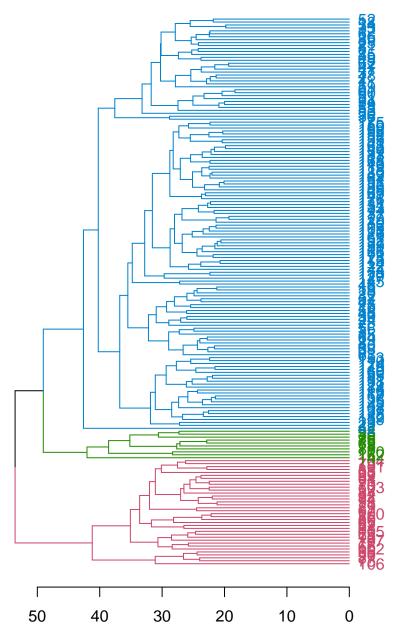


(f.) Use the filter command to create a new tibble leukemia\_subset by subsetting to include only rows for which Type is either T-ALL, TEL-AML1, or Hyperdip50. Compute a euclidean distance matrix between the subjects using the dist function and then run hierarchical clustering using complete linkage. Plot two dendrograms based on the hierarchical clustering result. In the first plot, force 3 leukemia types to be the labels of terminal nodes, color the branches and labels to have 3 groups and rotate the dendrogram counterclockwise to have all the terminal nodes on the right. In the second plot, do all the same things except that this time color all the branches and labels to have 5 groups. Please make sure library dendextend is installed. Hint: as.dendrogram, set\_labels, color\_branches, color\_labels and plot(..., horiz = TRUE) may be useful.

### library(dendextend)

##

```
## Attaching package: 'dendextend'
## The following object is masked from 'package:rpart':
##
##
       prune
## The following object is masked from 'package:stats':
##
##
       cutree
leukemia_subset <- leukemia_data_factor %>%
  filter(Type == "T-ALL" | Type == "TEL-AML1" | Type == "Hyperdip50")
leukemia_plot3 <- leukemia_subset %>%
  dist() %>%
  hclust(method = "complete") %>%
  as.dendrogram() %>%
  color_branches(k = 3) %>%
  color_labels(k = 3)
## Warning in dist(.): NAs introduced by coercion
set_labels(leukemia_plot3, labels=leukemia_subset$Type[order.dendrogram(leukemia_plot3)])
## 'dendrogram' with 2 branches and 186 members total, at height 53.56452
set(leukemia_plot3, "labels_cex", 0.1)
## 'dendrogram' with 2 branches and 186 members total, at height 53.56452
plot(leukemia_plot3, horiz = T)
```



```
leukemia_plot5 <- leukemia_subset %>%
  dist() %>%
  hclust(method = "complete") %>%
  as.dendrogram() %>%
  color_branches(k = 5) %>%
  color_labels(k = 5)
```

## Warning in dist(.): NAs introduced by coercion

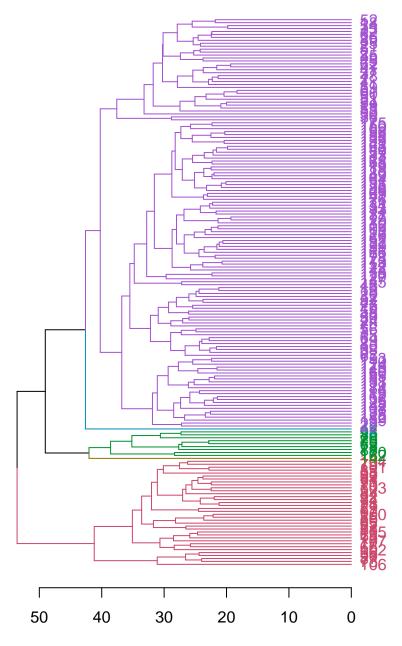
```
set_labels(leukemia_plot5, labels=leukemia_subset$Type[order.dendrogram(leukemia_plot5)])
```

 $\mbox{\tt \#\#}$  'dendrogram' with 2 branches and 186 members total, at height 53.56452

```
set(leukemia_plot5, "labels_cex", 0.1)
```

## 'dendrogram' with 2 branches and 186 members total, at height 53.56452

plot(leukemia\_plot5, horiz = T)



(g). (231 only). Use superheat to plot the distance matrix from the part above. Order the rows and columns by the hierarchical clustering you obtained in the previous part. You should see a matrix with a block diagonal structure. The labels (corresponding to leukemia types) will not be available to read on the plot. Print them out by looking at leukemia\_subset\$Type ordered by clustering order. Based on this plot which two leukemia types (of the three in the subset) seem more similar to one another? Hint: use heat.pal = c("dark red", "red", "orange", "yellow")) for colorbar specification in superheat.

(h). (231 only). You can also use superheat to generate a hierarchical clustering dendrogram or a kmeans clustering. First, use leukemia\_subset to run hierarchical clustering and draw the dendrogram. Second, use the same dataset to run kmeans clustering with three the optimal number of clusters, and order the genes (columns) based on hierarchical clustering.

Hint: arguments row.dendrogram, clustering.method, n.cluster.rows and pretty.order.cols may be useful, please read the argument descriptions before you attempt the problem. The package manual can be found here: https://cran.r-project.org/web/packages/superheat/superheat.pdf