$_nC_r$

1) $0! = 1$

2) $1! = 1$

3) $_nC_0 = {}_nC_n = 1$

4) $_nC_r = {}_nC_{n-r}$

5) $_nC_r = \dfrac{n!}{(n-r)!\,r!}$

6) pascal의 삼각형

1 r

1 2 1

… 1 3 3 1 … n-1

… 1 4 6 4 1 … n

r-1 r

$\boxed{{}_nC_r = {}_{n-1}C_{r-1} + {}_{n-1}C_r}$

$6 = 3 + 3$

7) $_{n-1}C_{r-1} = \dfrac{(n-1)!}{(n-r)!\,(r-1)!}$

$$_{n-1}C_{r-1} = \frac{(n-1)!}{(n-r)!\,\underbrace{(r-1)!\,r}}\cdot\frac{r\cdot n}{n}$$

$$= \frac{n!}{(n-r)!\,r!}\cdot\frac{r}{n}$$

$$= {}_nC_r\cdot\frac{r}{n}$$

$$\therefore \boxed{{}_nC_r = \frac{n}{r}\cdot{}_{n-1}C_{r-1}}$$

8) $_{n-1}C_r = \dfrac{(n-1)!}{(n-1-r)!\,r!}$

$$= \frac{(n-1)!\cdot n\cdot(n-r)}{(n-1-r)!\,(n-r)\cdot r!\,n}$$

$$= \frac{n!\,(n-r)}{(n-r)!\,r!\cdot n}$$

$$= \frac{n-r}{n}\cdot{}_nC_r$$

$$\therefore {}_nC_r = \frac{n}{n-r}\,{}_{n-1}C_r$$

9) $_nC_{r-1} = \dfrac{n!}{(n-r+1)!\,(r-1)!}$

$$= \frac{n!\,r}{(n-r+1)(n-r)!\,(r-1)!\,r}$$

$$= \frac{n!}{(n-r)!\,r!}\cdot\frac{r}{n-r+1}$$

$$\therefore = {}_nC_r\cdot\frac{r}{n-r+1}$$

$$\therefore \boxed{{}_nC_r = \frac{n-r+1}{r}\cdot{}_nC_{r-1}}$$

10) $_nP_r = \dfrac{n!}{(n-r)!}$

$$= \frac{n!\,r!}{(n-r)!\,r!} = {}_nC_r\cdot r!$$

$$\boxed{\therefore {}_nC_r = \frac{1}{r!}\cdot{}_nP_r}$$