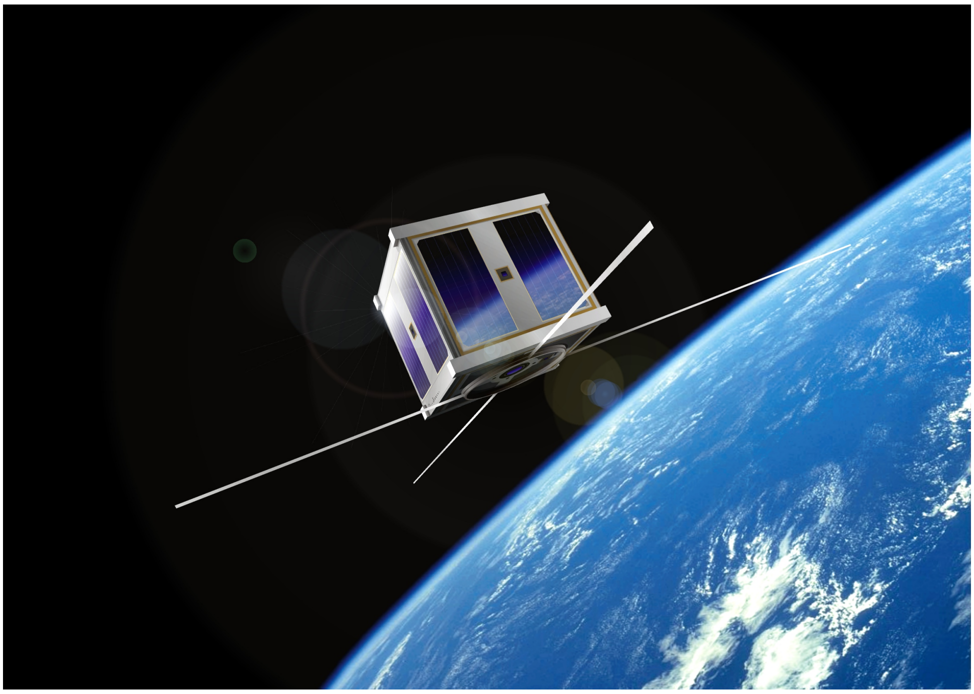
Formation flying using pico-satellites



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Synopsis

Abstract goes here

Preface

This report has been written by group 931 on third semester in Control and Automation on Aalborg University. References made before a full stop regards the sentence and reference after full stop regards the paragraph. Quotes are inside quotations marks and in cursive. Attached to report is a zip file with:

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Contents

1 | Introduction

[1] Intro [...]

1.1 Problem statement

Design and implement a controller for controlling the individual distance between satellites using the drag force.

1.2 Use-case

In this project the concept of a formation flight of satellites will be used for the purpose of monitoring. Denmark has a small island called Greenland, where the Danish Government needs to monitor it. One method is to have a formation of satellites going around the orbit and when they are located in the northern hemisphere, the satellites will point down and look towards Greenland.

One of the essentials in formation flight is choosing the number of satellites in orbit. Therefore, in order to have a continuous coverage, a distributed satellite system composed of six satellites equally distributed are chosen, compared with two or four satellites where communication between each other will be poor.

The task the satellite has to perform is acquiring data by flying around Greenland, using radio signals and taking pictures.

2 | System Description

The overall idea of the project is to consider more than one satellites flying in formation, with a certain distance in between and with the purpose of maintaining that distance by using the drag force. As a proof of concept, an AAU-CubeSat will be used, by choosing six AAU-CubeSat that orbit the Earth like is shown in *figure 2.1*. Therefore, a control system is developed, where the six satellites are nodes and they represent periods. Each satellite can only communicat with his two neighbour. In this project, all CubeSat's will be assumed identical, where each satellite needs to fulfill a few requirements stated in *chapter 3*. Moreover, a full-scale implementation of the system will not be possible, therefore, the whole system will be simulated using MATLAB and Simulink.

Figure 2.1: Six satellites in flying formation on orbit

About AAU-CubeSat

The AAU-CubeSat shown in *figure 2.2* is a pico-satellite developed by Stanford University, but assembled at Aalborg University by students and used mainly for Low Earth Orbit (LEO) tests.

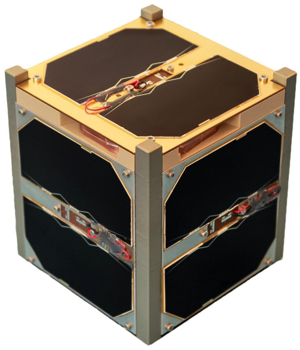

Figure 2.2: View of CubeSat satellite

The pico-satellite is designed for LEO, therefore a few constraints are imposed. The CubeSat is limited in size and weight. The dimensions of the satellite are $10cm \times 10cm \times 30cm$, while the weight is around 1 kg. [1]

In order place the CubeSat on the orbit, a deployment system is used, called P-POD [2] This system uses the force of a spring to launch the satellite into space. The satellite will be placed inside the launch rocket as payload. By using this system, an important advantage is reducing the cost of the launch.

AAU-CubeSat actuators

The selection of attitude control components is important in order to meet the performance requirements. For this project, three magnetorquers and three momentum wheels have been chosen as actuators. Initially, using only three momentum wheels has been considered, but the downside of using only momentum wheels is that some amount of momentum can be stored in the wheel, which will imply having a way to take back all that momentum and use it. Therefore, there are multiple ways to release that torque, and one is to use magnetorquers.

Magnetorquers are wire coils which generate an electromagnetic field. The field interacts with the Earth magnetic field and a torque is generated for stabilizing the satellite. An important aspect of the magnetorquer is when the momentum wheel reaches a maximum speed and can no longer produce the torque (this is referred as wheel saturation'), so a magnetorquer is used to extract the momentum from the wheel.

[1] FiXme Note: ref
[2] FiXme Note: ref

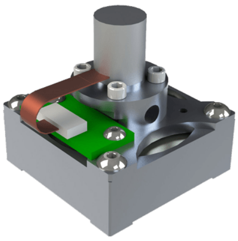

Figure 2.3: Example of a momentum wheel for CubeSat

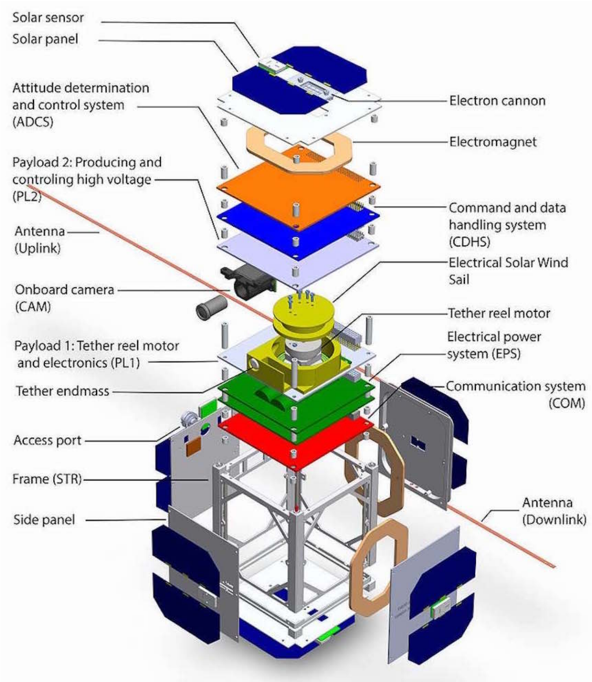

Figure 2.4: Expanded view for CubeSat

Momentum wheels shown in *figure 2.3* strength is that no information is needed about the magnetic field in order to control the CubeSat torque. These wheels are capable to store the momentum needed for maneuvering or pointing.

Thrusters ...description... Removing energy from the system it can be proved easily by using the drag force, but gaining energy it might be possible only if thrusters are used.

AAU-CubeSat sensors

The CubeSat can sustain itself using solar pannels [ref in fig 2.4] with in the middle a sun sensor, which provide a vector equal to the direction of the sun and also a magnetometer that gives a vector of the Earth's magnetic field. Whether the Earth's magnetic field is measured, or the sun vector, the objective is to use these sensors to deliver vector solutions for determining the satellite's pointing and rotation rates.

Magnetometer is a sensor used for attitude control, which measure the direction and intensity of the magnetic field. The atittude is determined from the magnetometer by comparing the measure magnetic field with a referance field.

Sun sensor is used for delivering a vector of measurements from the Sun. (ref to the fig 2.4)

Pointing accuracy

The required pointing accuracy when acquiring a photo is based on the a height from the picture is taken, in this case around 700 km above the Earth surface is going to cover approximately ?? km.

2.1 Coordinate frames

In order to determine the attitude in three-dimensional space, various coordinate frames are defined.

Reference Coordinate Systems

In order to define an orbit around Earth, two specific Earth coordinate systems are defined. Both of them have their origin in the geometrical center of Earth and are named the Earth Centered Inertial (ECI) coordinate frame and the Earth Centered Earth Fixed (ECEF) coordinate frame. These can be seen in *figure 2.5* and *figure 2.6*

Earth Centered Inertial frame(ECI)

In order to describe the orbit formation of the satellite, the ECI frame shown in *figure 2.5* is used, since it can be seen as a non-accelerating frame. The z axis is pointing through the geographical north pole, the x axis is crossing from the point where the equatorial of the earth and the vernal equinox met and the y axis is the cross product of x and z creating a right-handed coordinate system.

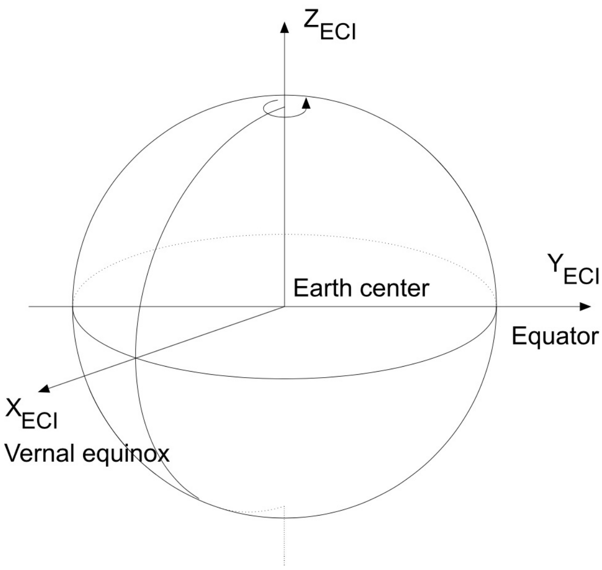

Figure 2.5: ECI coordinate frame

Earth Centered Earth Fixed Frame (ECEF)

Another coordinate frame is the Earth Centered Earth Fixed (ECEF) coordinate frame shown in *figure 2.6*. In this case the X-axis is passing through the zero longitude, also known as Greenwich meredian, and the Z-axis parallel with the rotational axis. In this way the ECEF frame is fixed to the earth itself and rotates around with it.

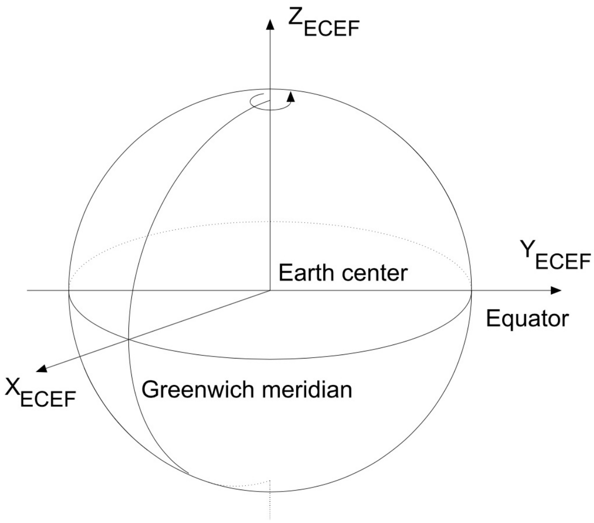

Figure 2.6: ECEF coordinate frame

Satellite Coordinate Systems

For the purpose of determining the attitude of the satellite, several coordinate systems are introduced. The attitude and position of the satellite is given as a rotation between the satellite fixed coordinate frames and the reference frames.

Orbit Reference frame(ORF)

The orbit reference shown in *figure 2.7* is a frame defined in Cartesian coordinates that can be seen as a non-changing frame with respect the earth and the satellite. The z axis always pointing at the Nadir point and it is parallel to the z_e axis o the inertial frame of the earth. The x_o axis, it is parallel to the orbit plane and y_o is the cross product of the x_o and z_o.

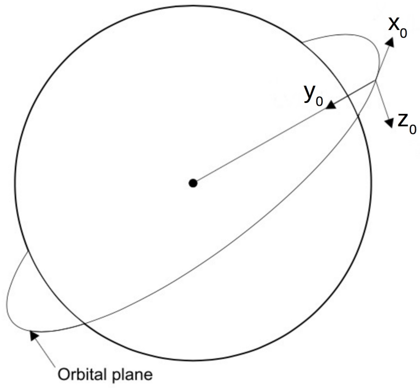

Figure 2.7: ORF coordinate frame

Satellite Body Frame(SBF)

The satellite body frame is placed in the center of mass of the satellite as shown in *figure 2.8*.

Satellite Controller frame(SCF)

In order to derive the kinematic equations, a controller reference frame seen in *figure 2.8* should be specified. It is located in the center of mass of the satellite and it is defined such that the axis of higher inertia z_c pointing in the center of ECI and the x_c axis with the smallest inertia, pointing along with the orbit's x_o

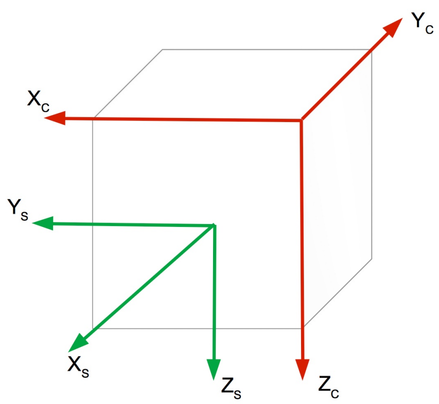

Figure 2.8: Satellite body frame and satellite controller frame

3 | Requirements

Based on the use-case introduced and the available system a set of requirements are formulated.

System requirements

1. **The formation shall be able to maintain a given distance within 60°**
2. **Each satellite shall be able to change its orientation**
3. **Each satellite shall be able to determine its own orientation and position**
4. **All satellites will be able to communicate to each other**

Part I

Distance control

4 | Modelling

In this part, we'll focus on the modelling and the control of the distance between two satellites using the drag force as the control input of the system. First, we'll considerer that the orientation of the satellite is instantaneous and therefore, the drag force can be modified instantaneous. The Earth and the satellite is assumed to be a point mass to simplify the system.
The Satellite is mainly subjected to three forces: the gravity, the drag force and the sun radiation. Thus, the second law of Newton gives:

$$\sum \mathbf{F} = m_{sat}\mathbf{a} = \mathbf{F_g} + \mathbf{F_D} + \mathbf{F_{rad}}$$

with the gravity can be modeled by:

$$\mathbf{F_g} = -G\frac{m_earth \cdot m_sat}{||\mathbf{p}||^3}\mathbf{p}$$

where $\mathbf{p}$ is the vector position of the satellite (vector from the earth center to the mass center of the satellite in the inertital frame). The modelization of the $\mathbf{F_D}$ and $\mathbf{F_{rad}}$ are explained in the next section.

4.1 Disturbance Models

Aerodynamic Drag Force

The satellite is subjected to a aerodynamic drag force due to the atmosphere. The collisions with the air caused a force in the opposite direction of the velocity of the satellite. the force was modeled by Lord Rayleigh[ref]:

$$\mathbf{F_D} = -\frac{1}{2}\rho \cdot C_D \cdot A_\perp ||\mathbf{v}||\mathbf{v}$$

where ρ is the density of the air, C_D is the drag coefficient, $A_\perp$ is the area that is perpendicular of the velocity of the satellite $\mathbf{v}$.
The drag coefficient C_D and the perpendicular area $A_\perp$ depend of the orientation of the satellite. Therefore, this force can be used as a input for the control of the position and the velocity of the satellite.
The density of the air depends of the altitude of the satellite, of the air temperature but we considered to be constant in our case to simplify the modelization. ρ is chosen to be equal to $1.454 \cdot 10^-13$ $[\frac{Kg}{m^3}]$ based to the empirical model of the Committee on Space Research (COSPAR) International Reference Atmosphere [Ref].
The drag coefficient as said before is orientation dependant. The maximum value of C_D is equal to 1.05 for a non tilited cubed as shown on the figure **??** and equal to 0.80 for an angled cubed. In our modelization, we will assume that the drag coefficient is constant

and equal to 1(not sure which value take) in order to simplified the equation.

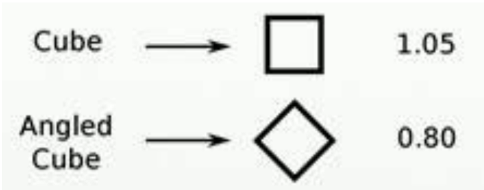

Figure 4.1: ORF coordinate frame

Therefore, the only parameter that we control is the perpendicular area $A_\perp$. The maximum and minimum value of $A_\perp$ are represented at the figure**??**. Thus, the minimum value is the surface of a square of 10cm of dimension ($A_\perp = 100cm^2$) and the maximum value is the surface of an hexagone of 10cm of dimension ($A_\perp = \frac{3\sqrt{3}}{2}100cm^2$).

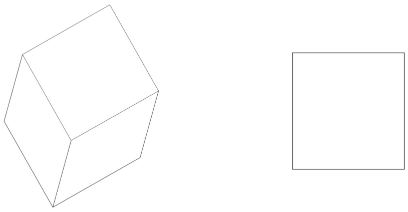

Figure 4.2: ORF coordinate frame

Thus, the drag force can be expressed as the following.

$$\mathbf{F_D} = -u||\mathbf{v}||\mathbf{v}$$

with u is the control input and it can take value between $7.27 \cdot 10^{-16}$ and $1.888 \cdot 10^{-15}$.

Solar radiation

Due to low earth orbit flying, the surface of the CubeSat will absorb or reflect the solar radiation, nevertheless, these two situations will alter the CubeSat, which will produce a torque about the satellite center of mass(CoM).

The torque around CoM is given by:

$$N_{rad} = F_{rad} \times R_{CoM} \tag{4.1}$$

where F_{rad} is the solar radiation and R_{CoM} is the vector from the centre of mass to the geometric centre of radiation

The solar radiation F_{rad} can be expressed as:

$$F_{rad} = C_a P A \tag{4.2}$$

where C_a is absorption constant of the radiated area and P is the solar flux, while A is the radiated area

4.2 State Space Representation

The state of the system is the vector position and the vecteur velocity in the inertia frame:

$$\mathbf{x} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \end{bmatrix}$$

The equation of (I don't remember the name of the equation xdot = f(x,u) + u) is given by:

$$\begin{aligned} \dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \mathbf{a} \end{bmatrix} && (4.3) \\ = \begin{bmatrix} \mathbf{v} \\ \frac{1}{m_{sat}}(-G\frac{m_{earth} \cdot m_{sat}}{||\mathbf{p}||^3}\mathbf{p}) - u||\mathbf{v}||\mathbf{v} + F_{rad} \end{bmatrix} && (4.4) \\ = \mathbf{f}(\mathbf{x}) + u \cdot \mathbf{g}(\mathbf{x}) + \boldsymbol{\delta}(\mathbf{x}, \mathbf{t}) && (4.5) \end{aligned}$$

with

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} \mathbf{v} \\ -G \cdot m_{earth} \frac{\mathbf{p}}{||\mathbf{p}||^3} \end{bmatrix}, \; \mathbf{g}(\mathbf{x}) = \begin{bmatrix} 0 \\ -\frac{1}{m_{sat}}||\mathbf{v}||\mathbf{v} \end{bmatrix}$$

and $\boldsymbol{\delta}(\mathbf{x}, \mathbf{t})$ reprensent the influence all the disturbances.

5 | Distance control design

Part II

Attitude control

6 | Modelling

This chapter provides a description of the dynamic and kinematic equations of motion which constitute the basis for further analysis and description of the forces and/or disturbances, which may affect a rigid body within Low Earth Orbit(LEO). The coordinate systems are defined first and then the model for the satellite is derived, based on rigid body dynamics and kinematics.

In order to control the distance between two or more satellites in orbit, a mathematical description of the governing equations should be derived. Since precious work have been made in previous projects, and all the measurements are available, in-depth analysis it is deemed not necessary.

6.1 Kinematics

This section will provide the orbit-attitude determination of the satellite using quaternion representation. Since the differential drag control method is based on the rotation of the satellite in order to achieve the effective cross-sectional area, a notation with respect the collaborating frames should be obtained.

6.2 Dynamic Model

In order to describe the behavior of the satellite a dynamic model based on reaction wheels and by using Euler's equation of motion has been derived. Euler's equation of motion describing the rotation of a rigid body is given by: [1]

$$\dot{L} = N_{tot} - \omega \times L \tag{6.1}$$

where N_{tot} represents all the external torques caused from the actuator and the disturbances, ω is the angular velocity of the satellite and L is the total angular momentum of the satellite and the reaction wheels, given by:

$$L = I_s \omega + h_{tot} \tag{6.2}$$

where h_{tot} is the vector of the angular momentum of the wheels $[h_1 h_2 h_3]^T$, all seen in the satellites coordinate system and I_s is the inertia matrix of the satellite. Inserting the equation *equation (6.2)* into *equation (6.1)* we obtain

$$\frac{d}{dt}(I_s \omega) + \dot{h}_{(tot)} = N_{tot} - \omega \times (I_s \omega + h_{tot}) \tag{6.3}$$

For three reaction wheels attached at the body coordinate system which are the axis roll, pitch and yaw, three equations shall be derived. The derivation of the three equations

[1] FiXme Note: ref

of motion along with the diagonal inertia matrix can be found in the *appendix A*). For the ease of notation, the cross product can be written as matrix operation using the $S()$ representing the skew symmetric matrix. Solving for $\dot{\omega}$ the dynamic equation can be written as

$$\dot{\omega} = -I_s^{-1}S(\omega)I_s^{-1}\omega - I_s^{-1}S(\omega)h_{tot} - I_s^{-1}\dot{h}_{(tot)} + I_s^{-1}N_{tot} \tag{6.4}$$

The rate of change in angular momentum $\dot{h_{tot}}$ can be absorbed from the controller. This can be written as:

$$\dot{h}_{(tot)} = -Nc \tag{6.5}$$

where the negative sign denotes the absorbed momentum. The total torque from external disturbances can be written as N_{dis}. Rearranging, equation *equation (6.4)* now reads

$$\dot{\omega}(t) = -I_s^{-1}S(\omega)I_s\omega(t) - I_s^{-1}S(\omega)h_{tot} + I_s^{-1}N_c(t) + I_s^{-1}N_{dis}(t) \tag{6.6}$$

which constitute the dynamics of the satellite with three reaction wheels. At the final equation *equation (6.6)* is shown the time dependency of the variables.

6.3 Disturbance Models

Gravitational torque

An unbalanced satellite in orbit is subjected to a torque due to the gravitational torque. Assumed that the earth is a point mass and the satellite is a rigid body, the gravitational torque can be estimated. Each infinitesimal element of the satellite of mass dm_i is subjected to an infinitesimal force dF_i that can be calculated thanks to Newton's law of universal gravitation.

$$dF_i = -G\frac{m_{earth}}{R_i^2}dm_i \cdot \hat{R}_i \tag{6.7}$$

where G is the gravitational constant, m_{earth} is the mass of the earth and R_i^2 is the vector from the earth to the infinitesimal element of the satellite.
The moment of the gravitational force about the geometric center is calculated as the formula:

$$N_{gra} = \int_{sat} r_i \times dF_i \tag{6.8}$$

with r_i is the vector from the geometric center to the infinitesimal element. r_i can be written as the sum of the vector from the geometric vector to the mass center $r_{g,m}$ and the vector from the mass center of the element $r_{m,i}$. Therefore, the expression of the gravitational torque is simplified:

$$N_{gra} = \int_{sat} r_{g,m} \times dF_i + \int_{sat} r_{m,i} \times dF_i \tag{6.9}$$

$$= \int_{sat} r_{g,m} \times -G\frac{m_{earth}}{R_i^2}dm_i \cdot \hat{R}_i + \int_{sat} r_{m,i} \times -G\frac{m_{earth}}{R_i^2}dm_i \cdot \hat{R}_i$$

We can assumed that $r_{m,g} << R_i$ and R_i can be supposed constant and equals to the vector from the center of the earth to the geometric center of the satellite $R_{e,g}$. Thus, The second term is null by definition of the mass center.

$$\Rightarrow N_{gra} = G\frac{m_{sat} \cdot m_{earth}}{R_{e,g}^2} \cdot (\hat{R}_i \times r_{g,m}) \tag{6.10}$$

The position of the center of mass was measured for the previous project and is eqals to [?;?;?] in the frame of the satellite. Therefore, $r_{g,m,i}$ can be expressed in the inertial frame as following:

$$[r_{g,m,i}; 0] = q_{i,s} \otimes [?; ?; ?.0] \otimes q_{i,s}* \tag{6.11}$$

where $q_{i,s}$ is the quaternion that represents the rotation of the satellite in the inertia frame and $\otimes$ is the quaternion multiplication. Thus, the moment of force can be calculated by this expression above.

Solar radiation

Due to low earth orbit flying, the surface of the CubeSat will absorb or reflect the solar radiation, nevertheless, these two situations will alter the CubeSat, which will produce a torque about the satellite center of mass(CoM).

The torque around CoM is given by:

$$N_{rad} = F_{rad} \times R_{CoM} \tag{6.12}$$

where F_{rad} is the solar radiation and R_{CoM} is the vector from the centre of mass to the geometric centre of radiation

The solar radiation F_{rad} can be expressed as:

$$F_{rad} = C_a P A \tag{6.13}$$

where C_a is absorption constant of the radiated area and P is the solar flux, while A is the radiated area

7 | Attitude control design

Part III

Test and implementation

8 | Acceptance test

The system is tested to see if it fulfills the requirements put up (*chapter 3*).

9 | Conclusion

9.1 Future work

A | Appendix A

A.1 Derivation of Equation of motion

The general Euler's rotation equation with three reaction wheels aligned on the satellite body axis are derived as

$$I_1\dot{\omega}_1 = (I_2 - I_3)\omega_2\omega_3 + N_1 - \omega_2 h_3 + \omega_3 h_2 \tag{A.1}$$

$$I_2\dot{\omega}_2 = (I_3 - I_1)\omega_1\omega_3 + N_2 - \omega_3 h_1 + \omega_1 h_3 \tag{A.2}$$

$$I_3\dot{\omega}_3 = (I_1 - I_2)\omega_1\omega_2 + N_3 - \omega_1 h_2 + \omega_2 h_1 \tag{A.3}$$

The equation in compact form has been written as

$$\dot{\omega} = -I_s^{-1}S(\omega)I_s^{-1}\omega - I_s^{-1}S(\omega)h_{tot} - I_s^{-1}\dot{h}_{(tot)} + I_s^{-1}N_{tot} \tag{A.4}$$

where $S(\omega)$ is the skew symmetric matrix given by

$$S\omega = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{A.5}$$

and the angular momentum of the reaction wheels as $h_{tot} = [h_1\ h_2\ h_3]^T$.

Inertia matrix

The inertia matrix for a solid cuboid of height z , width y, and depth x, amd mass m_i with respect the center of mass is given by

$$I_i = \begin{bmatrix} \frac{1}{12}m_i(z^2+y^2) & 0 & 0 \\ 0 & \frac{1}{12}m_i(z^2+x^2) & 0 \\ 0 & 0 & \frac{1}{12}m_i(x^2+y^2 \end{bmatrix} \tag{A.6}$$

It is assumed that the Cube have a symmetric mass distribution around the axis of rotation to simplify the inertia matrix. With the mass distributed evenly and the axis of rotation being around one of the tree axis, the off diagonal term of the inertia matrix are equal to zero. These terms are also referred to as cross products of inertia.

B | Appendix B

List of Corrections