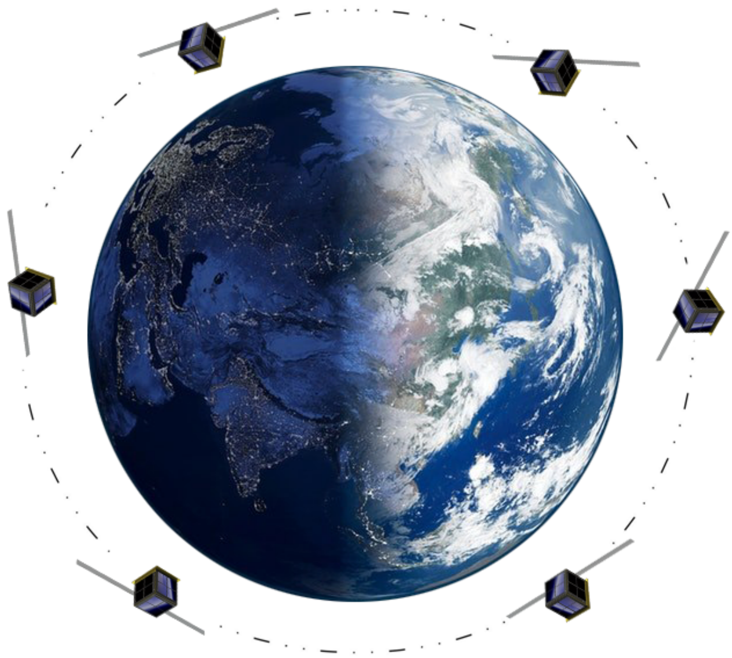
Formation flying using pico-satellites



Group 17gr931

Aalborg University
Control & Automation
Fredrik Bajers Vej 7
DK-9220 Aalborg

AALBORG UNIVERSITY
STUDENT REPORT

9th Semester, Project

School of Information and

Communication Technologies

Control and Automation

Fredrik Bajers Vej 7C

9220 Aalborg

en.aau.dk/education/master/control-automation

Title:
Formation flying using pico-satellites

Theme:
Complex systems

Project Period:
P9, Fall 2017
01/02/2017 - 20/10/2017

Project Group:
834

Participants:
- Thibaud Peers
- Nikolaos Biniakos
- Alexandru-Cosmin Nicolae

Supervisors:
Jesper Abilgaard Larsen

Prints: -
Pages: -
Appendices: - (- pages)
Attached: 1 zip file
Concluded: 20/10/2017

Synopsis

This report describes the design and implementation of a control system on an AAU-CubeSat, a pico- satellite used for Low Earth Orbit flight.
The objective is to use a flight formation for monitoring Greenland, by having six satellites equally distributed on orbit.
Two controllers must be design, one for controlling the distance between the satellites using the drag force, and one for attitude control.
Given the nonlinear nature of the system a SMC is implemented.

Preface

This report has been written by group 931 on third semester in Control and Automation on Aalborg University. References made before a full stop regards the sentence and reference after full stop regards the paragraph. Quotes are inside quotations marks and in cursive. Attached to report is a zip file with:

- -
- -
- -
- -

Report by:

Thibaud Peers

Nikolaos Biniakos

Alexandru-Cosmin Nicolae

Contents

1 | Introduction

In the last decades, the space technology is continuously growing. The reason for this is the increased deploying of satellites used in the numerous fields, in particular telecommunications and meteorology. [**SIDI**]

Missions containing satellites operating close to each other are commonly referred to as flying formation, which is known as a distributed satellite system. Two types of distributed space systems are identified as formations and constellations flying.

A distributed space system is defined by NASA Goddard Space Flight Center (GSFC) as *"an end-to-enpd system including two or more space vehicles and a cooperative infrastructure for scientific measurement, data acquisition, processing, analysis, and distribution".* [**SFF**]

Satellite formation flying is not having a precise definition, however, the definition proposed by NASA GSFC is that *"formation flight involves the use of an active control scheme to maintain the relative positions of the spacecraft "*. In contrast, a constellation is defined as *"two or more spacecraft in similar orbits with no active control by either to maintain a relative position"*. [**SF**]

Formation flying it might offer many possibilities for space exploration, such as surveillance, field measurements and atmospheric survey missions as well as on-orbit satellite inspection, maintenance, and recovery. This approach it has a few challenges which involve autonomous control of the satellites influenced by the different disturbing forces caused by gravity gradient, solar radiation pressure, aerodynamic drag, and Earth's oblateness effect, with a purpose of achieving it with minimum fuel consumption. Nevertheless, there is currently no formation flying satellites in orbit, however, two such missions are ESA's "Cluster" mission and the ESA/NASA "Grace" mission, which are in development stages. [**SF**]

The use of satellite formations is expected to rise in the next years. This makes it relevant to look at improving or adding functionalities to satellites. Based on this it has been decided to look at the case of a distributed space system consisting of a formation of six satellites equally distributed on the orbit and analyzing the behavior between them.

1.1 Problem statement

Design and implement a controller for controlling the individual distance between satellites using the drag force.

1.2 Use-case

In this project, the concept of a formation flight of satellites will be used for the purpose of monitoring. Denmark has a small island called Greenland, where the Danish Government needs to monitor it. One method is to have a formation of satellites going around the orbit and when they are located in the northern hemisphere, the satellites will point down and look towards Greenland.

One of the essentials in formation flight is choosing the number of satellites in orbit. Therefore, in order to have a continuous coverage, a distributed satellite system composed of six satellites equally distributed are chosen, compared with two or four satellites where communication between each other will be poor.

The task the satellite has to perform is acquiring data by flying around Greenland, using radio signals and taking pictures.

2 | System Description

The overall idea of the project is to consider more than one satellites flying in formation, with a certain distance in between and with the purpose of maintaining that distance by using the drag force. As a proof of concept, an AAU-CubeSat will be used, by choosing six AAU-CubeSat that orbit the Earth like is shown in *figure 2.1*. Therefore, a control system is developed, where the six satellites are nodes and they represent periods. In this project, all CubeSat's will be assumed identical. Moreover, a full-scale implementation of the system will not be possible, therefore, the whole system will be simulated using MATLAB and Simulink.

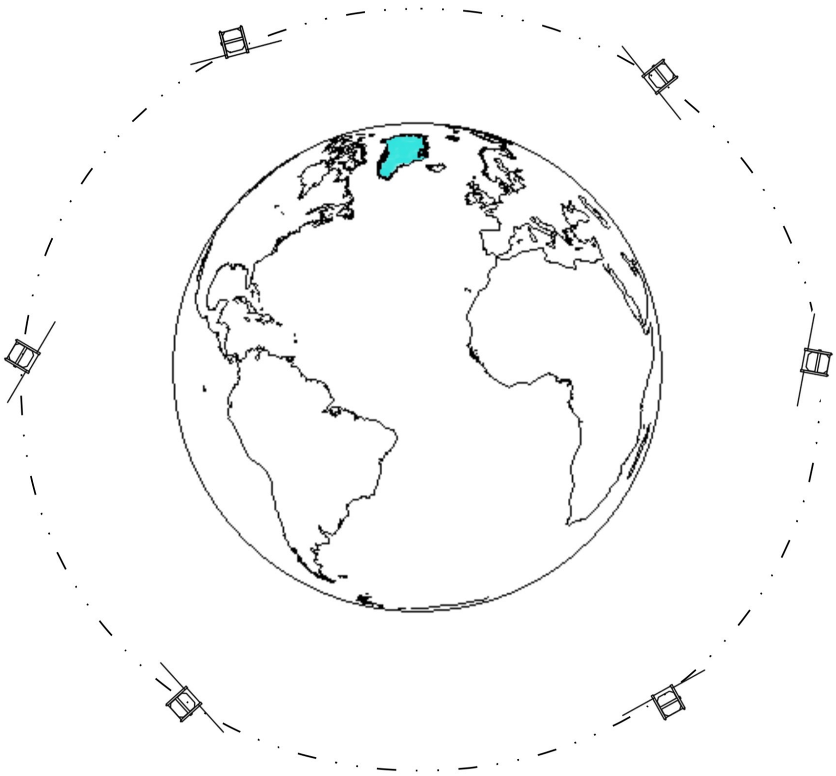


Figure 2.1: Six satellites in flying formation on orbit

2.1 About AAU-CubeSat

The AAU-CubeSat shown in *figure 2.2* is a pico-satellite developed by Stanford University, but assembled at Aalborg University by students and used mainly for Low Earth Orbit (LEO) tests.

Figure 2.2: View of CubeSat satellite [**cs**]

The pico-satellite is designed for LEO, therefore a few constraints are imposed. The CubeSat is limited in size and weight. The dimensions of the satellite are $10cm \times 10cm \times 30cm$, while the weight is around 1 kg.[**CDS**]

In order to place the CubeSat on orbit, a deployment system is used, called P-POD. This system uses the force of a spring to launch the satellite into space. The satellite will be placed inside the launch rocket as payload. By using this system, an important advantage is reducing the cost of the launch. [**PPOD**]

2.2 AAU-CubeSat actuators

The selection of attitude control components is important in order to meet the performance requirements. For this project, three magnetorquers and three momentum wheels have been chosen as actuators. Initially, using only three momentum wheels has been considered, but the downside of using only momentum wheels is that some amount of momentum can be stored in the wheel, which will imply having a way to take back all that momentum and use it. Therefore, there are multiple ways to release that torque, and one is to use magnetorquers.

Magnetorquers are wire coils which generate an electromagnetic field. The field interacts with the Earth magnetic field and a torque is generated for stabilizing the satellite. An important aspect of the magnetorquer is when the reaction wheel reaches a maximum speed and can no longer produce the torque this is referred as wheel saturation, so a magnetorquer is used to extract the momentum from the wheel.

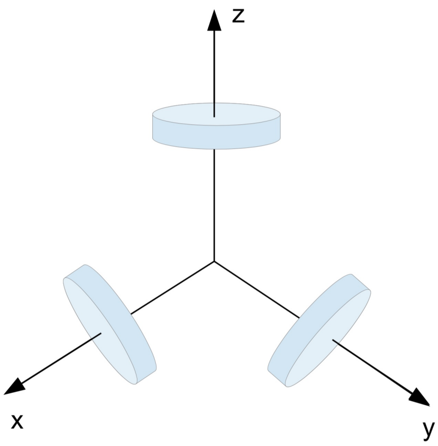


Figure 2.3: Example of three reaction wheel for CubeSat

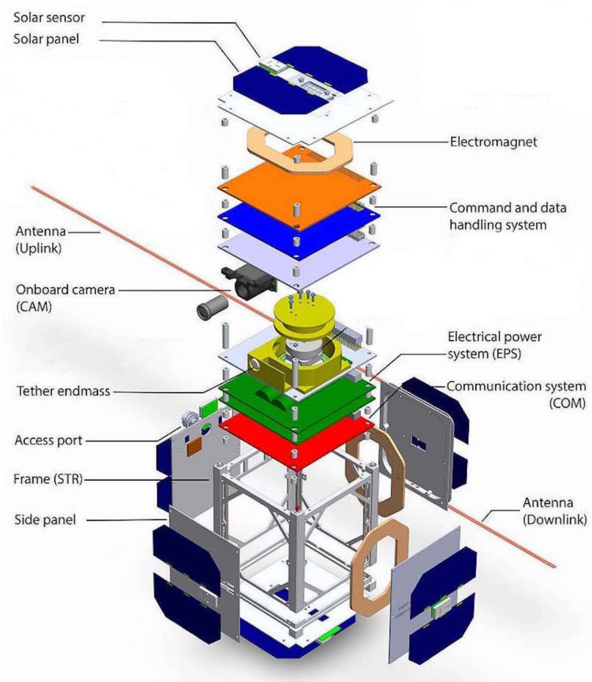


Figure 2.4: Expanded view for CubeSat [**view**]

Reaction wheels shown in *figure 2.3* strength is that no information is needed about the magnetic field in order to control the CubeSat torque. These wheels are capable to store the momentum needed for maneuvering or pointing.

Thrusters could represent a possibility for gaining energy because removing energy from the system it can be proved easily by using the drag force. Due to the weight of the trusters, they are not considered in this project.

2.3 AAU-CubeSat sensors

The CubeSat can sustain itself using solar pannels with in the middle a sun sensor similar in *figure 2.4* , which provide a vector equal to the direction of the sun and also a vector of the Earth's magnetic field measured by the magnetometer. Whether the Earth's magnetic field is measured, or the sun vector, the objective is to use these sensors to deliver vector solutions for determining the satellite's pointing and rotation rates.

Magnetometer is a sensor used for attitude control, which measure the direction and intensity of the magnetic field. The atittude is determined from the magnetometer by comparing the measure magnetic field with a referance field.

Sun sensor is used for estimating the position of the Sun and delivering a vector of measurements from the Sun.

Pointing accuracy

The required pointing accuracy when acquiring a photo is based on the a height from the picture is taken, in this case around 700 km above the Earth surface is going to cover approximately ?? km.

2.4 Coordinate frames

In order to determine the attitude in three-dimensional space, various coordinate frames are defined.

Reference Coordinate Systems

In order to define an orbit around Earth, two specific Earth coordinate systems are defined. Both of them have their origin in the geometrical center of Earth and are named the Earth Centered Inertial (ECI) coordinate frame and the Earth Centered Earth Fixed (ECEF) coordinate frame. These can be seen in *figure 2.5* and *figure 2.6*

Earth Centered Inertial frame(ECI)

In order to describe the orbit formation of the satellite, the ECI frame shown in *figure 2.5* is used, since it can be seen as a non-accelerating frame. The z axis is pointing through the geographical north pole, the x axis is crossing from the point where the equatorial of the earth and the vernal equinox met and the y axis is the cross product of x and z creating a right-handed coordinate system.

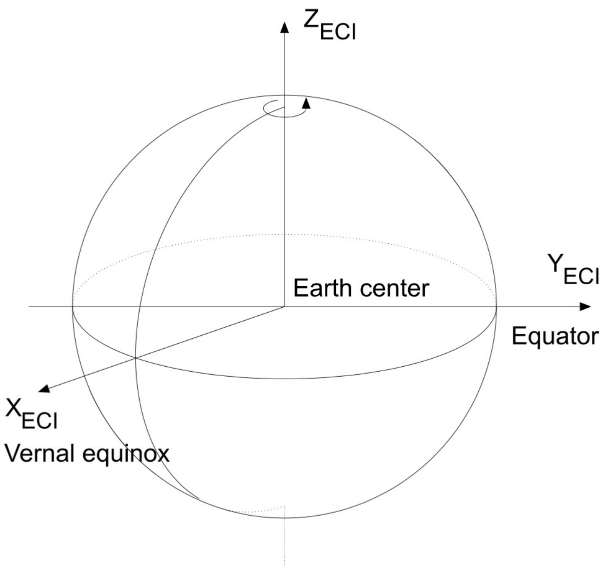


Figure 2.5: ECI coordinate frame

Earth Centered Earth Fixed Frame (ECEF)

Another coordinate frame is the Earth Centered Earth Fixed (ECEF) coordinate frame shown in *figure 2.6*. In this case the X-axis is passing through the zero longitude, also known as Greenwich meredian, and the Z-axis parallel with the rotational axis. In this way the ECEF frame is fixed to the earth itself and rotates around with it.

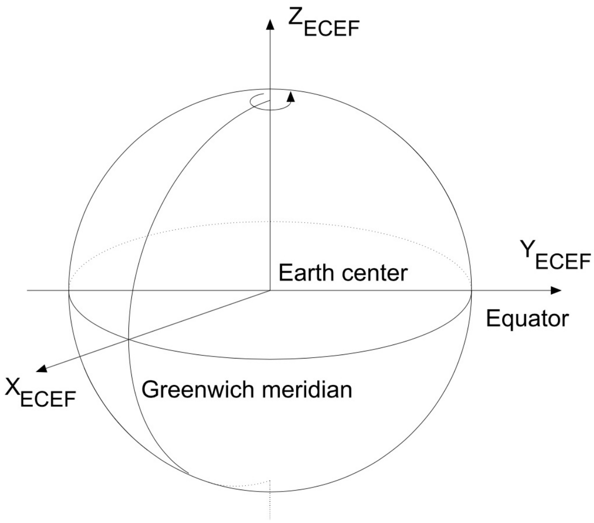


Figure 2.6: ECEF coordinate frame

Satellite Coordinate Systems

For the purpose of determining the attitude of the satellite, several coordinate systems are introduced. The attitude and position of the satellite is given as a rotation between the satellite fixed coordinate frames and the reference frames.

Orbit Reference frame(ORF)

The orbit reference shown in *figure 2.7* is a frame defined in Cartesian coordinates that can be seen as a non-changing frame with respect the earth and the satellite. The z axis always pointing at the Nadir point and it is parallel to the z_e axis o the inertial frame of the earth. The x_o axis, it is parallel to the orbit plane and y_o is the cross product of the x_o and z_o.

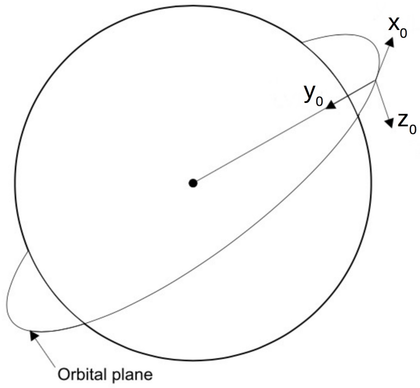


Figure 2.7: ORF coordinate frame

Satellite Body Frame(SBF)

The satellite body frame is placed in the center of mass of the satellite as shown in *figure 2.8*.

Satellite Controller frame(SCF)

In order to derive the kinematic equations, a controller reference frame seen in *figure 2.8* should be specified. It is located in the center of mass of the satellite and it is defined such that the axis of higher inertia z_c pointing in the center of ECI and the x_c axis with the smallest inertia, pointing along with the orbit's x_o

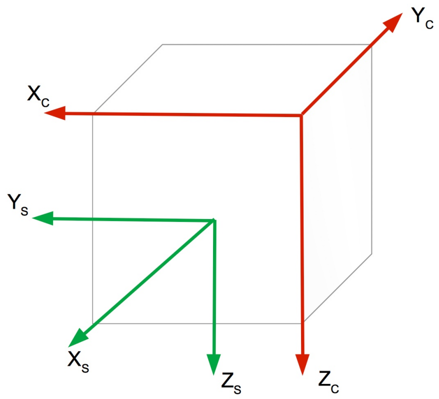


Figure 2.8: Satellite body frame and satellite controller frame

3 | Requirements

Based on the use-case introduced and the available system a set of requirements are formulated.

System requirements

1. **The formation shall be able to maintain a given distance within 60°**
2. **Each satellite shall be able to change its orientation**
3. **Each satellite shall be able to determine its own orientation and position**
4. **All satellites will be able to communicate to each other**

4 | Distance control

In this chapter, the focus will be on modeling and the control of the distance between two satellites using the drag force as the control input of the system. First, it is considered that the orientation of the satellite is instantaneous and therefore, the drag force can be modified instantaneously. The Earth and the satellite are assumed to be a point mass to simplify the system.

4.1 Modelling

The Satellite is mainly subjected to three forces: the gravity, the drag force and the sun radiation. Thus, the second law of Newton gives:

$$\sum F = m_{sat}\ a = F_g + F_D + F_{rad} \tag{4.1}$$

with the gravity modeled by:

$$F_g = -G\frac{m_{earth} \cdot m_{sat}}{||p||^3}p \tag{4.2}$$

where p is the vector position of the satellite (vector from the earth center to the mass center of the satellite in the inertial frame) and the expresion for F_D and F_{rad} are explained in the next section.

4.2 Disturbance Models

Aerodynamic Drag Force

The satellite is subjected to an aerodynamic drag force due to the atmosphere. The collisions with the air cause a force in the opposite direction of the velocity of the satellite. The force was modeled by Lord Rayleigh.[**FSA**]

$$F_D = -\frac{1}{2}\rho \cdot C_D \cdot A_\perp ||v||\ v \tag{4.3}$$

where ρ is the density of the air, C_D is the drag coefficient, $A_\perp$ is the area that is perpendicular of the velocity of the satellite v.

The drag coefficient C_D and the perpendicular area $A_\perp$ depend on the orientation of the satellite. Therefore, this force can be used as an input for the control of the position and the velocity of the satellite.

The density of the air depends on the altitude of the satellite, of the air temperature but we considered to be constant in our case to simplify the model. ρ **is chosen to be equal**

to $1.454 \cdot 10^{-13} Kg/m^3$ **based on the empirical model of the Committee on Space Research (COSPAR) International Reference Atmosphere [SADC].**

The drag coefficient as said before is orientation dependent. The maximum value of C_D is equal to 1.05 for a non tilted cubed as shown on the figure *figure 4.1* and equal to 0.80 for an angled cubed [**wik**]. **In our modelization, we will assume that the drag coefficient is constant and equal to 1(not sure which value take) in order to simplified the equation.**

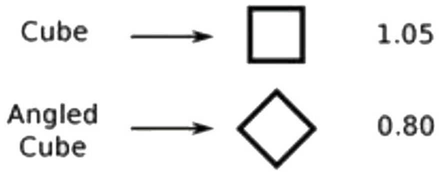


Figure 4.1: description needed

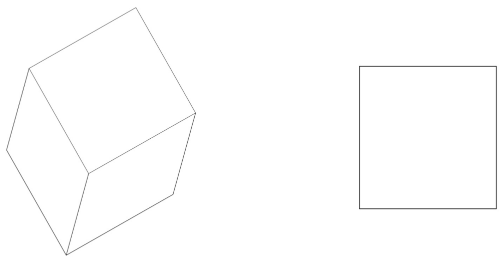


Figure 4.2: description needed

Therefore, the control parameter is the perpendicular area $A_\perp$. The maximum and minimum value of $A_\perp$ are represented in *figure 4.2*. Thus, the minimum value is the surface of a square of 10cm of dimension ($A_\perp = 100cm^2$) and the maximum value is the surface of an hexagone of 10cm of dimension ($A_\perp = \frac{3\sqrt{3}}{2}100cm^2$). Thus, the drag force can be expressed as the following.

$$F_D = -u||v||v \tag{4.4}$$

where u is the control input and it can take value between $7.27 \cdot 10^{-16}$ and $1.888 \cdot 10^{-15}$

Solar radiation

Tfhe surface of the CubeSat will absorb or reflect the solar radiation, nevertheless, these two situations will alter the CubeSat, which will produce a torque about the satellite center of mass (CoM). [**SADC**]

The torque around CoM is given by:

$$N_{rad} = F_{rad} \times R_{CoM} \tag{4.5}$$

where F_{rad} is the solar radiation and R_{CoM} is the vector from the centre of mass to the geometric centre of radiation

The solar radiation F_{rad} can be expressed as:

$$F_{rad} = C_a P A \tag{4.6}$$

where C_a is the surface's reflectance: 0 for a perfect absorber, 1 for a perfect reflector, while P is the solar flux and A is the radiated area

The solar flux can be computed as follows:

$$P = \frac{F_s}{c} \tag{4.7}$$

where F_s is the mean solar energy and it is equal with 1358 W/m^2 and c is the speed of light

J_2 gravity perturbation

A satellite orbiting the Earth encounter multiple perturbing forces. Some of these forces are the atmospheric drag, the gravity gradient, and the solar radiation. The influence of these forces upon the satellite is deemed to be negligible, but one perturbation produced by the oblateness of the Earth is taken into account because will provoke a change in the orientation of the orbit.

The force which the Earth is exerting upon a object outside its sphere is a conservative force and it can be written as follows:

$$U(r) = -\frac{\mu}{r} \tag{4.8}$$

Because the Earth is not a perfect sphere and also its mass distribution is not homogeneous, *equation (4.8)* is rewritten by adding the spherical harmonic expansion to correct the gravitational potential for the Earth:

$$U(r) = -\frac{\mu}{r} + B(r, \phi, \lambda) \tag{4.9}$$

where $B(r, \phi, \lambda)$ is the spherical harmonic expansion used to correct the gravitational potential for the Earth's nonsymmetric mass distribution seen in *figure 4.3*

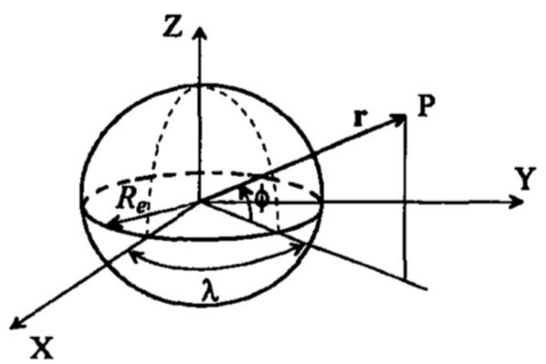


Figure 4.3: Coordinates for deriving the external gravitational potential of the Earth

In order to solve the problem regarding the oblatness, the gravitational potential of the Earth is extended into series of spherical harmonics: [ref]

$$B(r,\phi,\lambda) = \frac{\mu}{r}\left\{\sum_{n=2}^{\infty}\left[\left(\frac{R_e}{r}\right)^n J_n P_n sin(\phi)| + \sum_{m=1}^{n}\left(\frac{R_e}{r}\right)^n (C_{nm}cos(m\lambda) + S_{nm}sin(m\lambda))P_{nm}sin(\phi)\right]\right\} \tag{4.10}$$

where r, ϕ, λ are spherical coordinates and the parameteres from the function are defined as follows: r is the geocentric distance of point P, ϕ is the geocentric latitude, λ is the geographical longitude, R_e is the mean equatorial radius of the Earth, $cos(m\lambda)$ and $sin(m\lambda)$ are harmonics in λ, J_{nm} are the zonal harmonic coefficients, J_n zonal harmonic coefficients of order 0, P_{nm} associated Legendre polynomial of degree n and order m, P_n is Legendre polynomial degree n and order 0, C_{nm} is tesseral harmonic coefficients for $n \neq m$, S_{nm} is sectoral harmonic coefficients for $n = m$

The expression for gravitational potential of the Earth can be approximate as:

$$U \approx -\frac{\mu}{r}\left[1 - \sum_{n=2}^{\infty}\left(\frac{R_e}{r}\right)^n J_n P_n sin(\phi)\right] = \frac{\mu}{r}[U_0 + U_{J_2} + U_{J_3} + ...] \tag{4.11}$$

where U_0 = -1 and $U_{J_2} = \left(\frac{R_e}{r}\right)^2 J_2 \frac{1}{2}(3sin^2\phi - 1)$

The gravitational forces acting on the satellite are obtained from the relation:

$$F = -m\nabla U \tag{4.12}$$

and is obtaining the following:

$$F_x = -\frac{\partial U}{\partial x} = \mu\left[-\frac{x}{r^3} + A_{J_2}\left(15\frac{xz^2}{r^7} - 3\frac{x}{r^5}\right)\right] \tag{4.13}$$

$$F_y = -\frac{\partial U}{\partial y} = \mu\left[-\frac{y}{r^3} + A_{J_2}\left(15\frac{yz^2}{r^7} - 3\frac{y}{r^5}\right)\right] \tag{4.14}$$

$$F_z = -\frac{\partial U}{\partial z} = \mu\left[-\frac{z}{r^3} + A_{J_2}\left(15\frac{z^3}{r^7} - 3\frac{z}{r^5}\right)\right] \tag{4.15}$$

where $A_{J_2} = \frac{1}{2}J_2R_e^2$ and andR_e is the mean radius ofthe earth at the equator

4.3 State Space Representation

The state of the system is the vector position and the vector velocity in the inertial frame:

$$x = \begin{bmatrix} p \\ v \end{bmatrix} \tag{4.16}$$

The equation of (I don't remember the name of the equation xdot = f(x,u) + u) is given by:

$$\dot{x} = \begin{bmatrix} \dot{p} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ a \end{bmatrix} \tag{4.17}$$

$$= \begin{bmatrix} v \\ \frac{1}{m_{sat}}(-G\frac{m_{earth} \cdot m_{sat}}{||p||^3}p) - u||v||v + F_{rad} \end{bmatrix} \tag{4.18}$$

$$= f(x) + u \cdot g(x) + \delta(x,t) \tag{4.19}$$

with

$$f(x) = \begin{bmatrix} v \\ -G \cdot m_{earth}\frac{p}{||p||^3} \end{bmatrix}, \; g(x) = \begin{bmatrix} 0 \\ -\frac{1}{m_{sat}}||v||v \end{bmatrix}$$

and $\delta(x,t)$ reprensent the influence all the disturbances.

4.4 Relative dynamics

In order to analyse the distance between two satellites, the relative dynamics are analyzed. Furthermore, to simplify the system, satellites will be assumed to stay in the same plane. This assumption has also to be made due to the limitation of the direction of the input control (the drag force).

To compute the equations of the motion of one satellite compared to another, a new frame is used. The frame is illustrated in the figure *figure 4.4*, where the origin is the first satellite and the axis $\hat{x}$ is defined by $\hat{x} = \frac{R}{R}$ where R is the vector from the center of the Earth to the first satellite, the axis $\hat{y}$ is perpendicular to $\hat{x}$ and in the plane of motion of the satellites and $\hat{z}$ is defined by the right-hand law ($\hat{z} = \hat{x} \times \hat{y}$).

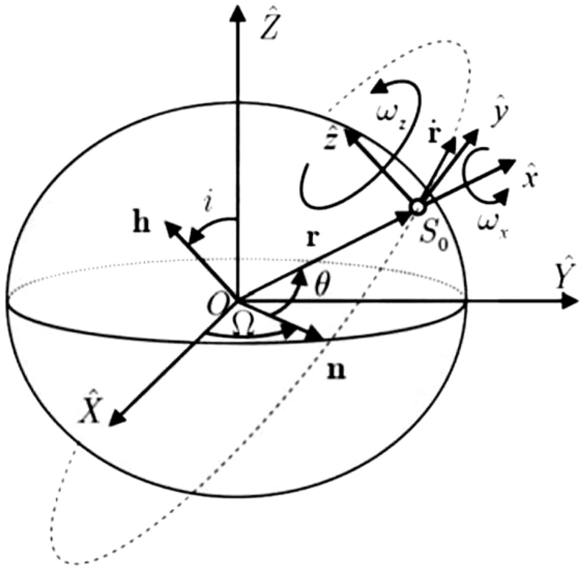


Figure 4.4: Frame for the relative dynamics

Therefore, the vector position from the Earth to the fist satellite and the second satellite can be expressed in this frame:

$$p_1 = R \cdot \hat{x} \tag{4.20}$$
$$p_2 = R \cdot \hat{x} + x \cdot \hat{x} + y \cdot \hat{y} \tag{4.21}$$

The equations of relative motions are derived in *appendix B*.

Relative state space representation

Since the equations of relative motion that have been derived in *appendix B* are not linear, an approximation of a linearization is made around the operating points, $x*$ and $y*$, by introducing the states with a new variable as $s = [x; \dot{x}; y; \dot{y}]$, more about this can be found in *appendix C*. In order to derive the linear state space model the assumptions that the radius is constant and that the angular velocity, $w = \sqrt{\frac{\mu}{R^3}}$ is constant, lead in the assumption that $\dot{R} = 0$ and $\dot{w} = 0$. Furthermore, using the approximation $\dot{x}$, $y << y^*$ and x, x^*, $\frac{\dot{y}}{w} << R$, the *equation (C.1)* has been derived. Finally assuming that $y* << R$ the linear system can be written in state space form

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2w \\ 0 & 0 & 0 & 1 \\ 0 & -2w & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -Rw \end{bmatrix}$$

$C = eye(4)$ and $D = 0$. With this assumptions and by using a control law $u = u_2 - u_1$ from the *equation (C.1)* it can be seen that when $u_2 > u_1$ then $u > 0$ and in this case y should have a decreasing attitude as seen in the *figure 4.5*

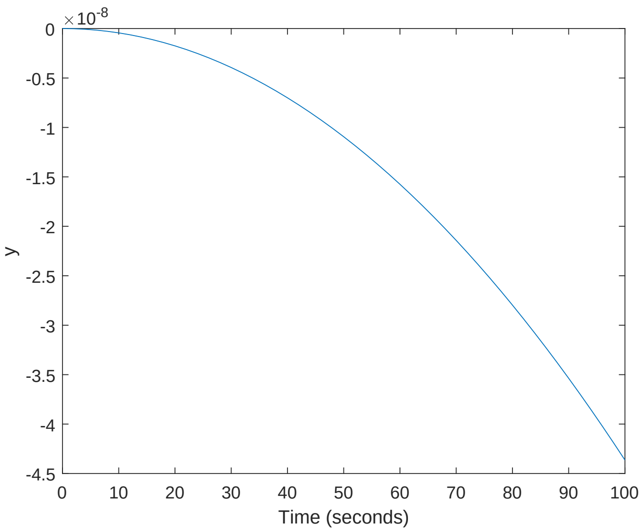


Figure 4.5: Theoretical decreasing attitude of y

Taking into account all the above assumptions, this theoretical attitude of y should be the same with the results from the actual model simulation, in order to proceed to the design of the controller. From the *figure 4.6*

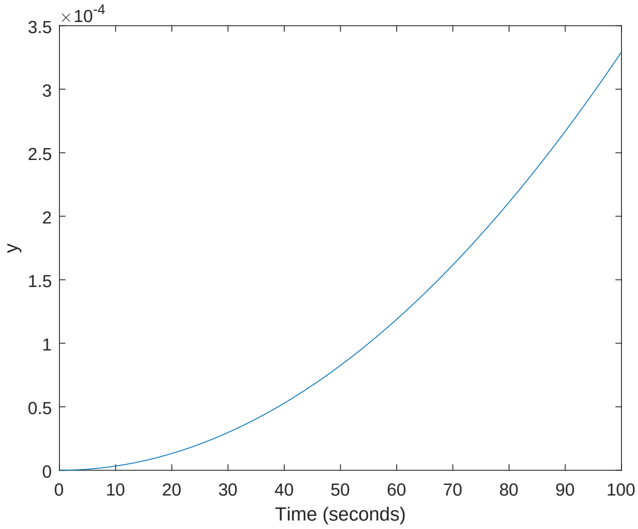


Figure 4.6: The practical attitude of y is increasing

it can be seen that the theoretical with the practical attitude of y are not the same. Since the approximations that have been made are not sufficient and there is not enough control authority, this set up has not deemed good enough for implementation.

4.5 Distance control design

5 | Attitude control

5.1 Modelling

This section provides a description of the dynamic and kinematic equations of motion which constitute the basis for further analysis and description of the forces and/or disturbances, which may affect a rigid body within LEO. The coordinate systems are defined first and then the model for the satellite is derived, based on rigid body dynamics and kinematics.

Kinematics

This section will provide the orbit-attitude determination of the satellite using quaternion representation. Since the differential drag control method is based on the rotation of the satellite in order to achieve the effective cross-sectional area, a notation with respect the collaborating frames have been obtained [1].

Quaternion parameterization it is deemed useful for the kinematic analysis of the satellite. Since the product of two quaternions gives the combined rotation, we shall specify the representation of rotation at time t of the collaborating frames in order to derive the combined rotation at time $t+\Delta t$. The orientation of the rigid body at time t is represented as $q(t)$ and at time $q(t+\Delta t)$ is the resulting quaternion at time $t+\Delta t$. The orientation of the controller reference frame $\hat{x_c}, \hat{y_c}, \hat{z_c}$ at time Δt with respect the orientation at time t can be represented as $q_c(\Delta_t$, then the orientation of the satellite at $t+\Delta t$ can found as

$$q(t+\Delta t) = q_c(\Delta_t) \otimes q(t) \tag{5.1}$$

with the components of the rotation axis unit vector along $\hat{x_c}, \hat{y_c}, \hat{z_c}$ at time t [**SADC**] written as $[e_x e_y e_z]$ respectively and $\Delta\Phi$ the rotation at time $\Delta(t)$, the parameters of the controller quaternion can be written[**SADC**] as

$$q_{1c} = e_x \sin\frac{\Delta\Phi}{2} \tag{5.2}$$

$$q_{2c} = e_y \sin\frac{\Delta\Phi}{2} \tag{5.3}$$

$$q_{3c} = e_z \sin\frac{\Delta\Phi}{2} \tag{5.4}$$

$$q_{4c} = \cos\frac{\Delta\Phi}{2} \tag{5.5}$$

[1] FiXme Note: chapter 2

combining the *equation (5.2)* - *equation (5.5)* with *equation (5.1)* we obtain

$$q(t+\Delta t) = \left\{ \cos\frac{\Delta\Phi}{2} I_{(4x4)} + \sin\frac{\Delta\Phi}{2} \begin{bmatrix} 0 & e_z & -e_y & e_x \\ -e_z & 0 & e_x & e_y \\ e_y & -e_x & 0 & e_z \\ -e_x & e_y & -e_z & 0 \end{bmatrix} \right\} q(t) \tag{5.6}$$

where I is the $4x4$ identity matrix. Using the small angle approximation [**SADC**] for infinitesimal $\Delta(t)$ and denoted ω the instantaneous change in angular velocity it is obtained

$$q(t+\Delta t) = \left[1 + \frac{1}{2}\Omega\Delta(t)\right] q(t) \tag{5.7}$$

with Ω be the skew symmetric matrix[**SADC**]

$$\Omega = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_1 & \omega_x \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix} \tag{5.8}$$

the angle approximations where taken as $\cos\frac{\Delta\Phi}{2} \simeq 1$ and $\sin\frac{\Delta\Phi}{2} \simeq \frac{1}{2}\omega\Delta(t)$

Dynamic Model

In order to describe the behavior of the satellite a dynamic model based on reaction wheels and by using Euler's equation of motion has been derived. Euler's equation of motion describing the rotation of a rigid body relates the time derivative of angular momentum to the applied torques[**Biezl**] and is given by:

$$\dot{L} = N_{tot} - \omega \times L \tag{5.9}$$

where N_{tot} represents all the external torques caused from the actuator and the disturbances, ω is the angular velocity of the satellite and L is the total angular momentum of the satellite including reaction wheels, given by[**Biezl**]:

$$L = I_s\omega + h_{tot} \tag{5.10}$$

where h_{tot} is the vector of the angular momentum of the wheels $[h_1\ h_2\ h_3]^T$, all seen in the satellites coordinate system and I_s is the inertia matrix of the satellite. Inserting the equation *equation (5.10)* into *equation (5.9)* we obtain

$$\frac{d}{dt}(I_s\omega) + \dot{h}_{(tot)} = N_{tot} - \omega \times (I_s\omega + h_{tot}) \tag{5.11}$$

For three reaction wheels attached at the body coordinate system which are the axis roll, pitch and yaw, three equations shall be derived. The derivation of the three equations of motion along with the diagonal inertia matrix can be found in the *appendix A*. For the ease of notation, the cross product can be written as matrix operation using the $S()$ representing the skew symmetric matrix. Solving for $\dot{\omega}$ the dynamic equation can be written as

$$\dot{\omega} = -I_s^{-1} S(\omega) I_s^{-1} \omega - I_s^{-1} S(\omega) h_{tot} - I_s^{-1} \dot{h}_{(tot)} + I_s^{-1} N_{tot} \tag{5.12}$$

The rate of change in angular momentum $\dot{h}_{(tot)}$ can be absorbed from the controller. This can be written as:

$$\dot{h}_{(tot)} = -Nc \tag{5.13}$$

where the negative sign denotes the absorbed momentum. The total torque from external disturbances can be written as N_{dis}. Rearranging, equation *equation (5.12)* now reads

$$\dot{\omega}(t) = -I_s^{-1} S(\omega) I_s \omega(t) - I_s^{-1} S(\omega) h_{tot} + I_s^{-1} N_c(t) + I_s^{-1} N_{dis}(t) \tag{5.14}$$

which constitute the dynamics of the satellite with three reaction wheels. At the final equation *equation (5.14)* is shown the time dependency of the variables.

5.2 Disturbance Models

Gravitational torque

An unbalanced satellite in orbit is subjected to a torque due to the gravitational torque. Assumed that the earth is a point mass and the satellite is a rigid body, the gravitational torque can be estimated. Each infinitesimal element of the satellite of mass dm_i is subjected to an infinitesimal force dF_i that can be calculated thanks to Newton's law of universal gravitation.

$$dF_i = -G \frac{m_{earth}}{R_i^2} dm_i \cdot \hat{R}_i \tag{5.15}$$

where G is the gravitational constant, m_{earth} is the mass of the earth and R_i^2 is the vector from the Earth to the infinitesimal element of the satellite.
The moment of the gravitational force about the geometric center is calculated as the formula:

$$N_{gra} = \int_{sat} r_i \times dF_i \tag{5.16}$$

with r_i is the vector from the geometric center to the infinitesimal element. r_i can be written as the sum of the vector from the geometric vector to the mass center $r_{g,m}$ and the vector from the mass center of the element $r_{m,i}$. Therefore, the expression of the gravitational torque is simplified:

$$N_{gra} = \int_{sat} r_{g,m} \times dF_i + \int_{sat} r_{m,i} \times dF_i \tag{5.17}$$

$$= \int_{sat} r_{g,m} \times -G\frac{m_{earth}}{R_i^2}dm_i \cdot \hat{R}_i + \int_{sat} r_{m,i} \times -G\frac{m_{earth}}{R_i^2}dm_i \cdot \hat{R}_i$$

We can assumed that $r_{m,g} << R_i$ and R_i can be supposed constant and equals to the vector from the center of the earth to the geometric center of the satellite $R_{e,g}$. Thus, The second term is null by definition of the mass center.

$$\Rightarrow N_{gra} = G\frac{m_{sat} \cdot m_{earth}}{R_{e,g}^2} \cdot (\hat{R}_i \times r_{g,m}) \tag{5.18}$$

The position of the center of mass was measured for the previous project and is eqals to [?;?;?] in the frame of the satellite. Therefore, $r_{g,m,i}$ can be expressed in the inertial frame as following:

$$[r_{g,m,i}; 0] = q_{i,s} \otimes [?; ?; ?.0] \otimes q_{i,s}* \tag{5.19}$$

where $q_{i,s}$ is the quaternion that represents the rotation of the satellite in the inertia frame and $\otimes$ is the quaternion multiplication. Thus, the moment of force can be calculated by this expression above.

Solar radiation

Tfhe surface of the CubeSat will absorb or reflect the solar radiation, nevertheless, these two situations will alter the CubeSat, which will produce a torque about the satellite center of mass(CoM). [**SADC**]

The torque around CoM is given by:

$$N_{rad} = F_{rad} \times R_{CoM} \tag{5.20}$$

where F_{rad} is the solar radiation and R_{CoM} is the vector from the centre of mass to the geometric centre of radiation

The solar radiation F_{rad} can be expressed as:

$$F_{rad} = C_a P A \tag{5.21}$$

where C_a is the surface's reflectance: 0 for a perfect absorber, 1 for a perfect reflector, while P is the solar flux and A is the radiated area

The solar flux can be computed as follows:

$$P = \frac{F_s}{c} \tag{5.22}$$

where F_s is the mean solar energy and it is equal with 1358 W/m^2 and c is the speed of light

5.3 Attitude control design

6 | Implementation and test

7 | Acceptance test

The system is tested to see if it fulfills the requirements put up (*chapter 3*).

8 | Conclusion

Future work

A | Derivation of equation of motion

The general Euler's rotation equation with three reaction wheels aligned on the satellite body axis are derived as

$$I_1\dot{\omega}_1 = (I_2 - I_3)\omega_2\omega_3 + N_1 - \omega_2 h_3 + \omega_3 h_2 \tag{A.1}$$

$$I_2\dot{\omega}_2 = (I_3 - I_1)\omega_1\omega_3 + N_2 - \omega_3 h_1 + \omega_1 h_3 \tag{A.2}$$

$$I_3\dot{\omega}_3 = (I_1 - I_2)\omega_1\omega_2 + N_3 - \omega_1 h_2 + \omega_2 h_1 \tag{A.3}$$

The equation in compact form has been written as

$$\dot{\omega} = -I_s^{-1}S(\omega)I_s^{-1}\omega - I_s^{-1}S(\omega)h_{tot} - I_s^{-1}\dot{h}_{(tot)} + I_s^{-1}N_{tot} \tag{A.4}$$

where $S(\omega)$ is the skew symmetric matrix given by

$$S(\omega) = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{A.5}$$

and the angular momentum of the reaction wheels as $h_{tot} = [h_1 \ h_2 \ h_3]^T$.

Inertia matrix

The inertia matrix for a solid cuboid of height z , width y, and depth x, amd mass m_i with respect the center of mass is given by

$$I_i = \begin{bmatrix} \frac{1}{12}m_i(z^2+y^2) & 0 & 0 \\ 0 & \frac{1}{12}m_i(z^2+x^2) & 0 \\ 0 & 0 & \frac{1}{12}m_i(x^2+y^2 \end{bmatrix} \tag{A.6}$$

It is assumed that the Cube have a symmetric mass distribution around the axis of rotation to simplify the inertia matrix. With the mass distributed evenly and the axis of rotation being around one of the tree axis, the off diagonal term of the inertia matrix are equal to zero. These terms are also referred to as cross products of inertia.

B | Derivation of relative dynamics equations

The vector position from the center of the Earth to the satellite 1 and the satellite 2 is given by

$$\mathbf{p_1} = R \cdot \hat{\mathbf{x}} \tag{B.1}$$
$$\mathbf{p_2} = R \cdot \hat{\mathbf{x}} + x \cdot \hat{\mathbf{x}} + y \cdot \hat{\mathbf{y}} \tag{B.2}$$

the first time derivative and second time relative of $\mathbf{p_1}$ and $\mathbf{p_2}$ is computed:

$$\dot{\mathbf{p_1}} = \dot{R} \cdot \hat{\mathbf{x}} + R(\mathbf{w} \times \hat{\mathbf{x}})$$

where $\mathbf{w}$ is the angular velocity vector and $\mathbf{w} = w \cdot \hat{\mathbf{z}}$ due to the fact the position of the satellites stay all over the time in the plan $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$. Therefore, the first time derivative and the second time derivative are given by:

$$\begin{aligned}
\dot{\mathbf{p_1}} &= \dot{R} \cdot \hat{\mathbf{x}} + wR \cdot \hat{\mathbf{y}} \\
\ddot{\mathbf{p_1}} &= \ddot{R} \cdot \hat{\mathbf{x}} + w\dot{R} \cdot \hat{\mathbf{y}} + \dot{w}R \cdot \hat{\mathbf{y}} + w\dot{R} \cdot \hat{\mathbf{y}} + wR \cdot (\mathbf{w} \times \hat{\mathbf{y}}) \\
&= \ddot{R} \cdot \hat{\mathbf{x}} + 2w\dot{R} \cdot \hat{\mathbf{y}} - w^2 R \cdot \hat{\mathbf{x}} \\
\dot{\mathbf{p_2}} &= \dot{\mathbf{p_1}} + \dot{x} \cdot \hat{\mathbf{x}} + xw \cdot \hat{\mathbf{y}} + \dot{y} \cdot \hat{\mathbf{y}} - yw \cdot \hat{\mathbf{x}} \\
&= \dot{\mathbf{p_1}} + (\dot{x} - yw) \cdot \hat{\mathbf{x}} + (xw + \dot{y}) \cdot \hat{\mathbf{y}} \\
\ddot{\mathbf{p_2}} &= \ddot{\mathbf{p_1}} + (\ddot{x} - \dot{y}w - y\dot{w}) \cdot \hat{\mathbf{x}} + (\dot{x} - yw)w \cdot \hat{\mathbf{y}} + (\dot{x}w + x\dot{w} + \ddot{y}) \cdot \hat{\mathbf{y}} - (xw + \dot{y})w \cdot \hat{\mathbf{x}} \\
&= \ddot{\mathbf{p_1}} + (\ddot{x} - 2\dot{y}w - y\dot{w} - xw^2) \cdot \hat{\mathbf{x}} + (\ddot{y} + 2\dot{x}w + x\dot{w} - yw^2) \cdot \hat{\mathbf{y}}
\end{aligned}$$

Furthermore, The Newton law gives:

$$m\ddot{\mathbf{p_1}} = \mathbf{F_{grav,1}} + \mathbf{F_{drag,1}} + \mathbf{F_{dist,1}} \tag{B.3}$$
$$m\ddot{\mathbf{p_2}} = \mathbf{F_{grav,2}} + \mathbf{F_{drag,2}} + \mathbf{F_{dist,2}} \tag{B.4}$$
$$\Rightarrow \ddot{\mathbf{p_2}} - \ddot{\mathbf{p_1}} = \frac{1}{m}(\Delta\mathbf{F_{grav}} + \Delta\mathbf{F_{drag}} + \Delta F_{dist}) \tag{B.5}$$

with m is the mass of both satellites. The gravity is given by the universal law of gravitation:

$$\begin{aligned}
\frac{\mathbf{F_{grav,1}}}{m} &= -G\frac{m_{earth}}{||\mathbf{R}||^3}\mathbf{R} \\
\frac{\mathbf{F_{grav,2}}}{m} &= -G\frac{m_{earth}}{||\mathbf{R}+\mathbf{r}||^3}(\mathbf{R}+\mathbf{r})
\end{aligned}$$

where $\mathbf{r} = (x, y)$ is the vector from the satellite 1 to the satellite 2. The denominateur can be approximated by(reference):

$$||\mathbf{R}+\mathbf{r}||^{-3} = ||\mathbf{r}||$$

and thus, the difference between the gravity force on satellite 2 and the the gravity force on 1 is:

$$\mathbf{F_{grav,2}} - \mathbf{F_{grav,1}} \approx -\frac{\mu}{R^3}\mathbf{r}$$

with $\mu = G \cdot m_{earth}$, The drag force can be modelling be using the formula(ref):

$$\begin{aligned}
\mathbf{F_{drag,1}} &= -u_1||\dot{\mathbf{p_1}}||\dot{\mathbf{p_1}} \\
&= -u_1||\dot{\mathbf{p_1}}||(\dot{R}\cdot\hat{\mathbf{x}} + wR\cdot\hat{\mathbf{y}}) \\
\mathbf{F_{drag,2}} &= -u_2||\dot{\mathbf{p_2}}||\dot{\mathbf{p_2}} \\
&= -u_2||\dot{\mathbf{p_2}}||((\dot{R} + \dot{x} - yw)\cdot\hat{\mathbf{x}} + (wR + xw + \dot{y})\cdot\hat{\mathbf{y}})
\end{aligned}$$

Therefore, the equation(reference B3 i don't know how to do it) becomes:

$$\begin{cases} \ddot{R} - w^2R = -\frac{\mu}{R^2} - \frac{u_1}{m}||\dot{\mathbf{p_1}}||\dot{R} + \frac{F_{dist,1,x}}{m} \\ 2w\dot{R} + \dot{w}R = -\frac{u_1}{m}||\dot{\mathbf{p_1}}||wR + \frac{F_{dist,1,y}}{m} \end{cases} \tag{B.6}$$

and the equation(reference B5) gives:

$$\begin{cases} \ddot{x} - 2\dot{y}w - y\dot{w} - xw^2 = -x\frac{\mu}{R^3} + \frac{u_1}{m}||\dot{\mathbf{p_1}}||\dot{R} - \frac{u_2}{m}||\dot{\mathbf{p_2}}||(\dot{R} + \dot{x} - yw) + \frac{\Delta F_{dist,x}}{m} \\ \ddot{y} + 2\dot{x}w + x\dot{w} - yw^2 = -y\frac{\mu}{R^3} + \frac{u_1}{m}||\dot{\mathbf{p_1}}||wR - \frac{u_2}{m}||\dot{\mathbf{p_2}}||(wR + xw + \dot{y}) + \frac{\Delta F_{dist,y}}{m} \end{cases} \tag{B.7}$$

The operating point is the position $(x*, y*)$ of the satellite 2 in the frame of satellite. $x*$ and $y*$ can be computed thanks to the *figure B.1*.

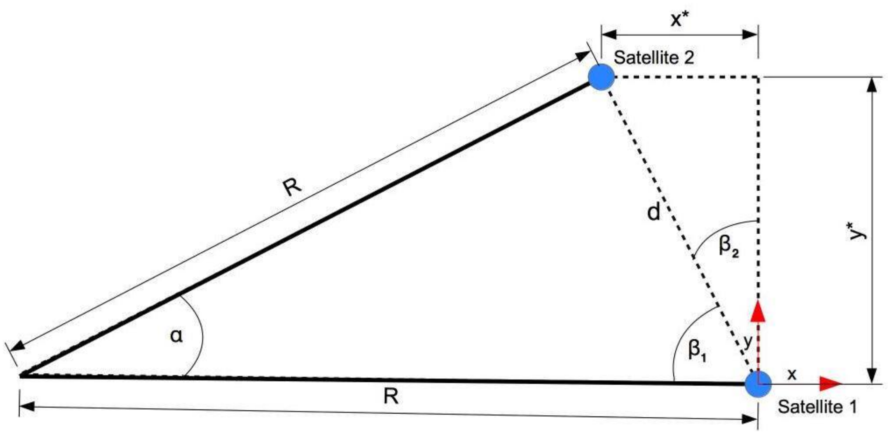


Figure B.1: Operating point

Thanks to basic trigonometry:

$$\begin{aligned} d &= 2R * sin(\frac{\alpha}{2}) \\ x* &= -d * sin(\beta_2) \\ &= -d * sin(\frac{\alpha}{2}) \\ &= -2R * sin(\frac{\alpha}{2})^2 \\ y* &= d * cos(\frac{\alpha}{2}) \\ &= 2R * sin(\frac{\alpha}{2}) * cos(\frac{\alpha}{2}) \\ &= R * sin(\alpha) \end{aligned}$$

with α is the desired angle between satellite and so $\beta_2 = 90° - \beta_1 = 90° - (90° - \frac{\alpha}{2}) = \frac{\alpha}{2}$. Therefore we change the coordinate reference as following:

$$\begin{aligned} x &\Leftarrow x - x* \\ y &\Leftarrow y - y* \end{aligned}$$

Thus, the equations (reference B7) become:

$$\begin{cases} \ddot{x} - 2\dot{y}w - (y + y*)\dot{w} - (x + x*)w^2 = \\ -(x + x*)\frac{\mu}{R^3} + \frac{u_1}{m}||\dot{\mathbf{p_1}}||\dot{R} - \frac{u_2}{m}||\dot{\mathbf{p_2}}||(\dot{R} + \dot{x} - (y + y*)w) + \frac{\Delta F_{dist,x}}{m} \\ \ddot{y} + 2\dot{x}w + (x + x*)\dot{w} - (y + y*)w^2 = \\ -(y + y*)\frac{\mu}{R^3} + \frac{u_1}{m}||\dot{\mathbf{p_1}}||wR - \frac{u_2}{m}||\dot{\mathbf{p_2}}||(wR + (x + x*)w + \dot{y}) + \frac{\Delta F_{dist,y}}{m} \end{cases} \tag{B.8}$$

C | Linearisation of the relative dynamics eqations

From the equations(ref B8) and assuming that the radius is constant and the angular velocity is equals to $w = \sqrt{\frac{\mu}{R^3}}$, a linearisation of the system can be derived using some apporximations. the state is defined as :

$$s = [x; \dot{x}; y; \dot{y}]$$

Moreover, the norm of the velocity of both satellite is assumed to be equal and to be constant ($||\dot{\mathbf{p_1}}|| = ||\dot{\mathbf{p_2}}|| = C$). Therefore, the nominal system is given by:

$$\begin{cases} \dot{s_1} = s_2 \\ \dot{s_2} = 2ws_4 - u_2 \dfrac{y^* wC}{m} \\ \dot{s_3} = s_4 \\ \dot{s_4} = -2ws_2 - (u_2 - u_1)\dfrac{wRC}{m} \end{cases} \tag{C.1}$$

using the approximation $\dot{x}$, $y << y^*$ and x, x^*, $\frac{\dot{y}}{w} << R$.

List of Corrections