Assignment 3

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Finite Element Analysis I

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Abstract

In this assignment we examine the behaviour of a beam after being subjected to an impulse. The goal is to extract the resonance frequency of the beam with the given dimensions and material coefficients.

Contents

1 Introduction

The goal of this assignment is the analysis of a beam attached to a wall, and how it is related to its resonance frequencies. We want to show the analytical solution and afterwards the model implemented in Abaqus. A possible application of this calculation can be used e.g. in dental implants, as those implants could begin to oscillate under given circumstances.

2 Methods

2.1 Analytical solution

Consider a beam with the following dimensions attached to a wall, in our case, on the lefthand end:

$$L = 150mm, h = 2.5mm, b = 20mm, E = 10GPa, v = 0.3, \rho = 7.0E3kg.m^{-3} \tag{1}$$

The resonance frequencies (inducing a bending in the direction of the dimension h) of such a model are given by the formula:

$$\frac{1}{2\pi\sqrt{12}}\alpha_i^2\sqrt{\frac{E}{\rho}\frac{h}{L^2}} \tag{2}$$

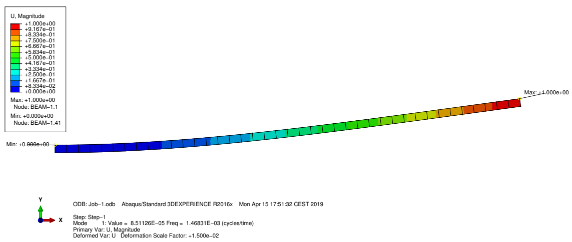
Where:

$\alpha_0 = 1.875, \alpha_1 = 4.695, \alpha_2 = 7.85, ..., \alpha_i = (2i+1)\frac{\pi}{2} \;\; for\, i > 2$
E = Young's modulus
ρ = density of the material
h = height of the cross-section of the beam
L = length of the beam

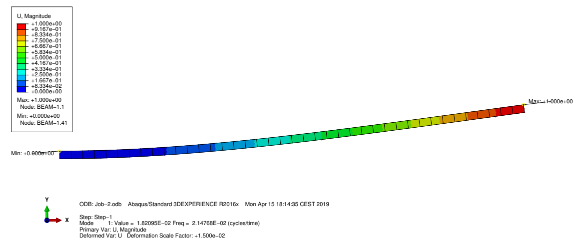
Using our values for α_0:

$$f_0 = \frac{1}{2\pi\sqrt{12}}1.875^2\sqrt{\frac{10\cdot 10^9 Pa}{7\cdot 10^3\frac{kg}{m^3}}\frac{2.5\cdot 10^{-3}\,m}{(150\cdot 10^{-3}\,m)^2}} = 21.451\,Hz \tag{3}$$

2.2 Frequency mode on Beam

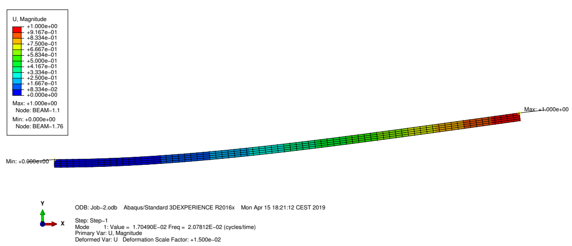


(a) calculated frequency with CPS4R: 14.6 Hz

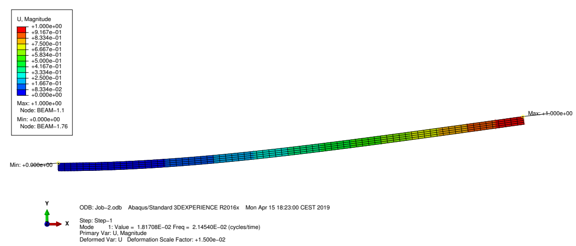


(b) calculated frequency with CPS8R: 21.4 Hz

Figure 1: Lower mesh count



(a) calculated frequency with CPS4R: 20.7 Hz



(b) calculated frequency with CPS8R: 21.4 Hz

Figure 2: Higher mesh count

Figure 1 shows us the results using a lower mesh count. With (a) using a linar meshing compared to a quadratic one, we immediately see the better results. This comparison was also discussed in assignment 1. With a higher mesh count (Figure 2), the differences between the linear and the quadratic method is less significant.

2.3 Generating an impulse

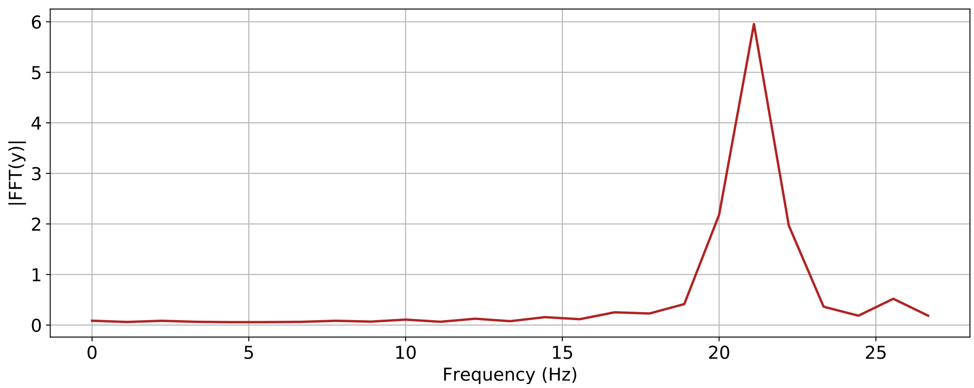


Figure 3: Fourier transformation

The impulse on the beam is generated by applying a concentrated force load on the extremity of the beam (the non-encastered end). This load is being applied for a short time period (1ms). Abaqus gives us the resonance frequency as one single number. We then export the X/Y data (obtained at the node where the force is applied) into Python to perform a forward Fourier Transform. The main advantage of this procedure is an output that shows not only one number, but the distribution of resonance frequencies. We observe a peak at around 21Hz.

3 Results and Discussion

...The fourier transform gives a much better analysis regarding the resonance frequencies.

References

[1] Michel Goossens, Frank Mittelbach, and Alexander Samarin. *The LaTeX Companion*. Addison-Wesley, Reading, Massachusetts, 1993.

[2] Albert Einstein. *Zur Elektrodynamik bewegter Körper*. (German) [*On the electrodynamics of moving bodies*]. Annalen der Physik, 322(10):891–921, 1905.

[3] Knuth: Computers and Typesetting, `http://www-cs-faculty.stanford.edu/~uno/abcde.html`