Najaf 2022EEB1192

INDIAN INSTITUTE OF TECHNOLOGY ROPAR

Department of Mathematics

Semester I, Academic Year 2023-2024

Quiz 1 for the course MA201 - Differential Equations

Date: September 09, 2023 | Time: 9 – 9:50 AM | Max. Marks:15

Instructions:

- Answer all questions as precisely as possible.
- If you are asked to find a solution using a specific method, then your answer will be considered for evaluation only when you answer using that specific method.
- Answer that is incorrect, but shows progress toward the correct answer may receive partial marks.
- All the notations used in this question paper are the standard notations used in the classroom.

1. Derive the differential equation for the given two parameter families of curves — 3 Marks

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where a, b are fixed positive real numbers.

2. Find the first four approximate solution to the following initial value problem — 3 Marks

$$y' = x + y^2$$

with the initial condition $y(0) = 0$ using Picard's successive approximation method.

3. Check whether the given differential equation is exact or not — 5 Marks

$$(3x^2y + 2xy + y^3)dx + (x^2 + y^2)dy = 0$$

and then solve it using a suitable method.

4. Solve the initial value problem — 4 Marks

$$6y' - 2y = xy^4$$

with the initial condition $y(0) = -2$. Also determine the interval in which the solution is valid.

Name: ......ms/cul...... Entry No.: ......1182......

INDIAN INSTITUTE OF TECHNOLOGY ROPAR

Department of Mathematics

Semester I, Academic Year 2023-2024

MA201 - Differential Equations

Mid Semester Exam | Duration: 2 Hrs | Max. Marks: 35

Instructions:

- Answer all questions as precisely as possible.
- If you are asked to find a solution using a specific method, then your answer will be considered for evaluation only when you answer using that specific method.
- Answers that are incorrect, but show progress toward the correct answer may receive partial credit.
- All the notations used in this question paper are the standard notations used in the classroom.

1. Solve the differential equation [3 Marks]

$$\frac{dy}{dx} = \sin(x+y) + \cos(x+y).$$

2. (a) When does a function $f(x,y)$ defined in a region R is said to satisfy Lipschitz condition (with respect to y) in R. [3 Marks]

   (b) Let $f(x,y) = e^{-x^2}y^2 \sin x$ and R be the strip $0 \le y \le 2$ in the xy-plane. Check whether $f(x,y)$ satisfies the Lipschitz condition on R or not.

3. With detailed justification and derivation, write the annihilator of the function $e^{3x} + 5\cos 2x$. [3 Marks]

4. Given ODE: $y''(x) - (2\alpha - 1)y'(x) + \alpha(\alpha - 1)y(x) = 0,\ \alpha \in \mathbb{R}$. [3 Marks]

   (a) Determine the value of α , if any, for which all the solutions tend to zero as $x \to \infty$.

   (b) Also, determine the value of α , if any, for which all (nonzero) solutions become unbounded as $x \to \infty$.

5. Given ODE: $y''(x) + 2y'(x) + 5y(x) = 4e^{-x}\cos 2x$. [6 Marks]

   (a) Find the solution of the corresponding homogeneous equation.

   (b) Find the particular solution of the nonhomogeneous equation using the method of undetermined coefficients.

   (c) Hence, find the general solution of the given nonhomogeneous equation.

Nafal 2022EEB1192

INDIAN INSTITUTE OF TECHNOLOGY ROPAR

Department of Mathematics

2^{nd} Semester of the Academic Year 2022–2023

MA 102: Linear Algebra, Integral Transforms, and Special Functions

Mid Sem. Exam.	Time: 2 Hrs	06 May, 2023	Marks: 35

Instructions:

- **Answer all the questions.**
- **Clearly show all the steps of your solution.**

---

1. (i) Find the point at which $\text{Span}\{(2,1,7)\}$ intersects the plane $5x - 2y + 3z = 0$.

   (ii) Let $M_{2\times 2}(\mathbb{R})$ be the vector space of 2×2 matrices over the field $\mathbb{R}$. Find a basis $\{M_1, M_2, M_3, M_4\}$ of $M_{2\times 2}(\mathbb{R})$ such that $M_i^2 = M_i$ for each $i = 1, 2, 3, 4$.

   (iii) Let W be a subspace of $\mathbb{R}^5$ defined as:

   $$W = \{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5 : 3x_1 = x_2 \text{ and } x_3 + x_4 + 3x_5 = 0\}.$$

   Find a basis and the dimension of W. [1+3+2]

2. Let a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by:

   $$T(x_1, x_2) = (x_1 + 2x_2, -x_1, 0).$$

   Find the matrix $[T]_{\beta_1}^{\beta_2}$ with respect to the bases $\beta_1 = \{u_1, u_2\}$ and $\beta_2 = \{v_1, v_2, v_3\}$, where $u_1 = (1,3)$, $u_2 = (-2,4)$, $v_1 = (1,1,1)$, $v_2 = (2,2,0)$, and $v_3 = (3,0,0)$. [2]

3. Find a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ whose null space $N(T)$ is given below:

   $$N(T) = \text{span}\{(1,2,3,,4), (0,1,1,1)\}.$$

   [5]

4. (i) Prove that there exist *infinitely many* linear transformations $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1,-1,1) = (1,2)$ and $T(-1,1,2) = (1,0)$.

   (ii) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(x,y) = (x+y, x)$. Show that T is invertible and find T^{-1}. [3+3]

5. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 0 \end{bmatrix}$ be a given matrix. Find row reduced echelon form R of A and an invertible matrix U such that $R = UA$. [2]

6. Let $A = \begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$ be a given matrix.

(a) Find a basis of the row space of A.

(b) Find a basis for the column space A.

(c) Find a basis for the null space of A.

[2 + 1 + 2]

7. Let A be a matrix with the characteristic polynomial:

$$f(\lambda) = \lambda^4(\lambda - 1)^5(\lambda - 2)^5.$$

Assume that A is diagonalizable. Then

(a) Find the dimension of the eigenspace E_2 corresponding to the eigenvalue 2.

(b) Find rank and nullity of A.

[3]

8. Let $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ be a given matrix.

(a) Find eigenvalues of the matrix A.

(b) Find eigenvectors corresponding to each eigenvalues.

(c) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

(d) Also diagonalize the matrix $A^3 - A^2 + I$, where I is the 2×2 identity matrix.

[1 + 2+ 2+ 1]

****** End ******

Name: Nakul Entry No.: 1192

INDIAN INSTITUTE OF TECHNOLOGY ROPAR
Department of Mathematics
Academic Year 2023-2024 Sem-I (End Semester Exam)
MA201 - Differential Equations

Duration: 3 Hrs | Full Marks 50 | 18 Nov 2023

Instructions:

- If you are asked to find a solution using a specific method, then your answer will be considered for evaluation only when you answer using that specific method.
- Answers that are incorrect, but show progress toward the correct answer may receive partial credit as per the marking scheme.
- All the notations used in this question paper are the standard notations used in the classroom.

---

1. Let $\phi_1(x)$, $\phi_2(x)$ be linearly independent solutions of [2 Marks]

$$L(y) = y'' + a_1(x)y' + a_2(x)y = 0$$

on an interval I. Suppose that $\psi_1(x)$, $\psi_2(x)$ are any other pairs of linearly independent solutions of the same differential equation $L(y) = 0$ on I, then prove that

$$W(\phi_1, \phi_2)(x) = k\,W(\psi_1, \psi_2)(x) \quad \text{for all } x \in I$$

for some fixed constant $k \neq 0$.

2. Write the definition and condition of exactness of second order equation [4 Marks]

$$P(x)y'' + Q(x)y' + R(x)y = 0.$$

Then, using the definition of exactness determine whether the following ODE is exact or not and solve the equation

$$x^2y'' + xy' - y = 0 \quad \text{for } x > 0.$$

3. Let ϕ be a function having a continuous derivative on $0 \le x < \infty$, satisfying [3 Marks]

$$\phi'(x) + 2\phi(x) \le 1 \quad \text{for all } x \ge 0,$$

and $\phi(0) = 0$. Show that $\phi(x) < \frac{1}{2}$ for all $x \ge 0$.

4. Given the power series $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$. With Proper justification [3 Marks]

i) Determine the radius of convergence.
ii) Determine the exact interval of convergence.
iii) Test whether it is convergent or divergent at the endpoints.

5. Given the DE: $x^2y'' + xy' + 2xy = 0$, [1+1+4=6 Marks]

(a) Find all the regular singular points of the given differential equation.

(b) Determine the indicial equation and the exponents at the singularity for each regular singular point.

(c) Using the method of Frobenius, find the two power series solutions of the given DE for $x > 0$.

6. Write the Legendre Differential equation. Without deriving the solution and with proper justification, find the lower bound for the radius of convergence of the series solution of the given differential equation about ordinary point $x = 0$. [2 Marks]

7. Using Laplace transform and the shifting property of Laplace transform, find the solution of the boundary value problem $y''' - 2y'' + 5y' = 0$ when $y(0) = 0,\ y'(0) = 1,\ y(\pi/8) = 1$. Further, find the value of $y''(0)$. [6 Marks]

8. Derive the Laplace transform of Dirac delta function $\delta(t)$ and hence solve the initial value problem $y'' + y = 4\delta(t - 2\pi)$ when $y(0) = 1,\ y'(0) = 0$. [5 Marks]

9. Consider the partial differential equation [1+2+4 = 7 Marks]

$$x^2\, u_{xx} - 2xy\, u_{xy} + y^2\, u_{yy} + xu_x + yu_y = 0$$

(i) Determine the region in which the given PDE is hyperbolic, parabolic or elliptic
(ii) Find the characteristics and characteristic coordinates.
(iii) Find the general solutions after reducing the given equation to canonical forms

10. Consider the partial differential equation [6 Marks]

$$u_{tt} - u_{xx} = 0, \qquad -\infty < x < \infty$$

$$u(x,0) = f(x) = \begin{cases} 0 & -\infty < x < -1 \\ x+1 & -1 \le x \le 0 \\ 1-x & 0 \le x \le 1 \\ 0 & 1 < x < \infty \end{cases}$$

$$u_t(x,0) = g(x) = \begin{cases} 0 & -\infty < x < -1 \\ 1 & -1 \le x \le 1 \\ 0 & 1 < x < \infty \end{cases}$$

Derive the d'Alembert's solution of the above initial value problem and evaluate the solution u at the point $(1, \frac{1}{2})$.

11. Consider the partial differential equation [6 Marks]

$$\begin{aligned} u_t - u_{xx} &= 0, & 0 \le x \le \pi,\ t > 0 \\ u(0,t) = u(\pi,t) &= 0 & t \ge 0 \\ u(x,0) = f(x) &= x & 0 \le x \le \pi/2 \\ &= \pi - x & \pi/2 \le x \le \pi \end{aligned}$$

Find the solution of the above initial and boundary value problem using method of separation of variables.

*************** Wish you all the Best! ***************

INDIAN INSTITUTE OF TECHNOLOGY ROPAR

MA 203/MA423: Probability and Stochastic Process

Second Semester of Academic Year 2023-24

Quiz-1

Duration: 45 Minutes | Max. Marks: 10 | Date: February 3, 2024

Answer all questions as precisely as possible with **justification**.

1. The probability that a teacher will give a surprise attendance during any class meeting is $1/5$. If a student is absent on two days, what is the probability that student will miss at least one attendance ? [2]

2. A and B are two very weak students of statistics and their chances of solving a problem correctly are $\frac{1}{8}$ and $\frac{1}{12}$ respectively. Let the probability of their making a common mistake is $\frac{1}{1001}$. If they obtain the same answer, find the probability that there answer is correct. [3]

3. The chance that a doctor will diagnose a certain disease correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of the doctor who had the disease dies. What is the probability that the disease was diagnosed correctly? [3]

4. Two fair dice are thrown independently. Three events A, B and C are defined as follows:

   A: Odd face with first die.

   B: Odd face with second die.

   C: Sum of points on two dice is odd.

   Verify that the events A, B and C mutually independent or not? [2]