Assignment 1

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Image blending

1. **Exercise 1**

$$\|\Delta u\|_2 \simeq \sum_{i,j} \sqrt{u[i+1,j] - u[i,j]^2 + (u[i,j+1] - u[i,j])^2}$$

2. **Exercise 2**

   1)

$$|u_C - g_C|^2_\Omega = \sum_{i,j} \Omega[i,j] |u_C[i,j] - g_C[i,j]|^2_2$$

$$\frac{\partial}{\partial u[p,q]} |u_C - g_C|^2_\Omega = \frac{\partial}{\partial u[p,q]} \sum_{i,j} \Omega[i,j] |u_C[i,j] - g_C[i,j]|^2_2$$

$$\text{if } p = i \text{ and } q = j :$$

$$\frac{\partial}{\partial u[p,q]} \sum_{i,j} \Omega[i,j] |u_C[i,j] - g_C[i,j]|^2_2 = 2 \cdot \Omega[i,j] \cdot (u_C[i,j] - g_C[i,j])$$

$$\frac{\partial |u_C - g_C|^2_\Omega}{\partial u[i,j]} = 2 \cdot \Omega[i,j] \cdot (u_C[i,j] - g_C[i,j])$$

2)

$$\begin{aligned}\frac{\partial |u_C - g_C|^2_\Omega}{\partial u[i,j]} &= \frac{\partial \tau[i,j]}{\partial u[i,j]} + \frac{\partial \tau[i-1,j]}{\partial u[i,j]} + \frac{\partial \tau[i,j-1]}{\partial u[i,j]} \\ &= \frac{u_C[i,j] - u_C[i-1,j]}{\sqrt{(u_C[i,j] - u_C[i-1,j])^2 + (u_C[i-1,j+1] - u_C[i-1,j])^2}} + \\ &\quad \frac{u_C[i,j] - u_C[i,j-1]}{\sqrt{(u_C[i+1,j-1] - u_C[i,j-1])^2 + (u_C[i,j] - u_C[i,j-1])^2}} + \\ &\quad \frac{2 \cdot u_C[i,j] - u_C[i+1,j] - u_C[i,j+1]}{\sqrt{(u_C[i+1,j] - u_C[i,j])^2 + (u_C[i,j+1] - u_C[i,j])^2}} + \end{aligned}$$

where

$$\tau[i,j] \doteq \sqrt{(u_C[i+1,j] - u_C[i,j])^2 + (u_C[i,j+1] - u_C[i-1,j])^2}$$

3. **Implementation.** For each of the 3 solvers (gradient descent, Linearization+Gauss-Seidel, Linearization+SOR):

   - Show images of the inputs
   - Show 5 images of the reconstruction as the method progresses iteration by iteration: The initial, the final image and 3 more images in between.
   - Show the energy against iteration time (we should see it decreasing over time).

4. **State which of the 3 solvers you choose. Show images obtained by very high, very low and manually-tuned (approximately optimal) λ.** In this section you should:

   - Display 3 images with different λ: one with very low, one with very high and one with the manually-tuned (approximately optimal) λ.
   - Describe the effect of λ on the solution.

5. **Image blending:**

   - Display your own image composition here along with the foreground, background and mask images.
   - Describe how you used or modified the code to create your image(s).