Optimization Techniques for Multi-Relay Multi-Cell Massive MIMO NOMA Systems

Index Terms

NOMA, WMMSE, SSUM.

I. System Model

We consider, as shown in Fig. 1, the downlink of an L-cell mMIMO NOMA system where in each cell, an N-antenna mMIMO BS serve clusters of single-antenna cellular users which via half-duplex single-antenna AF relays. These relays are installed at a high altitude such that the BS-relay and users-relay channels have both LoS and NLoS components and therefore the channels are Rician channels. The lth BS communicates with a cluster of $\mathcal{U}_{lk}$ users through the relay R_{lk} by employing NOMA. The users close to a relay form a clusters.

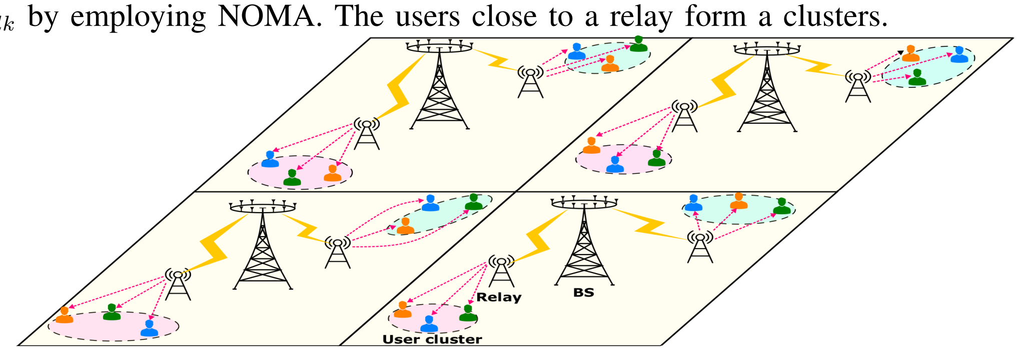

Fig. 1: Multi-cell relay-aided downlink mMIMO system model.

The communication operates in TDD mode, with τ_c symbols coherence interval is divided into channel estimation (CE) by transmitting pilots and data transmission (DT) phases of τ and (τ_c - τ) symbols respectively. In the CE phase, the pilots are transmitted from the relays to both BS and the Users for transmit beamforming and receive forming respectively. In DT phase, NOMA is deployed and users are served via relays.

A. Channel Modeling

As there are two hops, one from BS-relay and the other from relay-BS which channel modeling. As it based on the paper [1], We just provide the mathematical expression for the channel.

For the modelling of BS-relay channel, as mentioned earlier that channel contain LoS and NLos components and it is Rician channel, We denote the kth relay in the lth cell as R_{lk}. The channel from the l'th BS to R_{lk} is denoted as $\mathbf{h}_{lk}^{j} \in \mathbb{C}^{N\times 1}$. It is, accordingly, mathematically expressed as follows

$$\mathbf{h}_{lk}^{l'} = \overline{\mathbf{h}}_{lk}^{l'} + (\mathbf{R}_{lk}^{l'})^{\frac{1}{2}} \mathbf{h}_{lk}^{l',\text{NLoS}} \tag{1}$$

The matrix $\overline{\mathbf{R}}_{lk}^{k'}$ characterizes the spatial correlation of the NLoS component. The $\mathbf{h}_{lk}^{l',\text{NLoS}}$ is random variable which follows complex normal pdf. And the vector $\overline{\mathbf{h}}_{lk}^{l'}$ is the LoS component of the channel. The second-order statistics and its generation are explained in [1].

The relay-user channel is a scalar as both are single antennas. The channel pdf is a rician pdf as it contains Los and NLoS components. The channel scalar from the relay $R_{l'k'}$ to the nth user in the cluster $\mathcal{U}_{lk}$ is denoted as $g_{lk,n}^{l'k'} \in \mathbb{C}^{1\times 1}$ are mathematically expressed as:

$$g_{lk,n}^{l'k'} = \overline{g}_{lk,n}^{l'k'} + (\upsilon_{lk,n}^{l'k'})^{\frac{1}{2}} g_{lk,n}^{l'k',\text{NLoS}}. \tag{2}$$

The scalar $g_{lk,n}^{jk',\text{NLoS}}$ is random variable with complex normal distribution. The $\upsilon_{lk,n}^{l'k'}$ characterizes the spatial correlation of the NLoS component.

The CE phase: In this system model which follows [1], there is no direct link between users and BS as we considering they are far apart and causes huge shadowing and path loss and which leads to not having to estimate the BSs and user channel end-to-end. Therefore the relay helps to estimate the channel between user-relay and BS-relay, the local CSI. To estimate the local CSI, the K relays in the cluster R_{lk} transmits $K = \tau_p$ pilots (ψ_k) which are mutually orthogonal and with magnitude K to both lth BS and cluster of users $\mathcal{U}_{lk}$. The relays in each cell share the same pilot which leads to pilot contamination.

BS-Relay Estimate: The channel estimate of $\mathbf{h}_{lk}^{l}$ can be considered as a simple MMSE channel estimation between BS and relay. Without involving the mathematics involved in the estimation phase the MMSE estimate of the channel $\mathbf{h}_{lk}^{l}$ is therefore obtained from [1] as

$$\widehat{\mathbf{h}}_{lk}^{l} = \overline{\mathbf{h}}_{lk}^{l} + \sqrt{p_p}\mathbf{R}_{lk}^{l}\mathbf{\Psi}_{lk}\left[\tilde{\mathbf{y}}_{k}^{l} - \overline{\mathbf{y}}_{k}^{l}\right], \tag{3}$$

where $\mathbf{\Psi}_{lk} = \left(\mathbf{I}_N + \tau p_p \sum\limits_{l'=1}^{L} \mathbf{R}_{l'k}^{l}\right)^{-1}$, $\overline{\mathbf{y}}_{k}^{l} = \sum\limits_{l'=1}^{L} \sqrt{p_p}\tau\overline{\mathbf{h}}_{l'k}^{l}$ and $\tilde{\mathbf{y}}_{k}^{l} = \sum_{l'=1}^{L} \sqrt{p_p}\tau\mathbf{h}_{l'k}^{l} + \mathbf{N}_{k}^{l\,*}$ and p_p is the pilot power and $\boldsymbol{N}_{k}^{l*}$ is AWGN of the lth BS from kth relay.

Relay-User Estimate: The channel estimate of $g_{lk,n}^{lk}$, is can be formed similar to the MMSE channel estimate of D2D section mentioned in the previous chapter, The MMSE estimate of $g_{lk,n}^{lk}$ is obtained from [1] is expressed as:

$$\widehat{g}_{lk,n}^{lk} = \overline{g}_{lk,n}^{lk} + \frac{\sqrt{p_p}\gamma_{lk,n}^{l'k}}{1 + \sum\limits_{l'=1}^{L} \tau p_p \gamma_{lk,n}^{l'k}} \left[\tilde{y}_{lk,n}^{p} - \overline{y}_{lk,n}^{p} \right], \quad \text{where } \overline{y}_{lk,n}^{p} = \sum_{l'=1}^{L} \sqrt{p_p} \tau \overline{g}_{lk,n}^{l'k}. \tag{4}$$

where $\tilde{y}_{lk,n}^{p} = \mathbf{y}_{lk,n}^{p} \boldsymbol{\psi}_k^* = \sum\limits_{l'=1}^{L} \sqrt{p_p} \tau g_{lk,n}^{l'k} + \mathbf{n}_{lk,n}^{p} \boldsymbol{\psi}_k^*$

The DT phase: Data transmission from the BS to users via relay requires two time slots, whose functioning is explained as follows:

1) The 1st time slot: The First slot is used for BS-to-Relay transmission, and NOMA superposes user transmit signals in its cell, then precodes and sends them to relays. The precoded NOMA signal transmitted by the lth BS with $s_{lk,n}$ be the signal of the nth user in cluster $\mathcal{U}_{lk}$.

$$\mathbf{x}^l = \sum_{k=1}^{K} \mathbf{w}_{lk} \sum_{n=1}^{\mathcal{U}_{lk}} \sqrt{p_{lk,n}} s_{lk,n} \triangleq \sum_{k=1}^{K} \mathbf{w}_{lk} x_{lk}. \tag{5}$$

Here x_{lk} is the signal transmitted for the cluster $\mathcal{U}_{jk}$ to relay R_{lk} and the precoder $\mathbf{w}_{lk} \in \mathbb{C}^{N \times 1}$ is based on the channel estimate of $\mathbf{H}$ with unit norm due to power constraints. The received signal from the BS l to the kth relay in lth cell is given as

$$y_{R_{lk}} = \underbrace{\sum_{l''=1}^{L} \sum_{k''=1}^{K} (\mathbf{h}_{lk}^{l''})^T \mathbf{w}_{l''k''} x_{l''k''}}_{\tilde{y}_{R_{lk}}} + z_{R_{lk}}. \tag{6}$$

The scalar $z_{R_{lk}}$ with pdf $\mathcal{CN}(0,1)$ is the AWGN at the kth relay in lth cell.

2) The 2nd time slot: The second slot is used for Relay-BS transmission after the signal received to Relay k. The received signal is transmitted again after the amplification of the signal by AF relay to the users of the cluster $\mathcal{U}_{lk}$. Here $\overline{\mu}_{lk}$ is the amplification factor that limits the transmit relay power to q_{lk}. The $\overline{mu}_{lk}$ cited from [1] is given as:

$$\overline{\mu}_{lk}^2 \mathbf{E}(|y_{R_{lk}}|^2) = q_{lk} \implies \overline{\mu}_{lk} = \sqrt{\frac{q_{lk}}{\left(\sum\limits_{l''=1}^{L} \sum\limits_{k''=1}^{K} \rho_{l''k'',lk} p_{l''k''} + \sum\limits_{l''=1}^{K} \xi_{l''k,lk} p_{l''k} + 1 \right)}}. \tag{7}$$

where $p_{l'k'}$ is sum of the user transmit power from the cluster $\mathcal{U}_{l'k'}$. And the variables $\rho_{l''k'',lk}, \xi_{l''k,lk}$ are provided in the appendix A of [1]. The received NOMA signal at the nth user from the cluster $\mathcal{U}_{\updownarrow||}$ obtained from equation (11) from [1] is:

$$
\begin{aligned}
y_{lk,n} = &\underbrace{f_{lk,n} g_{lk,n}^{lk} \mu_{lk} \mathbf{h}_{lk}^{lT} \mathbf{w}_{lk} \sqrt{p_{lk,n}} s_{lk,n}}_{\text{desired signal}} + \underbrace{f_{lk,n} g_{lk,n}^{lk} \mu_{lk} \mathbf{h}_{lk}^{lT} \mathbf{w}_{lk} \sum_{n' \neq n}^{\mathcal{U}_{lk}} \sqrt{p_{lk,n'}} s_{lk,n'}}_{\text{intra-relay interference}} \\
&+ \underbrace{\sum_{l'' \neq l}^{L} \sum_{n'=1}^{\mathcal{U}_{lk}} f_{lk,n} g_{lk,n}^{lk} \mu_{lk} \mathbf{h}_{lk}^{l''T} \mathbf{w}_{l''k} \sqrt{p_{l''k,n'}} s_{l''k,n'}}_{\text{1st hop PS inter-relay interference}} + \underbrace{\sum_{l''=1}^{L} \sum_{k'' \neq k}^{K} \sum_{n'=1}^{\mathcal{U}_{lk}} f_{lk,n} g_{lk,n}^{lk} \mu_{lk} \mathbf{h}_{lk}^{l''T} \mathbf{w}_{l''k''} \sqrt{p_{l''k'',n'}} s_{l''k'',n'}}_{\text{1st hop nPS inter-relay interference}} \\
&+ \underbrace{\sum_{l' \neq l}^{L} f_{k,n} g_{lk,n}^{l'k} \mu_{l'k} \tilde{y}_{R_{l'k}}}_{\substack{\text{2nd hop PS} \\ \text{inter-relay interference}}} + \underbrace{\sum_{l'=1}^{L} \sum_{k' \neq k}^{K} f_{lk,n} g_{lk,n}^{l'k'} \mu_{l'k'} \tilde{y}_{R_{l'k'}}}_{\substack{\text{2nd hop nPS} \\ \text{inter-relay interference}}} + \underbrace{\sum_{l'=1}^{L} \sum_{k'=1}^{K} f_{lk,n} g_{lk,n}^{l'k'} \mu_{l'k'} z_{R_{l'k'}}}_{\text{forwarding noise}} + \underbrace{f_{k,n} z_{k,n}}_{\text{receiver noise}} .
\end{aligned} \tag{8}
$$

The explanation of the equation is provided in the system model section of the paper [1]. Users associated with the kth relay mitigate the intra-relay interference by performing SIC. To enable this, we assume similar to [2], that these users are ordered in the descending order of their path losses. Among the users associated with the kth relay, the nth user first cancels intra-relay interference from $\forall\ n' > n$ users by employing SIC [3], and then decodes its own signal while treating the signal from the first $n-1$ users as inherent intra-cluster interference [3]. The user employs the statistical value $\mathbb{E}\left[\mathbf{h}_k^T \mathbf{w}_k\right]$, along with the channel estimate $\hat{g}_{k,k,n}$, to perform SIC. After the SIC, the signal at the nth user associated with the kth relay experiences the following intra-relay interference (**??**) as shown below

$$
\underbrace{f_{lk,n} g_{lk,n}^{lk} \mu_{lk} \mathbf{h}_{lk}^{lT} \mathbf{w}_{lk} \sum_{n'=1}^{n-1} \sqrt{p_{k,n'}} s_{k,n'}}_{\text{inherent intra-relay interference}} + \underbrace{\sum_{n'=n+1}^{\mathcal{U}_{lk}} \mu_{lk} \left[f_{lk,n} g_{lk,n}^{lk} \mathbf{h}_{lk}^{lT} \mathbf{w}_{lk} - f_{lk,n} \widehat{g}_{lk,n}^{lk} \mathbb{E}\left[\mathbf{h}_{lk}^{lT} \mathbf{w}_{lk} \right] \right] \sqrt{p_{k,n'}} s_{k,n'}}_{\text{residual intra-relay interference due to imperfect SIC}} .
$$

After the SIC, the received NOMA signal at the n user of the cluster $\mathcal{U}_{lk}$ is obtained from equation (12) from [1] is expressed as:

$$
\begin{aligned}
\bar{y}_{lk,n} = &f_{lk,n} g_{lk,n}^{lk} \mu_{lk} \mathbf{h}_{lk}^{lT} \mathbf{w}_{lk} \sqrt{p_{lk,n}} s_{lk,n} + f_{lk,n} g_{lk,n}^{lk} \mu_{lk} \mathbf{h}_{lk}^{lT} \mathbf{w}_{lk} \sum_{n'=1}^{n-1} \sqrt{p_{lk,n'}} s_{lk,n'} \\
&+ \sum_{n'=n+1}^{\mathcal{U}_{lk}} \mu_{lk} \left[f_{lk,n} g_{lk,n}^{lk} \mathbf{h}_{lk}^{lT} \mathbf{w}_{lk} - f_{lk,n} \widehat{g}_{lk,n}^{lk} \mathbf{E}\left[\mathbf{h}_{lk}^{lT} \mathbf{w}_{lk} \right] \right] \sqrt{p_{lk,n'}} s_{lk,n'} \\
&+ \sum_{l'' \neq l}^{L} \sum_{n'=1}^{\mathcal{U}_{lk}} f_{lk,n} g_{lk,n}^{lk} \mu_{lk} \mathbf{h}_{lk}^{l''T} \mathbf{w}_{l''k} \sqrt{p_{l''k,n'}} s_{l''k,n'} + \sum_{l''=1}^{L} \sum_{k'' \neq k}^{K} \sum_{n'=1}^{\mathcal{U}_{lk}} f_{lk,n} g_{lk,n}^{lk} \mu_{lk} \mathbf{h}_{lk}^{l''T} \mathbf{w}_{l''k''} \sqrt{p_{l''k'',n'}} s_{l''k'',n'} \\
&+ \sum_{l' \neq l}^{L} f_{k,n} g_{lk,n}^{l'k} \mu_{l'k} \tilde{y}_{R_{l'k}} + \sum_{l'=1}^{L} \sum_{k' \neq k}^{K} f_{lk,n} g_{lk,n}^{l'k'} \mu_{l'k'} \tilde{y}_{R_{l'k'}} + \sum_{l'=1}^{L} \sum_{k'=1}^{K} f_{lk,n} g_{lk,n}^{l'k'} \mu_{l'k'} z_{R_{l'k'}} + f_{k,n} z_{k,n}.
\end{aligned} \tag{9}
$$

BS combiners: In this work, we analyze the system performance for three combining schemes namely MRC, ZF and MMSE. The MMSE and ZF combiners are designed to cancel the inter- and intra-cell interference experienced by the user. These techniques, however, cannot cancel the Relay to User interference, of the system. The ZF and MMSE schemes, by utilizing the statistics and the channel realizations of the relay transmitters, mitigates the intra-cell, inter-cell of the relay interference. The ZF and MMSE combiners are given as

$$\overline{\mathbf{W}_l} = \begin{cases} \mathbf{H}_l, & \text{for MRC} \\ \mathbf{H}_l \left[\mathbf{H}_l^H \mathbf{H}_j\right]^{-1}, & \text{for ZF} \\ \left[\sum\limits_{l'=1}^{L} \mathbf{H}_{l'} \overline{\mathbf{P}}_{l'}^{cd} \mathbf{H}_{l'}^H + \mathbf{I}_M\right]^{-1} \mathbf{H}_j \overline{\mathbf{P}}_j, & \text{for MMSE.} \end{cases} \tag{10}$$

Here $\overline{\mathbf{W}^l} = [\mathbf{w}_{l1}, \cdots, \mathbf{w}_{lK}] \in \mathbf{C}^{N\times K}$ denotes the set of combiners used by lth BS for the relays in the lth cell. The matrices $\mathbf{H}_l = [\mathbf{h}_{l1}^l, \cdots, \mathbf{h}_{lK}^l] \in \mathbf{C}^{N\times K}$ denote the set of channels from K relays in lth cell to the lth BS respectively. Further $\mathbf{P}_{l'} = \text{diag}(p_{l'1}, \cdots, p_{l'K}) \in \mathbb{R}_+^{K\times K}$ where $p_{l'k} = \sum\limits_{n'=1}^{\mathcal{U}_{l'k}} p_{l'k,n'}$contains the sum of transmit powers of the users of cluster $\mathcal{U}_{l'k}$.

II. SE ANALYSIS

1) Ergodic SE Analysis: From the equation (12) of the base paper [1], The Ergodic sum SE of the system using Genie-bound for a finite number of BS antennas with MMSE channel Estimation and with imperfect user SIC, is given as

$$R_{\text{sum}}^{\text{e}} = \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} \mathbb{E}\left[\frac{1}{2}\left(1-\frac{\tau}{\tau_c}\right)\log_2\left(1+\frac{\overline{\Delta}_{lk,n}^i}{\overline{\Omega}_{lk,n}^i}\right)\right], \text{ where } \overline{\Omega}_{lk,n}^i = \sum_{m=1}^{5} \hat{I}_{lk,n}^{(m)} + 1, \tag{11}$$

$$\hat{\Delta}_{lk,n} = \hat{A}_{lk,n}\overline{\mu}_{lk}^2 p_{lk,n}, \ \hat{I}_{lk,n}^{(1)} = \sum_{n'=1}^{n-1} \hat{C}_{lk,n}^{(1)} \overline{\mu}_{lk}^2 p_{lk,n'}, \ \hat{I}_{lk,n}^{(2)} = \sum_{n'=n+1}^{\mathcal{U}_{lk}} \hat{C}_{lk,n}^{(2)} p_{lk,n'} \overline{\mu}_{lk}^2,$$

$$\hat{I}_{lk,n}^{(3)} = \sum_{(l',k')\neq(l,k)} \sum_{n'=1}^{\mathcal{U}_{l'k'}} \hat{C}_{l'k',lk,n}^{(3)} p_{l'k',n'} \overline{\mu}_{l'k'}^2 \ \hat{I}_{lk,n}^{(4)} = \sum_{(l',k')} \sum_{(l'',k'')\neq(l',k')} \sum_{n'=1}^{\mathcal{U}_{l''k''}} C_{l''k'',l'k',lk,n}^{(4)} p_{l''k'',n'} \overline{\mu}_{l'k'}^2$$

$$\hat{I}_{k,n}^{(5)} = \sum_{l'=1}^{L}\sum_{k'=1}^{K} \hat{C}_{l'k',lk,n}^{(5)} \overline{\mu}_{l'k'}^2, \text{ and } \overline{\mu}_{lk} = \sqrt{\frac{q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K} \rho_{l''k'',lk} p_{l''k''} + \sum\limits_{l''=1}^{K} \xi_{l''k,lk} p_{l''k} + 1\right)}}.$$

The terms $\hat{A}_{lk,n}$, $\hat{C}^{(1)}_{lk,n}$, $\hat{C}^{(2)}_{lk,n}$, $\hat{C}^{(3)}_{l'k',lk,n}$, $\hat{C}^{(4)}_{\substack{l''k'',l'k',\\ lk,n}}$ and $\hat{C}^{(5)}_{l'k',lk,n}$ are functions of instantaneous channel realizations which are given as

$$\hat{A}_{lk,n} = |f_{lk,n}g^{lk}_{lk,n}\mathbf{h}^{l^H}_{lk}\mathbf{w}_{lk}|^2,\ \hat{C}^{(1)}_{lk,n} = |f_{lk,n}g^{lk}_{lk,n}\mathbf{h}^{l^H}_{lk}\mathbf{w}_{lk}|^2,\ \hat{C}^{(2)}_{lk,n} = |f_{lk,n}g^{lk}_{lk,n}\mathbf{h}^{l^H}_{lk}\mathbf{w}_{lk} - f_{lk,n}\hat{g}^{lk}_{lk,n}\mathbb{E}[\mathbf{h}^{l^H}_{lk}\mathbf{w}_{lk}]|^2,$$
$$\hat{C}^{(3)}_{l'k',lk,n} = |f_{lk,n}g^{l'k'}_{lk,n}\mathbf{h}^{l'^H}_{l'k'}\mathbf{w}_{l'k'}|^2,\ \hat{C}^{(4)}_{\substack{l''k'',l'k',\\ lk,n}} = |f_{lk,n}g^{l'k'}_{lk,n}\mathbf{h}^{l''^H}_{l'k'}\mathbf{w}_{l''k''}|^2,\ \hat{C}^{(5)}_{l'k',lk,n} = |f_{lk,n}g^{l'k'}_{lk,n}|^2 \tag{12}$$

2) UATF closed-form SE: The hardening bound technique to derive a UATF closed-form SE expression with MRC precoder for the multi-cell relay-aided mMIMO NOMA system for a finite number of BS antennas relying on MMSE channel estimation, and with imperfect user SIC which is derived in the base paper [1] is given as

$$R^c_{\text{sum}} = \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}}\frac{1}{2}\left(1-\frac{\tau}{\tau_c}\right)\log_2\left(1+\frac{\overline{\Delta}^c_{lk,n}}{\overline{\Omega}^c_{lk,n}}\right),\ \text{where}\ \ \overline{\Omega}_{lk,n} = \sum_{m=0}^{7}\overline{I}^{(m)}_{lk,n}+1, \tag{13}$$

$$\overline{\Delta}_{lk,n} = A_{lk,n}\overline{\mu}^2_{lk}p_{lk,n},\ \overline{I}^{(0)}_{lk,n} = C^{(0)}_{lk,n}p_{lk,n}\overline{\mu}^2_{lk},\ \overline{I}^{(1)}_{lk,n} = C^{(1)}_{lk,n}\sum_{n'=1}^{n-1}\overline{\mu}^2_{lk}p_{lk,n'},\ \overline{I}^{(2)}_{lk,n} = C^{(2)}_{lk,n}\sum_{n'=n+1}^{\mathcal{U}_{lk}}p_{lk,n'}\overline{\mu}^2_{lk},$$

$$\sum_{m=3}^{6}\overline{I}^{(m)}_{lk,n} = \sum_{(l',k')\neq(l,k)}\sum_{n'=1}^{\mathcal{U}_{l'k'}}C^{(3)}_{l'k',lk,n}p_{l'k',n'}\mu^2_{l'k'} + \sum_{(l',k')}\sum_{(l'',k'')\neq(l',k')}\sum_{n'=1}^{\mathcal{U}_{l''k''}}C^{(4)}_{\substack{l''k'',l'k',\\ lk,n}}p_{l''k'',n'}\overline{\mu}^2_{l'k'},$$

$$\overline{I}^{(7)}_{k,n} = \sum_{l'=1}^{L}\sum_{k'=1}^{K}C^{(5)}_{l'k',lk,n}\overline{\mu}^2_{l'k'},\ \text{and}\ \overline{\mu}_{lk} = \sqrt{\frac{q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}p_{l''k''} + \sum\limits_{l''=1}^{K}\xi_{l''k,lk}p_{l''k}+1\right)}}. \tag{14}$$

Here $p_{lk} = \sum_{n=1}^{\mathcal{U}_{lk}}p_{lk,n}$, $A_{lk,n} = \frac{\pi\upsilon^{lk}_{lk,n}\delta_{lk}}{4}\left[L_{1/2}\left(-\left|\overline{g}^{lk}_{lk,n}\right|^2/\upsilon^{lk}_{lk,n}\right)\right]^2$, with $L_{\frac{1}{2}}(\cdot)$ being Laguerre polynomial [4]. The terms $C^{(0)}_{lk,n}$, $C^{(1)}_{lk,n}$, $C^{(2)}_{lk,n}$, $C^{(3)}_{l'k',lk,n}$, $C^{(4)}_{\substack{l''k'',l'k',\\ lk,n}}$ and $C^{(5)}_{l'k',lk,n}$ are functions of long term channel statistics, which are given in Appendix A of paper [].

III. Preface to WMMSE Algorithm

The WMMSE algorithm was first proposed in [5] to optimize the weighted sum rate to design a linear transmit filter. The fundamental concept behind the approach is to transform the objective problem into a WMMSE maximisation, in which the weights are modified in an iterative fashion. The algorithm covers a wide range of issues, including sum rate use. The following fundamental equation serves as the foundation for the WMMSE method.

$$\text{SINR} = \max_u \gamma = \max_u \frac{1}{e} - 1 \tag{15}$$

where u can be considered as the receiving beamformer to the signal received and also acts as an auxiliary variable in the algorithm, and γ as the corresponding SINR with respect to the decoded signal and e is the *Mean Square Error*(MSE) of the decoded signal. This vital equation can be best explained by a special case of uplink single cell multi user SISO system. The signal received by the only BS is:

$$y_k = h_k \sqrt{p_k} s_k + \sum_{\substack{i=1 \\ i \neq k}}^{N} h_i \sqrt{p_i} s_i + n \tag{16}$$

where h is the channel matrix from Cellular User(CU) to the BS, s and p are the transmit symbol and power from the CU and n is the white Gaussian noise. The SE of the kth CU is:

$$\text{SE}_k^{CU} = C \log(1 + \text{SINR}_k^{CU}) = \frac{\Delta_k}{\Omega_k}, \text{ where}$$

$$\text{SINR}_k^{CU} = \frac{|h_k|^2 p_k}{\sum\limits_{\substack{i=1 \\ i \neq k}}^{N} |h_i|^2 p_i + 1}. \tag{17}$$

The received signal y_k at the BS of the CU k is decoded with a received beamformer u_k. The decoded signal is given as;

$$\hat{s_k} = u_k y_k = u_k h_k \sqrt{p_k} s_k + \sum_{\substack{i=1 \\ i \neq k}}^{N} u_k h_i \sqrt{p_i} s_i + u_k n, \tag{18}$$

the SINR of CU k of the decoded signal is:

$$\gamma_k = \frac{|u_k h_k|^2 p_k}{\sum_{\substack{i=1 \\ i \neq k}}^{N} |u_k h_i|^2 p_i + |u_k|^2}, \tag{19}$$

and the MSE of the decoded signal with the transmit symbol is

$$e_k = E(|\hat{s_k} - s_k|^2) = |1 - u_k h_k \sqrt{p_k}|^2 + \sum_{\substack{i=1 \\ i \neq k}}^{N} |u_k h_i|^2 p_i + |u_k|^2 \tag{20}$$

Upon minimizing the MSE e_k with respect to u_k. The equivalence one can observe is that:

$$\text{SINR}_k^{CU} \;=\; \frac{1}{\min\limits_{u_k} e_k} - 1 \;=\; \max_{u_k} \frac{1}{e_k} - 1 \tag{21}$$

which follows the equation mentioned earlier (15). In this part of the thesis, this equation serves as to manipulate the non-convex sum-rate utilization into pseudo concave functions for optimizations.

As the model becomes more complex and different, this relationship between MSE and SINR does not hold. Instead, it helps in understanding and providing relationship at a more general version which is provided in the below proposition.

Proposition 1: For any SINR with Δ and Ω as its numerator and denominator, can be reconstructed as,

$$\text{SINR} \;=\; \max_{u} \frac{1}{e} - 1 \text{ where,} \tag{22}$$

$$e \;=\; |1 - u\left(\sqrt{\Delta}\right)|^2 + |u|^2\,(\Omega) \tag{23}$$

Reason: If we look carefully the e_k variable in (20), it can be interpreted as,

$$e_k = |1 - u_k \underbrace{h_k\sqrt{p_k}}_{\sqrt{\Delta_k}}|^2 + |u_k|^2 \underbrace{\left(\sum_{\substack{i=1\\i\neq k}}^{N} |h_i|^2 p_i + 1\right)}_{\Omega_k} \tag{24}$$

For the sum-rate maximization, the WMMSE algorithm uses the auxiliary variables for optimization which is presented in the following proposition.

Proposition 2: For any SINR with Δ and Ω as its numerator and denominator, the SE can be reconstructed as:

$$\text{SE} \;=\; \log\left(1 + \frac{\Delta}{\Omega}\right) \;=\; \underset{w,u}{\text{Maximize}}\;\; \log(w) - we \text{ where,} \tag{25}$$

$$e \;=\; |1 - u\left(\sqrt{\Delta}\right)|^2 + |u|^2\,(\Omega) \;\text{ and} \tag{26}$$

$$w \;=\; \frac{1}{e}. \tag{27}$$

Proof : Using the proposition 1, the SE can be rewritten as:

$$\text{SE} \;=\; \log\left(1 + \frac{\Delta}{\Omega}\right) \;=\; \log\left(\max_{u} \frac{1}{e}\right) \;\text{ where,} \tag{28}$$

$$e \;=\; |1 - u\left(\sqrt{\Delta}\right)|^2 + |u|^2\,(\Omega) \tag{29}$$

As log is a monotonically increasing function, the log function can be brought inside the max function. Also using a epigrapgh trick on the e variable, the above SE is reconstructed as:

$$\text{SE} \;=\; \log\left(1 + \frac{\Delta}{\Omega}\right) \;=\; \max_{u} \log(w) \;\text{ where,} \tag{30}$$

$$w \;\leq\; \frac{1}{e}. \tag{31}$$

As the reconstructed SE is concave function with respect to e variable. The strong Duality holds for the Lagrangian dual problem which is:

$$\mathcal{L}(w, \lambda) \;=\; \log(w) - \lambda\left(1 - \frac{1}{we}\right) \tag{32}$$

Maximizing the dual problem with respect to w provides the optimal value of λ which is:

$$\lambda* \;=\; we \tag{33}$$

Therefore Applying the optimal $\lambda*$ and the optimal value of w which is $\frac{1}{e}$ to the dual problem the SE can be reconstructed as:

$$\text{SE} \;=\; \underset{w,u}{\text{Maximize}}\;\log(w) - we \text{ where,} \tag{34}$$

$$e \;=\; |1 - u\left(\sqrt{\Delta}\right)|^2 + |u|^2\left(\Omega\right) \text{ and} \tag{35}$$

$$w \;=\; \frac{1}{e}. \tag{36}$$

In the formulation a constant 1 is eliminated as it is the optimization and it has no responsibility in it. Hence the proof.

IV. Optimization of SUM SE

The SUM SE is a characterization of the Channel capacity of the homogeneous system. Furthermore, maximizing the SUM SE results in the improved data transmission between the BS and the User. However, it is impossible to ensure that each user will have improved data transfer. In the multi-cell multi-relay NOMA system, the sum SE is maximized by altering the power allocated to each user from the BS. In the sections that follow, we will attempt to maximise the sum SE through power allocation for UATF IV-A and Ergodic IV-B system models by iteratively optimising using closed-form solutions. *Previous works on NOMA mMIMO systems have not worked on optimizing the sum SE using iterative closed form solutions.* This work can be extended to optimizing GEE and WSEE which is a better characterization and also acts as the trade-off between Channel Capacity and the power consumption in the homogeneous and heterogeneous network respectively.

A. UATF: SUM SE problem formulation and Optimization

The optimization of sum SE consists of series of sub-problems which are solved iteratively with closed-from solutions. The objective function of the sum SE optimzation is:

$$\mathbf{P1}: \quad \underset{\mathbf{P},\mathbf{Q}}{\text{Maximize}}\; R^{c}_{\text{sum}} \tag{37}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} p_{lk,n} \le P_l\;\forall l,\;\; q_{lk} \le Q_{lk}\;\forall l,\;\forall k \text{ and } p_{lk,n}, q_{lk} \ge 0 \tag{38}$$

where P_l is the maximum transmit power of the base station of lth cell, Q_{lk} is the maximum power transmitted from kth relay of the lth cell of the cluster $\mathcal{U}_{lk}$ and here the matrix $\mathbf{P} = [\mathbf{p}_1, \cdots, \mathbf{p}_L] \in \mathbb{R}^{L\times\mathcal{U}_{lk}K}$ and $\mathbf{Q} = [\mathbf{q}_1, \cdots, \mathbf{q}_L] \in \mathbb{R}^{K\times L}$ where $\mathbf{p}_l = [p_{l1,1}, \cdots, p_{lK,\mathcal{U}_{lk}}]$ and $\mathbf{q}_l = [q_{l1}, \cdots, q_{lK}]$ and also

$$R^{c}_{\text{sum}} = \frac{1}{2}\left(\frac{\tau_c - \tau_p}{\tau_c}\right)\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}}\log_2\left(1 + \frac{\overline{\Delta}^{c}_{lk,n}}{\overline{\Omega}^{c}_{lk,n}}\right). \tag{39}$$

The constant term $\frac{1}{2}\left(\frac{\tau_c-\tau_p}{\tau_c}\right)$ is eliminated as it is irrelevant in the optimization. Also. the $\log_2$ function is converted into natural algorithm (*ln*) which removes any constant appearing while performing differentiation. The SINR expression in (96), contains fractional functions of transmit user power and transmit relay power sum, the optimization variables in both numerator and denominator. The optimization of sum SE comes under the ambit of multi-ratio(sum of log-ratios) fractional programming framework and are therefore challenging-convex problem.

The problem can be solved by novel optimization framework which approximates the non-convex functions into pseudo concave function at a point using the relationship between SINR and *mean square error*(MSE) where the algorithm knowingly termed as WMMSE algorithm. The algorithm converts non-convex hard problem like in **P1** and translates into pseudo concave function, which can be maximized using simple iterative closed form solutions.

For this optimization problem, instead of using $_{lk,n}$ as an optimization variable, we use $\bar{\mu}_{lk}$ as the replacement optimization variable, one can understand that this is an important replacement to attain closed form solutions from the solution for the following sub-problems. As $\hat{\mu}_{lk}$ depends on the power variables too, the constraints for $\bar{\mu}_{lk}$ are iteratively updated to satisfy all the constraints of the optimization. Therefore the problem **P1** is restructured as:

$$\textbf{P2}: \quad \underset{\mathbf{P},\boldsymbol{\mu}}{\text{Maximize}} \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} \log\left(1+\frac{\overline{\Delta}_{lk,n}^{c}}{\overline{\Omega}_{lk,n}^{c}}\right) \tag{40}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} p_{lk,n} \leq P_l\,\forall l,\ \bar{\mu}_{lk} \leq \hat{\mu}_{lk}\,\forall l,\,\forall k \quad \text{where,}$$

$$\hat{\mu}_{lk} = \sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}p_{l''k''} + \sum\limits_{l''=1}^{L}\xi_{l''k,lk}p_{l''k} + 1\right)}}. \tag{41}$$

With the restructured problem formulation **P2**, the optimization can proceed with modified WMMSE algorithm. Using Proposition 2, one can equivalently reconstruct the write the SE of the nth user of the cluster $\mathcal{U}_{lk}$ as:

$$\log(1+\text{SINR}_{lk,n}^{c}) = \log(1+\frac{\overline{\Delta}_{lk,n}^{c}}{\overline{\Omega}_{lk,n}^{c}}) = \max_{u_{lk,n}^{c},w_{lk,n}^{c}} \frac{1}{w_{lk,n}^{c}} - w_{lk,n}^{c}e_{lk,n}^{c} \text{ where,} \tag{42}$$

$$e_{lk,n}^{c} = |1-u_{lk,n}^{c}\sqrt{\overline{\Delta}_{lk,n}^{c}}|^2 + |u_{lk,n}^{c}|^2\overline{\Omega}_{lk,n}^{c} \tag{43}$$

The important aspect of $\mathrm{e}_{lk,n}$ is that the equation is concave in nature with respect to transmit power $\mathbf{P}$ and relay amplitude factor $\boldsymbol{\mu}$. Using the equation (42) to the Problem **P2**, the problem is reconstructed as:

$$\mathbf{P3}: \quad \underset{\mathbf{P},\boldsymbol{\mu}}{\text{Maximize}} \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} \underset{u^c_{lk,n},w^c_{lk,n}}{\text{Maximize}} \log\left(w^c_{lk,n}\right) - w^c_{lk,n}e^c_{lk,n} \tag{44}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} p_{lk,n} \le P_l \,\forall l,\ \bar{\mu}_{lk} \le \hat{\mu}_{lk}\,\forall l,\,\forall k \quad \text{where,}$$

$$\hat{\mu}_{lk} = \sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}p_{l''k''} + \sum\limits_{l''=1}^{L}\xi_{l''k,lk}p_{l''k}+1\right)}}.. \tag{45}$$

The auxiliary variables $u^c_{lk,n}, w^c_{lk,n}$ can be pushed out of the summation as the each $u^c_{lk,n}$ and $w^c_{lk,n}$ does not have any inter-dependencies for all l,k,n. Therefore the problem **P3** is reformulated as:

$$\mathbf{P4}: \quad \underset{\mathbf{P},\mathbf{U},\mathbf{W},\boldsymbol{\mu}}{\text{Maximize}} \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} \log\left(w^c_{lk,n}\right) - w^c_{lk,n}e^c_{lk,n} \tag{46}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} p_{lk,n} \le P_l \,\forall l,\ \bar{\mu}_{lk} \le \hat{\mu}_{lk}\,\forall l,\,\forall k \quad \text{where,}$$

$$\hat{\mu}_{lk} = \sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}p_{l''k''} + \sum\limits_{l''=1}^{L}\xi_{l''k,lk}p_{l''k}+1\right)}}. \tag{47}$$

Here the matrix $\mathbf{U} = [\mathbf{u}_1, \cdots, \mathbf{u}_L] \in \mathbb{R}^{L\times\mathcal{U}_{lk}K}$ where $\mathbf{u}_l = [u^c_{l1,1}, \cdots, u^c_{lK,\mathcal{U}_{lk}}]$ and $\mathbf{W} = [\mathbf{w}_1, \cdots, \mathbf{w}_L] \in \mathbb{R}^{L\times\mathcal{U}_{lk}K}$ where $\mathbf{w}_l = [w^c_{l1,1}, \cdots, w^c_{lK,\mathcal{U}_{lk}}]$. Now, expanding the auxiliary variable $e^t_{lk,n}$ by expanding the Numerator and Denominator of the SINR becomes:

$$\begin{aligned} e^c_{lk,n} &= 1 + |u^c_{lk,n}|^2 A_{lk,n}\overline{\mu}^2_{lk}p_{lk,n} - 2\left(u^c_{lk,n}\sqrt{A_{lk,n}\overline{\mu}^2_{lk}p_{lk,n}}\right) \\ &+|u^c_{lk,n}|^2\Bigg(C^{(0)}_{lk,n}\overline{\mu}^2_{lk}p_{lk,n} + \sum_{n'=1}^{n-1}C^{(1)}_{lk,n,n'}\overline{\mu}^2_{lk}p_{lk,n'} + \sum_{n'=n+1}^{\mathcal{U}_{lk}}C^{(2)}_{lk,n,n'}p_{lk,n'}\overline{\mu}^2_{lk} + \sum_{(l',k')\ne(l,k)}\sum_{n'=1}^{\mathcal{U}_{l'k'}}C^{(3)}_{l'k',lk,n}p_{l'k',n'}\mu^2_{l'k'} \\ &+\sum_{(l',k')}\sum_{(l'',k'')\ne(l',k')}\sum_{n'=1}^{\mathcal{U}_{l''k''}}C^{(4)}_{\substack{l''k'',l'k',\\ lk,n}}p_{l''k'',n'}\overline{\mu}^2_{l'k'} + \sum_{l'=1}^{L}\sum_{k'=1}^{K}C^{(5)}_{l'k',lk,n}\overline{\mu}^2_{l'k'} + 1\Bigg) \end{aligned} \tag{48}$$

Grouping the above equation transmit power-wise and multiplying with $w^t_{lk,n}$, the equation becomes:

$$w^c_{lk,n}e^c_{lk,n} = const(w^c_{lk,n}) + \sum_{l'=1}^{L}\sum_{k'=1}^{K}\sum_{n'=1}^{\mathbb{U}_{lk}}\alpha^c_{l'k',n',lk,n}p_{l'k',n'} + \beta^c_{lk,n}\sqrt{p_{lk,n}}. \tag{49}$$

All the constants including the amplitude factor $\boldsymbol{\mu}$ are combined and termed as α and β and are termed as:

$$\alpha^c_{l'k',n',lk,n} = w^c_{lk,n}|u^c_{lk,n}|^2 * \begin{cases} A_{lk,n}\overline{\mu}^2_{lk} + C^{(0)}_{lk,n}\overline{\mu}^2_{lk} + \sum\limits_{l''k''\neq lk} C^{(4)}_{\substack{lk,l''k'',\\ lk,n}}\overline{\mu}^2_{l''k''} & (l'k',n') = (l,k,n) \\ C^{(1)}_{lk,n,n'}\overline{\mu}^2_{lk} + \sum\limits_{l''k''\neq lk} C^{(4)}_{\substack{lk,l''k'',\\ lk,n}}\overline{\mu}^2_{l''k''} & (l'k') = (l,k) \;\&\; n' \leq n-1 \\ C^{(2)}_{lk,n,n'}\overline{\mu}^2_{lk} + \sum\limits_{l''k''\neq lk} C^{(4)}_{\substack{lk,l''k'',\\ lk,n}}\overline{\mu}^2_{l''k''} & (l'k') = (l,k) \;\&\; n' \geq n+1 \\ C^{(3)}_{l'k',lk,n}\overline{\mu}^2_{lk} + \sum\limits_{l''k''\neq l'k'} C^{(4)}_{\substack{l'k',l''k'',\\ lk,n}}\overline{\mu}^2_{l''k''} & (l'k') \neq (l,k) \end{cases} \tag{50}$$

$$\beta^c_{lk,n} = -2w^c_{lk,n}|u^c_{lk,n}|\sqrt{A_{lk,n}\overline{\mu}^2_{lk}} \tag{51}$$

Therefore, the objective function of problem **P4** becomes:

$$\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}} \log(w^c_{lk,n}) - w^c_{lk,n}e^c_{lk,n} =$$

$$\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}}\left(const(\mathbf{W}) - \sum_{l'=1}^{L}\sum_{k'=1}^{K}\sum_{n'=1}^{\mathbb{U}_{lk}} \alpha^c_{l'k',n',lk,n}p_{l'k',n'} + \beta^c_{lk,n}\sqrt{p_{lk,n}}\right) \tag{52}$$

Therefore the problem formulation **P4** is reconstructed as:

$$\textbf{P5}: \quad \underset{\mathbf{P},\mathbf{U},\mathbf{W},\boldsymbol{\mu}}{\text{Maximize}} \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}}\left(const(\mathbf{W}) - \sum_{l'=1}^{L}\sum_{k'=1}^{K}\sum_{n'=1}^{\mathbb{U}_{lk}} \alpha^c_{l'k',n',lk,n}p_{l'k',n'} + \beta^c_{lk,n}\sqrt{p_{lk,n}}\right) \tag{53}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} p_{lk,n} \leq P_l\,\forall l,\ \bar{\mu}_{lk} \leq \hat{\mu}_{lk}\,\forall l,\,\forall k \quad \text{where,}$$

$$\hat{\mu}_{lk} = \sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}p_{l''k''} + \sum\limits_{l''=1}^{L}\xi_{l''k,lk}p_{l''k} + 1\right)}}. \tag{54}$$

Now, after formulating the objective function **P5**, all the optimizing variables $\mathbf{P}, \mathbf{U}, \mathbf{W}, \boldsymbol{\mu}$ are iteratively optimized. Firstly, the auxiliary variable $u^c_{lk,n}$ is optimized by first order differentiation of $e^c_{lk,n}$ and equating to zero which is:

$$u^c_{lk,n}{}^* = \frac{\sqrt{\overline{\Delta}^c_{lk,n}}}{\overline{\Omega}^c_{lk,n}} \tag{55}$$

The optimization of the variable $w^c_{lk,n}$ can be easily known as it acts as the auxiliary variable for $e_{lk,n}$. Therefore, the optimal $w^c_{lk,n}$ is:

$$w^{c*}_{lk,n} = \frac{1}{e^{c*}_{lk,n}} \tag{56}$$

The only optimizing variables left are the transmit power and relay amplitude factor. As mentioned earlier that $e^c_{lk,n}$ is concave in nature with respect to the optimizing variables. *The variable*

$e^c_{lk,n}$ *couldn't have been concave in nature, if continued with variables* $\mathbf{P}$ *and* $\mathbf{Q}$. Therefore, with the auxiliary variables $w^{c*}_{lk,n}$ and $u^{c*}_{lk,n}$ acting as fixed point equations, the problem **P5** acts as a pseudo concave optimization with respect to the $\mathbf{P}$ and $\boldsymbol{\mu}$ and provides a optimal value. Therefore, the optimal $p_{lk,n}$ are obtained through first-order differentiation of the objective function **P5** and equating to zero which provides:

$$p^*_{lk,n} = \left\{ \frac{\boldsymbol{\beta}^c_{lk,n}}{\left(\sum\limits_{l'=1}^{L} \sum\limits_{k'=1}^{K} \sum\limits_{n'=1}^{\mathbb{U}_{lk}} \boldsymbol{\alpha}^c_{lk,n,l'k',n'} \right) + \lambda^*_l} \right\}^2 \tag{57}$$

where λ^*_l is an internal auxiliary variable which can optimized through bisection algorithm which helps to satisfy $\sum_{k=1}^{K} \sum_{n=1}^{\mathcal{U}_{lk}} p^*_{lk,n} \leq P_l$. Similarly, if the expanding the auxiliary variable $e^t_{lk,n}$ by expanding the Numerator and Denominator of the SINR and group with respect to the μ_{lk} and multiplying by $w^c_{lk,n}$ becomes:

$$w^c_{lk,n} e^c_{lk,n} = const(w^c_{lk,n}) + \sum_{l'=1}^{L} \sum_{k'=1}^{K} \sum_{n'=1}^{\mathcal{U}_{l'k'}} \gamma^c_{l'k'n',lk} \mu^2_{l'k'} + \omega^t_{lk} \mu_{lk}. \tag{58}$$

All the constants including the transmit power $\mathbf{P}$ are combined and termed as ω and γ and are termed as:

$$\gamma^c_{l'k'n',lk} = w^c_{lk,n} |u^c_{lk,n}|^2 * \begin{cases} A_{lk,n} p_{lk,n} + C^{(0)}_{lk,n} p_{lk,n} + C^{(5)}_{lk,lk,n} + \sum\limits_{l''k'' \neq lk} \sum\limits_{n'=1}^{\mathcal{U}_{l''k''}} C^{(4)}_{l''k'',lk,\atop lk,n} p_{l''k'',n'} & (l'k',n') = (l,k,n) \\ C^{(1)}_{lk,n,n'} p_{lk,n'} + C^{(5)}_{lk,lk,n} + \sum\limits_{l''k'' \neq lk} \sum\limits_{n'=1}^{\mathcal{U}_{l''k''}} C^{(4)}_{l''k'',lk,\atop lk,n} p_{l''k'',n'} & {(l'k')=(l,k) \atop n' \leq n-1} \\ C^{(2)}_{lk,n,n'} p_{lk,n'} + C^{(5)}_{lk,lk,n} + \sum\limits_{l''k'' \neq lk} \sum\limits_{n'=1}^{\mathcal{U}_{l''k''}} C^{(4)}_{l''k'',lk,\atop lk,n} p_{l''k'',n'} & {(l'k')=(l,k) \atop n' \geq n+1} \\ \sum\limits_{n'=1}^{\mathcal{U}_{l'k'}} C^{(3)}_{l'k',lk,n} p_{lk,n'} + C^{(5)}_{l'k',lk,n} + \sum\limits_{l''k'' \neq l'k'} \sum\limits_{n'=1}^{\mathcal{U}_{l''k''}} C^{(4)}_{l''k'',l'k',\atop lk,n} p_{l''k'',n'} & (l'k') \neq (l,k) \end{cases} \tag{59}$$

$$\omega^c_{lk} = -2 w^c_{lk,n} |u^c_{lk,n}| \sqrt{A_{lk,n} p_{lk,n}} \tag{60}$$

Therefore the problem formulation **P4** is reconstructed as:

$$\mathbf{P6}: \quad \underset{\mathbf{P},\mathbf{U},\mathbf{W},\boldsymbol{\mu}}{\text{Maximize}} \sum_{l=1}^{L} \sum_{k=1}^{K} \sum_{n=1}^{\mathbb{U}_{lk}} \left(const(\mathbf{W}) - \sum_{l'=1}^{L} \sum_{k'=1}^{K} \gamma^c_{l'k',lk} \mu^2_{lk} + \omega^t_{lk} \mu_{lk} \right) \tag{61}$$

$$s.t \quad \sum_{k=1}^{K} \sum_{n=1}^{\mathcal{U}_{lk}} p_{lk,n} \leq P_l \, \forall l, \;\; \bar{\mu}_{lk} \leq \hat{\mu}_{lk} \, \forall l, \forall k \quad \text{where,}$$

$$\hat{\mu}_{lk} = \sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L} \sum\limits_{k''=1}^{K} \rho_{l''k'',lk} p_{l''k''} + \sum\limits_{l''=1}^{L} \xi_{l''k,lk} p_{l''k} + 1 \right)}}. \tag{62}$$

Using the problem **P6**, the $\mu_{lk,n}$ is optimized through first-order differentiation and equated to zero as:

$$\bar{\mu}^{*}_{lk} = \min\left\{ \frac{\boldsymbol{\omega}^{c}_{lk}}{\sum\limits_{l'=1}^{L}\sum\limits_{k'=1}^{K}\sum\limits_{n'=1}^{\mathcal{U}_{lk}} \gamma^{c}_{lkn,l'k'}}, \hat{\mu_{lk}} \right\} \tag{63}$$

where,

$$\hat{\mu_{lk}} = \sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K} \rho_{l''k'',lk} p^{*}_{l''k''} + \sum\limits_{l''=1}^{L} \xi_{l''k,lk} p^{*}_{l''k} + 1\right)}}. \tag{64}$$

We observe that (57) and (63) are fixed-point equations, where the RHS expression depend themselves on $p_{lk,n}$ and μ_{lk}. The constraints $\hat{\mu_{lk}}$ are iteratively updated with the optimal power values. We therefore develop an iterative algorithm to solve the problem **P1** by starting from a feasible transmit power and relay transmit power and iteratively updating the auxiliary variables and transmit powers and relay amplitude factors with the solutions provided below. The resulting formal procedure to solve **P1** in (37) is provided in Algorithm 1.

Algorithm 1: sum SE maximization using Deterministic WMMSE approach

Input: Given a tolerance $\epsilon > 0$, the maximum number of iterations N and maximum power constraint P^{max}_{l} for UE U_{lk} and maximum power constraint Q^{max}_{lk} for the relay. Calculate the initial values $p_{lk,n}, \mu_{lk}$ with random power allocation for all relay and users i.e., $\mathbf{p}^{(1)}_{lk,n} \sim \mathbb{U}[0, P^{max}_{l}]$ and $\mu^{(1)}_{lk} \sim \mathbb{U}[0, Q^{lk}_{max}]$

Output: $p^{*}_{lk,n}$ and μ^{*}_{lk}.

for $n \leftarrow 1$ ***to*** N **do**
- Given a feasible $p^{(i)}_{lk,n}$ and $\mu^{(i)}_{lk}$, update auxiliary variable $u_{lk,n}$ using (55)
- Update the auxiliary variable $w_{lk,n}$ using (56)
- Compute $p^{(n+1)}_{jk}$ using (57)
- Update the $\hat{\mu}_{lk}$ constraint variable using (64)
- Compute $\mu^{(i+1)}_{lk}$ using (63)
- Do until convergence **if** $\sum\limits_{l=1}^{L}\sum\limits_{l=1}^{K}\sum\limits_{n=1}^{\mathbb{U}_{lk}} \left(p^{(i)}_{lk,n} - p^{(i-1)}_{lk,n}\right) < \epsilon$ **then** break.

return $\mathbf{p}^{*}$,$\mathbf{mu}^{*}$

B. Ergodic: SUM SE problem formulation and optimization

In the previous section of this chapter, we solved a deterministic SE issue by making use of a UATF SE expression. Although calculating these expectations in closed-form for MRC is a simple process, doing so for the ZF and MMSE combining schemes is a non-trivial endeavour. Inorder to solve it, with using deterministic algorithm, the optimum power allocation scheme for ZF and MMSE schemes makes use of statistical averages (see equation(15) of [1]), which necessitates the gathering of a large number of random channel realisations prior to the updating of the transmit powers. The deterministic sum SE maximisation has a larger computational complexity, and as a result, it calls for a greater amount of memory, as well as longer time to store the samples. We are going to now recast the sum SE problem that was in P1 as a stochastic optimization problem and then optimize it using a low-complexity stochastic modified WMMSE framework. The optimization process uses both the stochastic sequential upper-bound minimization technique (SSUM) algorithm [6] and the weighted minimum mean squared error (WMMSE) algorithm. This will help us lower the memory required as well as the computing complexity. The summary of this section is that, after every realization, the surrogate function is formed with instantaneous sum SE and the optimizing variables are updated. The problem formulation follows a pattern quite similar to that of the section before it. The following is the objective function for the optimization of the ergodic sum SE:

$$\mathbf{P1}_{sto}: \qquad \underset{\mathbf{P},\mathbf{Q}}{\text{Maximize}}\ \mathbf{E}\left[g(\mathbf{P},\mathbf{Q},\mathcal{F})\right] \triangleq R^{e}_{sum} \tag{65}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} p_{lk,n} \leq P_l\, \forall l,\ q_{lk} \leq Q_{lk}\, \forall l,\ \forall k \text{ and } p_{lk,n}, q_{lk} \geq 0. \tag{66}$$

Here $g(\mathbf{P},\mathbf{Q},\mathcal{F})$ denotes the instantaneous sum SE of the system, with $\mathcal{F} = \{\mathbf{h}^{l'}_{lk}, g^{l'k'}_{lk,n}\}, \forall l, k, l', k', n$. It is defined as

$$g(\mathbf{P},\mathbf{Q},\mathcal{F}) \;=\; \frac{1}{2}\left(\frac{\tau_c-\tau_p}{\tau_c}\right)\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}}\log_2\left(1+\frac{\overline{\Delta}^{i}_{lk,n}(\mathcal{F})}{\overline{\Omega}^{i}_{lk,n}(\mathcal{F})}\right) \tag{67}$$

where $\overline{\Delta}^{i}_{lk,n}(\mathcal{F}), \overline{\Omega}^{i}_{lk,n}(\mathcal{F})$ are instantaneous SINR, Numerator of SINR, Denominator of the SINR of the nth user of the cluster $\mathcal{U}_{lk}$. The expectation is due to the random channels $\mathcal{F}$ generated. While reconstructing **P2** of the deterministic optimization it is mentioned that the optimization variable q_{lk} is replaced by $\bar{\mu}_{lk}$ inorder to attain the concavity of the sub-objective functions. And the constraint $\hat{\mu}_{lk}$ is iteratively updated to satisfy the constraint of the relay transmit power. Considering the aforementioned reasons, and also eliminating the constants and converting $\log_2$ into log function the problem is reconstructed as:

$$\mathbf{P2}_{sto}: \qquad \underset{\mathbf{P},\boldsymbol{\mu}}{\text{Maximize}}\ \mathbf{E}\left[\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}}\log_2\left(1+\frac{\overline{\Delta}^{i}_{lk,n}((\mathcal{F})}{\overline{\Omega}^{i}_{lk,n}(\mathcal{F})}\right)\right] \tag{68}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} p_{lk,n} \leq P_l\, \forall l,\ \bar{\mu}_{lk} \leq \hat{\mu}_{lk}\, \forall l,\ \forall k \quad \text{where,}$$

$$\hat{\mu}_{lk} = \sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}p_{l''k''}+\sum\limits_{l''=1}^{L}\xi_{l''k,lk}p_{l''k}+1\right)}}. \tag{69}$$

To solve this stochastic non-convex optimization problem we propose a modified SSUM-WMMSE algorithm to solve $\mathbf{P2}_{sto}$. Using Proposition 2, we can reconstruct the instantaneous SE and therefore the problem $\mathbf{P2}_{sto}$ as:

$$\mathbf{P3}_{sto}: \quad \underset{\mathbf{P},\boldsymbol{\mu}}{\text{Maximize}} \lim_{T\Rightarrow\infty} \frac{1}{T}\sum_{t=1}^{T}\left[\underset{\mathbf{U}^t,\mathbf{W}^t}{\text{Maximize}} \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}} w^t_{lk,n}e^t_{lk,n} - \log(w^t_{lk,n})\right] \tag{70}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} p_{lk,n} \leq P_l\,\forall l,\ \bar{\mu}_{lk} \leq \hat{\mu}_{lk}\,\forall l,\,\forall k \quad \text{where,}$$

$$\hat{\mu}_{lk} = \sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}p_{l''k''} + \sum\limits_{l''=1}^{L}\xi_{l''k,lk}p_{l''k} + 1\right)}}. \tag{71}$$

where,

$$e^t_{lk,n} = |1 - u^t_{lk,n}\sqrt{\overline{\Delta}^i_{lk,n}(\mathcal{F}^t)}|^2 + |u^t_{lk,n}|^2\,\overline{\Omega}^i_{lk,n}(\mathcal{F}^t) \tag{72}$$

As there are no inter-dependence in the auxiliary variables $w^t_{lk,n}$ and $u^t_{lk,n}$ they are grouped together into matrices $\mathbf{W},\mathbf{U}$ respectively. As we know that $w^t_{lk,n}$ is the auxiliary variable to epigraph the $e^i_{lk,n}$ variable its optimal value is expressed as:

$$w^{t*}_{lk,n} = \frac{1}{e^{t*}_{lk,n}} \tag{73}$$

Minimizing $e^t_{lk,n}$ auxiliary variable under $u^t_{lk,n}$ through first order differentiation and equating to zero provides the optimal value of:

$$u^{t*}_{lk,n} = \frac{\sqrt{\overline{\Delta}^i_{lk,n}(\mathcal{F}^t)}}{\overline{\Omega}^i_{lk,n}(\mathcal{F}^t)} \tag{74}$$

After maximizing the inner optimization $(\mathbf{U}^t,\mathbf{W}^t)$ for each instantaneous SE, the problem $\mathbf{P3}_{sto}$ is deconstructed as:

$$\mathbf{P4}_{sto}: \quad \underset{\mathbf{P},\boldsymbol{\mu}}{\text{Maximize}} \lim_{T\Rightarrow\infty} \frac{1}{T}\sum_{t=1}^{T}\left[\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}} \log(w^{t**}_{lk,n}) - w^{t**}_{lk,n}e^{t**}_{lk,n}\right] \tag{75}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} p_{lk,n} \leq P_l\,\forall l,\ \bar{\mu}_{lk} \leq \hat{\mu}_{lk}\,\forall l,\,\forall k \quad \text{where,}$$

$$\hat{\mu}_{lk} = \sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}p_{l''k''} + \sum\limits_{l''=1}^{L}\xi_{l''k,lk}p_{l''k} + 1\right)}}. \tag{76}$$

where,

$$e^{t**}_{lk,n} = |1 - u^{t**}_{lk,n}\sqrt{\overline{\Delta}^i_{lk,n}(\mathcal{F}^t)}|^2 + |u^{t**}_{lk,n}|^2\,\overline{\Omega}^i_{lk,n}(\mathcal{F}^t) \tag{77}$$

and the auxiliary variables $u^{t**}_{lk,n}, w^{t**}_{lk,n}$ are the optimized values after inner optimization. The problem $\mathbf{P4}_{sto}$ is the outer optimization $(\mathbf{P},\boldsymbol{\mu})$ which is a case of stochastic optimization problem and is solved by the SSUM algorithm. The algorithm is summarized below.

Summary of the SSUM Algorithm: In the paper [6], the proposed SSUM algorithm is that, at each iteration t, a new realization of channel $\mathcal{F}^{\sqcup} = \mathbf{H}^t$,$\mathbf{g}^t$ are obtained and the surrogate function is update to tth realization $(\mathbf{R}^{1:t}(\mathbf{P}, \boldsymbol{\mu}, \mathbf{P}^t, \boldsymbol{\mu}^t))$. Using this surrogate function, the optimization variables are updated which is expressed as:

$$\mathbf{P5}_{sto} : [\mathbf{P}^{t+1}, \boldsymbol{\mu}^{t+1}] \Leftarrow \underset{\mathbf{P},\boldsymbol{\mu}}{\text{Argmax}} \ \frac{1}{t}\left[\mathbf{R}^{1:t}(\mathbf{P}, \boldsymbol{\mu}, \mathbf{P}^t, \boldsymbol{\mu}^t)\right] \tag{78}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} p_{lk,n} \leq P_l \, \forall l, \ \bar{\mu}_{lk} \leq \hat{\mu}_{lk} \, \forall l, \, \forall k \quad \text{where,}$$

$$\hat{\mu}_{lk} = \sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}p_{l''k''} + \sum\limits_{l''=1}^{L}\xi_{l''k,lk}p_{l''k} + 1\right)}}. \tag{79}$$

Formation of Surrogate function: At each iteration t, a new realization of channel $\mathcal{F}^{\sqcup} = \mathbf{H}^t$,$\mathbf{g}^t$ are obtained and the surrogate function upto $t-1$th realization $(\mathbf{R}^{1:t-1}(\mathbf{P}, \boldsymbol{\mu}))$ are updated with the convex approximate function $(\tilde{R}^i_{sum}(\mathbf{P}, \boldsymbol{\mu}, \mathcal{F}^t))$ of the instantaneous sum SE to surrogate function upto tth realization $(\mathbf{R}^{1:t}(\mathbf{P}, \boldsymbol{\mu}))$ through recursive surrogate function from [7], which also guarantee the convergence of the algorithm and expressed as:

$$\mathbf{R}^{1:t}(\mathbf{P}, \boldsymbol{\mu}, \mathbf{P}^t, \boldsymbol{\mu}^t) = \frac{1}{t}\left[\tilde{R}^i_{sum}(\mathbf{P}, \boldsymbol{\mu}, \mathcal{F}^t)\right] + \frac{t-1}{t}\left[\mathbf{R}^{1:t-1}(\mathbf{P}, \boldsymbol{\mu}, \mathbf{P}^{t-1}, \boldsymbol{\mu}^{t-1})\right] \tag{80}$$

Coming back to the stochastic optimization of Ergodic sum SE, we need to find the surrogate function and the convex approximate at the t realization and is found using the following proposition.

Proposition 3: The objective function in $\mathbf{P4}_{sto}$ can act as a convex approximate for the recursive surrogate function. In mathematical way,

$$\tilde{R}^i_{sum}(\mathbf{P}, \boldsymbol{\mu}, \mathcal{F}^t) = \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}} \log(w^{t**}_{lk,n}) - w^{t**}_{lk,n}e^{t**}_{lk,n} \tag{81}$$

Proof: Expanding the auxiliary variable $e^{t**}_{lk,n}$ by expanding the Numerator and Denominator of the SINR becomes:

$$\begin{aligned} e^t_{lk,n} &= 1 + |u^{t**}_{lk,n}|^2\hat{A}^t_{lk,n}\tilde{\mu}^2_{lk}p_{lk,n} - 2Re(u^{t**}_{lk,n}\sqrt{\hat{A}^t_{lk,n}\tilde{\mu}^2_{lk}p_{lk,n}}) \\ &+ |u^{t**}_{lk,n}|^2\Bigg(\sum_{n'=1}^{n-1}\hat{C}^{(1)^t}_{lk,n,n'}\tilde{\mu}^2_{lk}p_{lk,n'} + \sum_{n'=n+1}^{\mathcal{U}_{lk}}\hat{C}^{(2)^t}_{lk,n,n'}p_{lk,n'}\tilde{\mu}^2_{lk} + \sum_{(l',k')\neq(l,k)}\sum_{n'=1}^{\mathcal{U}_{l'k'}}\hat{C}^{(3)^t}_{l'k',lk,n}p_{l'k',n'}\mu^2_{l'k'} \\ &+ \sum_{(l',k')}\sum_{(l'',k'')\neq(l',k')}\sum_{n'=1}^{\mathcal{U}_{l''k''}} C^{(4)^t}_{\substack{l''k'',l'k',\\ lk,n}}p_{l''k'',n'}\tilde{\mu}^2_{l'k'} + \sum_{l'=1}^{L}\sum_{k'=1}^{K}\hat{C}^{(5)^t}_{l'k',lk,n}\tilde{\mu}^2_{l'k'} + 1\Bigg) \end{aligned} \tag{82}$$

Grouping the above equation transmit power-wise and multiplying with $w^{t**}_{lk,n}$, the equation becomes:

$$w_{lk,n}^{t**}e_{lk,n}^{t**} = const(w_{lk,n}^{t**}) + \sum_{l'=1}^{L}\sum_{k'=1}^{K}\sum_{n'=1}^{\mathbb{U}_{lk}} \alpha_{l'k',n',lk,n}^{t} p_{l'k',n'} + \beta_{lk,n}^{t}\sqrt{p_{lk,n}}. \tag{83}$$

All the constants including the amplitude factor $\boldsymbol{\mu}$ are combined and termed as α and β and are similar to (59). Therefore, the objective function of problem $\mathbf{P4}_{sto}$ becomes:

$$\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}} \log(w_{lk,n}^{t**}) - w_{lk,n}^{t**}e_{lk,n}^{t**} =$$

$$\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}} \left(const(\mathbf{W}) - \sum_{l'=1}^{L}\sum_{k'=1}^{K}\sum_{n'=1}^{\mathbb{U}_{lk}} \alpha_{l'k',n',lk,n}^{t} p_{l'k',n'} + \beta_{lk,n}^{t}\sqrt{p_{lk,n}} \right) \tag{84}$$

If the equation is carefully noticed, the function is concave in transmit power variables $\mathbf{P}$ and also the $t+1$th channel realization will also have the exact objective sum SE function. *The aforementioned reasons has proved that the instantaneous this objective function can be considered as the instantaneous convex approximate for the surrogate function.*

Hence the proof.

By induction method one can understand that the surrogate function upto tth realization $\mathbf{R}^{1:t}(\mathbf{P}, \boldsymbol{\mu}, \mathbf{P}^t, \boldsymbol{\mu}^t)$ will have similar kind of constants with them as in α and β. Therefore the surrogate function upto tth realization is written as:

$$\mathbf{R}^{1:t}(\mathbf{P}, \boldsymbol{\mu}, \mathbf{P}^t, \boldsymbol{\mu}^t) = \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}} \left(const(\mathbf{W}) - \sum_{l'=1}^{L}\sum_{k'=1}^{K}\sum_{n'=1}^{\mathbb{U}_{lk}} \boldsymbol{\alpha}_{l'k',n',lk,n}^{1:t} p_{l'k',n'} + \boldsymbol{\beta}^{\mathbf{1:t}}{}_{lk,n}\sqrt{p_{lk,n}} \right) \tag{85}$$

In order to update the constants of surrogate function $\boldsymbol{\alpha}_{l'k',n',lk,n}^{1:t}$ and $\boldsymbol{\beta}_{1:t}^{lk,n}$for the tth iteration, they are updated as:

$$\boldsymbol{\alpha}_{l'k',n',lk,n}^{1:t} = \frac{1}{t}\alpha_{l'k',n',lk,n}^{t} + \frac{t-1}{t}\boldsymbol{\alpha}_{l'k',n',lk,n}^{1:t-1} \tag{86}$$
$$\boldsymbol{\beta}_{lk,n}^{1:t} = \frac{1}{t}\beta_{lk,n}^{t} + \frac{t-1}{t}\boldsymbol{\beta}_{lk,n}^{1:t-1}$$

The surrogate functions can be viewed as a pseudo concave function, in hindsight, it can be said that WMMSE algorithm provides pseudo concave functions. At each iteration t, the optimal solution $p_{lk,n}^{t+1}$ and μ_{lk}^{t+1} is obtained.

For this optimization, it follows similar pattern of maximizing of deterministic sum SE but with the history of the previous terms. With the problem formulation $\mathbf{P5}_{sto}$ when update with the constants of surrogate function becomes

$$\mathbf{P6}_{sto}:[\mathbf{P}^{t+1},\boldsymbol{\mu}^{t+1}] \Leftarrow \underset{\mathbf{P},\boldsymbol{\mu}}{\text{Argmax}} \ -\frac{1}{t}\left[\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}}\left(\sum_{l'=1}^{L}\sum_{k'=1}^{K}\sum_{n'=1}^{\mathbb{U}_{lk}}\boldsymbol{\alpha}^{1:t}_{l'k',n',lk,n}p_{l'k',n'}+\boldsymbol{\beta^{1:t}}_{lk,n}\sqrt{p_{lk,n}}\right)\right] \tag{87}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}}p_{lk,n}\leq P_l\,\forall l,\ \bar{\mu}_{lk}\leq\hat{\mu}_{lk}\,\forall l,\,\forall k \quad \text{where,}$$

$$\hat{\mu}_{lk}=\sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}p_{l''k''}+\sum\limits_{l''=1}^{L}\xi_{l''k,lk}p_{l''k}+1\right)}}. \tag{88}$$

With the above objective function, the optimal $p_{lk,n}$ is provided through first order maximization of the objective function and equating to zero which is expressed as:

$$p^{t+1}_{lk,n}=\left\{\frac{\boldsymbol{\beta}^{1:t}_{lk,n}}{\left(\sum\limits_{l'=1}^{L}\sum\limits_{k'=1}^{K}\sum\limits_{n'=1}^{\mathbb{U}_{lk}}\boldsymbol{\alpha}^{1:t}_{lk,n,l'k',n'}\right)+\lambda^*_l}\right\}^2 \tag{89}$$

where λ^*_l found through bisection algorithm which satisfies $\sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}}p^{t+1}_{lk,n}\leq P_l$.

Similarly, instead of grouping the variable $\mathbf{P}$ wise, if the equation (116) is grouped with respect to $\boldsymbol{\mu}$ wise and multiplied by $w^{t**}_{lk,n}$ the term becomes:

$$w^{t**}_{lk,n}e^{t**}_{lk,n}=const(w^{t**}_{lk,n})+\sum_{l'=1}^{L}\sum_{k'=1}^{K}\sum_{n'=1}^{\mathcal{U}_{l'k'}}\gamma^{t}_{l'k'n',lk}\mu^2_{lk}+\omega^{t}_{lk}\mu_{lk}. \tag{90}$$

Using the similar steps of updating the surrogate function with the surrogate constants $\boldsymbol{\gamma}^{1:t}_{lk,l'k'}$ and $\boldsymbol{\omega}^{1:t}_{lk,l'k'}$ forming the objective function as in $\mathbf{P6}_{sto}$, as is constructed as:

$$\mathbf{P7}_{sto}:[\mathbf{P}^{t+1},\boldsymbol{\mu}^{t+1}] \Leftarrow \underset{\mathbf{P},\boldsymbol{\mu}}{\text{Argmax}} \ -\frac{1}{t}\left[\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}}\left(\sum_{l'=1}^{L}\sum_{k'=1}^{K}\sum_{n'=1}^{\mathbb{U}_{lk}}\boldsymbol{\gamma}^{1:t}_{l'k'n',lk}\overline{\mu}^2_{l'k'}+\boldsymbol{\gamma^{1:t}}_{lk}\overline{\mu}_{lk}\right)\right] \tag{91}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}}p_{lk,n}\leq P_l\,\forall l,\ \bar{\mu}_{lk}\leq\hat{\mu}_{lk}\,\forall l,\,\forall k \quad \text{where,}$$

$$\hat{\mu}_{lk}=\sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}p_{l''k''}+\sum\limits_{l''=1}^{L}\xi_{l''k,lk}p_{l''k}+1\right)}}. \tag{92}$$

With the above objective function, the μ_{lk} is optimized through first-order differentiation and equated to zero as:

$$\mu_{lk}^{t+1} = \min\left\{ \frac{\boldsymbol{\omega}_{lk}^{1:t}}{\sum\limits_{l'=1}^{L}\sum\limits_{k'=1}^{K}\sum_{n'=1}^{\mathcal{U}_{lk}} \gamma_{lkn,l'k'}^{1:t}}, \hat{\mu_{lk}} \right\} \quad \text{where,}$$

$$\hat{\mu_{lk}} = \sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K} \rho_{l''k'',lk} p_{l''k''}^{t+1} + \sum\limits_{l''=1}^{L} \xi_{l''k,lk} p_{l''k}^{t+1} + 1\right)}}. \tag{93}$$

Overall, the modified stochastic WMMSE algorithm is summarized in Algorithm 2.

Algorithm 2: sum SE maximization using Stochastic WMMSE approach

Input: Given a tolerance $\epsilon > 0$, the maximum number of iterations N and maximum power constraint P_l^{max} for UE U_{lk} and maximum power constraint Q_{lk}^{max} for the relay. Calculate the initial values $p_{lk,n}, \mu_{lk}$ with random power allocation for all relay and users i.e., $\mathbf{p}_{lk,n}^{(1)} \sim \mathbb{U}[0, P_l^{max}]$ and $\mu_{lk}^{(1)} \sim \mathbb{U}[0, Q_{lk}^{max}]$

Output: $p_{lk,n}^{*}$ and μ_{lk}^{*}.

for $i \leftarrow 1$ ***to*** N **do**
    Obtain the ith channel realization.
    for $j \leftarrow 1$ ***to*** N **do**
        Given a feasible $p_{lk,n}^{i}$ and μ_{lk}^{i}, update auxiliary variable $u_{lk,n}^{ij}$ using (74)
        Update the auxiliary variable $w_{lk,n}^{ij}$ using (73) Do until convergence **if** $\left(\hat{R}_{sum}^{i}(p,\mu) - \left\{\sum\limits_{l=1}^{L}\sum\limits_{k=1}^{K}\sum\limits_{n=1}^{\mathbb{U}_{lk}} w_{lk,n}^{ij+1} e_{lk,n}^{ij+1} - \log(w_{lk,n}^{ij+1})\right\}\right) < \epsilon$ **then**
            break.
        Output: $w_{lk,n}^{i**}$ and w_{lk}^{i**}.
    Compute p_{jk}^{i+1} using (57)
    Update the $\hat{\mu}_{lk}$ constraint variable using (64)
    Compute μ_{lk}^{i+1} using (63)
    Update $\bar{\alpha}^i, \bar{\beta}^i, \bar{\gamma}^i, \bar{\omega}^i$ using equations
    Do until convergence **if** $\sum\limits_{l=1}^{L}\sum\limits_{l=1}^{K}\sum\limits_{n=1}^{\mathbb{U}_{lk}} \left(p_{lk,n}^{(i)} - p_{lk,n}^{(i-1)}\right) < \epsilon$ **then**
        break.
return $\mathbf{p}^{*}$,μ^{*}

V. SUM SE MAXIMIZATION WITH OPTIMAL ALLOCATION OF PRECODER

In the previous sections of the multi-cell multi-relay downlink NOMA systems, we solved the optimization of sum SE with optimal power allocation deterministically and stochastically with different precoding combiners i.e MRC, Ia-ZF, Ia-MMSE. The precoding combiners with this approach depends on the instantaneous channel realizations of **H** (10). And especially precoding combiners such as Ia-ZF or Ia-MMSE requires a considerably large computation complexity and also requires a greater amount of memory. Therefore, the question arising that instead of using instantaneous percoder schemes, what if we observe deterministic precoder based on second-order statistics with samples of instantaneous channel realization. This section tries to understand the transmit signal representation, SE representation using the above transmitted signal precoders, the sum SE problem formulation and solving it all while optimally allocating precoders deterministacally with statistics from $\mathbf{H}$ and $\mathbf{g}$. *This is section is a initial study of precoder application and requires lot of further study into it. Previous works have never worked on stochastic sum SE precoder allocation.*

1) Data Transmission BS-Relay: In the data transmission mentioned in the above system model, it is considered that the precoder is unit norm and is used for decoding at relay w_{lk}. Also the transmitted signal is multiplied with the square root of power transmitter $p_{lk,n}$ of the users in the clusters (5).

In this section, we use the precoders which are for the the users $w_{lk,n}$ and we also eliminate the power variable $p_{lk,n}$ in the data transmission model by constraining the precoders with transmit powers. The precoded NOMA transmit signal with $s_{lk,n}$ being the signal of the nth user in cluster $\mathcal{U}_{lk}$ where broadcasted by the lth BS is

$$\mathbf{x}^l = \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{jk}} \mathbf{w}_{lk,n} s_{lk,n} \tag{94}$$

Here $\mathbf{w}_{lk,n} \in \mathbb{C}^{N\times 1}$ is the precoder for the NOMA signal for the users in cluster $\mathcal{U}_{jk}$.

2) Ergodic SE analysis: Using a simple traditional comparison method, it is possible to get the SINR and the SE of the user and of the system. Looking at the equations (5) and (94), the former can be converted to later by eliminating $\mathbf{w}_{lk}$ and replacing $\sqrt{p_{lk,n}}$ by $\mathbf{w}_{lk,n}$ which is given in the following equation:

$$\mathbf{x}^l = \sum_{k=1}^{K} \cancel{\mathbf{w}_{lk}} \sum_{n=1}^{\mathcal{U}_{jk}} \frac{\mathbf{w}_{lk,n}}{\cancel{\sqrt{p_{lk,n}}}} s_{lk,n} \tag{95}$$

Following the similar trend of eliminating and replacing the precoders in the (**??**), the Genie-Bounded Ergodic Sum SE of the user is obtained as:

$$\overline{R}^{p}_{lk,n\mathrm{sum}} = \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}} \mathbb{E}\left[\frac{1}{2}\left(1-\frac{\tau}{\tau_c}\right)\log_2\left(1+\frac{\overline{\Delta}^{p}_{lk,n}}{\overline{\Omega}^{p}_{lk,n}}\right)\right], \text{ where } \overline{\Omega}^{p}_{lk,n} = \sum_{m=0}^{7} \overline{I}^{(m)^t}_{lk,n} + 1, \tag{96}$$

$$\overline{\Delta}^{p}_{lk,n}=|\overline{A}_{lk,n}\tilde{\mu}_{lk}\mathbf{w}_{lk,n}|^2,\ \overline{I}^{(1)}_{lk,n}=\sum_{n'=1}^{n-1}|\overline{C}^{(1)}_{lk,n,n'}\tilde{\mu}_{lk}\mathbf{w}_{lk,n'}|^2,\ \overline{I}^{(2)}_{lk,n}=\sum_{n'=n+1}^{\mathcal{U}_{lk}}|\overline{C}^{(2)}_{lk,n,n'}\mathbf{w}_{lk,n'}\tilde{\mu}_{lk}|^2,$$

$$\sum_{m=3}^{6}\overline{I}^{(m)}_{lk,n}=\sum_{(l',k')\neq(l,k)}\sum_{n'=1}^{\mathcal{U}_{l'k'}}|\overline{C}^{(3)}_{l'k',lk,n}\mathbf{w}_{l'k',n'}\mu_{l'k'}|^2+\sum_{(l',k')}\sum_{(l'',k'')\neq(l',k')}\sum_{n'=1}^{\mathcal{U}_{l''k''}}|\overline{C}^{(4)}_{\substack{l''k'',l'k',\\ lk,n}}\mathbf{w}_{l''k'',n'}\tilde{\mu}_{l'k'}|^2,$$

$$\overline{I}^{(7)}_{k,n}=\sum_{l'=1}^{L}\sum_{k'=1}^{K}\overline{C}^{(5)}_{l'k',lk,n}\tilde{\mu}^2_{l'k'},\ \text{and}$$

$$\overline{\mu}_{lk}=\sqrt{\frac{q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}\sum_{n'=1}^{\mathcal{U}_{l''k''}}|\mathbf{w}_{l''k'',n'}|^2+\sum\limits_{l''=1}^{L}\xi_{l''k,lk}\sum_{n'=1}^{\mathcal{U}_{l''k}}|\mathbf{w}_{l''k,n'}|^2+1\right)}}. \tag{97}$$

where the terms $\overline{A}_{lk,n}$, $\overline{C}^{(1)}_{lk,n}$, $\overline{C}^{(2)}_{lk,n}$, $\overline{C}^{(3)}_{l'k',lk,n}$, $\overline{C}^{(4)}_{\substack{l''k'',l'k',\\ lk,n}}$ and $\overline{C}^{(5)}_{l'k',lk,n}$ are functions of instantaneous channel realizations which are given as

$$\overline{A}_{lk,n}=f_{lk,n}g^{lk}_{lk,n}\mathbf{h}^{l^H}_{lk},\ \overline{C}^{(1)}_{lk,n}=f_{lk,n}g^{lk}_{lk,n}\mathbf{h}^{l^H}_{lk},\ \overline{C}^{(2)}_{lk,n}=f_{lk,n}g^{lk}_{lk,n}\mathbf{h}^{l^H}_{lk}-f_{lk,n}\hat{g}^{lk}_{lk,n}\hat{\mathbf{h}}^{l^H}_{lk},$$
$$\overline{C}^{(3)}_{l'k',lk,n}=f_{lk,n}g^{l'k'}_{lk,n}\mathbf{h}^{l'^H}_{l'k'},\ \overline{C}^{(4)}_{\substack{l''k'',l'k',\\ lk,n}}=f_{lk,n}g^{l'k'}_{lk,n}\mathbf{h}^{l''^H}_{l'k'},\ \overline{C}^{(5)}_{l'k',lk,n}=|f_{lk,n}g^{l'k'}_{lk,n}|^2 \tag{98}$$

3) Sum SE formulation and Optimization: In this section, we solve the stochastic SE optimization by optimizing the precoder allocation, following the similar trend of the previous section. We use SSUM and WMMSE algorithm for optimization where at every realization the instantaneous sum SE is learnt through recursive surrogate function (SSUM) and the optimizing variables along with auxiliary variables are updated regularly. Before formulating the objective function, we also know that, inorder to attain the closed form solution, we replace the transmit relay power q_{lk} to amplitude factor μ_{lk} as the optimization variable to attain iterative closed form solutions. We also know that the stochastic optimization is based on the samples of the channel realization and the practical mean. Using all the known factors, the objective function for the optimization of the Ergodic sum SE is constructed as:

$$\mathbf{P1}_{pre}:\quad \underset{\overline{\mathbf{W}},\boldsymbol{\mu}}{\text{Maximize}}\ \lim_{T\Rightarrow\infty}\frac{1}{T}\sum_{t=1}^{T}\left[\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}}\log_2\left(1+\frac{\overline{\Delta}^{p}_{lk,n}(\mathcal{F}^t)}{\overline{\Omega}^{p}_{lk,n}(\mathcal{F}^t)}\right)\right] \tag{99}$$
$$s.t\quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}}|\mathbf{w}_{lk,n}|^2\leq P_l\,\forall l,\ \bar{\mu}_{lk}\leq\hat{\mu}_{lk}\,\forall l,\,\forall k\quad \text{where,}$$

$$\hat{\mu}_{lk}=\sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}\sum_{n'=1}^{\mathcal{U}_{l''k''}}|\mathbf{w}_{l''k'',n'}|^2+\sum\limits_{l''=1}^{L}\xi_{l''k,lk}\sum_{n'=1}^{\mathcal{U}_{l''k}}|\mathbf{w}_{l''k,n'}|^2+1\right)}}. \tag{100}$$

where $\mathcal{F}=\{\mathbf{h}^{l'}_{lk},g^{l'k'}_{lk,n}\},\forall l,k,l',k',n$. Using the methods followed to construct double optimization using auxiliary variables in $\mathbf{P4}_{sto}$ from $\mathbf{P3}_{sto}$, the above problem formulation $\mathbf{P1}_{pre}$ can reconstructed with auxiliary variables as:

$$\mathbf{P2}_{pre}: \quad \underset{\overline{\mathbf{W}},\boldsymbol{\mu}}{\text{Maximize}} \ \lim_{T\Rightarrow\infty}\frac{1}{T}\sum_{t=1}^{T}\left[\underset{\mathbf{U}^t,\mathbf{W}^t}{\text{Maximize}} \ \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}} w^t_{lk,n}e^t_{lk,n} - \log(w^t_{lk,n})\right] \tag{101}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}}|\mathbf{w}_{lk,n}|^2 \le P_l\,\forall l,\ \bar{\mu}_{lk}\le\hat{\mu}_{lk}\,\forall l,\,\forall k \quad \text{where,}$$

$$\hat{\mu}_{lk} = \sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}\sum_{n'=1}^{\mathcal{U}_{l''k''}}|\mathbf{w}_{l''k'',n'}|^2 + \sum\limits_{l''=1}^{L}\xi_{l''k,lk}\sum_{n'=1}^{\mathcal{U}_{l''k}}|\mathbf{w}_{l''k,n'}|^2+1\right)}}. \tag{102}$$

where the auxiliary variables $w^t_{lk,n}$ and $u^t_{lk,n}$ are grouped into matrix $\mathbf{W}$'$\mathbf{U}^t$ respectively. The auxiliary variable $e_{lk,n}$ is defined as:

$$e^t_{lk,n} = |1-u^t_{lk,n}\sqrt{\overline{\Delta}^p_{lk,n}(\mathcal{F}^t)}|^2 + |u^t_{lk,n}|^2\,\overline{\Omega}^p_{lk,n}(\mathcal{F}^t),\forall\, l,k,n.$$

And the optimal values of $w^t_{lk,n}$ and $u^t_{lk,n}$ are given as:

$$u^{t*}_{lk,n} = \frac{\sqrt{\overline{\Delta}^i_{lk,n}(\mathcal{F}^t)}}{\overline{\Omega}^i_{lk,n}(\mathcal{F}^t)} \tag{103}$$

$$w^{t*}_{lk,n} = \frac{1}{e^{t*}_{lk,n}} \ \text{where,} \tag{104}$$

$$e^{t*}_{lk,n} = |1-u^{t*}_{lk,n}\sqrt{\overline{\Delta}^p_{lk,n}(\mathcal{F}^t)}|^2 + |u^{t*}_{lk,n}|^2\,\overline{\Omega}^p_{lk,n}(\mathcal{F}^t)$$

Once the inner optimization (i.e optimizing the matrices $\mathbf{W}^t,\mathbf{U}^t$) is completed. With the optimal auxiliary variables $(\mathbf{W}^{t**},\mathbf{U}^{t**})$ the problem formulation $\mathbf{P2}_{pre}$ is deconstructed as:

$$\mathbf{P3}_{pre}: \quad \underset{\overline{\mathbf{W}},\boldsymbol{\mu}}{\text{Maximize}} \ \lim_{T\Rightarrow\infty}\frac{1}{T}\sum_{t=1}^{T}\left[\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}} w^{t**}_{lk,n}e^{t**}_{lk,n} - \log(w^{t**}_{lk,n})\right] \tag{105}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}}|\mathbf{w}_{lk,n}|^2 \le P_l\,\forall l,\ \bar{\mu}_{lk}\le\hat{\mu}_{lk}\,\forall l,\,\forall k \quad \text{where,}$$

$$\hat{\mu}_{lk} = \sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K}\rho_{l''k'',lk}\sum_{n'=1}^{\mathcal{U}_{l''k''}}|\mathbf{w}_{l''k'',n'}|^2 + \sum\limits_{l''=1}^{L}\xi_{l''k,lk}\sum_{n'=1}^{\mathcal{U}_{l''k}}|\mathbf{w}_{l''k,n'}|^2+1\right)}}. \tag{106}$$

where the variables $e^{t**}_{lk,n}$, $w^{t**}_{lk,n}$ and $u^{t**}_{lk,n}$ are the optimal variables after several iterations. As the above objective function (105) is pseudo concave function from the proposition 2. Therefore the problem $\mathbf{P3}_{pre}$ becomes stochastic convex optimization problem. Inorder to apply the SSUM algorithm, we require a convex approximation of the objective function of the tth realization which helps in forming the surrogate function. From the proposition 2, that:

$$\tilde{R}_{\text{sum}}(\overline{\mathbf{W}},\boldsymbol{\mu},\mathcal{F}^t) = \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}}\log(w^{t**}_{lk,n}) - w^{t**}_{lk,n}e^{t**}_{lk,n} \tag{107}$$

is a valid convex approximate of the objective function of $\mathbf{P2}_{pre}$ only at the t realization. Using the recursive surrogate function [], the surrogate function $\tilde{\mathbb{R}}^{1:t}_{\text{sum}}$ is expressed as:

$$\mathbf{R}^{1:t}(\bar{\mathbf{W}}, \boldsymbol{\mu}) = \frac{1}{t}\left[\mathbf{R}^{1:t-1}(\bar{\mathbf{W}}, \boldsymbol{\mu})\right] + \frac{t-1}{t}\left[\tilde{R}_{\text{sum}}(\overline{\mathbf{W}}, \boldsymbol{\mu}, \mathcal{F}^t)\right] \tag{108}$$

To make the above equation simpler, we try to group the auxiliary variable $e^{t**}_{lk,n}$ present in the equation (110), based on percoder $\mathbf{w}_{lk,n}$ which is presented as:

$$w^{t**}_{lk,n} e^{t**}_{lk,n} = cons(w^{t**}_{lk,n}) + \sum_{l'=1}^{L}\sum_{k'=1}^{K}\sum_{n'=1}^{\mathbb{U}_{lk}} \mathbf{w}^H_{l'k',n'} \boldsymbol{\alpha}^t_{l'k',n',lk,n} \mathbf{w}_{l'k',n'} + \boldsymbol{\beta}^t_{lk,n}\mathbf{w}_{lk,n}. \tag{109}$$

All the constants including the amplitude factor $\boldsymbol{\mu}$ are combined and termed as $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. With the above grouping the convex approximate function $\tilde{R}_{\text{sum}}(\overline{\mathbf{W}}, \boldsymbol{\mu}, \mathcal{F}^t)$ is written as:

$$\tilde{R}_{\text{sum}}(\overline{\mathbf{W}}, \boldsymbol{\mu}, \mathcal{F}^t) = \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}} \left(const(w) - \sum_{l'=1}^{L}\sum_{k'=1}^{K}\sum_{n'=1}^{\mathbb{U}_{lk}} \mathbf{w}^H_{l'k',n'} \boldsymbol{\alpha}^t_{l'k',n',lk,n} \mathbf{w}_{l'k',n'} + \boldsymbol{\beta}^t_{lk,n}\mathbf{w}_{lk,n} \right) \tag{110}$$

As one can understand that for any t realization, the convex approximate tend to be the same. Therefore using the mathematical induction and keeping the amplitude factor $\boldsymbol{\mu}$ as fixed and using the grouped auxiliary variable, one can simplify the surrogate function to be:

$$\mathbf{R}^{1:t}(\bar{\mathbf{W}}, \boldsymbol{\mu}) = \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}} \left(const(w) - \sum_{l'=1}^{L}\sum_{k'=1}^{K}\sum_{n'=1}^{\mathbb{U}_{lk}} \mathbf{w}^H_{l'k',n'} \check{\boldsymbol{\alpha}}^{1:t}_{l'k',n',lk,n} \mathbf{w}_{l'k',n'} + \check{\boldsymbol{\beta}}^{lk,n}_{1:t}\mathbf{w}_{lk,n} \right) \text{ where,} \tag{111}$$

$$\check{\boldsymbol{\alpha}}^{1:t}_{l'k',n',lk,n} = \frac{1}{t}\boldsymbol{\alpha}^t_{l'k',n',lk,n} + \frac{t-1}{t}\check{\boldsymbol{\alpha}}^{1:t-1}_{l'k',n',lk,n} \tag{112}$$

$$\check{\boldsymbol{\beta}}^{1:t}_{lk,n} = \frac{1}{t}\boldsymbol{\beta}^t_{lk,n} + \frac{t-1}{t}\check{\boldsymbol{\beta}}^{1:t-1}_{lk,n} \tag{113}$$

Using the construction of the objective function mentioned in $\mathbf{P6}_{sto}$, the problem formulation for the stochastic optimization with the surrogate function is constructed as:

$$\mathbf{P4}_{pre} : [\overline{\mathbf{W}}^{t+1}, \boldsymbol{\mu}^{t+1}] \Leftarrow \underset{\overline{\mathbf{W}}, \boldsymbol{\mu}}{\text{Argmax}} \ \frac{1}{t}\left[\mathbf{R}^{1:t-1}(\bar{\mathbf{W}}, \boldsymbol{\mu})\right] + \frac{t-1}{t}\left[\tilde{R}_{\text{sum}}(\overline{\mathbf{W}}, \boldsymbol{\mu}, \mathcal{F}^t)\right] \triangleq \mathbf{R}^{1:t}(\bar{\mathbf{W}}, \boldsymbol{\mu}) \tag{114}$$

$$s.t \quad \sum_{k=1}^{K}\sum_{n=1}^{\mathcal{U}_{lk}} |\mathbf{w}_{lk,n}|^2 \leq P_l \, \forall l, \ \bar{\mu}_{lk} \leq \hat{\mu}_{lk} \, \forall l, \forall k \quad \text{where,}$$

$$\hat{\mu}_{lk} = \sqrt{\frac{Q_{lk}}{\left(\sum\limits_{l''=1}^{L}\sum\limits_{k''=1}^{K} \rho_{l''k'',lk} \sum_{n'=1}^{\mathcal{U}_{l''k''}} |\mathbf{w}_{l''k'',n'}|^2 + \sum\limits_{l''=1}^{L} \xi_{l''k,lk} \sum_{n'=1}^{\mathcal{U}_{l''k}} |\mathbf{w}_{l''k,n'}|^2 + 1\right)}}.$$

the optimized $\mathbf{w}_{lk,n}$ is provided through the first order differentiation of the objective function of $\mathbf{P4}_{pre}$, which surprisingly follows the similar pattern to that of optimizing the Ergodic sum

SE and is given below:

$$\mathbf{w}_{lk,n}^{t+1} = \left\{ \left(\sum_{l'=1}^{L} \sum_{k'=1}^{K} \sum_{n'=1}^{\mathbb{U}_{lk}} \breve{\boldsymbol{\alpha}}_{lk,n,l'k',n'}^{1:t} \right) + \lambda_l^* I_N \right\}^{-1} \left\{ \left(\boldsymbol{\beta}_{lk,n}^{1:t} \right) \right\} \tag{115}$$

where λ_l^* found through bisection algorithm which satisfies $\sum_{k=1}^{K} \sum_{n=1}^{\mathcal{U}_{lk}} |\mathbf{w}_{lk,n}^{t+1}|^2 \leq P_l$.

Similarly, instead of grouping the variable $\overline{\mathbf{W}}$ wise, if the equation (109) is grouped with respect to $\boldsymbol{\mu}$ wise, the term becomes:

$$w_{lk,n}^{t**} e_{lk,n}^{t**} = const(w) + \sum_{l'=1}^{L} \sum_{k'=1}^{K} \gamma_{l'k',lk}^{t} \mu_{lk}^2 + \omega_{lk}^t \mu_{lk}. \tag{116}$$

Keeping the updated precoder fixed and following the similar steps from problem formulation $\mathbf{P3}_{pre}$ to $\mathbf{P4}_{pre}$ and by forming the simpler surrogate function (111) and updation of the surrogate variables (112), the $\mu_{lk,n}$ is optimized through first-order differentiation of new surrogate variable keeping precoder constant and equating to zeros is expressed as:

$$\overline{\mu}_{lk}^{t+1} = \min \left\{ \frac{\omega_{lk}^{1:t}}{\sum_{l'=1}^{L} \sum_{k'=1}^{K} \gamma_{lk,l'k'}^{1:t}}, \hat{\mu_{lk}} \right\} \text{ where,} \tag{117}$$

$$\hat{\mu}_{lk} = \sqrt{\frac{Q_{lk}}{\left(\sum_{l''=1}^{L} \sum_{k''=1}^{K} \rho_{l''k'',lk} \sum_{n'=1}^{\mathcal{U}_{l''k''}} |\mathbf{w}_{l''k'',n'}|^2 + \sum_{l''=1}^{L} \xi_{l''k,lk} \sum_{n'=1}^{\mathcal{U}_{l''k}} |\mathbf{w}_{l''k,n'}|^2 + 1 \right)}}. \tag{118}$$

The overall Algorithm, the modified stochastic successive convex approximation is summarized in Algorithm 3.

VI. Simulation results

In this section, We now evaluate the optimization techniques of the multiple-relay-aided mMIMO NOMA system, where the BS and users estimate CSI, and the users perform imperfect SIC. We consider a 20 MHz system with $N_T = 100$ BS antennas, $K = 5$ relays and a total of 20 users. We assume that i) relays and users are randomly allocated on a circle of the radius 500m, with the BS as center; and ii) each relay is allocated four users, i.e. $U_k = 4$. Each coherence interval is of $\tau_c = 200$ symbols, with a pilot transmission interval of $\tau = K$ symbols. The pilot power is $p_p = 20$ dBm. We model the large scale fading coefficient from the l'th BS to k th relay in lth cell and k' th relay in l'th cell to nth user in cluster $\mathcal{U}_{lk}$ as $\beta_{lk}^{l\prime}[\text{dB}] = \Upsilon - 10\alpha \log_{10}\left(d_{lk}^{l''}\right) + F_{lk}^{l''}$ and $\beta_{lk,n}^{l'k'}[\text{dB}] = \Upsilon - 10\alpha \log_{10}\left(d_{lk,n}^{l'k'}\right) + F_{lk,n}^{l'k'}$. Here Υ denotes the median channel gain at a reference distance of 30.18, α is the path loss exponent of 2.6, $d_{\underset{lk}{l\prime}}$ and $d_{lk,n}^{l'k'}$ is the separation distance between the BS-and the relay and relay and user in metres, and the scalars $F_{lk}^{l''}$ and $F_{lk,n}^{l'k'}$ are the shadow fading terms which models the log-normal random variations. The Rician factors of the random channels are modelled as $K_{lk}^{l'}$ and $K_{lk,n}^{l'k'}$

Algorithm 3: sum SE maximization with Precoder allocation

Input: Given a tolerance $\epsilon > 0$, the maximum number of iterations N and maximum power constraint P_l^{max} for UE U_{lk} and maximum power constraint Q_{lk}^{max} for the relay. Calculate the initial values $\mathbf{w}_{lk,n}, \mu_{lk}$ with random precoder allocation for all relay and users i.e., $|\mathbf{w}_{lk,n}^{(1)}|^2 \sim \mathbb{U}[0, P_l^{max}]$ and $\mu_{lk}^{(1)} \sim \mathbb{U}[0, Q_{lk}^{max}]$

Output: $\mathbf{w}_{lk,n}^*$ and μ_{lk}^*.

for $i \leftarrow 1$ ***to*** N **do**

  Obtain the ith channel realization.

  for $j \leftarrow 1$ ***to*** N **do**

    Given a feasible $\mathbf{w}_{lk,n}^i$ and μ_{lk}^i, update auxiliary variable $u_{lk,n}^{ij}$ using (**??**)

    Update the auxiliary variable $w_{lk,n}^{ij}$ using (104) Do until convergence **if**
    $\left\{\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}}\left(w_{lk,n}^{ij}e_{lk,n}^{ij} - \log(w_{lk,n}^{ij})\right)\right) - \left(w_{lk,n}^{ij+1}e_{lk,n}^{ij+1} - \log(w_{lk,n}^{ij+1})\right)\right\} < \epsilon$ **then**

     break.

    Output: $w_{lk,n}^{i**}$ and u_{lk}^{i**}.

  Compute p_{jk}^{i+1} using (115)

  Update the $\hat{\mu}_{lk}$ constraint variable using (118)

  Compute μ_{lk}^{i+1} using (117)

  Update $\bar{\alpha}^i, \bar{\beta}^i, \bar{\gamma}^i, \bar{\omega}^i$ using equations (112)

  Do until convergence **if** $\sum_{l=1}^{L}\sum_{l=1}^{K}\sum_{n=1}^{\mathbb{U}_{lk}}\left(|\mathbf{w}_{lk,n}^{(i)}|^2 - |\mathbf{w}_{lk,n}^{(i-1)}|^2\right) < \epsilon$ **then**

  break.

return $\mathbf{w}^*$,μ^*

can be calculated as $K_{lk}^{l'}[\mathrm{dB}] = 13 - 0.03d_{lk}^{l'}$ and $K_{lk,n}^{l'k'}[\mathrm{dB}] = 13 - 0.03d_{lk,n}^{l'k'}$ respectively.The other parameters is the assumption of ULA at the base station and the ASD of 10^o.

1) Deterministic sum SE optimization:

2) Stochastic sum SE optimization:

3) sum SE optimization with Precoder allocation:

VII. Conclusion

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