Glauber modeling of high energy nuclear collisions

Name: N. S. Chiranjeevi
Designation: 3rd year BS-MS student
Institute: Indian Institute of Science
. Education and Research Tirupati
Id Number: 201501030

PI: Dr.Chitrasen Jena

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Introduction

The Glauber model is a set of theoretical techniques used to model high energy heavy ion collisions. High energy heavy ion collisions produce thousands of particles through collisions of heavy nuclei. The heavy nuclei are made to collide in particle colliders. It is impossible to directly measure the properties of these collisions, like the impact parameter (b), the number of collisions (N_{part}) or the number of participants (N_{coll}). Hence we use the Glauber model to model these collisions and create theoretical estimates of these quantities and compare them to indirect experimental observations [1].

The Glauber model

In modelling these high energy collisions with the Glauber model we take two main approaches, they are the Optical Glauber Model approach and the Monte Carlo Glauber approach. The main inputs to both of these models are the nuclear charge densities and the energy dependence in the inelastic nucleon-nucleon cross section.

The nuclear charge density distribution is given by Eq.1.

$$\rho(r) = \rho_0 \frac{1}{1+\exp\left(\frac{r-R}{a}\right)} \tag{1}$$

ρ=nuclear charge density
ρ_0=nuclear charge density at centre of nucleus
r=distance from centre of nucleus
R=radius of nucleus
a=skin depth

The inelastic nucleon-nucleon cross section is dependent on the energy of the collision.

The Glauber model views the collision of two nuclei in terms of the individual interactions of the constituent nucleons. According to the model at sufficiently high energies they will carry enough momentum that they will pass right through each other un-deflected. It also assumes that each nucleon moves independently of each other in the nucleus, disregarding the nucleon-nucleon forces due to the size of the nucleus.

In the Optical Glauber Model the N_{part} and N_{coll} are calculated by probabilistic formulae based on the above approximations.

$$P(n,b) = \binom{AB}{n}[\hat{T}_{AB}(b)\sigma_{inel}^{NN}]^n[1-\hat{T}_{AB}(b)\sigma_{inel}^{NN}]^{AB-n} \tag{2}$$

$$N_{coll}(b) = \sum_{n=1}^{AB} nP(n,b) = AB\hat{T}_{AB}(b)\sigma_{inel}^{NN} \tag{3}$$

$$N_{part}(b) = A\int \hat{T}_A(s)\left\{1-\left[1-\hat{T}_B(s-b)\sigma_{inel}^{NN}\right]^B\right\}d^2s + B\int \hat{T}_B(s-b)\left\{1-\left[1-\hat{T}_A(s)\sigma_{inel}^{NN}\right]^A\right\}d^2s \tag{4}$$

A = number of nucleons in projectile A
B = number of nucleons in projectile B
b = impact parameter
s = displacement of flux tube from centre of target A
σ_{inel}^{NN} = inelastic nucleon-nucleon cross sectional area
$\hat{T}_{AB}$ = Effective overlap area for which a specific nucleon in A can interact with a given nucleon in B
$\hat{T}_{A}$ = Probability per unit transverse area of a given nucleon being located in the target flux tube A
$\hat{T}_{B}$ = Probability per unit transverse area of a given nucleon being located in the target flux tube B

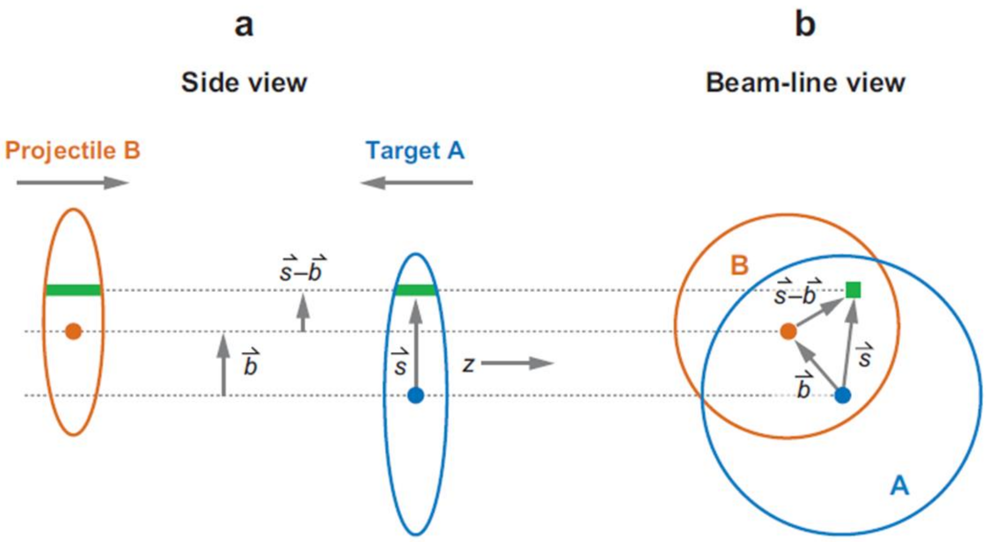


Figure 1. Schematic representation of Optical Glauber Model [1]

In the Monte Carlo Glauber simulation we generate data in a series of steps. The first step involves distributing the nucleons around the centre of the nucleus; this involves generating random values for spatial coordinates for each nucleon, and distributing them around the centre by means of a radial distribution defined by the Wood Saxon distribution Eq.1. There is also a minimum nucleon-nucleon distance that is also taken into account while distributing nuclei. This is done for both the target and the projectile nucleus. The centres of the two nuclei are moved to positions b/2 above and below the z axis (beam axis). Once this is done we can determine which two nucleons from each of the nucleus will collide with each other. To do this we measure the the ball diameter (D) given by Eq.5.

$$D = \sqrt{\frac{\sigma_{NN}}{\pi}} \qquad \textbf{(5)}$$

The ball diameter is the minimum transverse distance between two nucleons below which we can assume that a collision has occurred. Running this algorithm for every nucleon in each nucleus for a million cycles (total number of events), the number of participants, the number of collisions and the impact parameter distribution. These values are generated and stored as histograms that can be accessed for further processing.

Root

Root is an open source scientific data framework used to statistical analysis, big data processing and creating visualizations. It is used extensively in the field of high energy physics.
Over the course of the last four months I have learned how to do basic operations in this software, like creating histograms and graphs, generating random numbers and running code through the software. I have used the skills I have learned to draw graphs and histograms relevant to the Glauber model.

Analysis of data

The simulation was done for Au-Au collisions at $\sqrt{s_{NN}}$ = 200GeV using the Monte Carlo Glauber model [2]. The inelastic nucleon-nucleon cross section was σ_{inel}^{NN} = 42mb. The program was run for for 1 million events. The results from these simulations such as the N_{part} , N_{coll} and and Impact Parameter were saved as histograms as shown in Figure 2, 3 and 4 . These simulated results were used to construct two dimensional histograms of N_{part} and N_{coll} as shown in Figure 5. The charged particle multiplicity distribution was a then constructed using a two component model given by Eq.6 from randomly getting entries from the two dimensional histograms of N_{part} and N_{coll} [4]. The charge particle multiplicity data was then fitted to a Negative binomial distribution given by Eq.7.

$$\frac{dn}{d\eta} = (1-x)n_{pp}\frac{\langle N_{part}\rangle}{2} + xn_{pp}\langle N_{coll}\rangle \tag{6}$$

$$P_{NBD}(\mu,k;n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)}\cdot\frac{(\mu/k)^n}{(\mu/k+1)^{n+k}} \tag{7}$$

The simulated charge particle multiplicity distribution was normalised with the charge particle multiplicity distribution from the experimental data, ie RefMult, the uncorrected charge particle multiplicity for pseudorapidity $|\eta|<0.5$, and plotted together as shown in Figure 6. The 10 centrality bins were chosen and appropriate cuts were made to separate them according to the percentage of centrality they represented. Lines were drawn in the histogram at these cuts to separate different centrality bins.

Three 2 dimensional histograms of N_{part}, N_{coll} and Impact parameter(b) each with the simulated charge particle multiplicity were constructed as shown in Figure 7, 8 and 9 and the mean value of these quantities, N_{part}, N_{coll} and Impact parameter, were calculated for each centrality bin. These results were then compared with the experimental data as shown in Table 1 and 2.

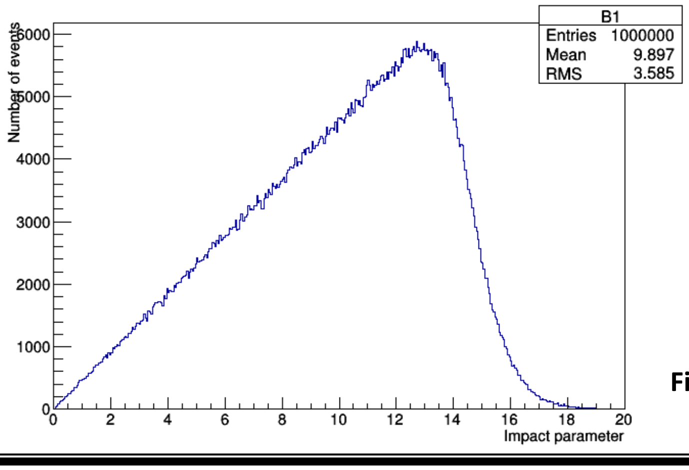
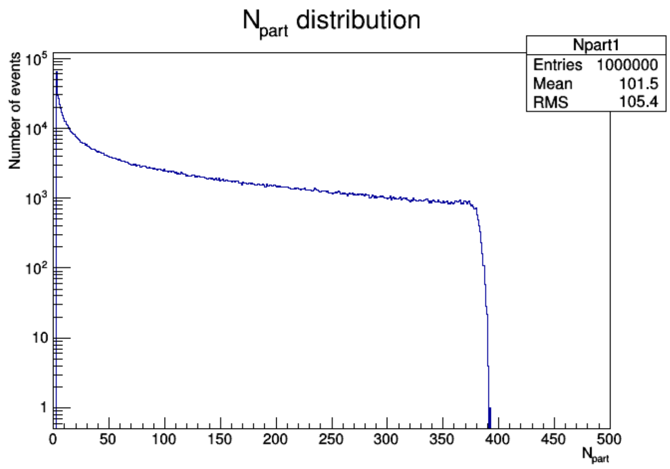


Figure 2. Distribution of the Impact Parameter (b)



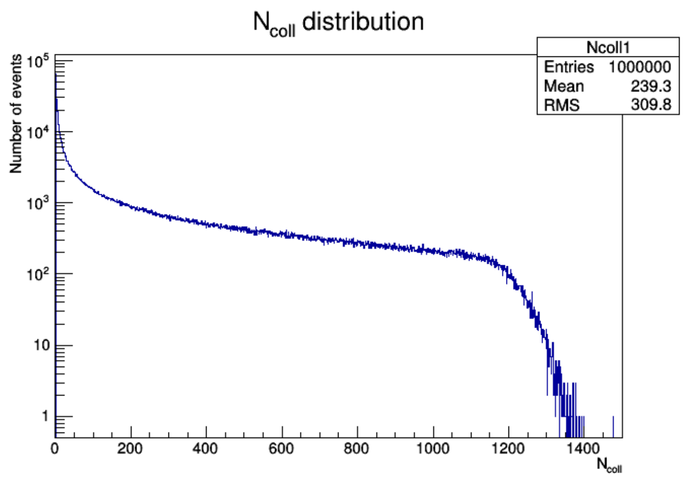


Figure 3. Distribution of the number of participants (N_{part})

Figure 4. Distribution of the number of collisions (N_{coll})

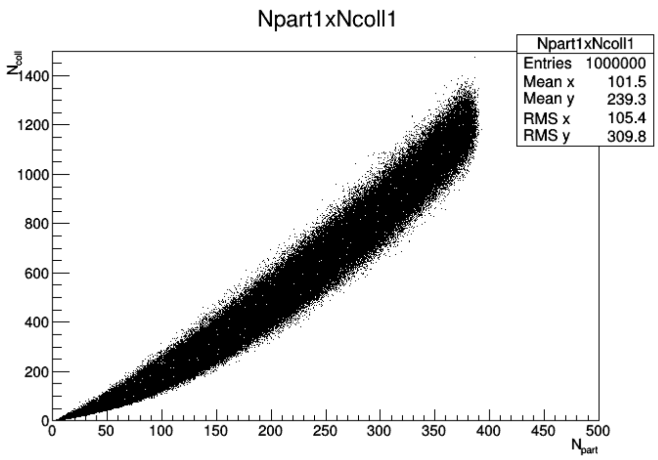


Figure 5. Two dimensional histogram of N_{part} vs N_{coll}

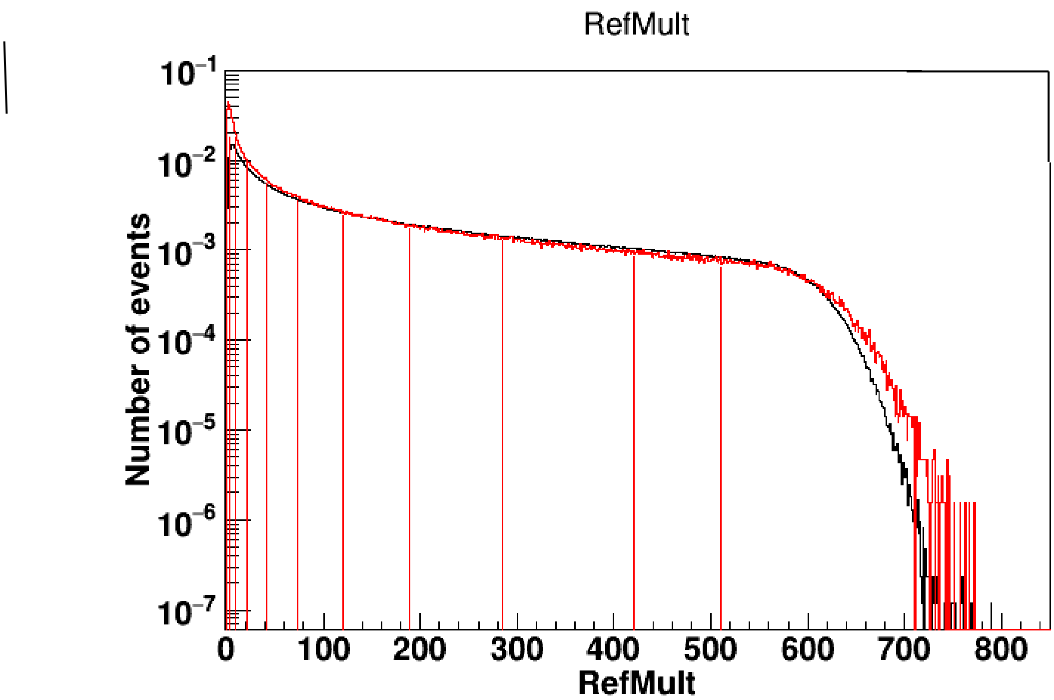


Figure 6. Histogram of simulated charged particle multiplicity(shown in red) and charged particle multiplicity data from experiment(shown in black) with centrality cuts (shown as vertical red lines)

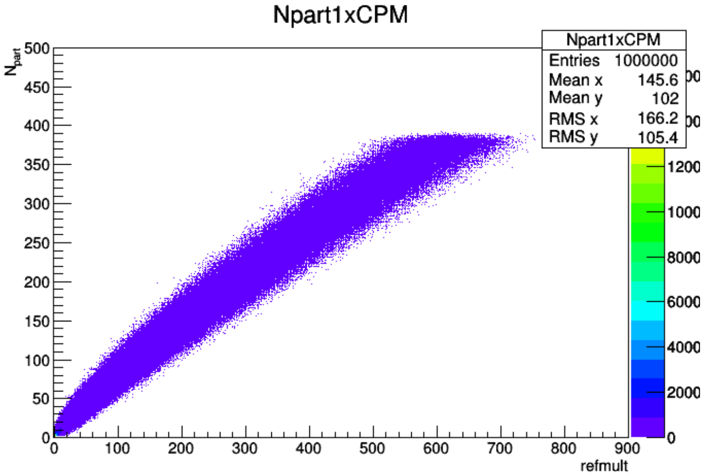


Figure 7. Two dimensional histogram of N_{part} vs charge particle multiplicity(refmult)

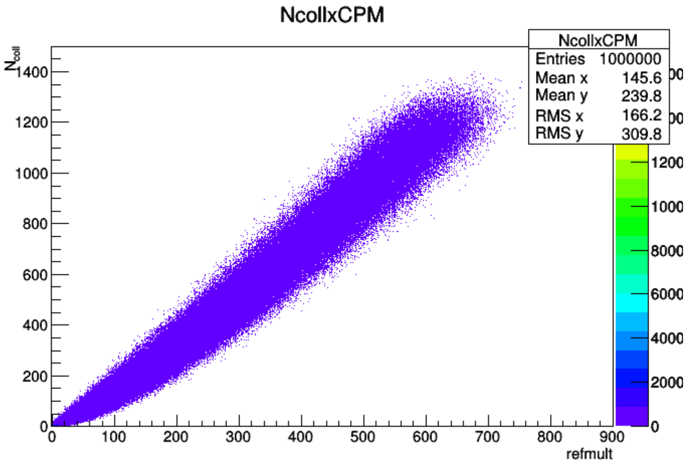


Figure 8. Two dimensional histogram of N_{coll} vs charge particle multiplicity(refmult)

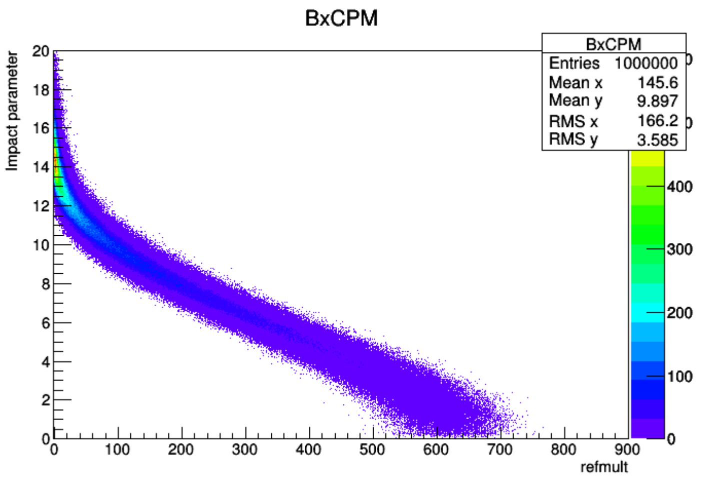


Figure 9. Two dimensional histogram of Impact parameter vs charge particle multiplicity(refmult)

Table.1. Mean values of Impact Parameter (b),Number of Participants(N_{part}) and the Number of Collisions(N_{coll}) for each centrality bin calculated from the simulated data compared with experimental data. [3]

Centrality	Multiplicity cut	Multiplicity cut (used in experiment)	Mean Impact Parameter (b)	Mean Impact Parameter (b) (published)
90-100%	3	-	14.54 ± 1.05	-
80-90%	9	-	14.03 ± 1.04	-
70-80%	21	>14	13.01 ± 0.86	12.3-13.2
60-70%	41	>30	11.97 ± 0.68	11.4-12.3
50-60%	73	>56	10.96 ± 0.59	10.5-11.4
40-50%	121	>94	9.89 ± 0.57	9.33-10.5
30-40%	189	>146	8.72 ± 0.56	8.10-9.33
20-30%	285	>217	7.36 ± 0.59	6.61-8.10
10-20%	421	>312	5.69 ± 0.7	4.66-6.61
5-10%	511	>431	4.10 ± 0.65	3.31-4.66
0-5%	>511	>510	2.31 ± 0.96	0-3.31

Table.2. Mean values of Impact Parameter (b),Number of Participants(N_{part}) and the Number of Collisions(N_{coll}) for each centrality bin calculated from Experimental data. [3]

Centrality	$\langle N_{part} \rangle$	$\langle N_{part} \rangle$ (published)[3]	$\langle N_{coll} \rangle$	$\langle N_{coll} \rangle$ (published)[3]
90-100%	3.47 ± 1.51	-	2.34 ± 1.34	-
80-90%	5.81 ± 3.19	-	4.43 ± 3.04	-
70-80%	13.26 ± 5.75	15.7 ± 2.6	12.05 6.71	15.0 ± 3.2
60-70%	26.96 ± 8.26	28.8 ± 3.7	29.65 ± 12.5	312.4 ± 5.5
50-60%	47.54 ± 11.18	49.3 ± 4.7	63.73 ± 21.52	66.8 ± 9.0
40-50%	76.79 ± 4.51	78.3 ± 5.3	124.4 ± 34.19	127 ± 13
30-40%	115.8 ± 18.13	117.1 ± 5.2	222.4 ± 50.23	221 ± 17
20-30%	167.4 ± 22.75	167.6 ± 4.4	373.8 ± 71.98	365 ± 24
10-20%	235.2 ± 28.39	234.3 ± 4.6	603.2 ± 101.6	577 ± 36
5-10%	299.7 ± 23.29	298.6 ± 4.1	847.3 ± 92.39	805 ± 50
0-5%	350 ± 21.44	350.6 ± 2.4	1066 ± 102.4	1012 ± 59

Table.3. The mean multiplicity from experimental data and from simulation for different

Centrality	Mean multiplicity	Mean multiplicity (published)[3]	% Error
10-20%	347.95	348.77	0.235666
20-30%	233.94	234.09	0.064119
30-40%	152.05	152.74	0.453798
40-50%	94.91	95.08	0.179117
50-60%	55.28	55.36	0.144718
60-70%	29.58	29.67	0.30426
70-80%	13.64	13.94	2.199413
80-90%	5.07	5.63	11.04536
90-100%	1.53	1.87	22.22222

Conclusion

A million events of Au-Au collisions at $\sqrt{s_{NN}}$ = 200 GeV were simulated using the Monte Carlo Glauber model and have found N_{part} and N_{coll} and Impact Parameter for each event. Using this data a charged particle multiplicity distribution was constructed. Centrality bins were constructed and the mean values of each of the above quantities we calculated for each centrality bin. These values were then compared with experimental results and most have shown to be consistent as they fall within the experimental range, ie. published results [3], of values as in Table 1 and 2.

The simulated multiplicity distribution is comparable with the experimental data at high multiplicity region whereas it deviates from experimental data at low multiplicity. This occurs due to the inefficiency of the detector. Some of the low multiplicity events are not detected in the experiment. Therefore it is important to use the simulated multiplicity distribution to correctly estimate the centrality of the collision.

References

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