constitutive relations
relations

DEFORMATION —

→ impact

for 1-D there is no shear strain

$\varepsilon_{yy} = \frac{\Delta v_y}{\Delta y}$, $\varepsilon_{xy} = \frac{\pi}{2} - (\theta_1 + \theta_2)$

(same as stress)

initial angle for non-rectangular coordinate system

$$\boxed{\varepsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}}$$

→ doesn't apply for fluids
→ does not tell the reason.

Volumetric strain:-

final dimensions : $l_x(1+\varepsilon_x)$, $l_y(1+\varepsilon_y)$, $l_z(1+\varepsilon_z)$

$$V = \underbrace{l_x l_y l_z}_{V_0}(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)$$

or $\frac{V}{V_0} = (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)$

or $\frac{V}{V_0} = 1 + \varepsilon_x + \varepsilon_y + \varepsilon_z + \varepsilon_x\varepsilon_y + \varepsilon_y\varepsilon_z + \varepsilon_z\varepsilon_x + \varepsilon_x\varepsilon_y\varepsilon_z$

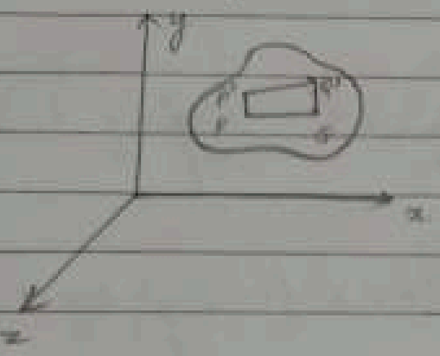
Neglecting higher order terms

$$\boxed{\frac{\Delta V}{V_0} = \varepsilon_x + \varepsilon_y + \varepsilon_z}$$

Line of zero deformation

* Strain is basically an average and displacement is a point function for that sense.

<u>Deformation in the neighbourhood of a point :-</u>



$$\varepsilon_{PQ} = \frac{\Delta s' - \Delta s}{\Delta s} \qquad \Delta s' = P'Q' \qquad \Delta s = PQ$$

$$P(x, y, z) \longrightarrow P'(x + u_x,\ y + u_y + z + u_z)$$

$$Q(x + \Delta x,\ y + \Delta y,\ z + \Delta z) \longrightarrow Q'(x + \Delta x + u_x + \Delta u_x,\ y + \Delta y + u_y + \Delta u_y,\ z + \Delta z + u_z + \Delta u_z)$$

Now,
$$\Delta u_x = \frac{\partial u_x}{\partial x}\Delta x + \frac{\partial u_x}{\partial y}\Delta y + \frac{\partial u_x}{\partial z}\Delta z$$

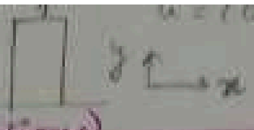
$$\Delta u_y = \frac{\partial u_y}{\partial x}\Delta x + \frac{\partial u_y}{\partial y}\Delta y + \frac{\partial u_y}{\partial z}\Delta z$$

$$\Delta u_z = \frac{\partial u_z}{\partial x}\Delta x + \frac{\partial u_z}{\partial y}\Delta y + \frac{\partial u_z}{\partial z}\Delta z$$

Displacement gradient

$$u_{i,j} = \begin{array}{c} \text{like } \varepsilon_{xx} \\ \begin{bmatrix} \dfrac{\partial u_x}{\partial x} & \dfrac{\partial u_x}{\partial y} & \dfrac{\partial u_x}{\partial z} \\[2ex] \dfrac{\partial u_y}{\partial x} & \dfrac{\partial u_y}{\partial y} & \dfrac{\partial u_y}{\partial z} \\[2ex] \dfrac{\partial u_z}{\partial x} & \dfrac{\partial u_z}{\partial y} & \dfrac{\partial u_z}{\partial z} \end{bmatrix} \end{array}$$

always prefer displacement to force (boundary conditions)



$PQ\ (\Delta x, \Delta y, \Delta z)$
$P'Q'\ (\Delta x', \Delta y', \Delta z')$

$$\Delta s = \Delta x^2 + \Delta y^2 + \Delta z^2$$

$$\Delta x' = \Delta x + \Delta u_x = \left(1 + \frac{\partial u_x}{\partial x}\right)\Delta x + \frac{\partial u_x}{\partial y}\Delta y + \frac{\partial u_x}{\partial z}\Delta z$$

$$\Delta y' = \Delta y + \Delta u_y = \frac{\partial u_y}{\partial x}\Delta x + \left(1 + \frac{\partial u_y}{\partial y}\right)\Delta y + \frac{\partial u_y}{\partial z}\Delta z$$

$$\Delta z' = \Delta z + \Delta u_z = \frac{\partial u_z}{\partial x}\Delta x + \frac{\partial u_z}{\partial y}\Delta y + \left(1 + \frac{\partial u_z}{\partial z}\right)\Delta z$$

Now,
$$\boxed{\begin{aligned}&\Delta s'^2 - \Delta s^2 \\ &= 2\left(E_{xx}\Delta x^2 + E_{yy}\Delta y^2 + E_{zz}\Delta z^2 + E_{xy}\Delta x\Delta y + E_{yz}\Delta y\Delta z + E_{zx}\Delta z\Delta x\right)\end{aligned}}$$

$$\boxed{\sigma = E\varepsilon}$$

ε is the independent ~~fret~~ variable.

* for small deformations, $\varepsilon << 1$

$$E_{xx} = \varepsilon_{xx} = \frac{\partial u_x}{\partial x}$$

$$E_{yy} = \varepsilon_{yy} = \frac{\partial u_y}{\partial y}$$

$$E_{zz} = \varepsilon_{zz} = \frac{\partial u_z}{\partial z}$$

$$\varepsilon_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

order of rotation and deformation

ROTATION $\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)$ [skew-symmetric]

$$\varepsilon_{yz} = \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}$$

$$\varepsilon_{xz} = \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}$$

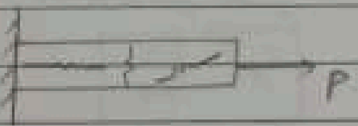
[neglecting higher order terms]

$$\boxed{\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}$$ (symmetric)

Tensorial | Engineering shear strain ← $\boxed{e_{xy} = \frac{1}{2}\varepsilon_{xy}}$

Strain gauge :-

$R = \frac{\rho l}{A}$



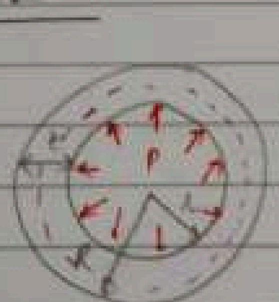
(forms WSB)

Photoelasticity :-

wire fringes (λ comparable to atomic spacing)

PRESSURE VESSELS -

↳ spherical stress
↳ hydrostatic



$\frac{r}{t} > 10 \rightarrow$ thin walled
else thick walled

$$\sigma \times 2\pi r_m t = p \times \pi r^2$$

or $$\boxed{\sigma = \frac{p r^2}{2 r_m t}}$$

outer surface is traction free

[similar to plane stress case]

for thin walled → $\left[\sigma = \frac{p r}{2t}\right]$

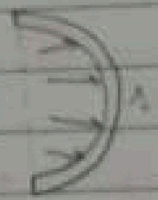
Cylindrical pressure vessel:-

* circumferential and longitudinal stress.

→ CIRCUMFERENTIAL-

$2 \times \sigma \times l \times t = p \times 2r \times l$

or $\boxed{\sigma_1 = \frac{pr}{t}}$



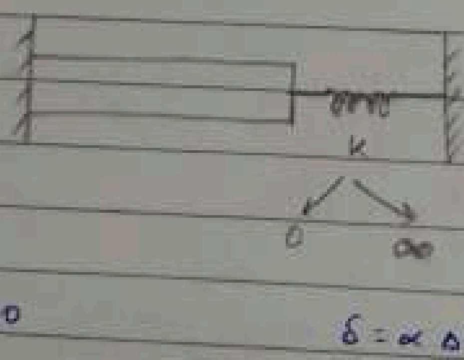
→ LONGITUDINAL-

$2\pi r t \times \sigma_2 = p \times \pi r^2$

or $\boxed{\sigma_2 = \frac{pr}{2t}}$

$\boxed{\sigma_1 = 2\sigma_2}$

THERMAL STRESSES-



$k = 0$

$\delta = 0 \Rightarrow \varepsilon = 0$

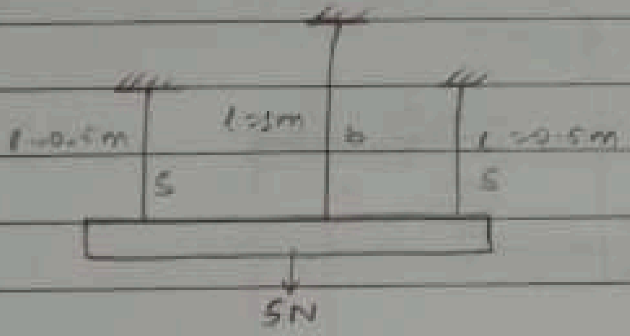
$\sigma = 0$

Relative distance b/w material points is same

$\delta = \alpha \Delta T l$

$\varepsilon = \alpha \Delta T$

$\boxed{\sigma = E \alpha \Delta T}$



symmetrically placed

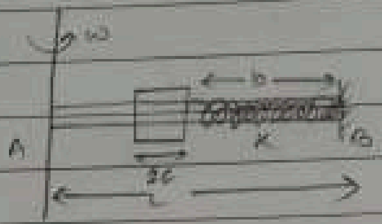
∴ The block is rigid, it must remain horizontal.
[If there was a thread there, thermal stresses will not develop].

$$2P_s + P_b = W \qquad [W = 5N]$$

$$\delta_s = \frac{P_s L_s}{A_s E_s} \qquad \delta_b = \frac{P_b L_b}{A_b E_b}$$

More supports not only make a structure stiffer, they also make it statically indeterminate.

Ques:



* $m\omega^2 r = k\delta$

or, $m\omega^2(L-b-c) = k\delta$

or, $\delta = \dfrac{m\omega^2(L-b-c)}{k}$

* put $\delta = \dfrac{b}{2}$

$\therefore \dfrac{b}{2} = \dfrac{m\omega^2(L-b-c)}{k}$

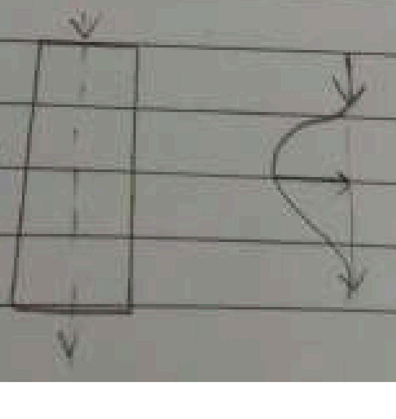
or, $\omega = \sqrt{\dfrac{bk}{2m(L-b-c)}}$

→ The beams/rods where the cross-sectional area does not change are called prismatic members, else non-prismatic.

→ Nut/bolt assembly

Most of the structures fail under bending, not axial loading.

centric loading



eccentric loading (has a moment + axial load) (wedges) O . . . .

a avoid single perk

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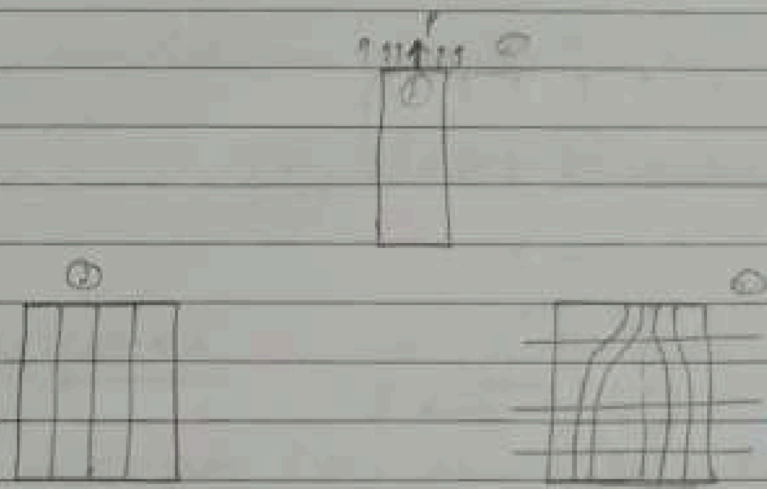
SAINT- VENANT'S PRINCIPLE -



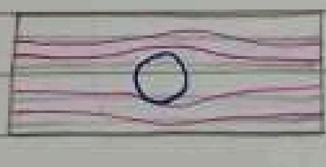
Stresses locally very high

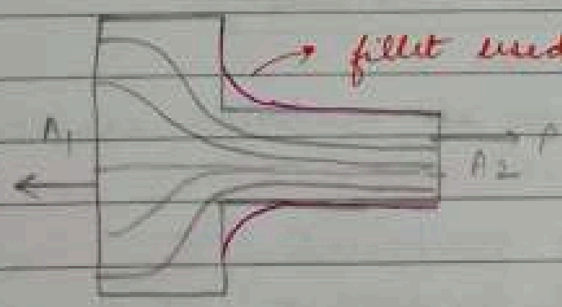
$\sigma_{avg} = P/A$

* Stress concentration

→ Stress is uniform away from the point of load application.

→ Any distortion is the point where fracture starts.





fillet used to reduce stress concentration.

eg: airplane components

→ dogbone shape of tensile test samples

$$\boxed{k = \frac{\sigma_{max}}{\sigma_{avg}}}$$

deformation has to be unique. compatibility conditions
↳ each point follows the way of points satisfying boundary conditions.

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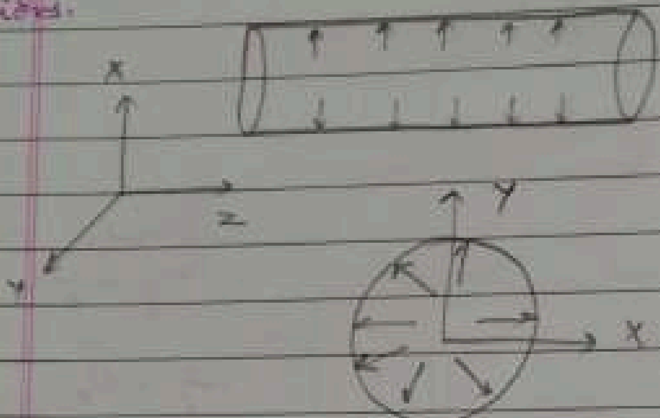
↳ follows lower boundary conditions.

PLANE STRAIN –



$z >> (x, y)$

(u is too long)

$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$, $\varepsilon_{yy} = \frac{\partial u_y}{\partial y}$, $\varepsilon_{zz} = \cancel{\frac{\partial u_z}{\partial z}} \to 0$

doesn't say anything about σ_{zz}.

* don't say $\sigma_z = 0$.

$$\boxed{\sigma_{ij} = f(\varepsilon_{ij}) = g(\omega)}$$

CONSTITUTIVE RELATIONS –

→ depends on ~~str~~ material constitution

σ_{ij} – 9 components
+ 6 independent (stress)
+ 6 independent (strain)
+ 3 displacement conditions

15 independent

- 3 eq^m eq^ns
- 3 strain conditions

strain gauge

gauge length

phenomenal model

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tensile test data

- 3 compatibility conditions (relates only strain)

⇒ 6 constitutive relations

$$\left[\sigma_{zz} = E\,\varepsilon_{zz}\right]$$ specialised constitutive relations

Tensile test is the most basic test.

* ASTM standards (for sample geometry in tensile test)

Threading avoids slipping but can generate additional stress concentration.

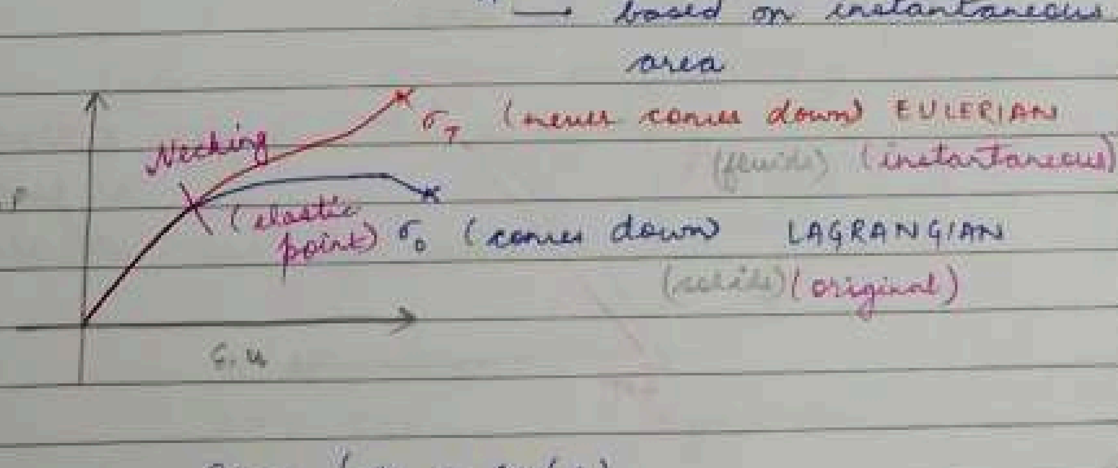
Engineering stress-strain: $\sigma_0 = \frac{P}{A_0}$ → seems constant but is not

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0}$$

in the linear part

$$[\sigma = E\varepsilon]$$

True stress-strain: $\sigma_T = \frac{P}{A}$ → based on instantaneous area



$$\varepsilon_T = \int \frac{dl}{l} = \ln\left(\frac{l}{l_0}\right)$$

$$\frac{l}{l_0} = 1 + \varepsilon_0 \Rightarrow \boxed{\varepsilon_T = \ln(1+\varepsilon_0)}$$

Ignore defect due to processing

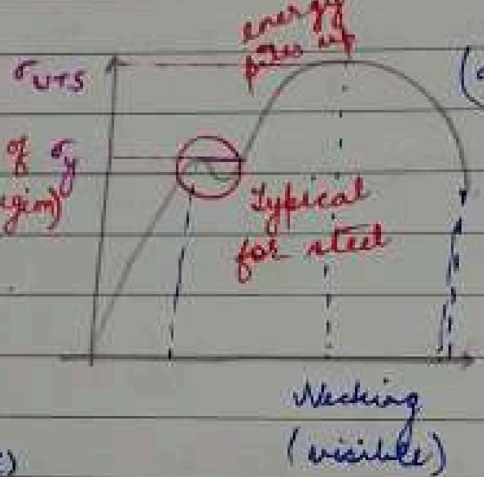
3rd quadrant

↳ weak in tension
↳ strong in compression

* for compressive test, curve is below σ_0.

* for shear test, it is the same as σ_0.

→ Isotropy and properties
→ different modulus in different dir^n's but random overall.
→ Anisotropy due to ~~extr~~ extrusion



Material 'feels' σ_T.

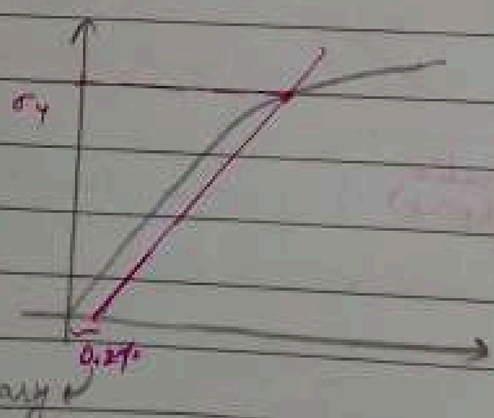
$$\boxed{\sigma = f(\varepsilon, \dot{\varepsilon}, T, \ldots)}$$

Rate effects

$\sigma = f(\varepsilon)$

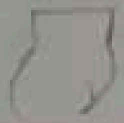
Measure of ductility :-

* % ~~elongati~~ elongation
* % reduction in area.



* Modulus at particular ~~at~~ strain

→ brittle ~~tetr~~ behaviour



(because of friction)

$\boxed{\sigma = E\varepsilon}$ $\quad [\varepsilon^T = \varepsilon^e + \varepsilon^p]$

* permanent set
* strain hardening
  - stronger material after cold working

→ Stronger material need not be tougher.
  - Stronger → higher UTS
  - Tougher → area under stress-strain curve (steel)

* ceramic tiles in armour

→ Energy is lost in overcoming friction b/w molecules, leading to heating.

→ multi-layered bow

* We neglect the plastic deformation in this course, as we want to recover the elastic energy.

- Assume quasi-static process for loading
  → Rate-dependent process/loading.

glass doesn't 'break' by bullet.

$$\Delta F = \sigma \times \text{Area}$$

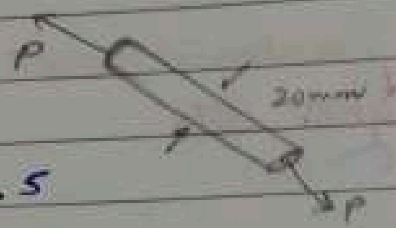
$$\Delta x = \varepsilon \Delta z$$

$$\frac{\Delta F}{2} \times \Delta x = \frac{\sigma}{2} \times \varepsilon \underbrace{\Delta x \Delta y \Delta z}_{V} \Rightarrow \boxed{\frac{E}{V} = \frac{1}{2}\sigma\varepsilon}$$

energy ~~Expos~~ imparted

→ Generally energy is used for comparisons as it is a scalar but stress is a ~~vector~~ tensor.

Ques: If $P = 150\,kN$ is applied and released, find the permanent deformation in the rod.

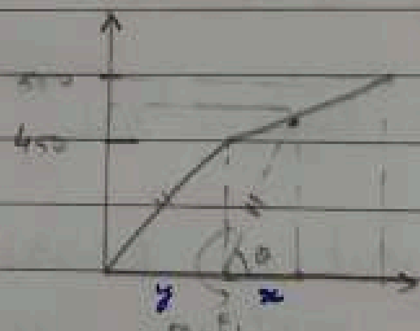


$$E_1 = \frac{450}{0.00225} = 2\times10^5$$

$$E_2 = \frac{50}{0.03-0.00225} = \cancel{08}\,1801.5$$

$$P = 150\times10^3$$

$$\sigma = \frac{150\,kN\times4}{20\times20\times\pi} = 477.7\,MPa$$



$$\tan\theta = E_1$$

$$\cancel{\frac{450}{x}}\quad \frac{477.7}{x} = E_1$$

$$x = \frac{477.7}{E_1} = 0.00238$$

$$y = \frac{450}{E_1} = 0.00225$$

~~y+x~~

Ans: 0.015 mm

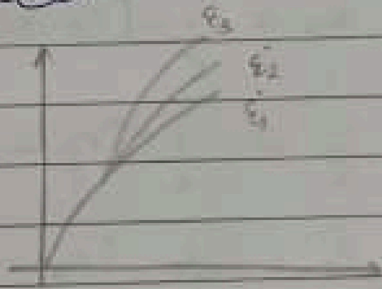
Material models:-

→ Linear
→ Rigid (∞ slope)
→ Perfectly plastic (0 slope) [eg: dough]
→ Rigid plastic material
→ Elastic perfectly plastic
→ Elastic Plastic material (most materials)

Linear vs non-linear [elastic behaviour]

$$\boxed{\sigma = k\varepsilon^m} \quad \left\{ m = \text{strain-hardening exponent} \right\}$$

Test at different rates → $\dot{\varepsilon}^n$



$(\dot{\varepsilon}_3 > \dot{\varepsilon}_2 > \dot{\varepsilon}_1)$

energy dissipation

$(T_3 < T_2 < T_1)$

$$E \rightarrow c_{1111} \quad [\sigma_{ij} = c_{ijkl}\,\varepsilon_{kl}]$$

Poisson's Ratio:-

-ve : quick sand

$$\boxed{\nu = -\frac{\varepsilon_T}{\varepsilon_L}}$$

$0.2 < \nu < 0.5$ → not possible

$\Rightarrow \boxed{0.2 < \nu < 0.45}$ Why?

$$\nu_{xy} = \frac{-\varepsilon_y}{\varepsilon_x}, \quad \nu_{xz} = \frac{-\varepsilon_z}{\varepsilon_x}$$

sliding of polymer chains in non-linear elastic

$\Rightarrow E$ at a given ε. | $\frac{\partial \sigma}{\partial \varepsilon}$ vs ε

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for isotropic case

$$\varepsilon_y = \varepsilon_z$$

$$\left[\nu_{xy} = \nu_{xz} = \nu\right]$$

$$\boxed{\varepsilon_{lateral} = -\nu\, \varepsilon_{longi}}$$

$$\varepsilon_x = \frac{\sigma_x}{E}$$

if loading is ~~dist~~ in all directions,

ε_x - loading in x dir^n

$-\nu\varepsilon_y$ - loading in y dir^n

$-\nu\varepsilon_z$ - loading in z ~~dist~~ dir^n

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

similarly, $\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

Normal strains are function of normal stress

$$\Rightarrow \quad E\varepsilon_x = \sigma_x - \nu(\sigma_y + \sigma_z)$$

$$E\varepsilon_y = \sigma_y - \nu(\sigma_x + \sigma_z)$$

$$E\varepsilon_z = \sigma_z - \nu(\sigma_x + \sigma_y)$$

$$\nu E\varepsilon_x = \nu\sigma_x - \nu^2\sigma_y - \nu^2\sigma_z$$

$$-\quad \nu E\varepsilon_y = \nu\sigma_y - \nu^2\sigma_x - \nu^2\sigma_z$$

$$\quad - \quad\quad + \quad\quad +$$

$$\nu E(\varepsilon_x - \varepsilon_y) = \nu(\sigma_x - \sigma_y) - \nu^2(\sigma_x - \sigma_y)$$

or $\nu E(\varepsilon_x - \varepsilon_y) = \nu(\sigma_x - \sigma_y)(1-\nu)$

compared based on energy.

$$\therefore \quad \sigma_x - \sigma_y = \frac{E(\varepsilon_x - \varepsilon_y)}{1-\nu}$$

$$\sigma_y - \sigma_z = \frac{E(\varepsilon_y - \varepsilon_z)}{1-\nu}$$

$$\sigma_z - \sigma_x = \frac{E(\varepsilon_z - \varepsilon_x)}{1-\nu}$$

$$\Rightarrow \quad \sigma_x - \sigma_y = \frac{E(\varepsilon_x - \varepsilon_y)}{1-\nu}$$

$$\sigma_y - \sigma_z = \frac{E(\varepsilon_y - \varepsilon_z)}{1-\nu}$$

$\cancel{\sigma_x - \sigma_y = E}$

$$\sigma_x = \frac{E\left[(1-\nu)\varepsilon_x + (\varepsilon_y + \varepsilon_z)\nu\right]}{(1-\nu-2\nu^2)}$$

$$1-\nu-2\nu^2 = (1+\nu)(1-2\nu)$$

$$\therefore \quad \boxed{\nu \neq -1,\ 1/2}$$

for plane stress, $\sigma_z = 0$

$$\left[\varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)\right]$$

Strain may not be zero even if stress is zero.

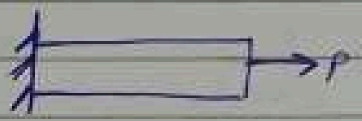
single crystals are anisotropic.

* Shear stress - strain diagram
  ↳ lap shear
  ↳ rotational degree of freedom

→ 81 elements in total
  ↳ reduces to 21 in triclinic

→ True stress - strain and engineering is same in case of SHEAR.

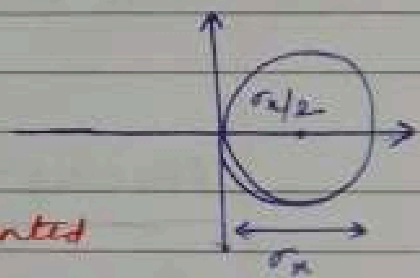
$$\sigma_{11} = \frac{P}{A}$$



$$C : \frac{\sigma_x + \sigma_y}{2} \quad , \quad R : \frac{\sigma_x - \sigma_y}{2}$$

$$\left[\sigma_N - \left(\frac{\sigma_x}{2}\right)\right]^2 = \left(\frac{\sigma_x}{2}\right)^2$$

here $\sigma_y = 0$



Shear stress is maximum on planes oriented at 45° to each other.

∴ SHEAR STRESS EXISTS [for uniaxial loading]

Derive.-

$$\boxed{\frac{E}{2G} = 1 + \nu}$$

$$\gamma_m = \frac{(1+\nu)\,\varepsilon_x}{1 + \left(\frac{1-\nu}{2}\right)\varepsilon_x} \qquad \text{if } \varepsilon_x \ll 1 \;\Rightarrow\; \gamma_m = (1+\nu)\,\varepsilon_x$$

$$\frac{\tau_m}{G} = (1+\nu)\frac{\sigma_x}{E} \;\Rightarrow\; \frac{E}{G} = (1+\nu)\frac{\sigma_x}{\tau_m} \qquad [\tau_m = P/2A]$$

$$\therefore \frac{E}{2G} = (1+\nu)$$

$\boxed{Z = GY}$

Alloy → components interact

composites → no interaction b/w phases; can be distinguished

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* Dilatation:-

$$V_1 = 1$$
$$V_2 = (1+\epsilon_x)(1+\epsilon_y)(1+\epsilon_z)$$

Neglecting higher orders

$$\epsilon = 1 + \epsilon_x + \epsilon_y + \epsilon_z + 0$$

$$\therefore \boxed{e = \epsilon_x + \epsilon_y + \epsilon_z}$$

$$k = \frac{E}{3(1-2\nu)}$$

$$E = 3k(1-2\nu) > 0$$
$$1-2\nu > 0$$
$$\left[\nu > \frac{1}{2}\right]$$

$$\therefore \nu > 0 \text{ or } \nu \leq \frac{1}{2}$$

BENDING-

* Pack of cards (slipping)

* Lines closer to centre compress whereas away from centre expand.

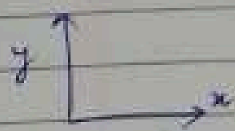
→ No exact analysis of bending possible yet.

* Points of discontinuity (eg- bricks)

Mixing of steel with concrete, to account for ductility ⇒ Primary load carrying members: ~~steel~~ steel

→ Aircraft wing design

SF and V's <u>maxima</u> ~~torque~~ torque



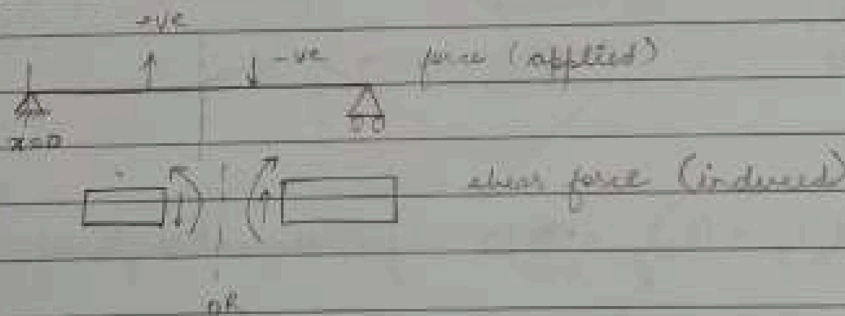
→ Supports
 – ~~Supp~~ Importance of inducing evolution.

Roller to give stability (vibration)

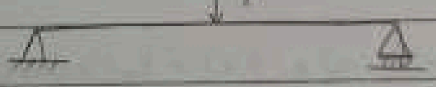
Pin joint release moment.

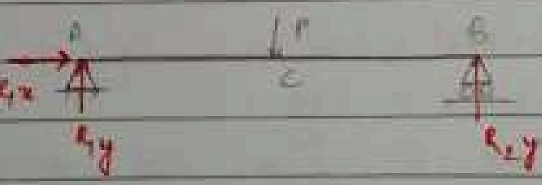
* vibration of bridge on passing heavy vehicle

<u>Sign convention for bending moment and shear force diagram :-</u>



<u>Ques-</u>

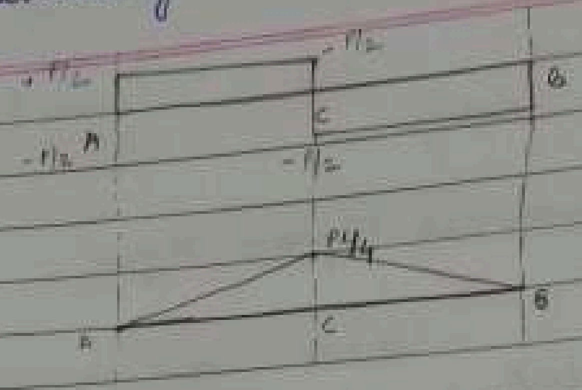




$R_{1y} + R_{2y} = P$

$R_{1x} = 0$

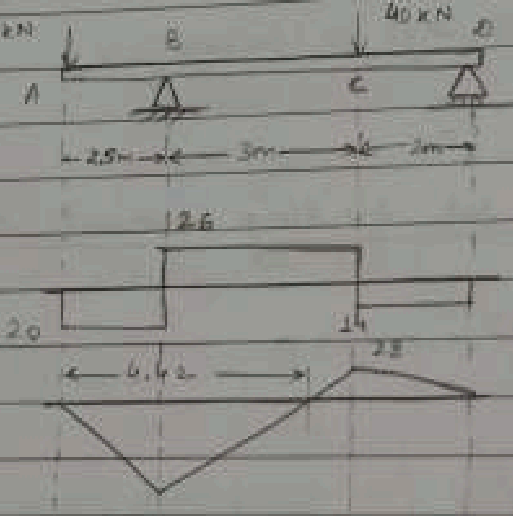
viscoelasticity $\dot{E}$



Maximum bending occurs at the point where shear force is ~~Maximum~~ changes its sign.

$$M_x = \frac{P}{2}x \;\Big|\; - P\left(x - \frac{L}{2}\right)$$

Ques:



$R_B = 46$
$R_D = 14$
$M_x = -20x \,|$
$+46(x-2.5) \,|$
$-40(x-5.5)$

$(R_B)_y + (R_D)_y = +60$ kN (for the entire beam)

$(R_B)_x = 0$

$R_B \times 2.5 - 20(0) + (-40)\times(5.5) + R_D(7.5) = 0$

or, $2.5R_B + 7.5R_D = 220$

or, $2.5(R_B + 3R_D) = 220$

or, $R_B + 3R_D = \frac{220}{2.5}$

or, $R_B + 3R_D = \frac{\cancel{220}\,88}{\cancel{25}}$ $\Rightarrow$ $R_B + R_D = +60$

$R_B + 3R_D = 88$

$\boxed{R_B = 46}$ $\boxed{R_D = 14}$

$-2R_D = -28$

$R_D = 14$

$R_B = 58$

Shear is zero as the boundary is a traction free surface.

$M_x = 0 \Rightarrow \quad -20x + 46(x-2.5) - 40(x-5.5) = 0$

or, $\quad -20x + 46x - 46\left(\frac{5}{2}\right) - 40x + 40\left(\frac{11}{2}\right) = 0$

or, $\quad -20x + 46x - 115 - 40x + 220 = 0$

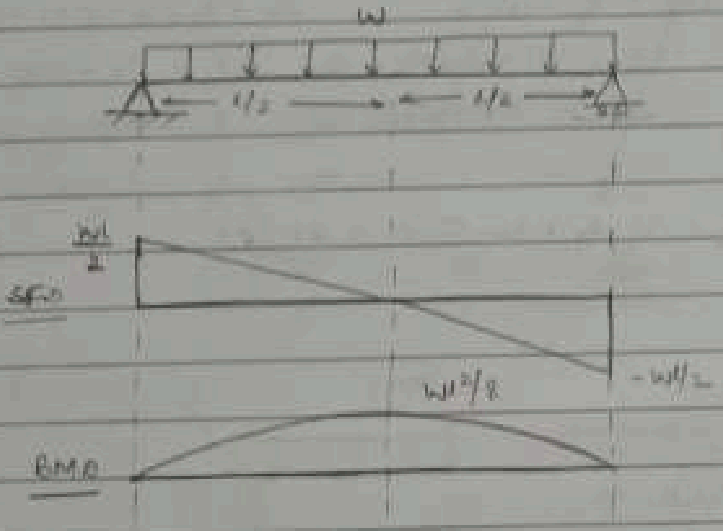
or, $\quad -14x = -105$

or, $\quad x = \frac{105}{14} = \underline{6.42}$

$\begin{array}{r} 220 \\ -115 \\ \hline 105 \end{array}$

$\begin{array}{r} 46 \\ \hline 115 \end{array}$

Ques.



$R_A + R_B = wl$

$\therefore R_A = R_B = \frac{wl}{2}$ (for uniform)

$V_x = \frac{wl}{2} - wx$

$M_x = \frac{wl}{2} \times x - wx \times \frac{x}{2}$

$= \frac{wlx}{2} - \frac{wx^2}{2}$

$\frac{dM_x}{dx} = 0 \Rightarrow \frac{wl}{2} - wx = 0 \Rightarrow x = \frac{l}{2}$

$M_x \big|_{x = l/2} = \frac{wl^2}{8}$

Ques:-

$(R_A)=(R_A)_y$; $(R_B)=(R_B)_y$, $(R_C)=(R_C)_y$

0 (not a support)

$\therefore \quad R_A + \cancel{R_B} + R_C = 40$

$\therefore \quad R_A + R_C = 40$

$\Sigma M_C = 0 \quad \cancel{-80 - 20(5) + R_C \times 10 = 0}$

or $\quad \cancel{10R_C = 75 + \ldots}$

or $\quad \cancel{R_C = \ldots}$

or $\quad \cancel{R_C = \cdot}$

$80 + R_A \times 10 - 15\times5 - 25\times2.5 = 0$

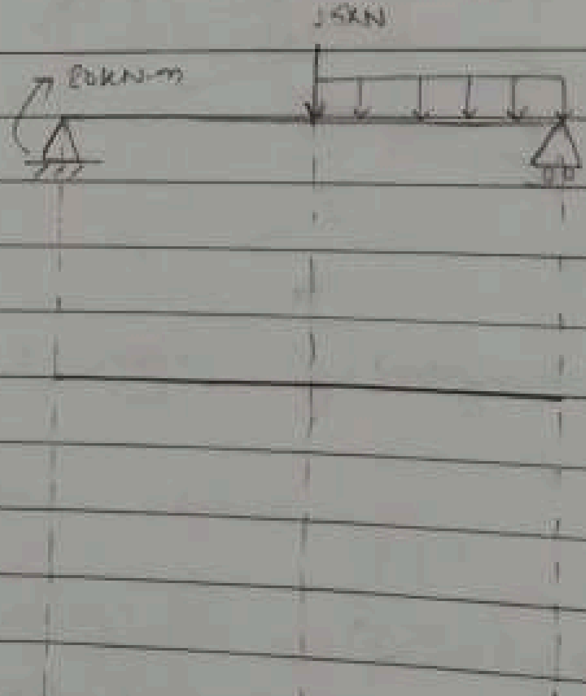
or $\quad 10R_A = 75 + 62.5 - 80$

or $\quad R_A = \frac{-5 + 62.5}{10}$

or $\quad R_A = \frac{62.5 - 5}{10} = 6.25 - 0.5$

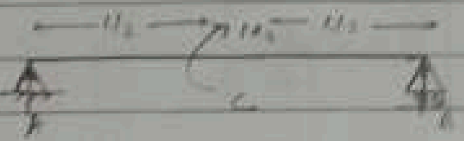
$= \underline{5.75}$

$R_B = R_C = 40 - R_A = 40 - 5.75 = \underline{34.25}$



graphical method

Ques:



SFD $R_A + R_B = 0$ | $[\Sigma F_y = 0]$

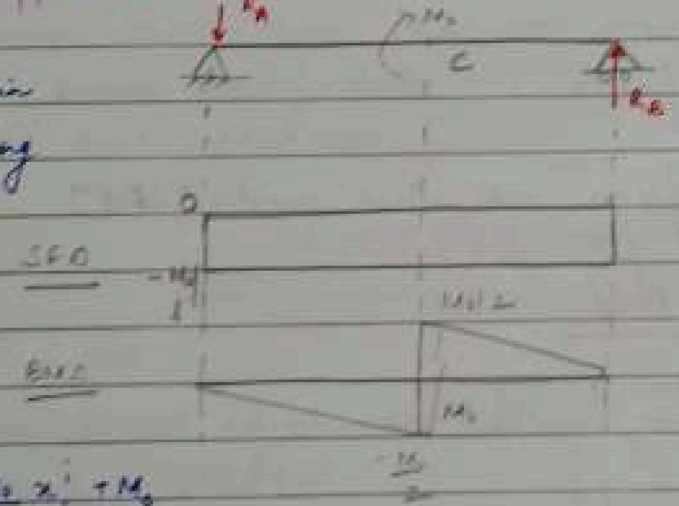
$\Sigma M_C = 0$ $\quad -M_0 - R_A\left(\frac{L}{2}\right) + R_B\left(\frac{L}{2}\right) = 0$

Or $\quad R_B - R_A = \frac{2M_0}{L}$

$\therefore \quad 2R_B = \frac{2M_0}{L} \Rightarrow \quad R_B = \frac{M_0}{L}, \quad R_A = \frac{-M_0}{L}$

Jump of moment should appear in BMD.

↳ P/2 to −P/2 in case of loading

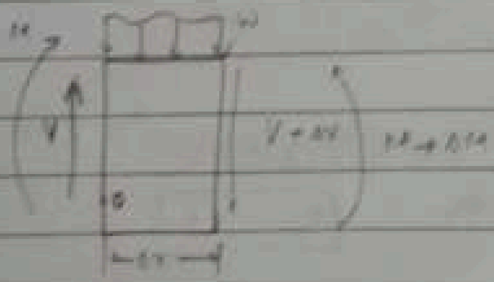


signature in moment diagram

$M_x = -\frac{M_0}{L}x + M_0$

Relation between load, shear and bending moment -

for distributed load



$V - (V + \Delta V) - w\Delta x = 0$

or $\quad -\Delta V - w\Delta x = 0 \Rightarrow \quad w = -\frac{\Delta V}{\Delta x}$

as $\Delta x \to 0$ $$\boxed{w = -\frac{dV}{dx}}$$

$\Sigma M_o = 0$

$$(w\Delta x)\frac{\Delta x}{2} + (V+\Delta V)\Delta x - (M+\Delta M) + M = 0$$

or $$w\frac{\Delta x^2}{2} + V\Delta x + \Delta V\Delta x - M - \Delta M + M = 0$$

as $\Delta x \to 0$

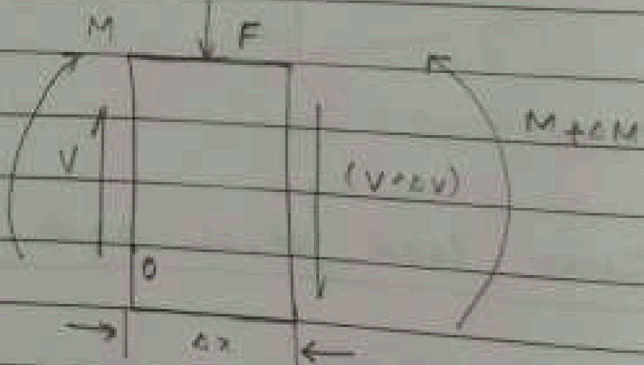
$$V\Delta x - \Delta M = 0$$

or $$\boxed{V = \frac{dM}{dx}}$$

$\int dV = -\int w\,dx \rightarrow$ Area under curve

$\int dM = \int V\,dx \rightarrow$ Area under SFD

for concentrated load



$\Sigma F_y = 0$ $\quad V - F - (V+\Delta V) = 0$

or, $$\boxed{\Delta V = -F}$$

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BM depends on reaction of one part of body on another

ending for signature

(applied moment in the cross-section)

$\underline{\Sigma M_0 = 0}$ $\quad M - (M+\Delta M) + M_0 + (V+\Delta V)\,\Delta x = 0$

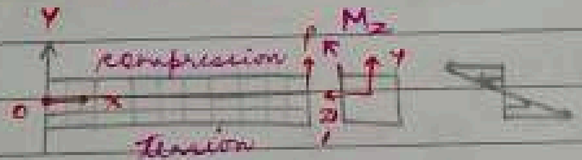
or $\quad -\Delta M + M_0 = 0 \quad$ (as $\Delta x \to 0$)

or $\quad \boxed{\Delta M = M_0}$ $\quad$ strength of jump

BENDING —

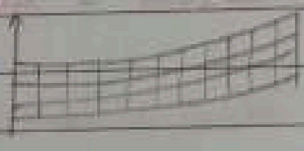
* We assume that beam has an axis of symmetry.

SPECIFY AXIS CAREFULLY.



The axis about which the stresses ~~is~~ are zero is known as NEUTRAL AXIS / SURFACE.

→ does not undergo a change in length.

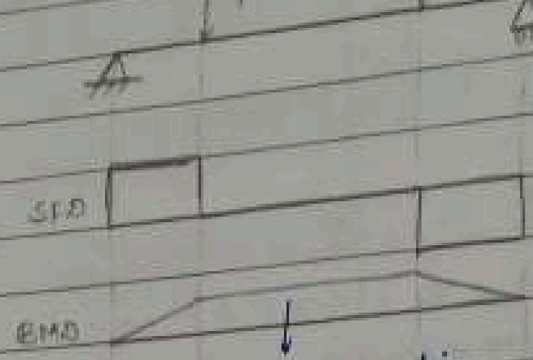


* no out of plane deformation.

→ Rotating but straight

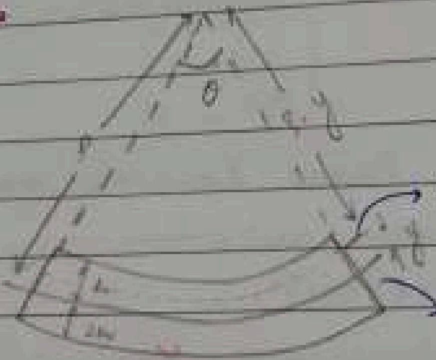
Pure bending :- → occurs under constant bending moment

→ member remains symmetric
→ uniform circular arc
→ other assumptions



pure bending assumption valid

Compressive towards the centre, tensile away from the centre.



$L = (\rho - y)\theta$

$L_0 = \rho\theta$

$$\Delta L = -y\theta$$

$$\varepsilon = \frac{y\theta}{\rho\theta} = \frac{-y}{\rho}$$

$$|\varepsilon_m| = \varepsilon^{C}_{max} = \frac{-h}{\rho}, \quad \varepsilon^{T}_{max} = \frac{h}{\rho} = |\varepsilon_m|$$

Sign of the bending stresses is along the length of the beam.

⇒ Bending induces axial stresses

$$\therefore \boxed{\varepsilon_x = \frac{-y}{h}\varepsilon_m}$$ varies linearly with y

[DIRECT MAPPING] $\sigma_x = \frac{-y}{h}\sigma_m = E\varepsilon_m$ (for isotropic beam)

* for isotropic material, neutral axis coincides with geometrical axis

To determine neutral axis: $\Sigma F = 0$

for any cross-section, $F_x = 0$

$$\text{or } \int \sigma_x \, dA = 0$$

$$\text{or } \int \frac{y}{h} \sigma_m \, dA = 0$$

cross-sectional area

$$\text{or } \boxed{\int y \, dA = 0}$$

passes through section centroid

for the moment, we should have, induced moment = applied moment

$$M = \int y \, dF \qquad \frac{\sigma_m}{h} \int y^2 \, dA$$

$$= \int y \cdot y \frac{\sigma_m}{h} \, dA \Rightarrow M_z = \frac{\sigma_m}{h} I_z$$

$$\therefore M_z = -\frac{\sigma_x}{y} I_z$$

$$\therefore \boxed{\sigma_x = -\frac{M_z y}{I_z}}$$

FLEXURE FORMULA

component-wise relation

at $y = \pm h$ $\quad \sigma_{xx} = \frac{M_z h}{I_z} = \frac{M_z}{\left(\frac{I_z}{h}\right)}$

$$\text{or } \left[\sigma_{xx} = \frac{M_z}{S}\right]$$

section modulus

we design members having more depth for a given cross-section [↑se S value to reduce σ]

eg: railway tracks, beams

$$\frac{M_z h}{I_z} = \frac{h}{\rho} E$$

or $$\boxed{\frac{M_z}{E I_z} = \frac{1}{\rho}}$$ Moment-curvature relationship

geometrical properties; independent of material

- * Mass moment of inertia (bending)
- * Polar second moment of inertia (twisting)
- * MOI for an area (bending) → along the beam

(does not depend on mass; A balanced)

→ We need quantities to describe resistance to motion/rotation in different axis

* Composite areas

Ques:-

M-1 Divide into 2 areas and solve.

M-2 Complete the square, and remove the remaining portion.

M-1 $(x_1, y_1) = (0, 11.5)$

$(x_2, y_2) = (0, 5)$

$$\bar{y} = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2} = \underline{\underline{8.545}}$$

* for parallel axis theorem, we need MOI about CENTROID (balance for area).

To find moment of inertia in the above case :-

$I_1 = \frac{1}{12} bh^3 = \frac{1}{12} \times 8 \times 3^3 = \frac{1}{12} \times 8 \times 27 = 18$

$I_2 = \frac{1}{12} \times 2 \times 10^3 = \frac{1000}{6} = 166.66$

$I = (I_1 + A_1 d_1^2) + (I_2 + A_2 d_2^2)$

$= [18 + [(8 \times 4.5) \times (4.45)^2]] + [166.66 + (20 \times (8.55)^2]$

$\sigma = \frac{M}{S} \Rightarrow \left[S = \frac{M}{\sigma_{max}} \right]$

Ques

$R_A + R_B = 5 \times 6 = 30$

$R_B \times 6 = 5 \times$

$R_A = R_B = 15$

$M_x = R_A \times x - \frac{wx^2}{2}$

$\frac{dM_x}{dx} = 0$ $R_A - wx = 0 \Rightarrow x = 3$

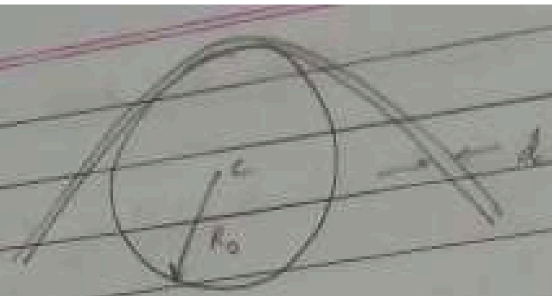
$A_1 = 20 \times 250$

$A_2 = 150 \times 20$

$I_1 = \frac{1}{12} \times 250 \times 20^3$

$I_2 = \frac{1}{12} \times 20 \times 150^3$

Ques:



$R_0 = 0.5\,m$

$d = 4\,mm$

$E = 200\,GPa$

$\sigma_1 = 250\,MPa$

$$\rho = R_0 + \frac{d}{2}$$

$$M = \frac{EI}{\rho} \quad ; \quad M = \frac{E \times \frac{\pi}{64} d^4}{\left(\frac{2R_0 + d}{2}\right)}$$

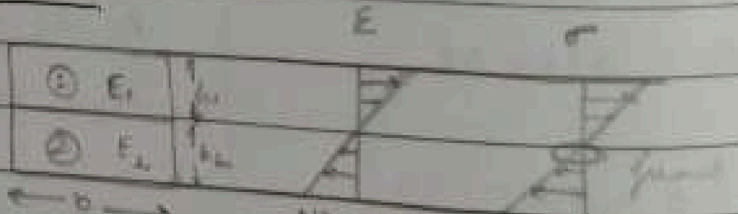
$$I = \frac{\pi}{64} d^4$$

$$\sigma = \frac{My}{I} = \frac{Md}{2I}$$

if $\sigma \le \sigma_{PL}$ ✓

if $\sigma \geq \sigma_{PL}$ go reverse to find M.

COMPOSITE BEAMS –

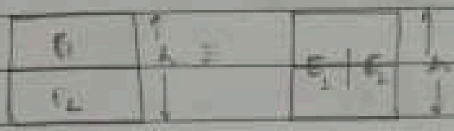


$$\varepsilon = -\frac{y}{c}\varepsilon_m$$

$$= -\frac{y}{c} \times \frac{c}{\rho} = \frac{-y}{\rho}$$ (does not depend on material property)

$\sigma = E\varepsilon$ (depends on material property) earthquake

Materials strong in tension / compression are not the same in the other



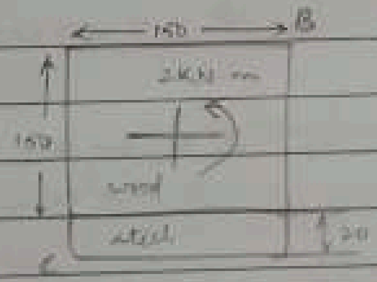
if we assume the entire beam to be ~~strapped~~ made of entirely same material, and then modify it geometrically, we get an equivalent beam.

We don't use an effective 'E' because in that case we have a beam which has nothing in common with the original one.

no physical meaning

Transformation factor $\boxed{n = \frac{E_1}{E_2} = \frac{\text{Material being transformed}}{\text{Reference material}}}$

Ques:



$E_W = 12\,GPa$

$E_{st} = 200\,GPa$

if we transform the entire beam into steel

$n = \frac{E_W}{E_{st}} = \frac{12}{200} = 0.06$

$b' = nb = 0.06 \times 150 = 9$

$\sigma_{B'} = \frac{My'}{I}$

$\sigma_B = \frac{My}{I}$

Add $\sigma_{xz} \Rightarrow \sigma_{xz}$, $\tau_{xy} \leftrightarrow \tau_{xy}$ } dimensional sort of

To find neutral axis

$$\bar{y} = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2}$$

$$= \frac{(10 \times 20 \times 150) + (9 \times 150 \times 95)}{(20 \times 150) + (150 \times 95)}$$

$$= 36.4$$

$$I_1 = \frac{1}{12} \times 150 \times 20^3 \qquad d_1 = 26.4$$

$$I_2 = \frac{1}{12} \times 9 \times 150^3 \qquad d_2 = 95 - 36.4 = 58.6$$

$$I = (I_1 + A_1 d_1^2) + (I_2 + A_2 d_2^2)$$

$$= \frac{1}{12}\left[150 \times 20^3 + 9 \times 150^3\right] + 20 \times 150 \times (26.4)^2 + 150 \times 95 \times (58.6)^2$$

$$\sigma_a' = \frac{2 \times 123.6}{I_2}, \quad \sigma_c = \frac{2 \times 36.4}{I_2}$$

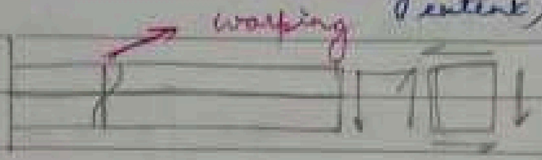
* concrete beam analysis (NA is in the air)

Instability of solution (existence of singularity)
↳ non-predictable solution

Compression (controlled)
Axial (uncontrolled)

$$\left[\sigma = \sigma_{ax} + \sigma_{bending}\right]$$

SHEAR STRESS-

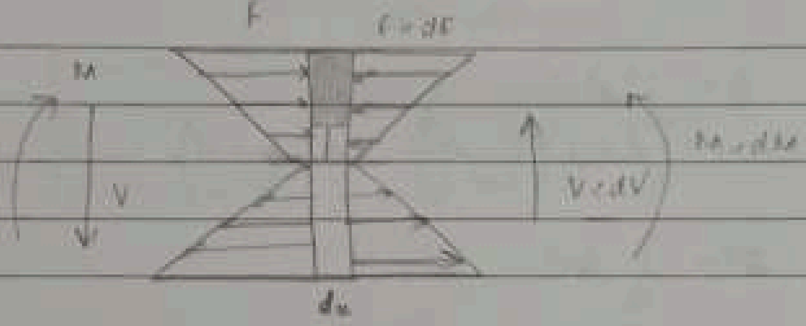


* neglect the deformation

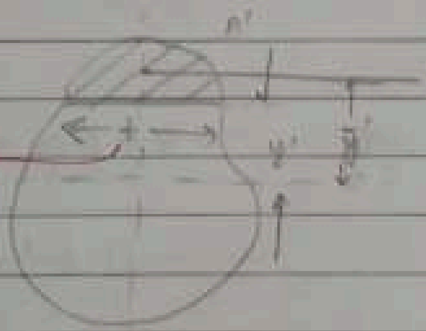
→ complimentary shear stress

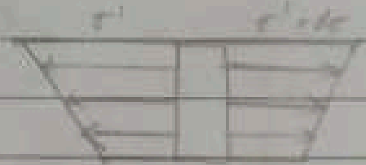
longitudinal as well as Transverse shear stresses.

* beam of glued together boards



taken at the point where τ is to be determined





τ (τ should be present to enable the body to remain in static equilibrium)

$$\Rightarrow \int \sigma' dA' + \tau - \int (\sigma' + d\sigma)\, dA = 0$$

or $$\int \frac{My'}{I} dA' - \int \left(\frac{M+dM}{I}\right) y'\, dA' + \tau\,(t \times dx) = 0$$

$$\therefore \tau\, t\, dx = \int \frac{dM\, y'\, dA'}{I}$$

or, $$\tau = \frac{1}{It}\left(\frac{dM}{dx}\right)\int y'\, dA'$$

$$\therefore \boxed{\tau = \frac{VQ}{It}}$$

or $$\tau = \frac{V}{It}\int y'\, dA' = \frac{VQ}{It}$$

$\bar{y}' \rightarrow$ dist. b/w the centroid of the chosen area A' and neutral axis of the entire cross-section

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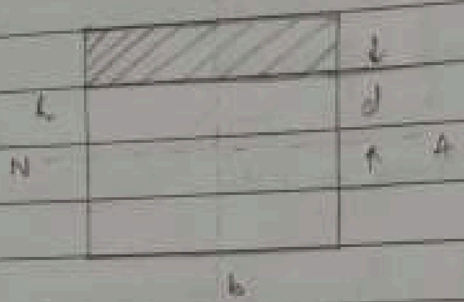
$\therefore \bar{y} = \dfrac{y_1 A_1 + y_2 A_2}{A_1 + A_2}$

or $\bar{y} = \dfrac{\int y' dA'}{A} \Rightarrow \int y' dA' = \bar{y} A$

eg:



for solid shaft, shear stress is zero at the centre.

$$Q = \bar{y}' A'$$
$$= \left(d + \frac{1}{2}\left(\frac{h}{2} - d\right)\right) b \times \left(\frac{h}{2} - d\right)$$

$$I = \frac{1}{12} bh^3$$

$$t = b$$

$$\tau = \frac{VQ}{It} = \frac{V \cancel{b} \left(\frac{h}{4} + \frac{d}{2}\right)\left(\frac{h}{2} - d\right)}{\frac{1}{12} bh^3 \times \cancel{b}}$$

$$= \frac{V\left(\frac{h^2}{8} - \frac{dh}{4} + \frac{dh}{4} - \frac{d^2}{2}\right)}{\frac{1}{12} bh^3}$$

$$= \frac{\frac{V}{2}\left(\frac{h^2}{4} - d^2\right)}{\frac{1}{12} bh^3}$$

$$\therefore \boxed{\tau = \frac{6V}{bh^3}\left(\frac{h^2}{4} - d^2\right)}$$

V is ...

$\tau = \tau_{max}$

$\tau = \tau_{min}$

eg:

$F = 6.5 - 6.5x$

$\therefore F = 0$

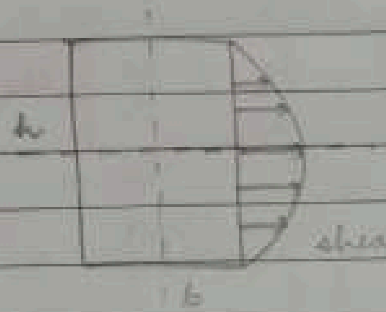
$\Rightarrow \boxed{6.5 = 1w ...}$

$\bar{y}$

V is constant along a cross section / face

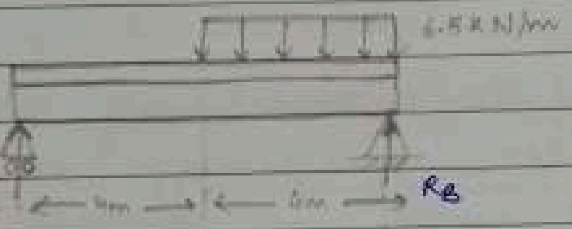
$\tau = \tau_{max}$ at $d = 0$; $\tau = \frac{3}{2}\frac{V}{bh}$

$\tau = \tau_{min}$ at $d = \pm\frac{h}{2}$; $\tau = 0$



∴ Beam fails at the middle

eg:



$R_A + R_B = 6.5 \times 4$
$= 26$

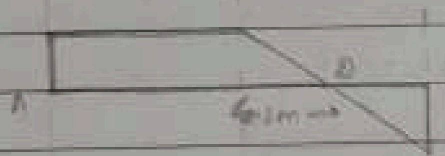
$\Sigma M_A = 0$

$\Rightarrow R_B \times 8 = 6.5 \times 4 \times 6$

or, $R_B = 6.5 \times 3$

or, $R_B = 19.5$

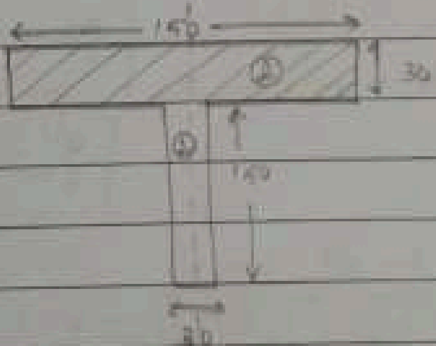
∴ $R_A = 6.5$



$F = 6.5 - 6.5x$

∴ $F = 0$

$\Rightarrow$ $\boxed{6.5 = 1m}$



$$\bar{y} = \frac{y_1A_1 + y_2A_2}{A_1 + A_2} = \frac{(75 \times 150 \times 30) + (165 \times 150 \times 30)}{2 \times 150 \times 30}$$

$I = (I_1 + A_1 d_1^2) + (I_2 + A_2 d_2^2)$

$I_1 = \frac{1}{12} \times 30 \times 150^3$; $d_1 = \bar{y} - 75$

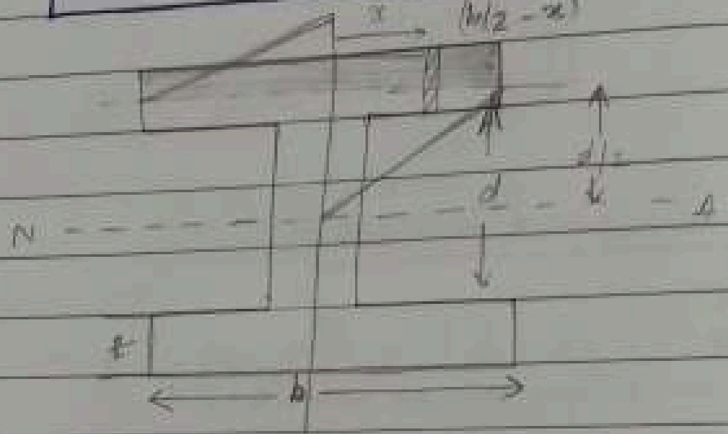
$I_2 = \frac{1}{12} \times 150 \times 30^3$; $d_2 = 165 - \bar{y}$

$$\tau = \frac{VQ}{It}$$

$t = 30$; $Q = \bar{y}' A'$
$= d_2 \times 150 \times 30$

SHEAR FLOW FOR THIN-WALLED MEMBERS

$$\boxed{q = \tau \times t = \frac{VQ}{I} = \frac{df}{dx}}$$ shear flow



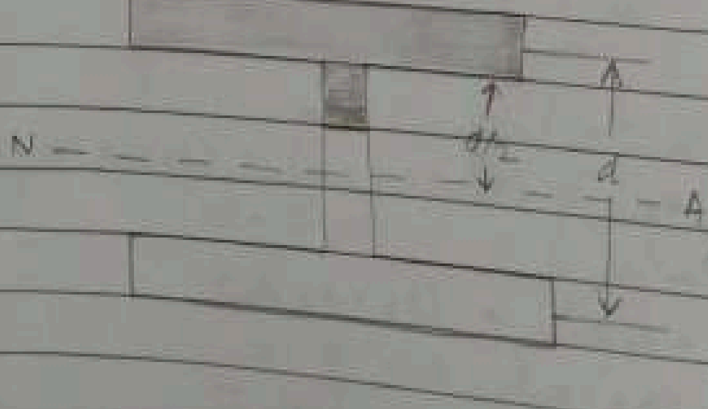
$Q = \frac{d}{2}\left(\frac{b}{2} - x\right)t$ $\left[Q = \bar{y}' A'\right]$

$q = \frac{V}{I}\frac{d}{2}\left(\frac{b}{2} - x\right)t$

or $\boxed{q = \frac{Vdt}{2I}\left(\frac{b}{2} - x\right)}$ max at $x = 0$
min at $x = b/2$

$\left[q_{max} = \frac{Vdt}{2I}\left(\frac{b}{2}\right)\right]$

$\left[q_{min} = 0\right]$



Con
sh

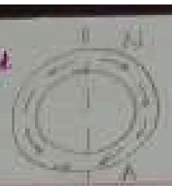
*

→

*
depends
on appli

F_f =

Concrete cannot withstand shear.



flying of birds

$$Q = \bar{y}'A'$$

$$= \frac{d}{2}(b\times t) + y + \frac{1}{2}\left(\frac{d}{2}-y\right)\left(\frac{d}{2}-y\right)t$$

$$\Rightarrow \boxed{Q = \frac{d}{2}(b\times t) + \frac{1}{2}\left(\frac{d^2}{4}-y^2\right)t + y}$$

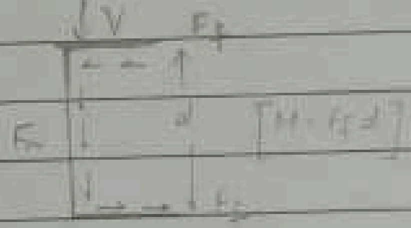
$$\left[q = \frac{VQ}{I} = \frac{V}{I}\left(\frac{d}{2}(b\times t) + \frac{1}{2}\left(\frac{d^2}{4}-y^2\right)t + y\right) \right]$$

↓

parabolic

* It is called 'flow' because it is something that enters in the upper flange, through the web and out through the lower flange.

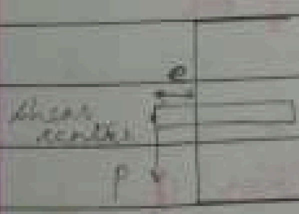
→ We are concerned about the forces in the flanges because they are capable of producing a moment and also cause twisting.



* If the forces pass through the SHEAR CENTRE, there is bending without twisting, otherwise there is both.

depends on application

Shear centre lies on the axis of symmetry



$$P\times e = F_f \times d$$

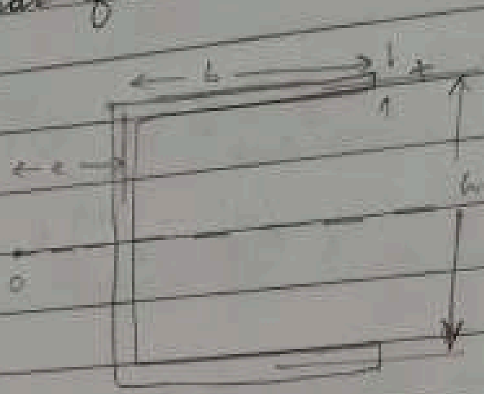
or $\boxed{e = \dfrac{F_f \times d}{P}}$

→ Bending and twisting always occurs for unsymmetric bodies.

$$F_f = \int q\,dx$$

$\hookrightarrow \dfrac{VQ}{I}$

Ques: Find shear flow and shear centre.



$$I = I_{web} + 2 I_{flange}$$

$$= \frac{1}{12} th^3 + 2\left[\frac{1}{12} bt^3 + bt\left(\frac{h}{2}\right)^2\right]$$

$$= \frac{1}{12} th^2 (6b + h)$$

$Q = \bar{y}' A'$
$\bar{y} = \frac{h}{2}$

$$q = \frac{Vht}{2I}\left(\frac{b}{2} - x\right)$$

$$= \frac{12Vht}{th^2(6b+h)}\left(\frac{b}{2} - x\right)$$

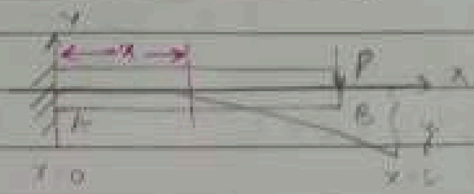
$$= \frac{12V}{h(6b+h)}\left(\frac{b}{2} - x\right)$$

$$F_f = \int_0^b \frac{Vht}{2I} (b - x)\, dx = \frac{P \times e}{d}$$

Symmetry depends on the geometry of the cross-section.

'o' corresponds to point of intersection of the axes of symmetry.

DEFLECTION OF BEAMS -



we define the beam as $L >> w$

$$\frac{1}{\rho} = \frac{M}{EI}$$ → bending rigidity / flexural rigidity

Also
$$\frac{1}{\rho} = \frac{d^2y/dx^2}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

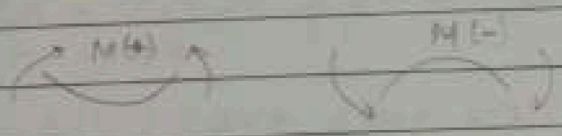
$\because$ $L >> w \Rightarrow \frac{dy}{dx} \to 0$

$$\therefore \quad \frac{1}{\rho} = \frac{d^2y}{dx^2}$$

or, $$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

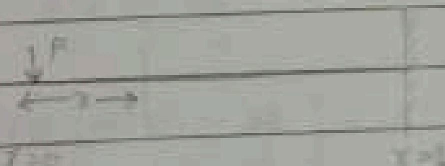
or, $$\boxed{EI\frac{d^2y}{dx^2} = M}$$ Euler - Bernoulli beam eq^n

Sign convention :-



$R_A = P$, $M_A = PL$ (for the above diagram)

In this case,



$$EI\frac{d^2y}{dx^2} = -Px$$

or, $$EI\frac{dy}{dx} = \frac{-Px^2}{2} + C_1$$

$$\Rightarrow \quad EIy = \frac{-Px^3}{6} + C_1x + C_2$$

* Rotation at fixed end is zero. [$\theta = 0$, $y' = \tan\theta = 0$]

Boundary conditions: $x = L,\ y = 0$

$x = L,\ \frac{dy}{dx} = 0$

$$\therefore\quad 0 = -\frac{PL^3}{6} + C_1 L + C_2$$

$$EI(0) = -\frac{PL^2}{2} + C_1 \quad \Rightarrow C_1 = \frac{PL^2}{2}$$

$$C_2 = \frac{PL^3}{6} - \frac{PL^2}{2} \cdot L$$

$$= \frac{PL^3}{6} - \frac{PL^3}{2}$$

$$= -\frac{2PL^3}{6} = -\frac{PL^3}{3}$$

$$\Rightarrow\quad EIy = -\frac{Px^3}{6} + \frac{PL^2}{2}(x) - \frac{PL^3}{3}$$

$$\boxed{EIy = -\frac{Px^3}{6} + \frac{PL^2}{2}(x) - \frac{PL^3}{3}}$$

$\Rightarrow$ roots of $-\frac{Px^3}{6} + \frac{PL^2}{2}(x) - \frac{PL^3}{3} = 0$

one root: $x = L$

$$\frac{x^3}{6} - \frac{L^2 x}{2} + \frac{L^3}{3} = 0$$

or $\quad x^3 - 3L^2 x + 2L^3 = 0$

$$x^2(x-L) + Lx(x-L) - 2L^2(x-L) = 0$$

or $\quad (x^2 + Lx - 2L^2)(x-L) = 0$

$$x = \frac{-L \pm \sqrt{L^2 + 8L^2}}{2} = \frac{-L \pm 3L}{2} = -2L, L$$

$$\boxed{x = L, L, -2L}$$

MACAULAY METHOD

$\sum F_y = 0$ $R_A + R_B = P$

$\sum M_A = 0$ $\quad P(2l) = R_B(3l) \Rightarrow R_B = \frac{2P}{3}$; $R_A = \frac{P}{3}$

$$M_x = R_A x \;|\; + P(2l - x)$$

$$= \frac{P}{3}x \;|+\; P(2l) - Px = 2Pl - \frac{2}{3}Px$$

$\therefore$ ~~EIy = ...~~

$$EI\frac{d^2y}{dx^2} = \frac{Px}{3} \;\Big|\; + P(2l - x)$$

$$\Rightarrow EI\frac{dy}{dx} = \frac{Px^2}{6} \;\Big|\; + P\left[2lx - \frac{x^2}{2}\right] + C_1$$

$$EI\,y = \frac{Px^3}{18} \;\Big|\; + P\left[lx^2 - \frac{x^3}{6}\right] + C_1 x + C_2$$

Boundary conditions :- (i) $x = 0,\ y = 0$

$$EI(0) = 0 + P(0) + C_1(0) + C_2$$

$[C_1 = -2Pl^2]$ $\quad \Rightarrow \quad C_2 = 0$

(ii) $x = L,\ y = 0$

$$EI(0) = \frac{PL^3}{18} \;\Big|\; + P\left[l(L^2) - \frac{L^3}{6}\right] + C_1 L$$

or $$0 = \frac{PL^3}{18} \;\Big|\; + P\left[L^2 l - \frac{L^3}{6}\right] + C_1 L$$

$L = 3l$

or $$0 = \frac{PL^3}{18} + PL^2 l - \frac{PL^3}{6} + \frac{C_1 L}{\cancel{L}}$$

$C_1 = \frac{PL}{6} - \frac{PL}{3} - \frac{PL}{18}$

or $$0 = \frac{PL}{18} + Pl - \frac{PL}{6} + \frac{C_1 L}{L^2} \Rightarrow C_1 = \left(\frac{PL}{6} - Pl - \frac{PL}{18}\right)L$$

$= \frac{(3PL - 6PL - PL)L}{18} = -\frac{4PL}{18}L = -\frac{2PL^2}{9} = \left(-\frac{2}{9}P\right)L$

$\therefore \quad EI\,y = \frac{PL^3}{12} + P\left[aL^2 - \frac{L^3}{6}\right] + C_1L$

or, $\quad EI(0) = \frac{PL^3}{12} + P\left[aL^2 - \frac{L^3}{6}\right] + C_1L$

or, $\quad 0 = \frac{PL^2}{12} - P\left[aL - \frac{L^2}{6}\right] + C_1$

$\Rightarrow \quad C_1 = -\frac{PL^2}{12} + PaL - \frac{PL^2}{6}$

$= -\frac{PL^2}{12} - \frac{2PL^2}{12} + PaL$

$= \frac{-4PL^2}{12} + PaL$

$= -\frac{2}{6}PL^2 + PaL$

$a = \frac{L}{2} \quad \Rightarrow \quad C_1 = -\frac{2}{6}PL^2 + \frac{PL^2}{2} = (-2+3)\frac{PL^2}{6}$

$= \frac{PL^2}{6}$

Principle of superposition → applicable only for elastic limit

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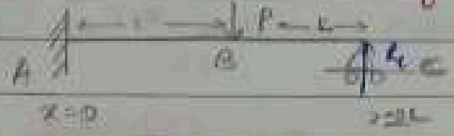
25 11 22

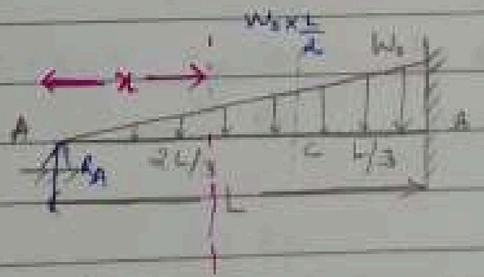


$R_A + R_C = P \Rightarrow \quad PL = R_C(2L)$

$\Rightarrow R_C = \frac{P}{2} \quad ; \quad R_A = \frac{P}{2}$

$$EI\frac{d^2y}{dx^2} = M_A + R_A x \; | - P(x-L)$$

Ques:

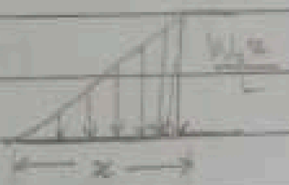


$R_A + R_B = \frac{w_0 \times L}{2}$

$\frac{w_0 \times L}{2} \times \frac{2L}{3} = R_B \times L \Rightarrow R_B = \frac{w_0 L}{3}$

$R_A = \frac{w_0 L}{2} - \frac{w_0 L}{3} = \frac{w_0 L}{6}$

$\underline{M_x} = R_A \times x - \frac{w_0 x}{2L} x \frac{x}{3}$



$= \frac{w_0 L x}{6} - \frac{w_0 x^2}{6L}$

$$EI\frac{d^2y}{dx^2} = \frac{W_0 L x}{6} - \frac{W_0 L x^3}{6L}$$

Here we cannot use this approach as the problem is statically indeterminate and we have to account for the moment at B.

$$\Rightarrow R_A + R_B = \frac{W_0 L}{2}$$

$\Sigma M_A = 0$

$$\frac{W_0 \times L}{2} \times \frac{2L}{3} = M_B + R_B \times \frac{L}{3}$$

or,
$$\frac{W_0 L}{2} = \frac{3M_B}{L} + R_B$$

$$\therefore EI\frac{d^2y}{dx^2} = R_A x - \frac{W_0 x^3}{6L}$$

$$\Rightarrow EI\frac{dy}{dx} = R_A\frac{x^2}{2} - \frac{W_0}{6L}\frac{x^4}{4} + C_1$$

$$\Rightarrow EIy = R_A\frac{x^3}{6} - \frac{W_0}{6L}\frac{x^5}{20} + C_1 x + C_2$$

at $x=0$, $y=0$

$$EI(0) = R_A(0) - \frac{W_0}{6L}(0) + C_1(0) + C_2$$

$$\Rightarrow C_2 = 0$$

at $x=L$, $y=0$

$$EI(0) = \frac{R_A L^3}{6} - \frac{W_0}{6L} \times \frac{L^5}{20} + C_1 L.$$

or,
$$0 = \frac{R_A L^3}{6} - \frac{W_0 L^4}{120} + C_1 L$$

$$EI\frac{d^2y}{dx^2} = \frac{w_0 L x}{6} - \frac{w_0 x^3}{6L}$$

Here we cannot use this approach as the problem is statically indeterminate and we have to account for the moment at B.

$$\Rightarrow R_A + R_B = \frac{w_0 L}{2}$$

$\Sigma M_A = 0$

$$\frac{w_0 L}{2} \times \frac{2L}{3} = M_B + R_B \times \frac{L}{3}$$

or

$$\frac{w_0 L}{2} = \frac{3M_B}{L} + R_B$$

$$\therefore EI\frac{d^2y}{dx^2} = R_A x - \frac{w_0 x^3}{6L}$$

$$\Rightarrow EI\frac{dy}{dx} = R_A\frac{x^2}{2} - \frac{w_0}{6L}\frac{x^4}{4} + C_1$$

$$\Rightarrow EIy = R_A\frac{x^3}{6} - \frac{w_0}{6L}\frac{x^5}{20} + C_1 x + C_2$$

at $x=0,\ y=0$

$$EI(0) = R_A(0) - \frac{w_0}{6L}(0) + C_1(0) + C_2$$

$$\Rightarrow C_2 = 0$$

at $x=L,\ y=0$

$$EI(0) = \frac{R_A L^3}{6} - \frac{w_0}{6L} \times \frac{L^5}{20} + C_1 L.$$

or

$$0 = \frac{R_A L^3}{6} - \frac{w_0 L^4}{120} + C_1 L$$

'glue' is the weakest layer

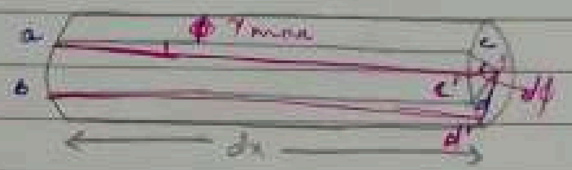
TORSION-

→ Twisting moment / torque.

* draw a grid

AVOID shear deformation in all directions
↳ assumptions

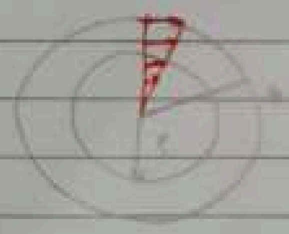
only in elastic region



$$\gamma_{max} = \frac{cc'}{ac}$$

$$= \frac{r\, d\phi}{dx} \rightarrow \theta$$ (independent of length, hence constant)

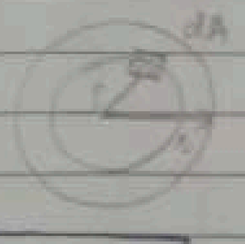
$$\therefore \left[\gamma_{max} = r\theta\right]$$



$$\gamma = \rho\theta$$

or $$\boxed{\gamma = \frac{\rho}{r}\gamma_{max}}$$

There are only shear strains in this case.



$$dF = \tau\, dA$$

$$dM = \tau\, dA \times \rho$$

$$dM = \frac{\tau_{max}}{r}\rho \times \rho\, dA$$

$$\therefore \quad T = \int dM = \frac{\tau_{max}}{r}\int \rho^2\, dA \Rightarrow \boxed{T = \frac{\tau_m}{r} J}$$

Torsion formula

for circle $J = \frac{\pi}{32} d^4$

$$T = \frac{2\tau_m}{d} \times \frac{\pi}{32} d^4 = \frac{\pi \tau_m d^3}{16}$$

$$\Rightarrow \quad \tau = \frac{\rho}{r} \tau_m$$

$$\text{or} \quad \frac{\tau}{G} = \frac{\rho}{r} \frac{\tau_m}{G}$$

$$= \frac{\rho}{r} \frac{16T}{\pi d^3}$$

$$= \cancel{\frac{\rho}{r}}$$

State of pure shear induced by ~~shear~~ pure torsion.

Ques:- Solid bar / tube example.

$\theta_{allow} = \frac{\phi}{L} \Rightarrow T = \theta I G \quad [\tau = G(\rho\theta)]$

or, $\theta = \frac{T}{JG}$

$\tau_{allow} = 40 MPa \Rightarrow T = \frac{J}{r} \tau_{allow}$

$$J = \frac{T \times r}{\tau_{allow}}$$

Find d from here.

for hollow $d_1 = ?$

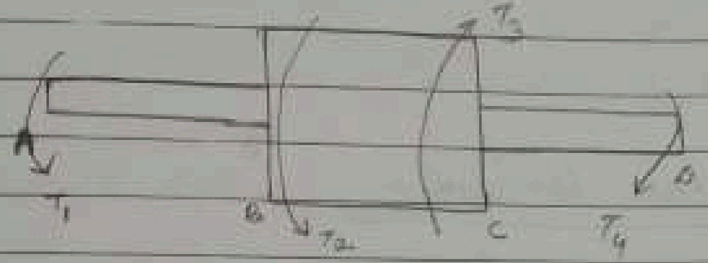
$d_0 = d_1 + 2t$

$\hookrightarrow$ substitute and find

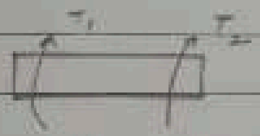
Rectangular UDL → self weight

Non-uniform torsion :-

→ similar to axial load.



$$T_1 + T_2 + T_3 + T_4 = 0$$



check individually

$$T = \theta J G = \frac{\phi}{L} J G \Rightarrow \left[\phi = \frac{TL}{JG} \right]$$

$\boxed{\text{Total } \angle \text{le of twist = algebraic sum of individual } \angle\text{les}}$

* for beam bounded at both ends

$\hookrightarrow \Sigma \phi = 0$

* for composite beam

$\hookrightarrow \phi_1 = \phi_2$

Transmission of power :-

$$P = \frac{T \times \theta}{t}$$

$$\therefore \boxed{P = \frac{dW}{dt} = T\omega = 2\pi f T = \frac{2\pi n T}{60}}$$

$K \rightarrow$ multiplication factor

15 11 2?

tooth design

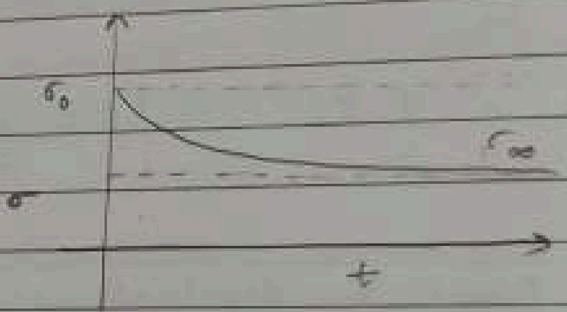
* Torsion of non-circular members

$\rightarrow$ equal stress distribution / equal strength distribution

CREEP AND FATIGUE -

$\rightarrow$ time dependent phenomena

Spring vs elastic band



$\tau = \frac{\sigma_0}{e}$ $\downarrow$ Relaxation time

$E = E_0 \, e^{-t/\tau}$

at $t = \tau$

$\left[E = \frac{E_0}{e} \right]$

visco-elastic material

$\rightarrow$ ex dependence.

(brings time dependence in the picture)

$\sigma = E\varepsilon$ (time independent)

$\tau = \eta \dot{\gamma}$ (time dependent)

Time temperature superposition

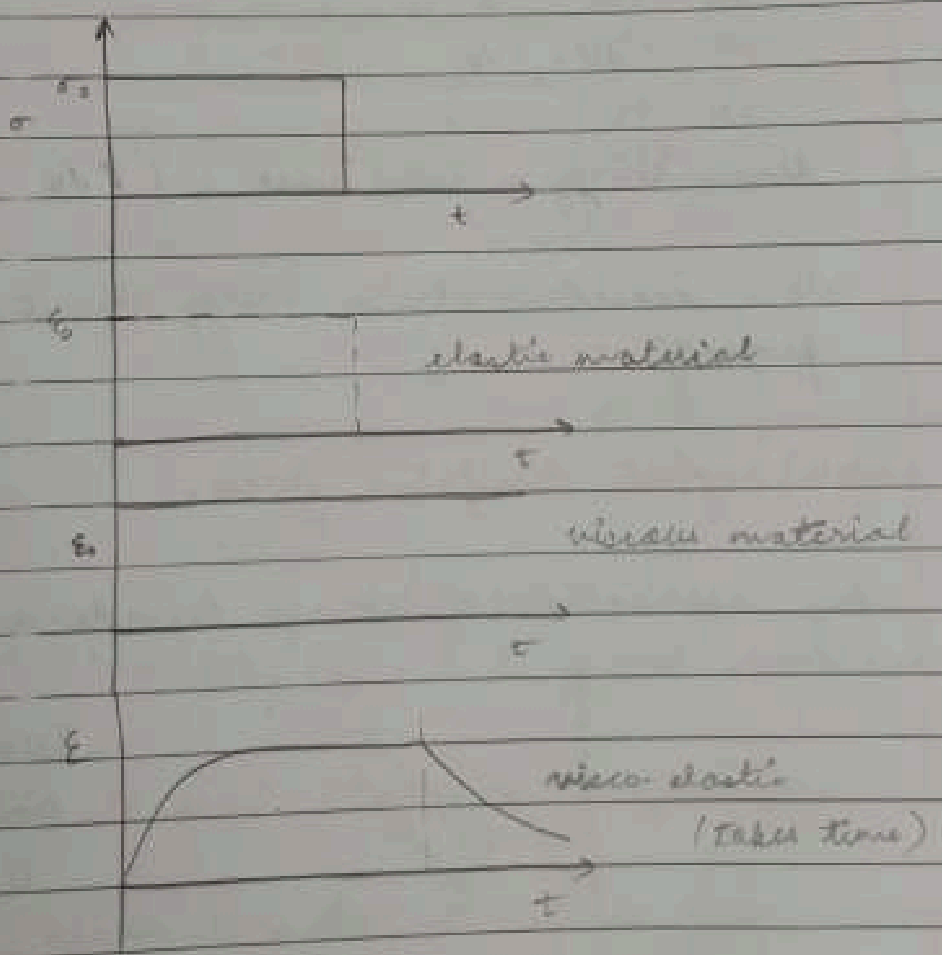
Creep strength

$\sigma \propto \dot{\varepsilon}$ (visco-elastic material).

* Polymer can behave like steel at high strain rate
↳ brittle material.

Every material comes out like powder.

Relaxation experiment and creep experiment



load varies with ~~time~~ time.

$\left[\sigma \propto \frac{1}{\sqrt{t}}\right]$ for dislocations $\rightarrow \frac{1}{t}$

aircraft failure.

$P = P_0 \sin \omega t$
$\varepsilon = \varepsilon_0 \sin \omega t$ }} → cyclic loading

S-N graph

ENERGY METHODS-

→ Handling scalars is easier
→ energy is more of a 'global' phenomenon

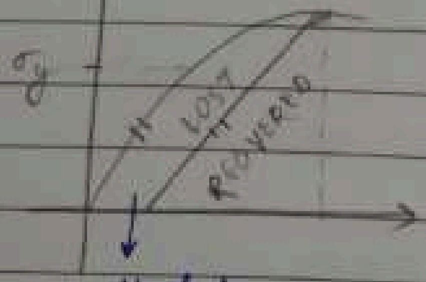
$$dU = P\,dx$$

$$\text{Strain energy} = \text{Total work} = \int_0^{x_1} P\,dx$$

We consider strain energy as it is an intensive property.

* We neglect dissipative forces.

↳ Any application of springs depends on strain energy.



Modulus of resilience

Below the proportional limit

$$U = \int \frac{\sigma^2}{2E}\, dV$$

* σ varies as per the case and can be found for axial loading, bending, twisting etc.

<u>dummy load</u> /
fictitious load

corresponding work in stresses

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for axial $\sigma = \frac{P}{A}$

for bending $\sigma = \frac{My}{I}$

for torsion $\tau = \frac{Tr}{I_p}$

in <u>bending</u> $U_b = \int \frac{M^2 y^2}{2EI^2} \, dV$

$$= \int \frac{M^2 y^2}{2EI^2} \, dA \cdot dx$$

$$= \int \frac{M^2 I}{2EI^2} \, dx = \int \frac{M^2}{2EI} \, dx$$

$$= \int \frac{P^2}{2EA} \, dx$$

for bending couple $\Rightarrow U = \frac{1}{2} M_1 \theta_1$

average for quasi-static process

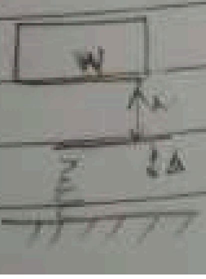
Impulse loading

$U = \frac{1}{2} kx^2 \rightarrow U = \frac{1}{2} Px$ $[P = kx]$ [P is not constant]

- Use superposition to find general strain energy.

for ductile materials, ~~dev~~ deviatoric part dominates and vice-versa.

<u>Ques</u>



$mg(h + \Delta) = \frac{1}{2} k\Delta^2$

$\Rightarrow \cancel{mg(h+\Delta)} = \quad \cancel{mg} \; \cancel{mg} \; \Delta \; \cancel{st} - start$

$$mgh + mgh\Delta = \cancel{\tfrac{1}{2}k\Delta^2} \; \frac{1}{2}k\Delta^2$$

or $\quad k\Delta^2 - 2mgh - 2mg\Delta = 0$

$$\Delta = \frac{2mg \pm \sqrt{4mg^2 + 8kmg}}{2k}$$

$$\text{or } \Delta = \frac{mg}{k} \pm \sqrt{\frac{mg^2}{k^2} \cancel{+ \frac{2mgh}{k}} \; \frac{2mgh}{k}}$$

$$\boxed{\Delta = \Delta_{st} \pm \sqrt{\Delta_{st}^2 + 2\Delta_{st} h}}$$

$$\boxed{\text{Impact force} = 2 \times \text{Quasi-static force}}$$

~~As~~ Uncontrolled lateral deformation under axial load is known as buckling.

Aspect ratio.

↳ Wave modes

↳ Eigen vectors and eigen values (poles)