Assignment 3

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Finite Element Analysis I

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Abstract

In this assignment we examine the behaviour of a beam after being subjected to an impulse. The goal is to extract the resonance frequency of the beam with the given dimensions and material coefficients.

Contents

1 Introduction

The goal of this assignment is the analyzation of a beam attached to a wall and how it is related to its resonance frequencies. We want to show the analytical solution and after the model implemented in Abaqus. An example of this calculation can be used in dental implants, as the beginns to oscillate under given circumstances.

2 Methods

2.1 Analytical solution

Consider a beam with the following dimensions attached to a wall in our case on the left end:

$$L = 150mm, h = 2.5mm, b = 20mm, E = 10GPa, v = 0.3, \rho = 7.0E3kg.m^{-3} \tag{1}$$

The resonance frequencies (inducing a bending in the direction of the dimension h) of such a model are given by the formula:

$$\frac{1}{2\pi\sqrt{12}}\alpha_i^2\sqrt{\frac{E}{\rho}\frac{h}{L^2}} \tag{2}$$

Where:

$\alpha_0 = 1.875, \alpha_1 = 4.695, \alpha_2 = 7.85, ..., \alpha_i = (2i+1)\frac{\pi}{2} \;\; for\, i > 2$
E = Young's modulus
ρ = density of the material
h = height of the cross-section of the beam
L = length of the beam

Using our values for α_0:

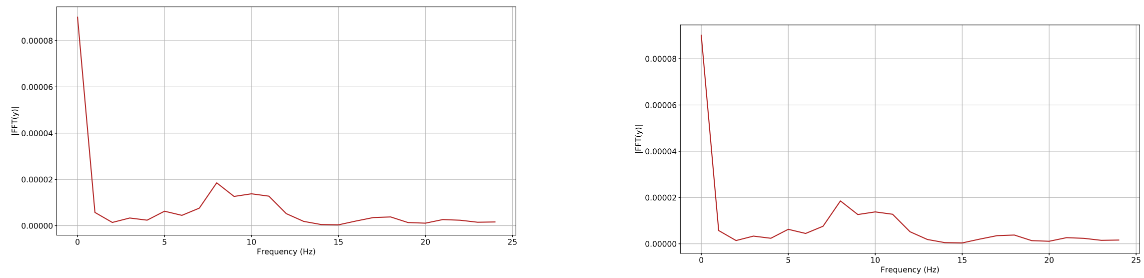
$$f_0 = \frac{1}{2\pi\sqrt{12}}1.875^2\sqrt{\frac{10 \cdot 10^3 Pa}{7 \cdot 10^3 \frac{kg}{m^3}}\frac{2.5 \cdot 10^3 m}{(150 \cdot 10^3 m)^2}} = 21.451\,Hz \tag{3}$$

2.2 Generating an impulse

The impulse on the beam is generated by applying a concentrated force load on the extremity of the beam (the non-encastered end). This load is being applied for a short time period (1ms).

2.3 tbd

3 Results and Discussion

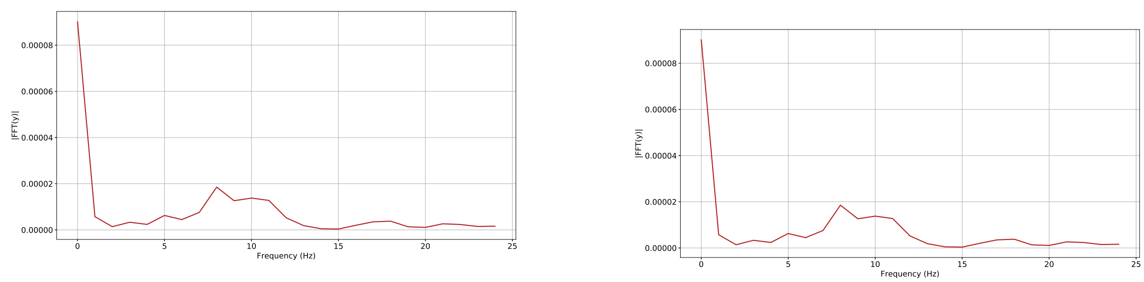


(a) maximal stress: 603 MPa
minimal stress: 1,24 MPa

(b) Stress (Mises) in N/m^2 max. 603 N/m^2

Figure 1: Global model with no fillet

We observe, that the maximum stress value for this setup exceeds the yield stress of 500 MPa. We, therefore, expect the beam to be plastically deformed in the region of the inner corner.

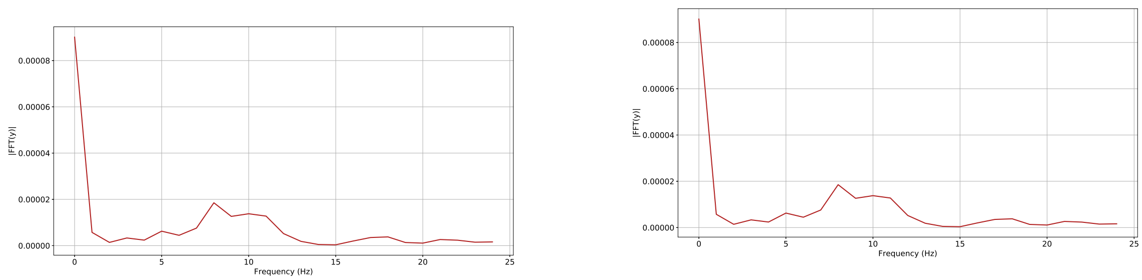


(a) maximal stress: 553 MPa
minimal stress: 0.21 MPa

(b) Stress (Mises) in N/m^2 max. 553 N/m^2

Figure 2: Submodel with 1mm fillet

In the submodel with the 1mm fillet, we observe an evenly distributed stress pattern. Also, the maximum value for stress has decreased by about 50 MPa. From a mechanical point of view, the round corner offers a smoother distribution of internal stresses because the lines of forces in a material are not interrupted. However, the maximum stress is still the yield stress of our material, we still expect it to deform.

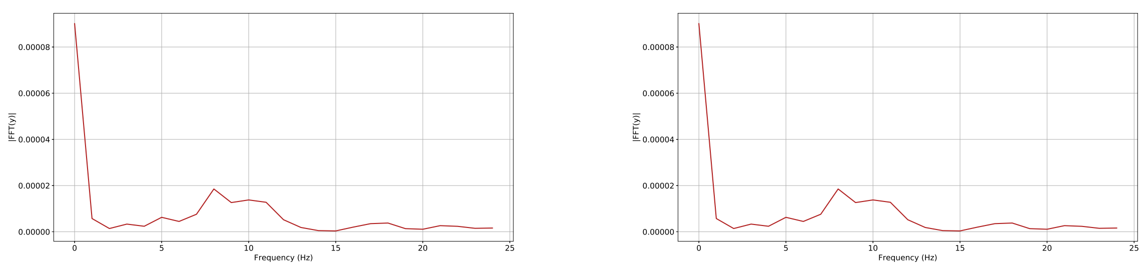


(a) Path around the inner corner

(b) Stress (Mises) in N/m^2 around the inner corner max. approx. 550 N/m^2

Figure 3: Submodel with 1mm fillet

We observe that the stresses are highest in the apex of the corner and decrease rapidly with distance. The max stress of 553 Mpa is not reached in this plot because it represents only one force line.



(a) maximal stress: 549 MPa
minimal stress: 0.23 MPa

(b) Stress (Mises) in N/m^2 around the inner corner max. approx. 500 N/m^2

Figure 4: Submodel with 5mm fillet

The 5mm fillet causes a minor decrease in maximum stress value. However, the region of high stresses is elongated and doesn't reach as deep into the beam compared to the smaller fillet.

3.1 Conclusion

The introduction of a fillet in a sharp corner offers a significant decrease in maximum stresses. Even a small fillet as a big impact on the model, respectively on the result output. Even though the yield stress of the material is still below the maximum stresses (in this exercise), we recognize the notable impact to be crucial for any construction project. In order to achieve a suitable mesh around big fillets, a repartitioning of the model can be necessary.

References

[1] Michel Goossens, Frank Mittelbach, and Alexander Samarin. *The LaTeX Companion*. Addison-Wesley, Reading, Massachusetts, 1993.

[2] Albert Einstein. *Zur Elektrodynamik bewegter Körper*. (German) [*On the electrodynamics of moving bodies*]. Annalen der Physik, 322(10):891–921, 1905.

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