Review Assignment 1

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1 Linear algebra review

1. S = $\{v_1, ..., v_n\}$ be an orthogonal set of non-zero vectors in R^n. Prove that the vectors in S are linearly independent.

We assume a linear combination

$$c_1 v_1 + c_2 v_2 + ... + c_k v_k = 0$$

We want to show that

$$c_1 = c_2 = ... = 0$$

The dot product of v_i for each $i = 1, 2, ..., k$:

$$\begin{aligned} 0 &= v_i \cdot 0 \\ &= v_i \cdot (c_1 v_1 + c_2 v_2 + ... + c_k v_k) \\ &= c_1 v_i \cdot v_1 + c_2 v_i \cdot v_2 + ... + c_k v_i \cdot v_k \end{aligned}$$

S is an orthogonal set, we have $v_i \cdot v_j = 0 \, \text{if} \, i \neq j$, then we have:

$$0 = c_i v_i \cdot v_i = c_i \|v_i\|^2$$

v_i is nonzero and length $\|v_i\|$ is nonzero, following that $c_i = 0$
We conclude that $c_1 v_1 + c_2 v_2 + ... + c_k v_k = 0$ for every $i = 1, 2, ..., k$, so S is **linearly independent**

2. Given a square matrix $A \in \mathbb{R}^{n \times n}$ and a vector $x \in \mathbb{R}^n$ show that $x^\intercal A x = x^\intercal(\frac{1}{2}A + \frac{1}{2}A^\intercal)x$.

We assume that:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} x = \begin{pmatrix} b_1 & b_2 & \dots & b_m \end{pmatrix} \quad where\, m = n$$

The transposed values are:

$$A^\intercal = \begin{bmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{bmatrix} x^\intercal = \begin{pmatrix} b_1 & b_2 & \dots & b_m \end{pmatrix}$$

We want to show that this equation is true:

$$x^\intercal A x = x^\intercal (\frac{1}{2} A + \frac{1}{2} A^\intercal) x$$

If we insert the matrices:

$$\begin{pmatrix} b_1 & b_2 & \dots & b_m \end{pmatrix} \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{pmatrix} b_1 & b_2 & \dots & b_m \end{pmatrix} = \begin{pmatrix} b_1 & b_2 & \dots & b_m \end{pmatrix} (\frac{1}{2} \begin{bmatrix} a_{11} & a_{12} & \dots \\ a_{21} & a_{22} & \dots \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots \end{bmatrix}$$