# Summer project report

# Controlling rogue waves in graded index fibre

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# INDEX

# Introduction

Rogue waves are large amplitude waves that spontaneously appear and disappear in oceans. They are large enough to topple ships and are a huge risk to ocean liners. Unlike regular ocean waves they are not governed by linear dynamics; they do not obey the superposition principle. Their generation and propagation is unique and unpredictable. They are extremely rare phenomena and for years have befuddled scientists and sailors alike.

Recent studies have shown that these waves cannot only be generated but also controlled. These waves are modelled by nonlinear equations. With the advent of nonlinear dynamics and chaos our understanding of these waves have grown enormously. Rogue waves are being extensively studied in the field of optics. There are used in the generation of a supercontinuum in optical fibre. They are studied in different fields ranging from Raman fibre amplifiers to spatiotemporal structures and as parametric processes. They have been generated not only on fibre optics cables and on the surface of water but also in Bose Einstein condensates.

My study primarily revolves around rogue waves in nonlinear optical fibres specifically using one such method of modelling these waves, the nonlinear Schrodinger wave equation(NLSE).

# Method

We start by using the equation for the beam propagation in tapered graded-index nonlinear fibre amplifiers with refractive index given by

$$n(x) = n_o + n_1 F(z) x^2 + n_2 |u|^2$$

is described by the generalized nonlinear equation Schrödinger equation(GNLSE)

$$i\frac{\partial u}{\partial z} + \frac{1}{2k_o}\frac{\partial^2 u}{\partial x^2} + \frac{1}{2}k_o n_1 F(z) x^2 u - \frac{i[g(z) - \alpha(z)]}{2}u + k_o n_2 |u|^2 u = 0$$

Where the terms are:

| | |
|---|---|
| u(x,y) | is the complex of the electrical field |
| $k_o$ | $= 2\pi n_o / l$ |
| g and α | are the linear gain and loss respectively |
| l | is the wavelength of the optical source generating the beam |
| $\mathbf{n}_1$ $\mathbf{n}_2$ | Is the linear defocusing parameter and the Kerr type linear defocusing parameter respectively |
| F(z) | Is a dimensionless profile function that can be either positive or negative depending on the weather the graded index medium acts as a focusing or defocusing linear lens |

We then make the following substitutions to the equation so that it can be written in a dimensionless form. We do this by introducing 4 new normalized variables.

$$X = x / w_\circ$$

$$Z = z / L_D$$

$$G = [g(z) - \alpha(z)]L_D$$

$$U = (k_\circ \mid n_2 \mid L_D)^{1/2} u$$

Where

$$L_D = k_\circ w_\circ^2$$

is the diffraction length associated with the characteristic transverse scale

$$w_\circ = (k_\circ^2 n_1)^{-1/4}$$

Thus making the following substitutions it can easily be shown that Eq. (2) and be rewritten in a dimensionless form as

$$i\frac{\partial U}{\partial Z} + \frac{1}{2}\frac{\partial^2 U}{\partial X^2} + F(Z)\frac{X^2}{2}U - \frac{i}{2}G(Z)U + |U|^2 U = 0 \qquad (3)$$

The solution to the above equation can be be found by transforming it into a standard NLSE using gauge and similarity transformations together with generalized scaling of the Z variable.

$$U(X,Z) = \frac{1}{W(Z)}\psi[\frac{X - X_C(Z)}{W(Z)}, \zeta(Z)]e^{i\phi(X,Z)}$$

Where

| | |
|---|---|
| $W(Z)$ | Dimensionless width of self similar wave centre |

| | |
|---|---|
| $X_c(Z)$ | Dimensionless position of self similar wave centre |

The quadratically chirped phase is given by.

$$\phi(X,Z) = C_1(Z)\frac{X^2}{2} + C_2(Z)X + C_3(Z)$$

Where

| | |
|---|---|
| $C_1(Z)$ | Parameter related to phase front curvature |
| $C_2(Z)$ | Parameter related to the frequency shift |
| $C_3(Z)$ | Parameter related to the phase offset |

$\psi(\zeta, X)$

Satisfies the equation

$$i\frac{\partial\psi}{\partial\zeta} + \frac{1}{2}\frac{\partial^2\psi}{\partial\chi^2} + |\psi|^2\psi = 0$$

Where the effective propagation distance is.

$$\zeta(Z) = \zeta_0 + \int_0^z \frac{dS}{W^2(S)}$$

Similarity variable is

$$\chi(X,Z) = \frac{X - X_c(Z)}{W(Z)}$$

Guiding-centre position is

$$X_c = W(Z)[C_{02}\int_0^z \frac{dS}{W^2(S)} + X_0]$$

Phase is

$$\phi(X,Z)=\frac{X^2}{2W(Z)}\frac{dW(Z)}{dZ}+\frac{C_{02X}}{W(Z)}-\frac{C_{02}^2}{2}\int_0^Z\frac{dS}{W^2(S)}$$

With

$C_0(0)=C_{02} \qquad X_C(0)=X_0 \qquad W(0)=1$

The phase front curvature is related to the gain function as

$C_1(Z)=-G(Z)$

# Calculation

With the variables defined above we verify Eq.(3)

$$i\frac{\partial U}{\partial Z}+\frac{1}{2}\frac{\partial^2 U}{\partial X^2}+F(Z)\frac{X^2}{2}U-\frac{i}{2}G(Z)U+|U|^2U=0$$

$$U(X,Z)=\frac{1}{W(Z)}\psi[\frac{X-X_C(Z)}{W(Z)},\zeta(Z)]e^{i\phi(X,Z)}$$

$$\frac{\partial U}{\partial Z}=-\frac{1}{W^2(Z)}\frac{dW}{dZ}\psi e^{i\phi}+\frac{1}{W^3}\frac{\partial\phi}{\partial\zeta}e^{i\phi}+\frac{1}{W}\frac{\partial\psi}{\partial\chi}\frac{\delta\chi}{\delta Z}+\frac{1}{W}\psi i\frac{d\phi}{dZ}e^{i\phi}$$

$$\frac{\partial\zeta}{\partial Z}=\frac{1}{W^2(Z)}$$

$$i\frac{\partial U}{\partial Z}=-\frac{i}{W^2}\frac{dW}{dZ}\psi+\frac{i}{W^2}\frac{\partial\psi}{\partial\zeta}+\frac{i}{W}\frac{\partial\psi}{\partial\chi}\frac{\partial\chi}{\partial Z}+\frac{1}{2W^3}\psi X^2(\frac{dW}{dZ})^2-\frac{\psi X^2}{2W^2}\frac{d^2W}{dZ^2}+\frac{\psi XC_{02}}{W^3}\frac{dW}{dZ}+\frac{\psi C_{02}^2}{2W^3}$$

$$\frac{\partial\phi}{\partial Z}=-\frac{X^2}{2W^2}[\frac{dW}{dZ}]^2+\frac{X^2}{2W}\frac{d^2W}{dZ^2}-\frac{C_{02}X}{W^2}\frac{dW}{dZ}-\frac{C_{02}{}^2}{2}\frac{1}{W^2}$$

$$\frac{\partial^2}{\partial X^2}=\frac{1}{W^2}\frac{\partial^2}{\partial\chi^2}$$

$$\frac{d\phi}{dX}=\frac{X}{W}\frac{dW}{dZ}+\frac{C_{02}}{W}$$

$$\frac{\partial U}{\partial X}=\frac{1}{W^2}\frac{\partial\psi}{\partial\chi}e^{i\phi}+\frac{1}{W}\psi i\frac{d\phi}{dX}e^{i\phi}$$

$$\frac{d^2\phi}{dX^2}=\frac{1}{W}\frac{dW}{dZ}$$

$$\frac{\partial^2U}{\partial X^2}=\frac{1}{W^3}\frac{\partial^2\psi}{\partial\chi^2}+\frac{2i}{W^2}\frac{\partial\psi}{\partial\chi}[\frac{X}{W}\frac{dW}{dZ}+\frac{C_{02}}{W}]+\frac{i\psi}{W^2}\frac{dW}{dZ}-\frac{1}{W}\psi[\frac{X^2}{W^2}(\frac{dW}{dZ})^2+\frac{C_{02}^2}{W^2}+\frac{2XC_{02}}{W^2}\frac{dW}{dZ}]$$

$$-\frac{i}{W^2}\frac{dW}{dZ}\psi+\frac{i}{W^3}\frac{\partial\psi}{\partial\zeta}+\frac{i}{W}\frac{\partial\psi}{\partial\chi}\frac{\partial\chi}{\partial Z}+\frac{1}{2W^3}\psi X^2(\frac{dW}{dZ})^2-\frac{\psi X^2}{2W^2}\frac{d^2W}{dZ^2}+\frac{\psi XC_{02}}{W^3}\frac{dW}{dZ}+\frac{\psi C_{02}^2}{2W^3}+\frac{1}{2W^3}\frac{\partial^2\psi}{\partial\chi^2}$$

$$+\frac{i}{W^3}X\frac{\partial\psi}{\partial\chi}\frac{dW}{dZ}+\frac{i}{W^3}C_{02}\frac{\partial\psi}{\partial\chi}+\frac{i\psi}{2W^2}\frac{dW}{dZ}-\frac{1}{2}\frac{\psi X^2}{W^3}(\frac{dW}{dZ})^2-\frac{\psi C_{02}^2}{2W^3}-\frac{\psi C_{02}X}{W^3}\frac{dW}{dZ}+F(Z)\frac{X^2}{2W}\psi$$

$$-\frac{i}{2}G(Z)\frac{\psi}{W}+\frac{1}{W^3}|\psi|^2\psi=0$$

Collecting like terms and equating tem to zero; we obtain the following eqauations.

The tapering function, gain and the similarton width W(Z) are related as

$$\frac{d\ W}{dZ^2}- \quad =$$

$$[-\frac{i}{W}\frac{dW}{dZ}-\frac{iG\ Z}{W} \quad =$$

$$G\ Z\ W\ Z\ =-\frac{dW}{dZ}$$

$$G\ Z\ =-\frac{1}{W\ Z}\frac{dW}{dZ}=-\frac{d}{dZ}\quad W\ Z$$

$$\frac{i}{W^3}\frac{\partial\psi}{\partial\zeta}+\frac{1}{2W^3}\frac{\partial^2\psi}{\partial\chi^2}+\frac{1}{W^3}|\psi|^2\psi+\frac{i}{W}\frac{\partial\psi}{\partial\chi}\frac{\partial\chi}{\partial Z}+\frac{i}{W^3}X\frac{\partial\psi}{\partial\chi}\frac{dW}{dZ}+\frac{i}{W^3}C_{02}\frac{\partial\psi}{\partial\chi}=0$$

$$\frac{dX_c}{dZ}=\frac{dW}{dZ}(C_{02}\int\frac{dS}{W^2(S)}+X_c)+\frac{C_{02}}{W}\quad\frac{\partial\chi}{\partial Z}=-\frac{1}{W^2}(X-X_c(Z))\frac{dW}{dZ}-\frac{1}{W}\frac{dX_c(Z)}{dZ}$$

$$\frac{dX}{dZ}=-\frac{X}{W^2}\frac{dW}{dZ}+\frac{X_c}{W^2}\frac{dW}{dZ}-\frac{X_c}{W^2}\frac{dW}{dZ}-\frac{C_{02}}{W^2}$$

After further simplification we are left with this equation

$$i\frac{\partial\psi}{\partial\zeta}+\frac{1}{2}\frac{\partial^2\psi}{\partial\chi^2}+|\psi|^2\psi=0$$

It is well know that the NLSE admits bright and dark soiton solutions as well as Ma solutions and Akhmediev breather solution. As a limiting case these solutions reduce reduce to the lower order rational solutions called "Peregrine solitons" or rogue waves.

The expression of intensity for a first-order self-similar optical rogue wave is given as

$$I = \quad = \frac{1}{W\ Z} \quad + \quad \frac{1+ \quad -}{(1+ \quad +}$$

The general expression of intensity for a second-order self-similar optical rogue wave is given as

$$I = \frac{1}{W\ Z}\ \frac{D-}{D} + \frac{H}{D}$$

Where

$$K = \quad + \quad + \frac{3}{4} \quad + \quad + \frac{3}{4} - \frac{3}{4}$$

$$H = \quad - \quad + \quad + \quad - \frac{15}{8}$$

And

$$D = \frac{1}{3} \quad + \quad + \frac{1}{4} \quad - \quad + \frac{3}{64} \quad + \quad +$$