The Hitchhiker's Guide to Probability and Optimization

MAT 2233

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Abstract

This course covers probability and optimization from a practical lens. We will first cover the basics of counting, before moving on the probability, where we will learn varied topics such as Bayes' Theorem and Random variables. Finally, we will move on to optimization, which covers some of the most common algorithms and methods for function optimization.

Syllabus[1]

1. **Permutation and Combination**	**8 Hours**
(a) Basic Methods	
(b) Generating Functions	
(c) Distributions	
(d) Partition and Composition	
2. **Probability**	**15 Hours**
(a) Bayes' Theorem	4 Hours
(b) Random Variables	5 Hours
(c) Distributions and Functions of Random Variables	6 Hours
3. **Optimization**	**7 Hours**
(a) Vector Valued Functions	3 Hours
(b) Back Propagation	4 Hours

Grading[2]

Internal Assessment — 50 Marks

1. Quiz/Assignment	5 Marks
2. Quiz/Assignment	5 Marks
3. Quiz/Assignment	5 Marks
4. Quiz/Assignment	5 Marks
5. Mid Semester Examination	30 Marks

External Assessment — 50 Marks

[1] This syllabus needs to be updated as it only adds up to 30 hours of course material

[2] More details need to be added for grading.

Contents

Chapter 1

Combinatorics

Abstract

Combinatorics is a field of mathematics that deals with the counting of discrete objects. Any applications of mathematics that deals with selection or arrangements of objects comes under the purview of combinatorics. Combinatorics is essential for calculating probability, as it is often necessary to know the total number of possibilities to calculate probability.

Whether you use permutation or combination is irrelevant in this case.

Theorem 1.1 (The Fundamental Theorem Of Counting): If there are two events that happen in succession, and there are m ways to do the first one; and n ways to do the second one, then there are $m \times n$ ways of completing the tasks in succession

Example 1.1 (Throwing 2 Dice): Suppose we have two dice which are thrown successively, and the readings are noted. there are 6 possibilities in each die, and 36 possibilities in total. Each of those possibilities are represented in the following matrix.

$$\begin{pmatrix} (1,1) & (1,2) & \dots & (1,6) \\ (2,1) & (2,2) & \dots & (2,6) \\ \vdots & \vdots & \ddots & \vdots \\ (6,1) & (6,2) & \dots & (6,6) \end{pmatrix}$$

1.1 Permutation

Permutation is the study of arrangement of objects. Formally, it is defined as the act of arranging members of a given set into a sequence or order.

Intuition 1.1 (Factorials in Combinatorics): Factorials are omnipresent in combinatorics. This is because factorials allow us to elegantly represent a particular scenario that occurs often in problems of combinatorics. Suppose you are given the responsibility of arranging 5 people into 5 seats. Logically, you have 5 choices for seat 1, 4 for seat 2 and so on. Thus you have stumbled upon 5!.

$$\boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} = 5! = 120 \text{ ways}$$