Assignment 3

Nalet Meinen and Pascal Wyss
Finite Element Analysis I

April 29, 2019

Abstract

In this assignment we examine the behaviour of a beam after being subjected to an impulse. The goal is to extract the resonance frequency of the beam with the given dimensions and material coefficients.

Contents

1 Introduction

Measuring the uniaxial true stress/strain relation is the most fundamental task before any plastic analysis can be made. It defines the strain hardening of a material and determines the general stress-strain relation when plastic deformation is taking place. Plane strain test is a method of choice to obtain this information.

During plane strain compression test, a narrow band of the material is compressed by narrow plates (see Figure). The constraints on each side of the plate prevent deformations of the material in the width dimension.

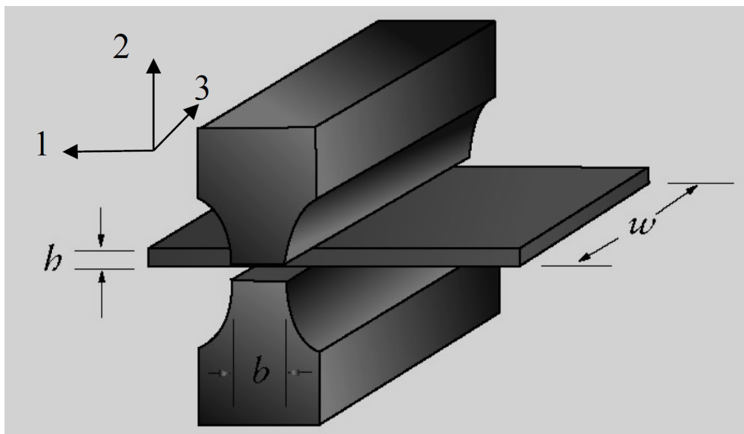


Figure 1: Compression of the aluminum plate

2 Methods

2.1 Analytical solution

In order to ensure the validity of the plane strain assumption, the following relationships have to be fulfilled:

$$w > 5h,\ w > 5b,\ 2h < b < 4h \tag{1}$$

The variables in (1) are the same as in Figure 2

In our situation, the true stress and true strain are given by:

$$\varepsilon_1 = -\varepsilon_2,\ \varepsilon_2 = ln(\frac{h}{h_x}),\ \varepsilon_3 = 0,\ \sigma_1 = 0,\ \sigma_2 = \frac{P}{wb},\ \sigma_3 = \frac{\sigma_2}{2} \tag{2}$$

And the equivalent (Mises) stress and strain are defined as:

$$\bar{\varepsilon} = \frac{2}{\sqrt{3}}\varepsilon_2,\ \bar{\sigma} = \frac{\sqrt{3}}{2}\sigma_2 \tag{3}$$

Plastic deformation of aluminium. In this case, the stress-strain relation is given by:

$$\bar{\varepsilon} = \begin{cases} \frac{\bar{\sigma}}{E} & ,\bar{\sigma} < \sigma_0 \\ \frac{\sigma_0}{E} + \frac{\sigma_0}{B}(\frac{\bar{\sigma}}{\sigma_0} - 1)^n & ,\bar{\sigma} >= \sigma_0 \end{cases} \tag{4}$$

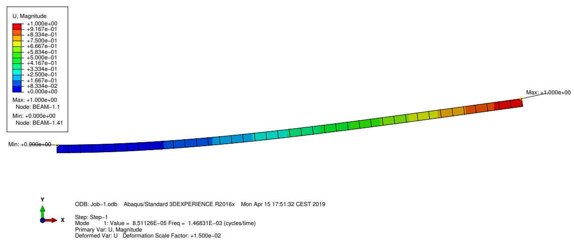
Where:

E = 70 GPa
σ_0 = 220 MPa
B = 3 GPa
n = 3.2
v = 0.3

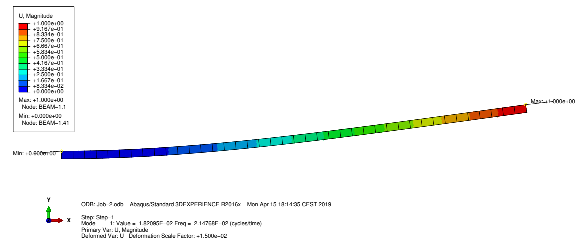
Using our values for $\bar{\sigma} >= \sigma_0$ in equation 4:

$$\frac{2}{\sqrt{3}} ln(\frac{h}{h_x}) = \frac{220 \cdot 10^6 Pa}{70 \cdot 10^9 Pa} + \frac{220 \cdot 10^6 Pa}{3 \cdot 10^9 Pa}(\frac{\bar{\sigma}}{220 \cdot 10^6 Pa} - 1)^{3.2} \tag{5}$$

2.2 Frequency mode on Beam

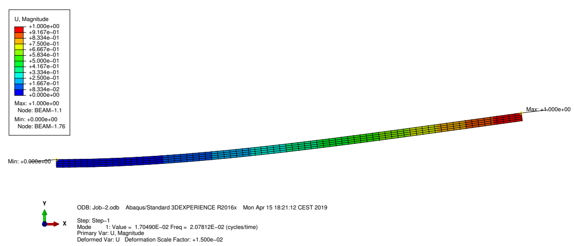


(a) calculated frequency with CPS4R: 14.6 Hz

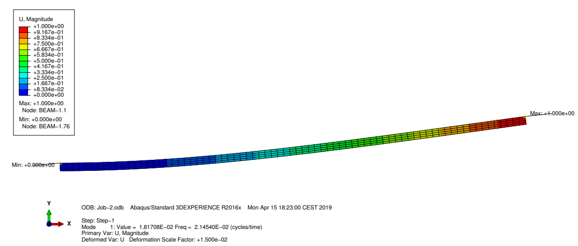


(b) calculated frequency with CPS4R: 21.4 Hz

Figure 2: Lower mesh count



(a) calculated frequency with CPS4R: 20.7 Hz



(b) calculated frequency with CPS4R: 21.4 Hz

Figure 3: Higher mesh count

Figure 2 shows us the lower mesh count. With (a) using a linar meshing compared to a quadratic one, we immediately see the better results. This comperation was also discussed in assignment 1. With a higher mesh count, the differences between the linear and the quadratic method is no so different anymore.

2.3 Generating an impulse

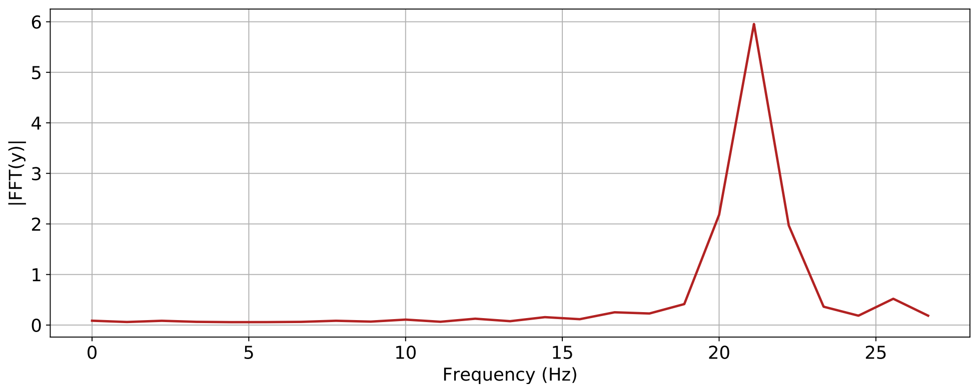


Figure 4: Fourier transformation

The impulse on the beam is generated by applying a concentrated force load on the extremity of the beam (the non-encastered end). This load is being applied for a short time period (1ms). The main advantage is the output, we can not only see one number. With the Fourier transformation we can also see the distribution of the frequencies and seeing the correct resonannce frequency in the peek of the plot.

3 Results and Discussion

In the submodel with the 1mm fillet, we observe an evenly distributed stress pattern. Also, the maximum value for stress has decreased by about 50 MPa. From a mechanical point of view, the round corner offers a smoother distribution of internal stresses because the lines of forces in a material are not interrupted. However, the maximum stress is still the yield stress of our material, we still expect it to deform.

References

[1] Michel Goossens, Frank Mittelbach, and Alexander Samarin. *The LaTeX Companion*. Addison-Wesley, Reading, Massachusetts, 1993.

[2] Albert Einstein. *Zur Elektrodynamik bewegter Körper*. (German) [*On the electrodynamics of moving bodies*]. Annalen der Physik, 322(10):891–921, 1905.

[3] Knuth: Computers and Typesetting, `http://www-cs-faculty.stanford.edu/~uno/abcde.html`