

Homework 1

Nalet Meinen
Introduction to Signal and Image Processing

March 19, 2019

1 Regular Tessellation

1.1

There are three shapes that satisfy the two conditions above: triangles, squares and regular hexagon.

1.2

Theorem 1. *There are three shapes that satisfy the two conditions above: triangles, squares and regular hexagon.*

Proof. The sum of the angles in a polygon is $180(a - 2)$, a is the number of angles. Using the three polygons from above we know that in all three polygons the sum of the angles where the vertices meet is 360, also point b . By that circumstances we can use

$$\frac{180(a - 2)}{a}b = 360$$

In a simple matter this leads us to

$$(a - 2)b = 2a$$

The result of the equation above leads us to 5 solutions:

$$a = -2, b = 1; a = 1, b = -2; a = 3, b = 6; a = 4, b = 4; a = 6, b = 3$$

As can only use the positives integer solutions, we have (b corresponds number of edges) the polygons with 3, 4 and 6 edges. \square

2 Lloyd-Max quantization

2.1

Following the notes quantization from the lecture: We can compute the partial derivatives with respect to z_k

$$\begin{aligned}\delta &= \sum_{k=1}^K \int_{z_k}^{z_{k+1}} (z - q_k)^2 p(z) dz \\ \frac{\partial \delta}{\partial z_k} &= (z_k - q_{k+1})^2 p(z_k) - (z_k - q_k)^2 p(z_k) = 0 \\ (z_k - q_{k+1})^2 p(z_k) - (z_k - q_k)^2 p(z_k) &= 0 \\ (z_k - q_{k+1})^2 - (z_k - q_k)^2 &= 0 \\ z_k^2 - 2q_{k+1}z_k + q_{k+1}^2 - z_k^2 - q_k z_k + q_k^2 &= 0 \\ 2z_k(q_{k+1} - q_k) + q_{k+1}^2 + q_k^2 &= 0 \\ 2z_k &= \frac{q_{k+1}^2 + q_k^2}{q_{k+1} - q_k} \implies 2z_k = \frac{q_{k-1}^2 + q_k^2}{q_{k-1} + q_k} \\ 2z_k &= \frac{q_{k-1} + q_k}{1} \\ z_k &= \frac{q_{k-1} + q_k}{2}\end{aligned}$$

2.2

$$\begin{aligned}\delta &= \sum_{k=1}^K \int_{z_k}^{z_{k+1}} (z - q_k)^2 p(z) dz \\ \frac{\partial \delta}{\partial q_k} &= \int_{z_k}^{z_{k+1}} (z - q_k)^2 p(z) dz = 0 \\ \frac{\partial \delta}{\partial q_k} \int_{z_k}^{z_{k+1}} z^2 p(z) - 2q_k z p(z) + q_k^2 p(z) dz &= 0 \\ \frac{\partial \delta}{\partial q_k} (q_k \int_{z_k}^{z_{k+1}} -2z p(z) dz + q_k^2 \int_{z_k}^{z_{k+1}} p(z) dz) &= 0 \\ \int_{z_k}^{z_{k+1}} -2z p(z) dz + 2q_k \int_{z_k}^{z_{k+1}} p(z) dz &= 0 \\ q_k &= \frac{\int_{z_k}^{z_{k+1}} 2z p(z) dz}{2 \int_{z_k}^{z_{k+1}} p(z) dz} \\ q_k &= \frac{\int_{z_k}^{z_{k+1}} z p(z) dz}{\int_{z_k}^{z_{k+1}} p(z) dz}\end{aligned}$$

2.3

lloyd will give use the converge, but the solution could be a local minimum. The goal is a optimum quantizer, even distributed groups.

3 Chamfer distances

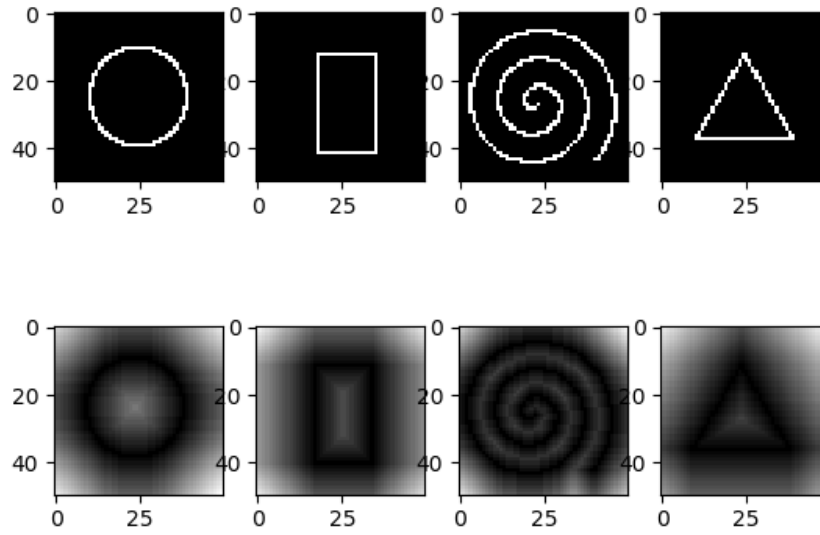


Figure 1: Result of the chamfer distances in given images.

4 Bilinear interpolation

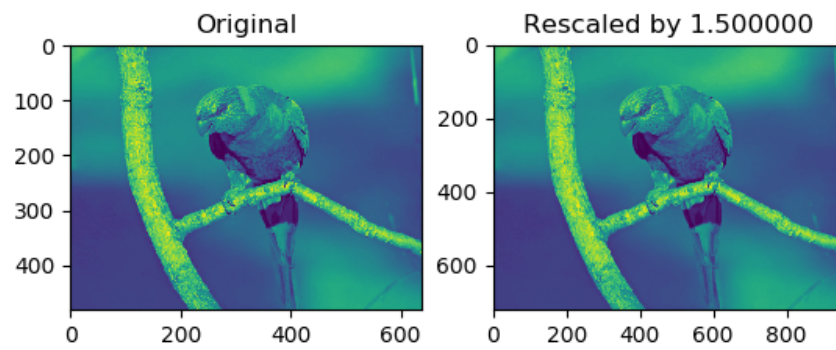


Figure 2: Result of the bilinear interpolation in the image bird.jpg.