



Fig. 5.11: **Training density forest on a “non-function” dataset.** (a) Input *unlabelled* training points on a 2D space. (b,c,d) Three density forests are trained on such data, and the corresponding densities shown in the figures. Dark pixels correspond to small density and vice-versa. The original points are overlaid in green. Visually reasonable results are obtained in this dataset for $D = 4$.

(see also [63]).

We repeat a variant of this experiment in fig. 5.11. However, this time a *density* forest is trained on the “S-shaped” training set. In contrast to the regression approach in chapter 4, here the data points are represented as pairs (x, y) , with both dimensions treated equally as *input features*. Thus, now the data is thought of as *unlabelled*. Then, the joint generative density function $p(x, y)$ is estimated from the data. The density forest for this 2D dataset remains defined as

$$p(x, y) = \frac{1}{T} \sum_{t=1}^T p_t(x, y)$$

with t indexing the trees. Individual tree densities are

$$p_t(x, y) = \frac{\pi_l}{Z_t} \mathcal{N}((x, y); \boldsymbol{\mu}_l, \Lambda_l),$$

where $l = l(x, y)$ denotes the leaf reached by the point (x, y) . For each leaf l in the t^{th} tree we have $\pi_l = |\mathcal{S}_l|/|\mathcal{S}_0|$, the mean $\boldsymbol{\mu}_l = (\mu_x, \mu_y)$ and the covariance

$$\Lambda_l = \begin{pmatrix} \sigma_{xx}^2 & \sigma_{xy}^2 \\ \sigma_{xy}^2 & \sigma_{yy}^2 \end{pmatrix}.$$