Analysis of Supply chain outcomes with network effects: The case of differentiated goods, quantity and price competition

September 8, 2022

1. The Model 1

We consider an economy with a network goods sector with two retailers (R_1 and R_2) and one manufacturer (M) . Manufacturer produces good at constant marginal cost of production c(≥ 0) and charges a wholesale price w to retailer . Retailer R_1 and R_2 makes the good differentiated and then sell it to consumer at price p_1 and p_2.
We consider following demand function for Retailer 1:

$$\mathrm{x}_1 = a + ny_1 - p_1 + bp_2 \tag{1}$$

where x_1 and p_1 denote quantity and price, respectively, of goods sell by retailer 1 and y_1 denotes consumers expectation about retailer 1's total sale. a($>$ c), b $\in$ (0,1) and n $\in$ [0,1) are demand parameter.
We consider following demand function for Retailer 2:

$$\mathrm{x}_2 = a + ny_2 - p_2 + bp_1 \tag{2}$$

where x_2 and p_2 denote quantity and price, respectively, of goods sell by retailer 2 and y_2 denotes consumers expectation about retailer 2's total sale. a($>$ c), b $\in$ (0,1) and n $\in$ [0,1) are demand parameter.

Lower value of parameter β corresponds to the case of higher degree of product differentiation by retailers. The parameter n ($= \frac{\partial x_i}{\partial y_i}$) measures the strength of network externalities -lower value n indicates weaker network externalities.

From 1, we get the corresponding inverse demand functions of Retailer 1 as follows.

$$p_1 = \frac{a(1+b) - x_1 - bx_2}{1-b^2} + \frac{n(y_1 + by_2)}{1-b^2} \tag{3}$$

From 2, we get the corresponding inverse demand functions of Retailer 2 as follows.

$$p_2 = \frac{a(1+b) - x_2 - bx_1}{1-b^2} + \frac{n(y_2 + by_1)}{1-b^2} \tag{4}$$

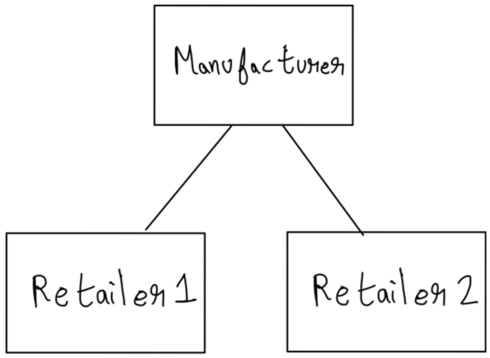

Figure 1: Model 1

1.1 Bertrand competition

Stage 1:

M chooses 'w' to maximize its profit.
Profit of Manufacturer M as follows:

$$\pi_M = (w - c)(x_1 + x_2) \tag{5}$$

Stage 2:

Each R_i chooses p_i to maximize their profit.
Profit of Retailer R_1 as follows:

$$\pi_{R_1} = (p_1 - w)x_1 \tag{6}$$
$$\pi_{R_1} = (p_1 - w)(a + ny_1 - p_1 + bp_2) \tag{7}$$

Profit of Retailer R_2 as follows:

$$\pi_{R_2} = (p_2 - w)x_2 \tag{8}$$
$$\pi_{R_2} = (p_2 - w)(a + ny_2 - p_2 + bp_1) \tag{9}$$

By using Backward induction, we will solve for equilibra.
In stage 2,
Retailer 1 sets its price p_1, taking p_2, y_1 and y_2 as given, to maximize its profit π_{R_1}.

$$\frac{\partial \pi_{R_1}}{\partial p_1} = 0 \tag{10}$$
$$\frac{\partial((p_1 - w)(a + ny_1 - p_1 + bp_2))}{\partial p_1} = 0 \tag{11}$$
$$\tag{12}$$

By solving above equation 11,we obtain retailer 1 price reaction function (RF_1^B) as follow.

$$p_1 = \frac{a + w + ny_1 + bp_2}{2} \tag{13}$$

Also,

$$\frac{\partial^2 \pi_{R_1}}{\partial^2 p_1} = \frac{\partial(a - 2p_1 + bp_2 + w + ny_1)}{\partial p_1} = -2 \tag{14}$$

$$\frac{\partial^2 \pi_{R_1}}{\partial^2 p_1} < 0 \tag{15}$$

Retailer 2 sets its price p_2, taking p_1, y_2 and y_1 as given, to maximize its profit π_{R_2}.

$$\frac{\partial \pi_{R_2}}{\partial p_2} = 0 \tag{16}$$

$$\frac{\partial((p_2 - w)(a + ny_2 - p_2 + bp_1))}{\partial p_2} = 0 \tag{17}$$

By solving above equation 17,we obtain retailer 2 price reaction function (RF_2^B) as follow.

$$p_2 = \frac{a + w + ny_2 + bp_1}{2} \tag{18}$$

Also,

$$\frac{\partial^2 \pi_{R_2}}{\partial^2 p_2} = \frac{\partial(a - 2p_2 + bp_1 + w + ny_2)}{\partial p_2} = -2 \tag{19}$$

$$\frac{\partial^2 \pi_{R_2}}{\partial^2 p_2} < 0 \tag{20}$$

We consider that consumers form rational expectation, which implies in equilibrium $y_1 = x_1$ for retailer 1 and $y_2 = x_2$ for retailer 2.
Solving RF_1^B, RF_2^B, $y_1 = x_1$ and $y_2 = x_2$, we obtain the stage 2 equilibrium prices and quantities in term of w.

$$p_1 = p_2 = \frac{a + (1-n)w}{2-n-b}, \quad x_1 = x_2 = \frac{a-(1-b)w}{2-n-b} \tag{21}$$

Now, taking this values of x_1 and x_2 in Manufacturer profit(π_M), we obtain following

$$\pi_M = (w - c)(\frac{a - (1-b)w}{2-n-b} + \frac{a - (1-b)w}{2-n-b}) \tag{22}$$

By maximizing profit π_M of manufacturer M.

$$\frac{\partial \pi_M}{\partial w} = 0 \tag{23}$$

$$\frac{\partial(2(w-c)(\frac{a-(1-b)w}{2-n-b})}{\partial w} = 0 \tag{24}$$

By solving above equation 24, we obtain equilibrium wholesale price as follows.

$$w^B = \frac{a + c(1-b)}{2(1-b)} \tag{25}$$

Also,

$$\frac{\partial^2 \pi_M}{\partial^2 w} = \frac{(-4(1-b))}{(2-b-n)} \tag{26}$$

$$\frac{\partial^2 \pi_M}{\partial^2 w} < 0 \tag{27}$$

Now by taking w^B, we obtain bertrand equilibrium prices , quantities ,profits of retailer and manufacturer profit as follows.

$$p_1^B = p_2^B = p^B = \frac{2a(1-b) + (1-n)a + c(1-b)(1-n)}{2(1-b)(2-n-b)} \tag{28}$$

$$x_1^B = x_2^B = x^B = \frac{a-(1-b)c}{2(2-n-b)} \tag{29}$$

$$\pi_{R1}^B = \pi_{R2}^B = \pi^B = \frac{(a-(1-b)c)^2}{4(2-n-b)^2} \tag{30}$$

$$\pi_M^B = \frac{(a-(1-b)c)^2}{2(1-b)(2-n-b)} \tag{31}$$

1.2 Cournot competition

Stage 1:

M chooses 'w' to maximize its profit.
Profit of Manufacturer M as follows:

$$\pi_M = (w-c)(x_1 + x_2) \tag{32}$$

Stage 2:

Each R_i chooses x_i to maximize their profit.
Profit of Retailer R_1 as follows:

$$\pi_{R_1} = (p_1 - w)x_1 \tag{33}$$

$$\pi_{R_1} = (\frac{a(1+b) - x_1 - bx_2}{1-b^2} + \frac{n(y_1 + by_2)}{1-b^2} - w)x_1 \tag{34}$$

Profit of Retailer R_2 as follows:

$$\pi_{R_2} = (p_2 - w)x_2 \tag{35}$$

$$\pi_{R_2} = (\frac{a(1+b) - x_2 - bx_1}{1-b^2} + \frac{n(y_2 + by_1)}{1-b^2} - w)x_2 \tag{36}$$

By using Backward induction, we will solve for equilibra.
In stage 2,
Retailer 1 decides its quantity x_1, taking x_2, y_1 and y_2 as given, to maximize its profit π_{R_1}.

$$\frac{\partial \pi_{R_1}}{\partial x_1} = 0 \tag{37}$$

$$\frac{\partial((\frac{a(1+b)-x_2-bx_1}{1-b^2} + \frac{n(y_2+by_1)}{1-b^2} - w)x_2)}{\partial x_1} = 0 \tag{38}$$

By solving above equation 38,we obtain retailer 1 quantity reaction function (RF_1^C) as follow.

$$x_1 = \frac{[a - w(1-b)](1+b) + n(y_1 + by_2) - bx_2}{2} \tag{39}$$

Also,

$$\frac{\partial^2 \pi_{R_1}}{\partial^2 x_1} = \frac{-2}{1-b^2} \tag{40}$$

$$\frac{\partial^2 \pi_{R_1}}{\partial^2 x_1} < 0 \tag{41}$$

Retailer 2 decides its quantity x_2, taking x_1, y_1 and y_2 as given, to maximize its profit π_{R_1}.

$$\frac{\partial \pi_{R_2}}{\partial x_2} = 0 \tag{42}$$

$$\frac{\partial((\frac{a(1+b)-x_1-bx_2}{1-b^2} + \frac{n(y_1+by_2)}{1-b^2} - w)x_1)}{\partial x_2} = 0 \tag{43}$$

By solving above equation 43,we obtain retailer 2 quantity reaction function (RF_2^C) as follow.

$$x_2 = \frac{[a - w(1-b)](1+b) + n(y_2 + by_1) - bx_1}{2} \tag{44}$$

Also,

$$\frac{\partial^2 \pi_{R_2}}{\partial^2 x_2} = \frac{-2}{1-b^2} \tag{45}$$

$$\frac{\partial^2 \pi_{R_2}}{\partial^2 x_2} < 0 \tag{46}$$

We consider that consumers form rational expectation, which implies in equilibrium $y_1 = x_1$ for retailer 1 and $y_2 = x_2$ for retailer 2.
Solving RF_1^C, RF_2^C, $y_1 = x_1$ and $y_2 = x_2$, we obtain the stage 2 equilibrium prices and quantities in term of w.

$$p_1 = p_2 = \frac{a + (1-n)(1-b^2)w}{(1-b)(2-n+(1-n)b)}, \quad x_1 = x_2 = \tfrac{(1+b)(a-(1-b)w)}{2-n+(1-n)b} \tag{47}$$

Now, taking this values of x_1 and x_2 in Manufacturer profit(π_M), we obtain following

$$\pi_M = (w-c)(\frac{(1+b)(a-(1-b)w)}{2-n+(1-n)b} + \frac{(1+b)(a-(1-b)w)}{2-n+(1-n)b}) \tag{48}$$

By maximizing profit π_M of manufacturer M.

$$\frac{\partial \pi_M}{\partial w} = 0 \tag{49}$$

$$\frac{\partial(2(w-c)(\frac{(1+b)(a-(1-b)w)}{2-n+(1-n)b})}{\partial w} = 0 \tag{50}$$

By solving above equation 50, we obtain equilibrium wholesale price as follows.

$$w^C = \frac{a + c(1-b)}{2(1-b)} \tag{51}$$

Also,

$$\frac{\partial^2 \pi_M}{\partial^2 w} = \frac{-((2(1+b)(a-(-1+b)(c-2w)))}{(-2+b(-1+n)+n))} \tag{52}$$

$$\frac{\partial^2 \pi_M}{\partial^2 w} < 0 \tag{53}$$

Now by taking w^C, we obtain cournot equilibrium prices , quantities ,profits of retailer and manufacturer profit as follows.

$$p_1^C = p_2^C = p^C = \frac{a(2+(1-n)(1+b)) + c(1-b^2)(1-n)}{2(1-b)(2-n+(1-n)b)} \tag{54}$$

$$x_1^C = x_2^C = x^C = \frac{(a-(1-b)c)(1+b)}{2(2-n+(1-n)b)} \tag{55}$$

$$\pi_{R1}^C = \pi_{R2}^C = \pi^C = \frac{(a-(1-b)c)^2(1+b)}{4(1-b)(2-n+(1-n)b)^2} \tag{56}$$

$$\pi_M^C = \frac{(a-(1-b)c)^2(1+b)}{2(1-b)(2-n+(1-n)b)} \tag{57}$$

1.3 Cournot versus Bertrand equilibria

Lemma 1: $p^B < p^C$, $x^B > x^C$ and $\pi_M^B > \pi_M^C$, all n $\in$ [0,1).

Proof:

From 28 and 54, we get:

$$p^C - p^B = \frac{b^2(a-(1-b)c)(1-n)}{2(1-b)(2-b-n)(2-n+(1-n)b)} \tag{58}$$

$$\tag{59}$$

From 29 and 55, we get:

$$x^B - x^C = \frac{b^2(a-(1-b)c)}{2(2-b-n)(2-n+(1-n)b)} \tag{60}$$

$$\tag{61}$$

From 31 and 57, we get:

$$\pi_M^B - \pi_M^C = \frac{b^2(a-(1-b)c)^2}{2(1-b)(2-b-n)(2-n+(1-n)b)} \tag{62}$$

$$\tag{63}$$

Also, we have $0 \leq n < 1, 0 < b < 1$ and $0 \leq c < a$
Therefore, $p^B < p^C$, $x^B > x^C$ and $\pi_M^B > \pi_M^C$, all n $\in$ [0,1).

Proposition 1: In the presence of strong network externalities (n $> n_0$), profits of retailer under Bertrand equilibrium are higher compared with that under Cournot equilibrium; where $n_0 = 1 - \sqrt{\frac{1-b}{1+b}}$, $0 < n_0 < 1$, for all b $\in$ [0,1). Otherwise, if network externalities are weak(n $< n_0$), the reverse is true.

Proof:

From 30 and 56, we get:

$$\pi_R^B - \pi_R^C = \frac{((a-(1-b)c)^2)[(1-b)(2+b-n(1+b))^2 - (2-n-b)^2(1+b)]}{4(1-b)(2-b-n)^2(2-n+(1-n)b)^2} \tag{64}$$

$$\text{sign}(\pi_R^B - \pi_R^C) = sign[(1-b)(2+b-n(1+b))^2 - (2-n-b)^2(1+b)] \tag{65}$$

$$= sign(n - (1 - \sqrt{\frac{1-b}{1+b}})) \tag{66}$$

let $n_0 = 1 - \sqrt{\frac{1-b}{1+b}}$
Therefore,
If $n > n_o$, then $\pi_R^B > \pi_R^C$
and If $n < n_o$, then $\pi_R^B < \pi_R^C$

2. The Model 2

We consider an economy with a network goods sector with two Manufacturer (M_1 and M_2) and one retailer (R) . Manufacturer M_1 and M_2 produces differentiated good at constant marginal cost of production $c_1(\geq 0$) and $c_2(\geq 0$) respectively. M_1 and M_2 charges a wholesale price w_1 and w_2 respectively to retailer R. Retailer R sell the goods of M_1 and M_2 to consumer at price p_1 and p_2 respectively.
We consider following demand function for Network goods produce by Manufacturer 1:

$$\mathrm{x}_1 = a + ny_1 - p_1 + bp_2 \tag{67}$$

where x_1 and p_1 denote quantity and price, respectively, of goods sell by retailer produced by manufacturer 1 and y_1 denotes consumers expectation about retailer total sale through manufacturer 1 goods. a(> c), b ∈ (0,1) and n ∈ [0,1) are demand parameter.
We consider following demand function for Network goods produce by Manufacturer 2:

$$\mathrm{x}_2 = a + ny_2 - p_2 + bp_1 \tag{68}$$

where x_2 and p_2 denote quantity and price, respectively, of goods sell by retailer produced by manufacturer 2 and y_2 denotes consumers expectation about retailer total sale through manufacturer 2 goods. a(> c), b ∈ (0,1) and n ∈ [0,1) are demand parameter.

Lower value of parameter β corresponds to the case of higher degree of product differentiation by retailers. The parameter n ($= \frac{\partial x_i}{\partial y_i}$) measures the strength of network externalities -lower value n indicates weaker network externalities.

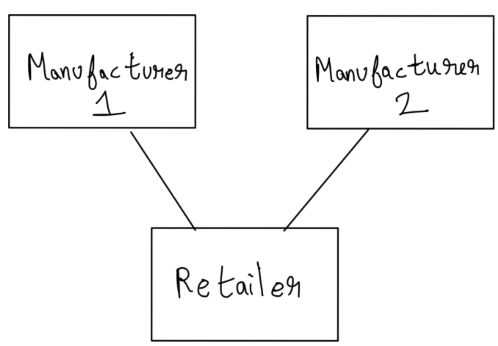

Figure 2: Model 2

From 67, we get the corresponding inverse demand functions for Network goods produce by Manufacturer 1:

$$p_1 = \frac{a(1+b) - x_1 - bx_2}{1-b^2} + \frac{n(y_1 + by_2)}{1-b^2} \tag{69}$$

From 68, we get the corresponding inverse demand functions for Network goods produce by Manufacturer 2:

$$p_2 = \frac{a(1+b) - x_2 - bx_1}{1-b^2} + \frac{n(y_2 + by_1)}{1-b^2} \tag{70}$$

2.1 Bertrand competition

Stage 1:

Each M_i chooses w_i to maximize their profit.
Profit of Manufacturer M_1 as follows:

$$\pi_{M_1} = (w_1 - c_1)x_1 \tag{71}$$

Profit of Manufacturer M_2 as follows:

$$\pi_{M_2} = (w_2 - c_2)x_2 \tag{72}$$

Stage 2:

R chooses p_1 and p_2 to maximize its profit.

$$\pi_R = (p_1 - w_1)x_1 + (p_2 - w_2)x_2 \tag{73}$$

$$\pi_R = (p_1 - w_1)(a + ny_1 - p_1 + bp_2) + (p_2 - w_2)(a + ny_2 - p_2 + bp_1) \tag{74}$$

By using Backward induction, we will solve for equilibra.
In stage 2, retailer sets its price p_1 and p_2 for goods of M_1 and M_2, to maximize its profit π_R.

$$= \frac{\partial \pi_R}{\partial p_1} = 0 \tag{75}$$

$$= \frac{\partial \pi_R}{\partial p_2} = 0 \tag{76}$$

From 75

$$a - 2p_1 + 2bp_2 + w_1 - bw_2 + ny_1 = 0 \tag{77}$$

From 76

$$a - 2p_2 + 2bp_1 + w_2 - bw_1 + ny_2 = 0 \tag{78}$$

Also,

$$= (\frac{\partial^2 \pi_R}{\partial^2 p_1})(\frac{\partial^2 \pi_R}{\partial^2 p_2}) - (\frac{\partial^2 \pi_R}{\partial p_2 \partial p_1}) \tag{79}$$

$$= (-2)(-2) - (2b) \tag{80}$$

$$= 2(2-b) > 0 \tag{81}$$

$$\tag{82}$$

And,

$$\frac{\partial^2 \pi_R}{\partial^2 p_1} = -2 < 0 \tag{83}$$

We consider that consumers form rational expectation, which implies in equilibrium $y_1 = x_1$ for manufacturer 1 produced goods and $y_2 = x_2$ for manufacturer 2 produced goods.
By solving equation (76), (77) ,$y_1 = x_1$,$y_2 = x_2$,we obtain as follow.

$$p_1 = \frac{a + (b-1)(n-1)w_1}{(b-1)(n-2)} \tag{84}$$

$$p_2 = \frac{a + (b-1)(n-1)w_2}{(b-1)(n-2)} \tag{85}$$

By using above equation, $y_i = x_i$, (77) and (78), we obtain x_i as follow

$$x_1 = \frac{a - w_1 + bw_2}{2-n} \tag{86}$$

$$x_2 = \frac{a - w_2 + bw_1}{2-n} \tag{87}$$

Taking this values of x_i in (stage 1) Manufacturer M_i profit(π_{M_i}),

$$\pi_{M_1} = (w_1 - c_1)(\frac{a - w_1 + bw_2}{2-n} \tag{88}$$

$$\pi_{M_2} = (w_2 - c_2)(\frac{a - w_2 + bw_1}{2-n} \tag{89}$$

Manufacturer 1 sets its wholeprice w_1, taking w_2 as given, to maximize its profit

$$= \frac{\partial \pi_{M_1}}{\partial w_1} = 0 \tag{90}$$

$$= \frac{\partial((w_1 - c_1)(\frac{a-w_1+bw_2}{2-n}))}{\partial w_1} = 0 \tag{91}$$

By solving above equation,we obtain manufacturer 1 wholeprice reaction function (RF_1^B) as follow.

$$\frac{a + c_1 - 2w_1 + bw_2}{2-n} = 0 \tag{92}$$

Also,

$$= \frac{\partial^2 \pi_{M1}}{\partial^2 w_1} = \frac{-2}{2-n} \tag{93}$$

$$= \frac{\partial^2 \pi_{M1}}{\partial^2 w_1} < 0 \tag{94}$$

Manufacturer 2 sets its wholeprice w_2, taking w_1 as given, to maximize its profit

$$= \frac{\partial \pi_{M_2}}{\partial w_2} = 0 \tag{95}$$

$$= \frac{\partial((w_2 - c_2)(\frac{a-w_2+bw_1}{2-n}))}{\partial w_2} = 0 \tag{96}$$

By solving above equation,we obtain manufacturer 2 wholeprice reaction function (RF_2^B) as follow.

$$\frac{a + c_2 - 2w_2 + bw_1}{2-n} = 0 \tag{97}$$

Also,

$$= \frac{\partial^2 \pi_{M2}}{\partial^2 w_2} = \frac{-2}{2-n} \tag{98}$$

$$= \frac{\partial^2 \pi_{M2}}{\partial^2 w_2} < 0 \tag{99}$$

Solving RF_1^B, RF_2^B, we obtain the Bertrand equilibrium wholeprices in term of c_1 and c_2.

$$w_1^B = \frac{a(2+b) + 2c_1 + bc_2}{b^2 - 4} \tag{100}$$

$$w_2^B = \frac{a(2+b) + 2c_2 + bc_1}{b^2 - 4} \tag{101}$$

$$\tag{102}$$

Now by taking w_i^B, we obtain bertrand equilibrium prices , quantities ,profits of retailer and manufacturer profit as follows.

$$p_1^B = \frac{a(b^2-4) - ((-1+b)(a(2+b) + 2c_1 + bc_2)(-1+n))}{(-4+b^2)(-1+b)(-2+n)} \tag{103}$$

$$p_2^B = \frac{a(b^2-4) - ((-1+b)(a(2+b)+2c_2+bc_1)(-1+n))}{(-4+b^2)(-1+b)(-2+n)} \tag{104}$$

$$x_1^B = \frac{a(2+b)+(-2+b^2)c_1+bc_2}{(-4+b^2)(-2+n)} \tag{105}$$

$$x_2^B = \frac{a(2+b)+(-2+b^2)c_2+bc_1}{(-4+b^2)(-2+n)} \tag{106}$$

$$\pi_R^B = \frac{(2a^2(2+b)^2 + 2a(-1+b)(2+b)^2(c1+c2) + (-1+b)((-4+3b^2)c1^2 + 2b^3c1c2 + (-4+3b^2)c2^2))}{(1-b)(-4+b^2)^2(-2+n)^2} \tag{107}$$

$$\pi_{M_1}^B = \frac{(a(2+b)+(-2+b^2)c_1+bc_2)^2}{(-4+b^2)^2(2-n)} \tag{108}$$

$$\pi_{M_2}^B = \frac{(a(2+b)+(-2+b^2)c_2+bc_1)^2}{(-4+b^2)^2(2-n)} \tag{109}$$

2.2 Cournot competition

Stage 1:

Each M_i chooses w_i to maximize their profit.
Profit of Manufacturer M_1 as follows:

$$\pi_{M_1} = (w_1 - c_1)x_1 \tag{110}$$

Profit of Manufacturer M_2 as follows:

$$\pi_{M_2} = (w_2 - c_2)x_2 \tag{111}$$

Stage 2:

R chooses x_1 and x_2 to maximize its profit.

$$\pi_R = (p_1 - w_1)x_1 + (p_2 - w_2)x_2 \tag{112}$$

$$\pi_R = (\frac{a(1+b) - x_1 - bx_2}{1-b^2} + \frac{n(y_1+by_2)}{1-b^2} - w_1)x_1 + (\frac{a(1+b) - x_2 - bx_1}{1-b^2} + \frac{n(y_2+by_1)}{1-b^2} - w_2)x_2 \tag{113}$$

By using Backward induction, we will solve for equilibra.
In stage 2, retailer sets its quantity x_1 and x_2 for goods of M_1 and M_2, to

maximize its profit π_R.

$$= \frac{\partial \pi_R}{\partial x_1} = 0 \tag{114}$$

$$= \frac{\partial \pi_R}{\partial x_2} = 0 \tag{115}$$

From 114,

$$\frac{a + ab - w_1 + b^2 w_1 - 2x_1 - 2bx_2 + ny_1 + bny_2}{1 - b^2} = 0 \tag{116}$$

From 115,

$$\frac{a + ab - w_2 + b^2 w_2 - 2x_2 - 2bx_1 + ny_2 + bny_1}{1 - b^2} = 0 \tag{117}$$

Also,

$$= \left(\frac{\partial^2 \pi_R}{\partial^2 x_1}\right)\left(\frac{\partial^2 \pi_R}{\partial^2 x_2}\right) - \left(\frac{\partial^2 \pi_R}{\partial x_2 \partial x_1}\right) \tag{118}$$

$$= \left(\frac{-2}{1 - b^2}\right)\left(\frac{-2}{1 - b^2}\right) - \left(\frac{2b}{1 - b^2}\right) \tag{119}$$

$$= \frac{2(2 - b)}{1 - b^2} > 0 \tag{120}$$

$$\tag{121}$$

And,

$$\frac{\partial^2 \pi_R}{\partial^2 x_1} = \frac{-2}{1 - b^2} < 0 \tag{122}$$

We consider that consumers form rational expectation, which implies in equilibrium $y_1 = x_1$ for manufacturer 1 produced goods and $y_2 = x_2$ for manufacturer 2 produced goods.
By solving equation (115), (116) ,$y_1 = x_1$,$y_2 = x_2$,we obtain as follow.

$$x_1 = \frac{a - w_1 + bw_2}{2 - n} \tag{123}$$

$$x_2 = \frac{a - w_2 + bw_1}{2 - n} \tag{124}$$

$$\tag{125}$$

By using above equation, $y_i = x_i$, (116) and (117), we obtain p_i as follow

$$p_1 = \frac{a + (b - 1)(n - 1)w_1}{(b - 1)(n - 2)} \tag{126}$$

$$p_2 = \frac{a + (b - 1)(n - 1)w_2}{(b - 1)(n - 2)} \tag{127}$$

Taking this values of x_i in (stage 1) Manufacturer M_i profit(π_{M_i}),

$$\pi_{M_1} = (w_1 - c_1)(\frac{a - w_1 + bw_2}{2 - n}) \tag{128}$$
$$\pi_{M_2} = (w_2 - c_2)(\frac{a - w_2 + bw_1}{2 - n}) \tag{129}$$

Manufacturer 1 sets its wholeprice w_1, taking w_2 as given, to maximize its profit

$$= \frac{\partial \pi_{M_1}}{\partial w_1} = 0 \tag{130}$$
$$= \frac{\partial((w_1 - c_1)(\frac{a-w_1+bw_2}{2-n}))}{\partial w_1} = 0 \tag{131}$$

By solving above equation,we obtain manufacturer 1 wholeprice reaction function (RF_1^C) as follow.

$$\frac{a + c_1 - 2w_1 + bw_2}{2 - n} = 0 \tag{132}$$

Also,

$$= \frac{\partial^2 \pi_{M1}}{\partial^2 w_1} = \frac{-2}{2 - n} \tag{133}$$
$$= \frac{\partial^2 \pi_{M1}}{\partial^2 w_1} < 0 \tag{134}$$

Manufacturer 2 sets its wholeprice w_2, taking w_1 as given, to maximize its profit

$$= \frac{\partial \pi_{M_2}}{\partial w_2} = 0 \tag{135}$$
$$= \frac{\partial((w_2 - c_2)(\frac{a-w_2+bw_1}{2-n}))}{\partial w_2} = 0 \tag{136}$$

By solving above equation,we obtain manufacturer 2 wholeprice reaction function (RF_2^C) as follow.

$$\frac{a + c_2 - 2w_2 + bw_1}{2 - n} = 0 \tag{137}$$

Also,

$$= \frac{\partial^2 \pi_{M2}}{\partial^2 w_2} = \frac{-2}{2 - n} \tag{138}$$
$$= \frac{\partial^2 \pi_{M2}}{\partial^2 w_2} < 0 \tag{139}$$

Solving RF_1^C, RF_2^C, we obtain the Cournot equilibrium wholeprices in term of c_1 and c_2.

$$w_1^C = \frac{a(2+b)+2c_1+bc_2}{b^2-4} \tag{140}$$

$$w_2^C = \frac{a(2+b)+2c_2+bc_1}{b^2-4} \tag{141}$$

$$\tag{142}$$

Now by taking w_i^C, we obtain Cournot equilibrium prices , quantities ,profits of retailer and manufacturer profit as follows.

$$p_1^C = \frac{a(b^2-4)-((-1+b)(a(2+b)+2c_1+bc_2)(-1+n))}{(-4+b^2)(-1+b)(-2+n)} \tag{143}$$

$$p_2^C = \frac{a(b^2-4)-((-1+b)(a(2+b)+2c_2+bc_1)(-1+n))}{(-4+b^2)(-1+b)(-2+n)} \tag{144}$$

$$x_1^C = \frac{a(2+b)+(-2+b^2)c_1+bc_2}{(-4+b^2)(-2+n)} \tag{145}$$

$$x_2^C = \frac{a(2+b)+(-2+b^2)c_2+bc_1}{(-4+b^2)(-2+n)} \tag{146}$$

$$\pi_R^C = \frac{(2a^2(2+b)^2+2a(-1+b)(2+b)^2(c1+c2)+(-1+b)((-4+3b^2)c1^2+2b^3c1c2+(-4+3b^2)c2^2))}{(1-b)(-4+b^2)^2(-2+n)^2} \tag{147}$$

$$\pi_{M_1}^C = \frac{(a(2+b)+(-2+b^2)c_1+bc_2)^2}{(-4+b^2)^2(2-n)} \tag{148}$$

$$\pi_{M_2}^C = \frac{(a(2+b)+(-2+b^2)c_2+bc_1)^2}{(-4+b^2)^2(2-n)} \tag{149}$$

1.3 Cournot versus Bertrand equilibria

Lemma:

$$p_1^B = p_1^C \tag{150}$$

$$p_2^B = p_2^C \tag{151}$$

$$x_1^B = x_1^C \tag{152}$$

$$x_2^B = x_2^C \tag{153}$$

$$\pi_R^B = \pi_R^C \quad \pi_{M_1}^B = \pi_{M_1}^C \tag{154}$$

$$\pi_{M_2}^B = \pi_{M_2}^C \tag{155}$$