

# Assignment 4

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Finite Element Analysis I

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## Abstract

In this assignment we examine the behaviour of a beam after being subjected to an impulse. The goal is to extract the resonance frequency of the beam with the given dimensions and material coefficients.

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# 1 Introduction

Measuring the uniaxial true stress/strain relation is the most fundamental task before any plastic analysis can be made. It defines the strain hardening of a material and determines the general stress-strain relation when plastic deformation is taking place. Plane strain test is a method of choice to obtain this information.

During plane strain compression test, a narrow band of the material is compressed by narrow plates (see Figure). The constraints on each side of the plate prevent deformations of the material in the width dimension.

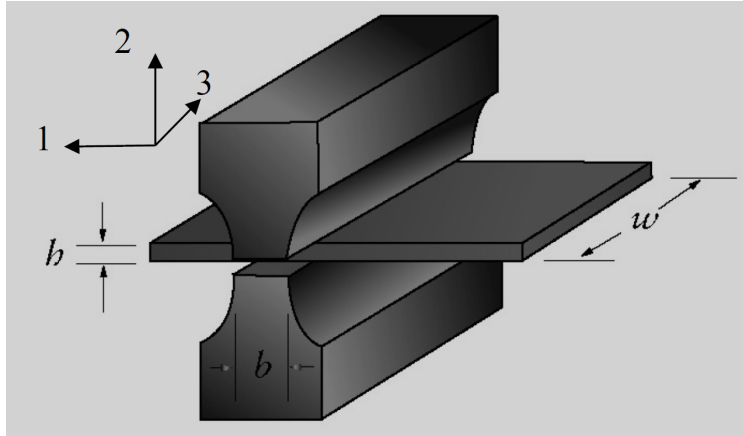


Figure 1: Compression of the aluminum plate

## 2 Methods

### 2.1 Analytical solution

In order to ensure the validity of the plane strain assumption, the following relationships have to be fulfilled:

$$w > 5h, w > 5b, 2h < b < 4h \quad (1)$$

The variables in (1) are the same as in Figure 1

With those constraints we assume the following proportions for our model in the analytical solution and later with abaqus:

$$w = 2m, b = 0.3m, h = 0.2m \quad (2)$$

In our situation, the true stress and true strain are given by:

$$\varepsilon_1 = -\varepsilon_2, \varepsilon_2 = \ln\left(\frac{h}{h_x}\right), \varepsilon_3 = 0, \sigma_1 = 0, \sigma_2 = \frac{P}{wb}, \sigma_3 = \frac{\sigma_2}{2} \quad (3)$$

And the equivalent (Mises) stress and strain are defined as:

$$\bar{\varepsilon} = \frac{2}{\sqrt{3}}\varepsilon_2, \bar{\sigma} = \frac{\sqrt{3}}{2}\sigma_2 \quad (4)$$

Plastic deformation of aluminium. In this case, the stress-strain relation is given by:

$$\bar{\varepsilon} = \begin{cases} \frac{\bar{\sigma}}{E} & , \bar{\sigma} < \sigma_0 \\ \frac{\sigma_0}{E} + \frac{\sigma_0}{B} \left( \frac{\bar{\sigma}}{\sigma_0} - 1 \right)^n & , \bar{\sigma} \geq \sigma_0 \end{cases} \quad (5)$$

Where:

$$\begin{aligned} E &= 70 \text{ GPa} \\ \sigma_0 &= 220 \text{ MPa} \\ B &= 3 \text{ GPa} \\ n &= 3.2 \\ v &= 0.3 \end{aligned}$$

Using our values for  $\bar{\sigma} \geq \sigma_0$  in equation 5:

$$\frac{2}{\sqrt{3}} \ln\left(\frac{200 \cdot 10^{-3}m}{h_x}\right) = \frac{220 \cdot 10^6 Pa}{70 \cdot 10^9 Pa} + \frac{220 \cdot 10^6 Pa}{3 \cdot 10^9 Pa} \left( \frac{\bar{\sigma}}{220 \cdot 10^6 Pa} - 1 \right)^{3.2} \quad (6)$$

Or more simple:

$$\left( \sqrt[n]{\left( \bar{\varepsilon} - \frac{\sigma_0}{E} \right) \frac{B}{\sigma_0}} + 1 \right) \sigma_0 = \bar{\sigma} \quad (7)$$

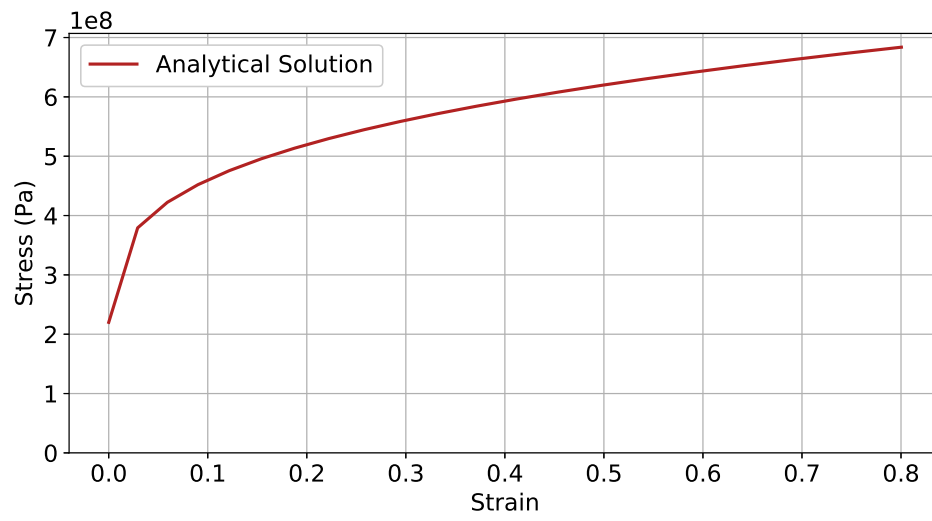


Figure 2: Result of the analytical calculations

## 2.2 Frequency mode on Beam

### 3 Results and Discussion

In the submodel with the 1mm fillet, we observe an evenly distributed stress pattern. Also, the maximum value for stress has decreased by about 50 MPa. From a mechanical point of view, the round corner offers a smoother distribution of internal stresses because the lines of forces in a material are not interrupted. However, the maximum stress is still the yield stress of our material, we still expect it to deform.

## References

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