Assignment 3

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Finite Element Analysis I

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Abstract

In this assignment we examine the behaviour of a beam after being subjected to an impulse. The goal is to extract the resonance frequency of the beam with the given dimensions and material coefficients.

Contents

1 Introduction

The goal of this assignment is the analyzation of a beam attached to a wall and how it is related to its resonance frequencies. We want to show the analytical solution and after the model implemented in Abaqus. An example of this calculation can be used in dental implants, as the beginns to oscillate under given circumstances.

2 Methods

2.1 Analytical solution

Consider a beam with the following dimensions attached to a wall in our case on the left end:

$$L = 150mm, h = 2.5mm, b = 20mm, E = 10GPa, v = 0.3, \rho = 7.0E3kg.m^{-3} \tag{1}$$

The resonance frequencies (inducing a bending in the direction of the dimension h) of such a model are given by the formula:

$$\frac{1}{2\pi\sqrt{12}}\alpha_i^2\sqrt{\frac{E}{\rho}}\frac{h}{L^2} \tag{2}$$

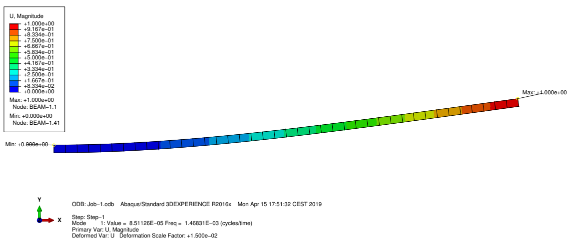
Where:

$\alpha_0 = 1.875, \alpha_1 = 4.695, \alpha_2 = 7.85, ..., \alpha_i = (2i+1)\frac{\pi}{2} \ \ for \, i > 2$
E = Young's modulus
ρ = density of the material
h = height of the cross-section of the beam
L = length of the beam

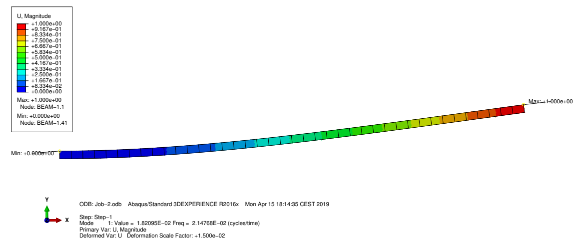
Using our values for α_0:

$$f_0 = \frac{1}{2\pi\sqrt{12}}1.875^2\sqrt{\frac{10 \cdot 10^3 Pa}{7 \cdot 10^3 \frac{kg}{m^3}}}\frac{2.5 \cdot 10^3 m}{(150 \cdot 10^3 m)^2} = 21.451\, Hz \tag{3}$$

2.2 Frequency mode on Beam

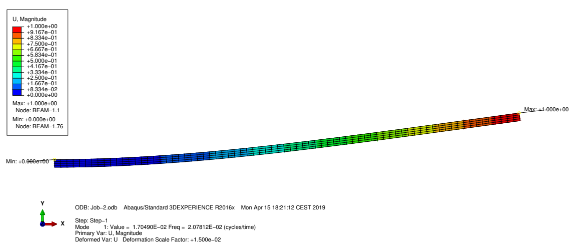


(a) calculated frequency with CPS4R: 14.6 Hz

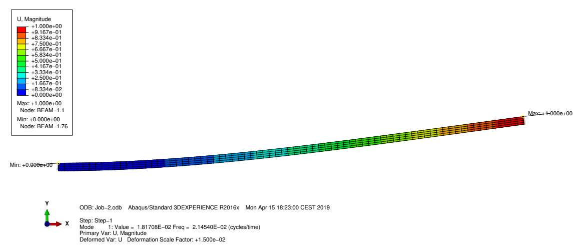


(b) calculated frequency with CPS4R: 21.4 Hz

Figure 1: Lower mesh count



(a) calculated frequency with CPS4R: 20.7 Hz



(b) calculated frequency with CPS4R: 21.4 Hz

Figure 2: Higher mesh count

Figure 1 shows us the lower mesh count. With (a) using a linar meshing compared to a quadratic one, we immediately see the better results. This comperation was also discussed in assignment 1. With a higher mesh count, the differences between the linear and the quadratic method is no so different anymore.

2.3 Generating an impulse

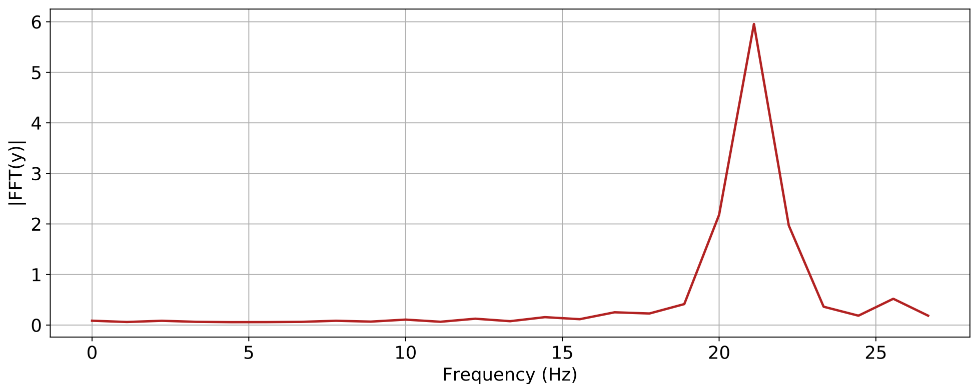


Figure 3: Fourier transformation

The impulse on the beam is generated by applying a concentrated force load on the extremity of the beam (the non-encastered end). This load is being applied for a short time period (1ms). The main advantage is the output, we can not only see one number. With the Fourier transformation we can also see the distribution of the frequencies and seeing the correct resonannce frequency in the peek of the plot.

3 Results and Discussion

In the submodel with the 1mm fillet, we observe an evenly distributed stress pattern. Also, the maximum value for stress has decreased by about 50 MPa. From a mechanical point of view, the round corner offers a smoother distribution of internal stresses because the lines of forces in a material are not interrupted. However, the maximum stress is still the yield stress of our material, we still expect it to deform.

References

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