Price and quantity Competition in network goods for one manufacturer and two retailer system

Prof. Vinay Ramani[1] **and Nakul Patel**[2]

[1]**Address of first author**
[2]**Address of second author**

ABSTRACT

Keywords: Keyword1, Keyword2, Keyword3

1. INTRODUCTION

Cournot vs Bertrand under Network Effects

2. THE MODEL

We consider an economy with a network goods sector with two retailers (R1 and R2) and one manufacturer (M) . Manufacturer produces good at constant marginal cost of production c(≥ 0) and charges a wholesale price w to retailer . Retailer R1 and R2 makes the good differentiated and then sell it to consumer at price p_i and p_2.
We consider following demand function for network goods:

$$x_i = a + ny_i - p_i + bp_j,\ i, j = 1,2,\ i \neq j$$

where x_i and p_i denote quantity and price, respectively, of good of firm i and y_i denotes consumers expectation about firm i's total sale. a($>$ c), b $\in$ (0,1) and n $\in$ [0,1) are demand parameter.

2.1 BERTRAND COMPETITION

Stage 1:
M chooses 'w' to maximize its profit.

Stage 2:
Each R_i chooses p_i to maximize their profit.

By using Backward induction, we will solve for equilibra.
In stage 2, each retailer i sets its price p_i, taking p_j, y_i and y_j as given, to maximize its profit π_{Ri} = (p_i - c)x_i . Solving retailer i problem, we obtain its price reaction function (RF_i^B) as follow.

$$p_i = \frac{a+w+ny_i+bp_j}{2},\ i, j = 1,2,\ i \neq j$$

We consider that consumers form rational expectation, which implies in equilibrium $y_i = x_i$.
Solving RF_1^B,RF_2^B, $y_1 = x_1$ and $y_2 = x_2$, we obtain the Bertrand equilibrium prices and quantities in term of w.

$$p_1 = p_2 = \frac{a+(1-n)w}{2-n-b},\ x_1 = x_2 = \frac{a-(1-b)w}{2-n-b}$$

Now taking this values of x_i in stage 1, and by maximizing profit $\pi_M = (w-c)(x_1+x_2)$ of manufacturer M. We obtain equilibrium wholesale price as follows.

$$w^B = \frac{a+c(1-b)}{2(1-b)}$$

Now by taking w^B, we obtain bertrand equilibrium prices , quantities ,profits of retailer and manufacturer profit as follows.

$$p_1^B = p_2^B = p^B = \frac{2a(1-b)+(1-n)a+c(1-b)(1-n)}{2(1-b)(2-n-b)},\ x_1^B = x_2^B = x^B = \frac{a-(1-b)c}{2(2-n-b)}$$
$$\pi_{R1}^B = \pi_{R2}^B = \pi^B = \frac{(a-(1-b)c)^2}{4(2-n-b)^2},\ \pi_M^B = \frac{(a-(1-b)c)^2}{2(1-b)(2-n-b)}$$

2.2 COURNOT COMPETITION

Stage 1:
M chooses 'w' to maximize its profit.

Stage 2:
Each R_i chooses x_i to maximize their profit.

By using Backward induction, we will solve for equilibra.
In stage 2, each retailer i decides its quantity x_i, taking x_j, y_i and y_j as given, to maximize its profit π_{Ri} = (p_i - c)x_i . Solving retailer i problem, we obtain its price reaction function (RF_i^C) as follow.
$$x_i = \frac{[a-w(1-b)](1+b)+n(y_i+by_j)-bx_j}{2},\ i,j = 1,2,\ i \neq j$$
We consider that consumers form rational expectation, which implies in equilibrium $y_i = x_i$.
Solving RF_1^C, RF_2^C, $y_1 = x_1$ and $y_2 = x_2$, we obtain the cournot equilibrium prices and quantities in term of w.
$$p_1 = p_2 = \frac{a+(1-n)(1-b^2)w}{(1-b)(2-n+(1-n)b)},\ x_1 = x_2 = \frac{(1+b)(a-(1-b)w)}{2-n+(1-n)b}$$
Now taking this values of x_i in stage 1, and by maximizing profit $\pi_M = (w-c)(x_1+x_2)$ of manufacturer M. We obtain equilibrium wholesale price as follows.
$$w^C = \frac{a+c(1-b)}{2(1-b)}$$
Now by taking w^C, we obtain cournot equilibrium prices , quantities ,profits of retailer and manufacturer profit as follows.
$$p_1^C = p_2^C = p^C = \frac{a(2+(1-n)(1+b))+c(1-b^2)(1-n)}{2(1-b)(2-n+(1-n)b)},\ x_1^C = x_2^C = x^C = \frac{(a-(1-b)c)(1+b)}{2(2-n+(1-n)b)}$$
$$\pi_{R1}^C = \pi_{R2}^C = \pi^C = \frac{(a-(1-b)c)^2(1+b)}{4(1-b)(2-n+(1-n)b)^2},\ \pi_M^C = \frac{(a-(1-b)c)^2(1+b)}{2(1-b)(2-n+(1-n)b)}$$

3. COURNOT VERSUS BERTRAND EQUILIBRIA

Lemma 1: $p^B < p^C$, $x^B > x^C$ and $\pi_M^B > \pi_M^C$, all n ∈ [0,1).

Proposition 1: In the presence of strong network externalities (n > n_0), profits under Bertrand equilibrium are higher compared with that under Cournot equilibrium; where $n_0 = 1 - \sqrt{\frac{1-b}{1+b}}$, $0 < n_0 < 1$, for all b ∈ [0,1). Otherwise, if network externalities are weak(n < n_0), the reverse is true.