Assignment 3

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Finite Element Analysis I

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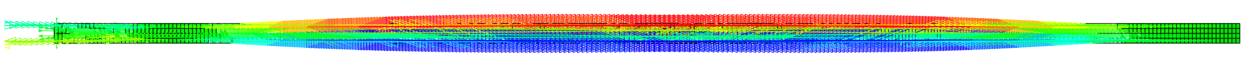


Abstract

In this assignment we examine the behaviour of a beam after being subjected to an impulse. The goal is to extract the resonance frequency of the beam with the given dimensions and material coefficients.

Contents

1 Introduction

The goal of this assignment is the analysis of a beam attached to a wall, and how it is related to its resonance frequencies. We want to show the analytical solution and afterwards the model implemented in Abaqus. A possible application of this calculation can be used e.g. in dental implants, as those implants could begin to oscillate under given circumstances.

2 Methods

2.1 Analytical solution

Consider a beam with the following dimensions attached to a wall, in our case, on the lefthand end:

$$L = 150mm, h = 2.5mm, b = 20mm, E = 10GPa, v = 0.3, \rho = 7.0E3kg.m^{-3} \tag{1}$$

The resonance frequencies (inducing a bending in the direction of the dimension h) of such a model are given by the formula:

$$\frac{1}{2\pi\sqrt{12}}\alpha_i^2\sqrt{\frac{E}{\rho}\frac{h}{L^2}} \tag{2}$$

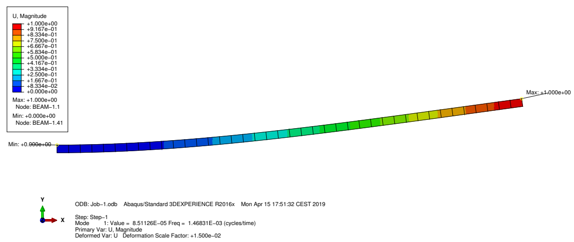
Where:

$\alpha_0 = 1.875, \alpha_1 = 4.695, \alpha_2 = 7.85, ..., \alpha_i = (2i+1)\frac{\pi}{2} \;\; for\, i > 2$
E = Young's modulus
ρ = density of the material
h = height of the cross-section of the beam
L = length of the beam

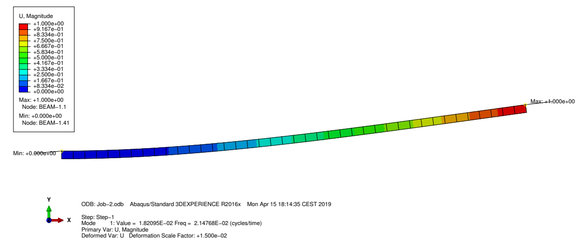
Using our values for α_0:

$$f_0 = \frac{1}{2\pi\sqrt{12}}1.875^2\sqrt{\frac{10 \cdot 10^9 Pa}{7 \cdot 10^3 \frac{kg}{m^3}}\frac{2.5 \cdot 10^{-3}\, m}{(150 \cdot 10^{-3}\, m)^2}} = 21.451\, Hz \tag{3}$$

2.2 Frequency mode on Beam

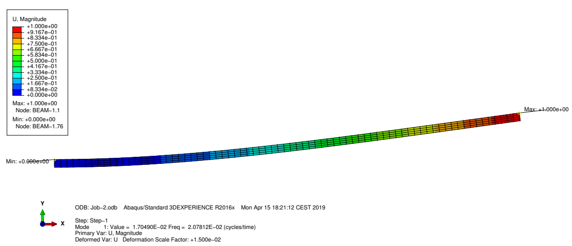


(a) calculated frequency with CPS4R: 14.6 Hz

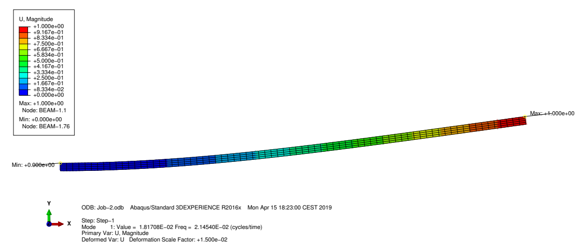


(b) calculated frequency with CPS8R: 21.4 Hz

Figure 1: Lower mesh count



(a) calculated frequency with CPS4R: 20.7 Hz



(b) calculated frequency with CPS8R: 21.4 Hz

Figure 2: Higher mesh count

Figure 1 shows us the results using a lower mesh density. With (a) using a linar meshing compared to a quadratic one, we immediately see the better results. This comparison was also discussed in assignment 1. With a higher mesh density (Figure 2), the differences between the linear and the quadratic method is less significant.

2.3 Generating an impulse

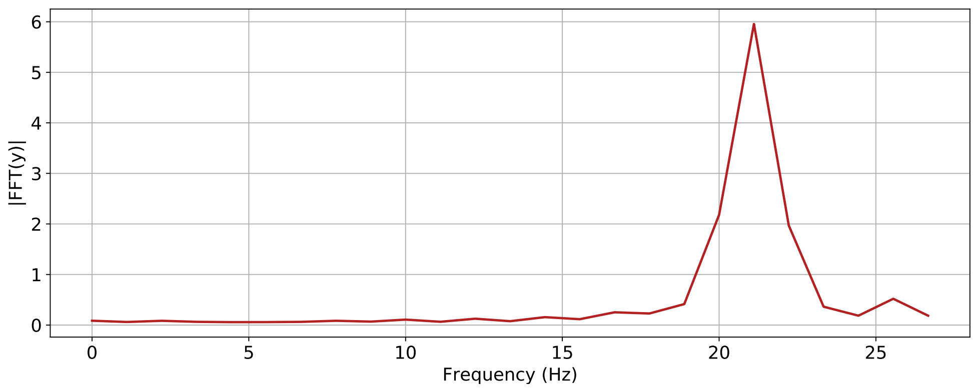


Figure 3: Fourier transformation

The impulse on the beam is generated by applying a concentrated force load on the extremity of the beam (the non-encastered end). This load is being applied for a short time period (1ms). Abaqus gives us the resonance frequency as one single number. We then export the X/Y data (obtained at the node where the force is applied) into Python to perform a forward fourier transform. The main advantage of this procedure is an output that shows not only one number, but the distribution of resonance frequencies. We observe a peak at around 21Hz. This supports our analytical solution from section 2.1.

3 Results and Discussion

The fourier transform gives a much better analysis regarding the resonance frequencies, than just a single number output from Abaqus. Possible secondary frequency-peaks can also be identified. The FE simulation in comparison offers only a limited insight. The obtainable and exportable data however depends highly on the elements used and the applied mesh density. Additionaly, we observed that the natural frequency of the beam does not change with impulse intensity[1]. However the ackquired data shows a more distinguished peak.

References

[1] Barry J. Goodno, James McGere *Mechanics of Materials (2018)*. (English)

[2] Michel Goossens, Frank Mittelbach, and Alexander Samarin. *The LaTeX Companion*. Addison-Wesley, Reading, Massachusetts, 1993.