Assignment 1

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Finite Element Analysis I

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1 Regular Tesselation

1.1

There are three shapes that satisfy the two conditions above: triangles, squares and regular hexagon.

1.2

Theorem 1. *There are three shapes that satisfy the two conditions above: triangles, squares and regular hexagon.*

Proof. The sum of the angles in a poligon is $180(a-2)$, a is the number of angles. Using the three polygons from above we know that in all three polygons the sum of the angles where the vertices meet is 360, also point b. By that circumstances we can use

$$\frac{180(a-2)}{a}b = 360$$

In a simple matter this leads us to

$$(a-2)b = 2a$$

The result of the equation above leads us to 5 solutions:

$$a = -2, b = 1; a = 1, b = -2, a = 3, b = 6; a = 4, b = 4; a = 6, b = 3$$

As can only use the positives integer soultions, we have (b corresponds number of edges) the polygons with 3,4 and 6 edges. $\square$

2 Lloyd-Max quantization

2.1

Following the notes quanzization from the lecture: We can compute the partial derivatives with respect to z_k

$$\delta = \sum_{k=1}^{K} \int_{z_k}^{z_{k+1}} (z - q_k)^2 p(z) dz$$

$$\frac{\partial \delta}{\partial z_k} = (z_k - q_{k+1})^2 p(z_k) - (z_k - q_k)^2 p(z_k) = 0$$

$$(z_k - q_{k+1})^2 p(z_k) - (z_k - q_k)^2 p(z_k) = 0$$

$$(z_k - q_{k+1})^2 - (z_k - q_k)^2 = 0$$

$$z_k^2 - 2q_{k+1} z_k + q_{k+1}^2 - z_k^2 - q_k z_k + q_k^2 = 0$$

$$2z_k(q_{k+1} - q_k) + q_{k+1}^2 + q_k^2 = 0$$

$$2z_k = \frac{q_{k+1}^2 + q_k^2}{q_{k+1} - q_k} \implies 2z_k = \frac{q_{k-1}^2 + q_k^2}{q_{k-1} + q_k}$$

$$2z_k = \frac{q_{k-1} + q_k}{1}$$

$$z_k = \frac{q_{k-1} + q_k}{2}$$

2.2

$$\delta = \sum_{k=1}^{K} \int_{z_k}^{z_{k+1}} (z - q_k)^2 p(z) dz$$

$$\frac{\partial \delta}{\partial q_k} = \int_{z_k}^{z_{k+1}} (z - q_k)^2 p(z) dz = 0$$

$$\frac{\partial \delta}{\partial q_k} \int_{z_k}^{z_{k+1}} z^2 p(z) - 2q_k z p(z) + q_k^2 p(z) dz = 0$$

$$\frac{\partial \delta}{\partial q_k} (q_k \int_{z_k}^{z_{k+1}} -2zp(z)dz + q_k^2 \int_{z_k}^{z_{k+1}} p(z)dz) = 0$$

$$\int_{z_k}^{z_{k+1}} -2zp(z)dz + 2q_k \int_{z_k}^{z_{k+1}} p(z)dz = 0$$

$$q_k = \frac{\int_{z_k}^{z_{k+1}} 2zp(z)dz}{2\int_{z_k}^{z_{k+1}} p(z)dz}$$

$$q_k = \frac{\int_{z_k}^{z_{k+1}} zp(z)dz}{\int_{z_k}^{z_{k+1}} p(z)dz}$$

2.3

lloyd will give use the converge, but the solution could be a local minimum. The goal is a optimum quantizer, even distributed groups. Both equations converge in a coordinate descen fashion.

3 Chamfer distances

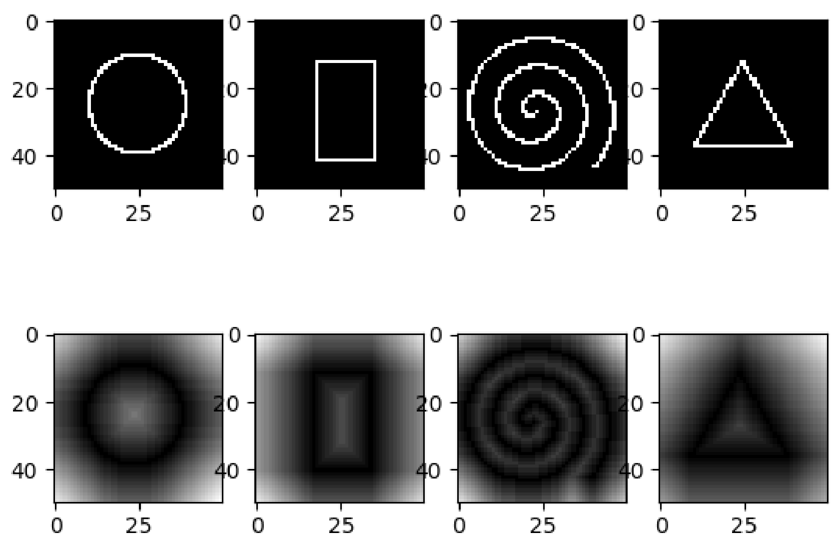

Figure 1: Result of the chamfer distances in given images.

4 Bilinear interpolation

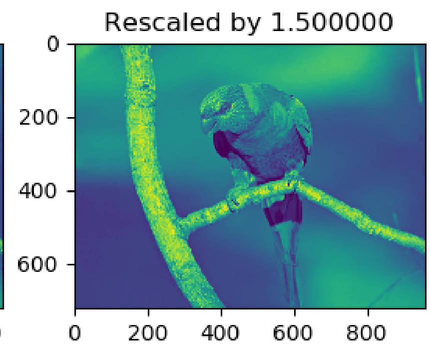

Figure 2: Result of the bilinear interpolation in the image bird.jpg.