

## Binary arithmetic

① addition  $\rightarrow$ 

$0+0=0$

$0+1=1$

$1+0=1$

$1+1=0 \text{ \& carry } 1$

$(37)_{10} + (29)_{10}$

$(10101)_2 + (11101)_2$

$$\begin{array}{r}
 10101 \\
 11101 \\
 \hline
 100010
 \end{array}$$

$$\begin{array}{r}
 100101 \\
 011101 \\
 \hline
 1000010
 \end{array}$$

## Binary sub.

$0-0=0$

$0-1=1 \text{ and borrow}$

$1-0=1$

$1-1=0$

eg.  $(13)_{10} + (11)_{10}$

$(1101)_2 + (1011)_2$

$$\begin{array}{r}
 01101 \\
 01011 \\
 \hline
 00000
 \end{array}$$

$$\begin{array}{r}
 2 \overline{) 29} \text{ ①} \\
 \underline{2} \phantom{0} 14 \text{ ②} \\
 \underline{2} \phantom{0} 7 \phantom{0} -1 \\
 \underline{2} \phantom{0} 3 \phantom{0} -1 \\
 \underline{2} \phantom{0} 1 \phantom{0} -1 \\
 \hline
 1
 \end{array}$$

$$\begin{array}{r}
 2 \overline{) 37} \text{ ①} \\
 \underline{2} \phantom{0} 18 \phantom{0} \\
 \underline{2} \phantom{0} 9 \phantom{0} -1 \\
 \underline{2} \phantom{0} 4 \phantom{0} -0 \\
 \underline{2} \phantom{0} 2 \phantom{0} -1 \\
 \hline
 1
 \end{array}$$

$(12)_{10} - (3)_{10}$

$1000 - 0011$

=

$$\begin{array}{r}
 1100 \\
 0011 \\
 \hline
 1001
 \end{array}$$

$$\begin{array}{r}
 10001 \\
 \hline
 1001
 \end{array}$$

$$\begin{array}{r}
 11101 \rightarrow 29 \\
 10011 \rightarrow 19 \\
 \hline
 01010 \text{ ①}
 \end{array}$$



Multiplication →

Eg.  $(12)_{10} \times (8)_{10}$

$\boxed{13 \times 11}$

$$\left. \begin{array}{l} 0 \times 0 = 0 \\ 0 \times 1 = 0 \\ 1 \times 0 = 0 \\ 1 \times 1 = 1 \end{array} \right\}$$

$$\begin{array}{r} \phantom{00} 1100 \\ \times \phantom{00} 1000 \\ \hline \phantom{00} 0000 \\ \phantom{00} 1100 \times \times \times \\ \hline 110000 \end{array}$$

$$\begin{array}{r} 1101 \\ 1011 \\ \hline 1101 \end{array}$$

1101 X

0000 X X

~~1101~~ X X X  
1000 1 1 1

128 + 8 + 4 + 2 + 1

138  
5  
143

13  
1 4 3

54

32 + 16

28  
64  
32 + 2  
128

$$\begin{array}{r} 99999 \\ 012398 \\ \hline 87601 \end{array}$$

$$\begin{array}{r} 999999 \\ 546700 \\ \hline 453299 \end{array}$$

$(1010)_2 = 10$

$(0101)_2 = 5$

$(0101)_2$  is comp.

101

88

$(11)_2$

$(00)$

99  
10  
100  
011

$$\begin{array}{r} 99 \\ 10 \\ \hline 89 \end{array}$$

$$\begin{array}{r} 999 \\ 010 \\ \hline 989 \end{array}$$

1010  
 $(0101)_2$

5



Q.  $2467_{10} \leftarrow 8\text{'s comp.}$

$\boxed{753300} \rightarrow 10\text{'s comp.}$

subtraction with complement  $\rightarrow$

$$\begin{array}{r} (13)_{10} \\ - (7)_{10} \\ \hline (6)_{10} \end{array}$$

$$\begin{array}{r} 1101 \\ - 0111 \rightarrow \text{comp} \\ \hline 0010 \\ \text{---} \\ 110 \end{array}$$

$$\begin{array}{r} + 1101 \\ 1000 \\ \hline 10101 \end{array}$$

$$\begin{array}{r} 0101 \\ 1 \\ \hline 0110 \rightarrow 6 \end{array}$$

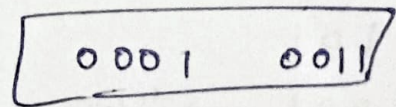
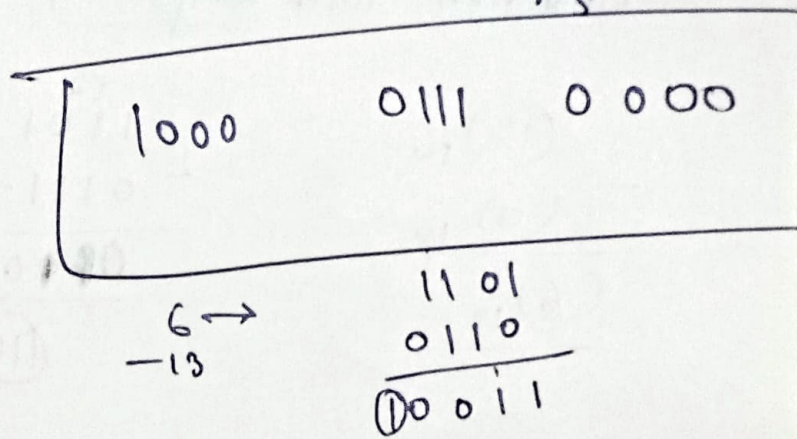
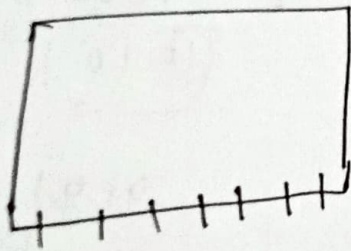
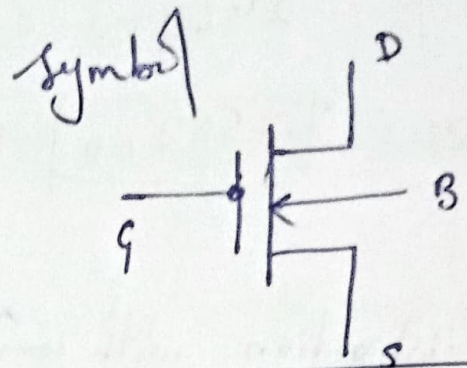
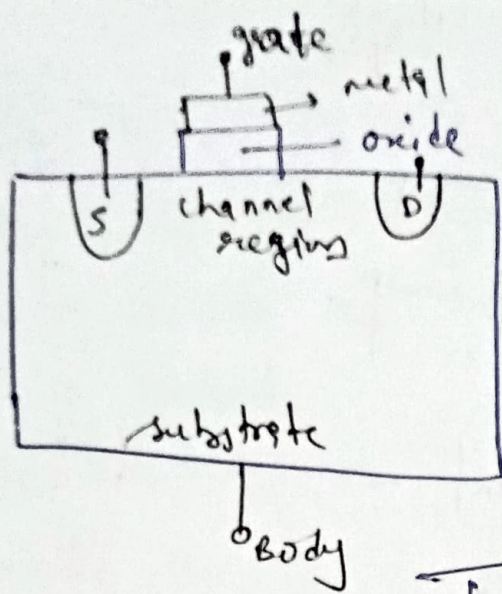
$$\begin{array}{r} 1101 \\ 1001 \rightarrow 2\text{'s comp} \\ \hline 0010 \end{array}$$

$$\begin{array}{r} 10110 \\ \text{---} \\ \boxed{X} \end{array}$$

if we are adding in 1's complement then @ MSB carry  
add to LSB

in 2's complement just discard <sup>MSB</sup> carry.





2/2/23

Binary Codes.

476

394

71610

870

0100 00111 0110

0010 01001 0000

0111 10000 0010

7

1 0110 0000

0100

0111

0110

0000

0011

1001

0100

0011

0111

10000

0001

0000

0001

0001

0001

0000

0000

00110

0000

0110

0000

0000

10111



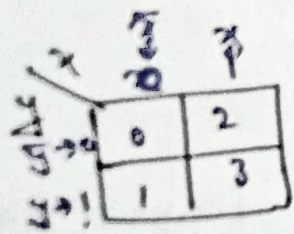
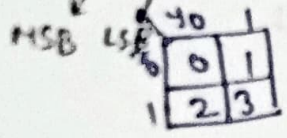
K-Map: →

- modified form of truth table
- for  $n$ -variable fun<sup>n</sup> than in k-map  $2^n$  shells will be present.
- Gray Code sequence is used in k-map representation.

2-variable k-map: →

$f(x, y) = 4$ -shell.

make square if it is possible.



Gray Code sequence

var.	gray code seq.
1	0, 1
2	0, 1, 3, 2
3	0, 1, 3, 2, 6, 7, 5, 4

to minimize

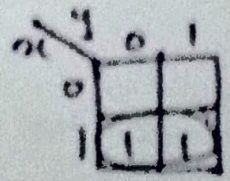
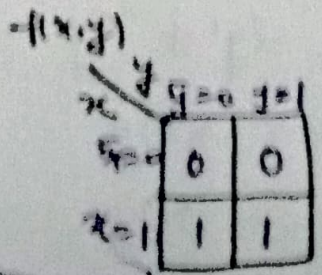
our aim to find  $f(x, y) = \sum(a, b)$

for ex.

x	y	f(x, y)
0	0	0
0	1	0
1	0	1
1	1	1

$f(x, y) = \sum(2, 3) = \pi(0, 1)$

$f(x, y) = \sum(2, 3) = \pi(0, 1)$



$f(x, y) = x$  (This is fully mino form).



'1' की grouping करते हैं और sop find करते हैं तो  $f$  मिलता है '0' की करते हैं और sop find करते हैं तो  $f'$  मिलता है।

(1) का pos grouping =  $f'$  , '1' का sop =  $f$   
 '0' की sop grouping =  $f$  '0' का sop =  $f'$

$x \backslash y$	0	1
0		
1		

$$f(x, y) = 0$$

$x \backslash y$	0	1
0	1	1
1	1	1

$$f(x, y) = 1$$

$x \backslash y$	0	1
0	1	1
1	1	1

$$f(x, y) = \bar{x} + \bar{y}$$

$$f'(x, y) = (x)(y) = xy$$

$x \backslash y$	0	1
0		
1		0

$$f(x, y) = \bar{x} + \bar{y}$$

$$f'(x, y) = xy$$

$x \backslash y$	0	1
0	1	0
1	0	1

$$f(x, y) = \bar{x}y + x\bar{y}$$

$$= \bar{x} \oplus \bar{y}$$

$$= x \oplus y$$

$x \backslash y$	0	1
0		1
1	1	

$$f(x, y) =$$

$$\begin{aligned} & \bar{x}y + x\bar{y} \\ & \text{prime Implicants (P.I.)} \\ & \uparrow \quad \uparrow \\ & \bar{x}y + x\bar{y} \\ & = x \oplus y \end{aligned}$$



- Max. no. of P.I Can be possible in 2-variable K-map.

Q1 max. no. of P.I =  $\frac{2^n}{2}$

for  $n=2$

P.I = 2  $\rightarrow$  max. (sop based only not pos)

3 variable K-map  $\rightarrow$

$f(x,y,z)$ , shells =  $2^n = 2^3 = 8$

$yz$	00	01	11	10
$x$				
0	0	1	3	2
1	4	5	7	6

or

$xy$	0	1
$z$		
00	0	1
01	2	3
11	6	7
10	4	5

Eg  $\rightarrow f(x,y,z) = \sum (3,5,6,7)$   
minimize the boolean fun"

$yz$	00	01	11	10
$x$				
0			1	
1		1	1	1

$f(x,y,z) = xz + xy + yz \rightarrow \text{sop}$

$f'(x,y,z) = (\bar{x} + \bar{z}) \cdot (\bar{x} + \bar{y}) \cdot (\bar{y} + \bar{z}) \rightarrow \text{pos}$

literals (always from sop) = 6 (no. of char" repeated also counted)

less no. of literals = less no. of logic gate required.



3 variable k-map by 2 variable →

$x \backslash z$	00	01	11	10
0	0	1	3	2
1	4	5	7	6

$x \backslash z$	0	1
0	0	1
1	4	5

$y=0$

$x \backslash z$	0	1
0	2	3
1	6	7

$y=1$

4 variable k-map

$f(w, x, y, z)$ , shells =  $2^4 = 16$

$w \backslash x \backslash z$	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Ques. minimise the boolean fun<sup>n</sup>

$w \backslash x \backslash z$	00	01	11	10
00	1			
01	1	1	1	
11		1	1	
10	1	1		

$$f(w, x, y, z) = \sum (0, 1, 4, 5, 7, 8, 9, 13, 15)$$

→ for

$$\text{Sop } f(w, x, y, z) = \underline{\underline{xz + \bar{x}\bar{y}z}}$$

$$= \underset{\substack{\downarrow \\ \text{E.P.I.}}}{xz} + \underset{\substack{\downarrow \\ \text{P.I.}}}{w\bar{x}\bar{y}} + \underset{\substack{\downarrow \\ \text{P.I.}}}{w\bar{y}z}$$

no. of literals = 8

$w \backslash x \backslash z$	00	01	11	10
00		0	0	0
01				0
11	0			0
10		0	0	

↓  
for pos