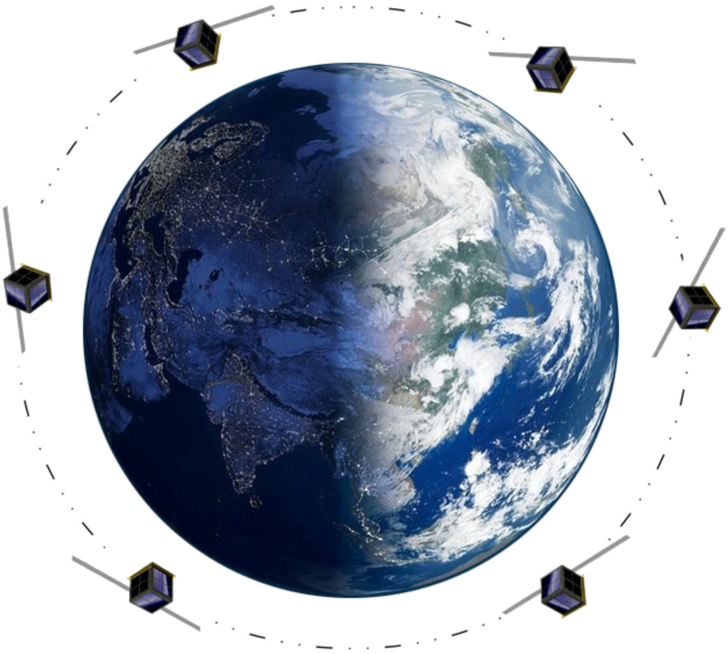
Formation flying using pico-satellites



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Synopsis

This report describes the design and implementation of a control system on an AAU-CubeSat, a pico- satellite used for Low Earth Orbit flight.
The objective is to use a flight formation for monitoring Greenland, by having six satellites equally distributed on orbit.
Two controllers must be design, one for controlling the distance between the satellites using the drag force, and one for attitude control.
Given the nonlinear nature of the system a SMC is implemented.

Preface

This report has been written by group 931 on third semester in Control and Automation on Aalborg University. References made before a full stop regards the sentence and reference after full stop regards the paragraph. Quotes are inside quotations marks and in cursive. Attached to report is a zip file with:

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1 | Introduction

In the last decades, the space technology is continuously growing. The reason for this is the increased deploying of satellites used in the numerous fields, in particular telecommunications and meteorology. [**SIDI**]

Missions containing satellites operating close to each other are commonly referred to as flying formation, which is known as a distributed satellite system. Two types of distributed space systems are identified as formations and constellations flying.

A distributed space system is defined by NASA Goddard Space Flight Center (GSFC) as *"an end-to-enpd system including two or more space vehicles and a cooperative infrastructure for scientific measurement, data acquisition, processing, analysis, and distribution"*. [**SFF**]

Satellite formation flying is not having a precise definition, however, the definition proposed by NASA GSFC is that *"formation flight involves the use of an active control scheme to maintain the relative positions of the spacecraft "*. In contrast, a constellation is defined as *"two or more spacecraft in similar orbits with no active control by either to maintain a relative position"*. [**SF**]

Formation flying it might offer many possibilities for space exploration, such as surveillance, field measurements and atmospheric survey missions as well as on-orbit satellite inspection, maintenance, and recovery. This approach it has a few challenges which involve autonomous control of the satellites influenced by the different disturbing forces caused by gravity gradient, solar radiation pressure, aerodynamic drag, and Earth's oblateness effect, with a purpose of achieving it with minimum fuel consumption. Nevertheless, there is currently no formation flying satellites in orbit, however, two such missions are ESA's "Cluster" mission and the ESA/NASA "Grace" mission, which are in development stages. [**SF**]

The use of satellite formations is expected to rise in the next years. This makes it relevant to look at improving or adding functionalities to satellites. Based on this it has been decided to look at the case of a distributed space system consisting of a formation of six satellites equally distributed on the orbit and analyzing the behavior between them.

1.1 Problem statement

Design and implement a controller for controlling the individual distance between satellites using the drag force.

1.2 Use-case

In this project, the concept of a formation flight of satellites will be used for the purpose of monitoring. Denmark has a small island called Greenland, where the Danish Government needs to monitor it. One method is to have a formation of satellites going around the orbit and when they are located in the northern hemisphere, the satellites will point down and look towards Greenland.

One of the essentials in formation flight is choosing the number of satellites in orbit. Therefore, in order to have a continuous coverage, a distributed satellite system composed of six satellites equally distributed are chosen, compared with two or four satellites where communication between each other will be poor.

The task the satellite has to perform is acquiring data by flying around Greenland, using radio signals and taking pictures.

2 | System Description

The overall idea of the project is to consider more than one satellites flying in formation, with a certain distance in between and with the purpose of maintaining that distance by using the drag force. As a proof of concept, an AAU-CubeSat will be used, by choosing six AAU-CubeSat that orbit the Earth like is shown in *figure 2.1*. Therefore, a control system is developed, where the six satellites are nodes and they represent periods. In this project, all CubeSat's will be assumed identical. Moreover, a full-scale implementation of the system will not be possible, therefore, the whole system will be simulated using MATLAB and Simulink.

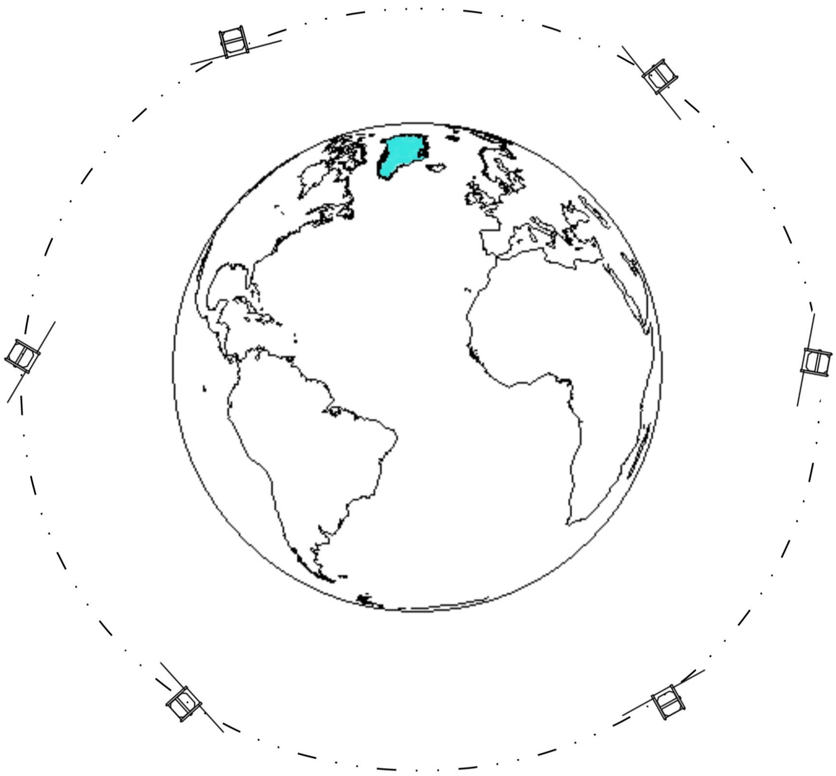

Figure 2.1: Six satellites in flying formation on orbit

2.1 About AAU-CubeSat

The AAU-CubeSat shown in *figure 2.2* is a pico-satellite developed by Stanford University, but assembled at Aalborg University by students and used mainly for Low Earth Orbit (LEO) tests.

Figure 2.2: View of CubeSat satellite [**cs**]

The pico-satellite is designed for LEO, therefore a few constraints are imposed. The CubeSat is limited in size and weight. The dimensions of the satellite are $10cm \times 10cm \times 30cm$, while the weight is around 1 kg.[**CDS**]

In order to place the CubeSat on orbit, a deployment system is used, called P-POD. This system uses the force of a spring to launch the satellite into space. The satellite will be placed inside the launch rocket as payload. By using this system, an important advantage is reducing the cost of the launch. [**PPOD**]

2.2 AAU-CubeSat actuators

The selection of attitude control components is important in order to meet the performance requirements. For this project, three magnetorquers and three momentum wheels have been chosen as actuators. Initially, using only three momentum wheels has been considered, but the downside of using only momentum wheels is that some amount of momentum can be stored in the wheel, which will imply having a way to take back all that momentum and use it. Therefore, there are multiple ways to release that torque, and one is to use magnetorquers.

Magnetorquers are wire coils which generate an electromagnetic field. The field interacts with the Earth magnetic field and a torque is generated for stabilizing the satellite. An important aspect of the magnetorquer is when the reaction wheel reaches a maximum speed and can no longer produce the torque this is referred as wheel saturation, so a magnetorquer is used to extract the momentum from the wheel.

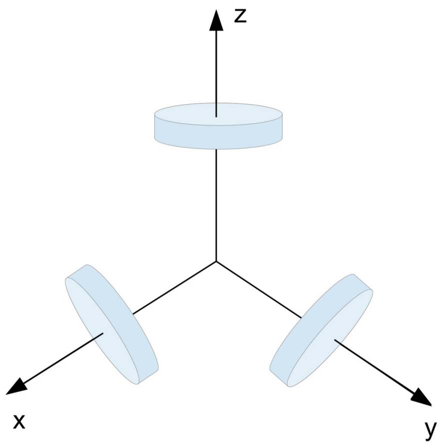

Figure 2.3: Example of three reaction wheel for CubeSat

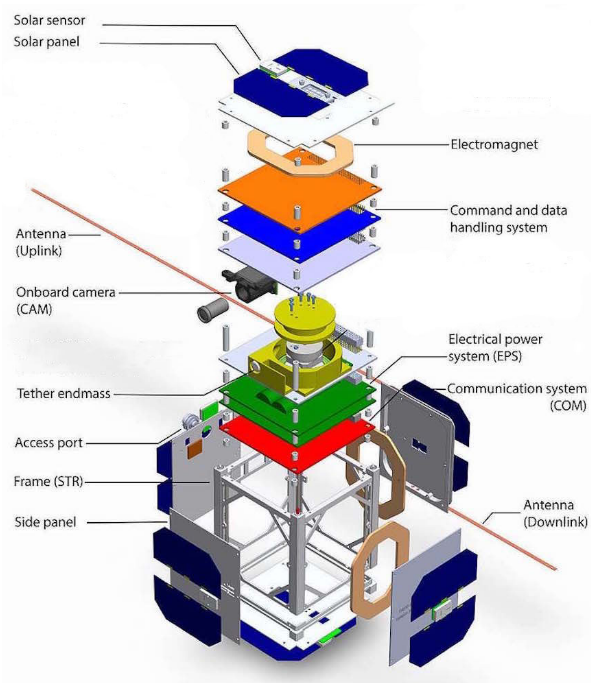

Figure 2.4: Expanded view for CubeSat [**view**]

Reaction wheels shown in *figure 2.3* strength is that no information is needed about the magnetic field in order to control the CubeSat torque. These wheels are capable to store the momentum needed for maneuvering or pointing.

Thrusters could represent a possibility for gaining energy because removing energy from the system it can be proved easily by using the drag force. Due to the weight of the trusters, they are not considered in this project.

2.3 AAU-CubeSat sensors

The CubeSat can sustain itself using solar pannels with in the middle a sun sensor similar in *figure 2.4* , which provide a vector equal to the direction of the sun and also a vector of the Earth's magnetic field measured by the magnetometer. Whether the Earth's magnetic field is measured, or the sun vector, the objective is to use these sensors to deliver vector solutions for determining the satellite's pointing and rotation rates.

Magnetometer is a sensor used for attitude control, which measure the direction and intensity of the magnetic field. The atittude is determined from the magnetometer by comparing the measure magnetic field with a referance field.

Sun sensor is used for estimating the position of the Sun and delivering a vector of measurements from the Sun.

Pointing accuracy

The required pointing accuracy when acquiring a photo is based on the a height from the picture is taken, in this case around 700 km above the Earth surface is going to cover approximately ?? km.

2.4 Coordinate frames

In order to determine the attitude in three-dimensional space, various coordinate frames are defined.

Reference Coordinate Systems

In order to define an orbit around Earth, two specific Earth coordinate systems are defined. Both of them have their origin in the geometrical center of Earth and are named the Earth Centered Inertial (ECI) coordinate frame and the Earth Centered Earth Fixed (ECEF) coordinate frame. These can be seen in *figure 2.5* and *figure 2.6*

Earth Centered Inertial frame(ECI)

In order to describe the orbit formation of the satellite, the ECI frame shown in *figure 2.5* is used, since it can be seen as a non-accelerating frame. The z axis is pointing through the geographical north pole, the x axis is crossing from the point where the equatorial of the earth and the vernal equinox met and the y axis is the cross product of x and z creating a right-handed coordinate system.

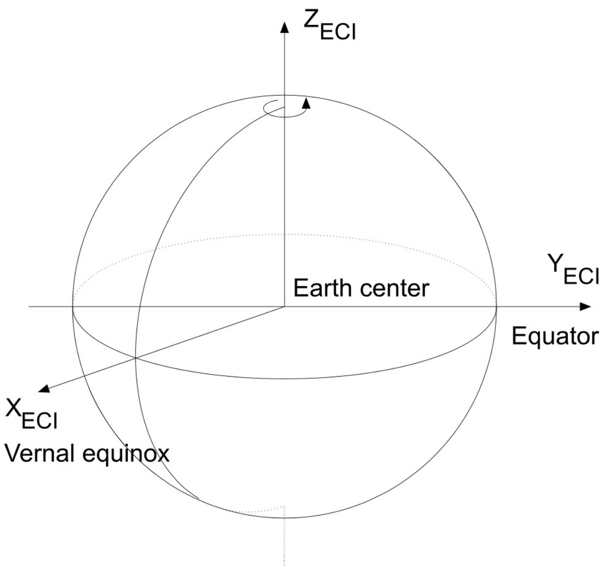

Figure 2.5: ECI coordinate frame

Earth Centered Earth Fixed Frame (ECEF)

Another coordinate frame is the Earth Centered Earth Fixed (ECEF) coordinate frame shown in *figure 2.6*. In this case the X-axis is passing through the zero longitude, also known as Greenwich meredian, and the Z-axis parallel with the rotational axis. In this way the ECEF frame is fixed to the earth itself and rotates around with it.

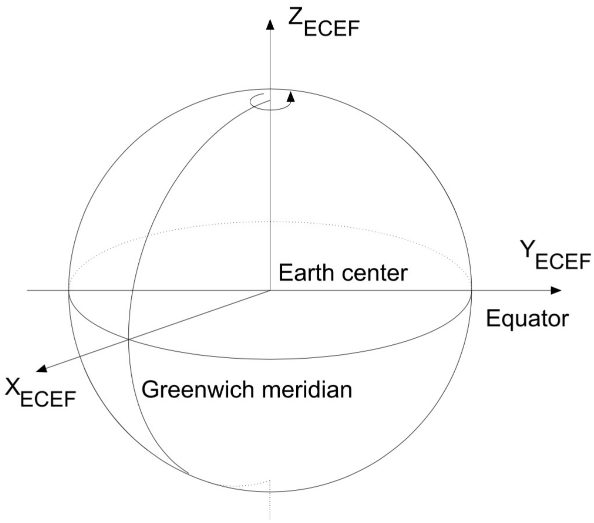

Figure 2.6: ECEF coordinate frame

Satellite Coordinate Systems

For the purpose of determining the attitude of the satellite, several coordinate systems are introduced. The attitude and position of the satellite is given as a rotation between the satellite fixed coordinate frames and the reference frames.

Orbit Reference frame(ORF)

The orbit reference shown in *figure 2.7* is a frame defined in Cartesian coordinates that can be seen as a non-changing frame with respect the earth and the satellite. The z axis always pointing at the Nadir point and it is parallel to the z_e axis o the inertial frame of the earth. The x_o axis, it is parallel to the orbit plane and y_o is the cross product of the x_o and z_o.

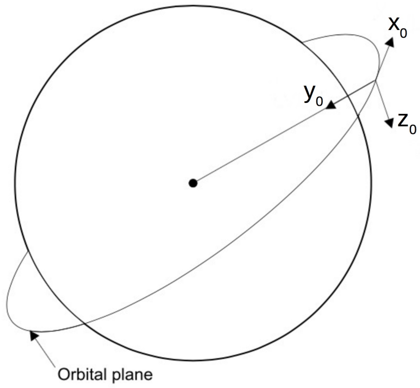

Figure 2.7: ORF coordinate frame

Satellite Body Frame(SBF)

The satellite body frame is placed in the center of mass of the satellite as shown in *figure 2.8*.

Satellite Controller frame(SCF)

In order to derive the kinematic equations, a controller reference frame seen in *figure 2.8* should be specified. It is located in the center of mass of the satellite and it is defined such that the axis of higher inertia z_c pointing in the center of ECI and the x_c axis with the smallest inertia, pointing along with the orbit's x_o

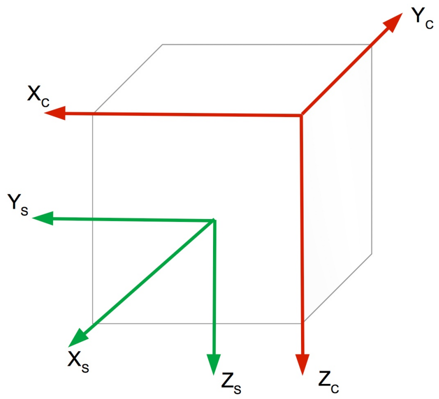

Figure 2.8: Satellite body frame and satellite controller frame

3 | Requirements

Based on the use-case introduced and the available system a set of requirements are formulated.

System requirements

1. **The formation shall be able to maintain a given distance within 45°**
2. **Each satellite shall be able to change its orientation**

4 | Angle control between satellites

In this chapter, the focus will be on modelling and the control of the angle between two satellites using the drag force as the control input of the system. In this section the satellite is able to change its orientation instantaneously, therefore the drag force can be modified instantaneously. The Earth and the satellite are assumed to be a point mass to simplify the system.

4.1 Modelling

The satellite is mainly subjected to three forces: the gravity, the drag force and the solar radiation. Thus, the second law of Newton gives:

$$\sum \mathbf{F} = m_{sat}\ \mathbf{a} = \mathbf{F_g} + \mathbf{F_D} + \mathbf{F_{rad}} \tag{4.1}$$

with the gravity modeled by:

$$\mathbf{F}_g = -G\frac{m_{earth}m_{sat}}{||\mathbf{p}||^3}\mathbf{p} \tag{4.2}$$

where $\mathbf{p}$ is the vector position of the satellite from the Earth centre to the mass centre of the satellite in the inertial frame and the expression for $\mathbf{F}_D$ and $\mathbf{F}_{rad}$ are explained in the next section.

4.2 Disturbance Models

Aerodynamic Drag Force

The satellite is subjected to an aerodynamic drag force due to the atmosphere. The collisions with the air cause a force in the opposite direction of the velocity of the satellite. The force was modelled by Lord Rayleigh.[**FSA**]

$$\mathbf{F_D} = -\frac{1}{2}\rho\ C_D\ A_\perp||\mathbf{v}||\mathbf{v} \tag{4.3}$$

where ρ is the density of the air, C_D is the drag coefficient, $A_\perp$ is the area that is perpendicular of the velocity of the satellite $\mathbf{v}$.

The drag coefficient C_D and the perpendicular area $A_\perp$ depend on the orientation of the satellite. Therefore, this force can be used as an input for the control of the position and the velocity of the satellite.

The density of the air depends on the altitude of the satellite and the air temperature, but it is considered to be constant in this case for simplifying the model. ρ is chosen to be equal to $1.454 \cdot 10^{-13} Kg/m^3$ based on the empirical model of the Committee on Space Research (COSPAR) International Reference Atmosphere [**FSA**].

The drag coefficient is dependent on the orientation and the maximum value of C_D is equal to 1.05 for a non tilted cubed as shown on the *figure 4.1* and equal to 0.80 for an angled cubed [**wik**]. The drag coefficient is assumed to be constant and equal to 1 in order to simplify the equation.

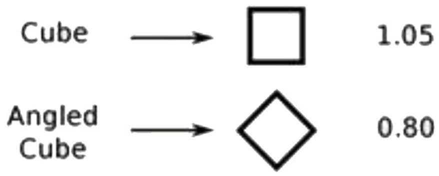

Figure 4.1: description needed

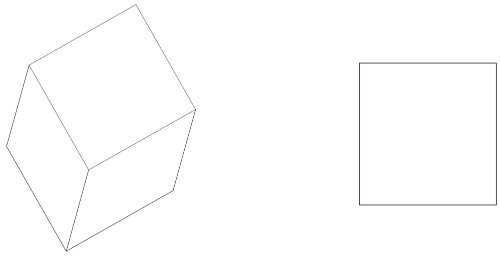

Figure 4.2: description needed

Therefore, the control parameter is the perpendicular area $A_\perp$. The maximum and minimum value of $A_\perp$ is represented in *figure 4.2*. Thus, the minimum value is the surface of a square of 10cm of dimension ($A_\perp = 100cm^2$) and the maximum value is the surface of an hexagon of 10cm of dimension ($A_\perp = \sqrt{3}\ 100cm^2$). Thus, the drag force can be expressed as the following.

$$\mathbf{F_D} = -u||\mathbf{v}||\mathbf{v} \tag{4.4}$$

where u is the control input and it can take value between $7.27 \cdot 10^{-16}$ and $1.888 \cdot 10^{-15}$

Solar radiation

Solar radiation is emitted constantly by the Sun, which illuminates the surface of the CubeSat. The surface of the satellite will absorb or reflect the solar radiation, nevertheless, these two situations will alter the CubeSat and produce a radiation force. [**SADC**]

The solar flux can be computed as follows:

$$P = \frac{F_s}{c} \tag{4.5}$$

where F_s is the mean solar energy and it is equal with 1358 W/m^2 and c is the speed of light

The solar radiation $\mathbf{F}_{rad}$ can be expressed as:

$$\mathbf{F_{rad}} = C_a P A \frac{\mathbf{r_{sun,sat}}}{||\mathbf{r_{sun,sat}}||} \tag{4.6}$$

where C_a is the surface's reflectance: 0 for a perfect absorber, 2 for a perfect reflector, P is the solar flux, A is the radiated area, $r_{sun,sat}$ is the vector from the sun to the satellite and the norm of $||r_{sun,sat}||$ is equal to 1

J_2 gravity perturbation

The force which the Earth is exerting upon a object outside its sphere is a conservative force and its potential energy can be written as follows:

$$U(r) = -\frac{\mu}{r} \tag{4.7}$$

Because the Earth is not a perfect sphere and also its mass distribution is not homogeneous, *equation (4.7)* is rewritten by adding the spherical harmonic expansion to correct the gravitational potential for the Earth:

$$U(r) = -\frac{\mu}{r} + B(r, \phi, \lambda) \tag{4.8}$$

where $B(r, \phi, \lambda)$ is the spherical harmonic expansion used to correct the gravitational potential for the Earth's nonsymmetric mass distribution seen in *figure 4.3*

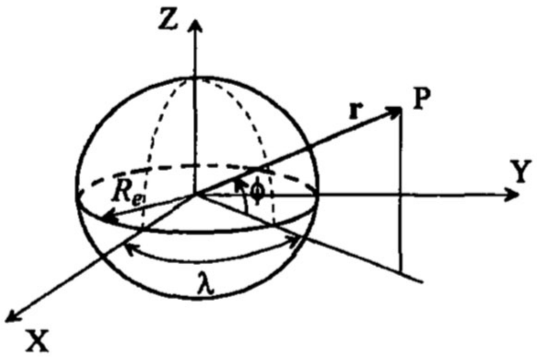

Figure 4.3: Coordinates for deriving the external gravitational potential of the Earth

The expression for gravitational potential of the Earth can be approximate as:

$$U \approx -\frac{\mu}{r}\left[1 - \sum_{n=2}^{\infty}\left(\frac{R_e}{r}\right)^n J_n P_n sin(\phi)\right] = \frac{\mu}{r}[U_0 + U_{J_2} + U_{J_3} + ...] \tag{4.9}$$

where U_0 = -1 and $U_{J_2} = \left(\frac{R_e}{r}\right)^2 J_2 \frac{1}{2}(3sin^2\phi - 1)$

The gravitational forces acting on the satellite are obtained from the relation:

$$F = -m\nabla U \tag{4.10}$$

and is obtaining the following [**SIDI**]:

$$F_x = -\frac{\partial U}{\partial x} = \mu\left[-\frac{x}{r^3} + A_{J_2}\left(15\frac{xz^2}{r^7} - 3\frac{x}{r^5}\right)\right] \tag{4.11}$$

$$F_y = -\frac{\partial U}{\partial y} = \mu\left[-\frac{y}{r^3} + A_{J_2}\left(15\frac{yz^2}{r^7} - 3\frac{y}{r^5}\right)\right] \tag{4.12}$$

$$F_z = -\frac{\partial U}{\partial z} = \mu\left[-\frac{z}{r^3} + A_{J_2}\left(15\frac{z^3}{r^7} - 3\frac{z}{r^5}\right)\right] \tag{4.13}$$

where $A_{J_2} = \frac{1}{2}J_2R_e^2$ and and R_e is the mean radius ofthe earth at the equator

4.3 State Space Representation

The state of the system is the vector position and the vector velocity in the inertial frame:

$$x = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \end{bmatrix} \tag{4.14}$$

The equation is given by:

$$\dot{x} = \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} v \\ a \end{bmatrix} \tag{4.15}$$

$$= \begin{bmatrix} \mathbf{v} \\ \frac{1}{m_{sat}}(-G\frac{m_{earth}m_{sat}}{||\mathbf{p}||^3}p - u||\mathbf{v}||\mathbf{v}) + \delta(x,t) \end{bmatrix} \tag{4.16}$$

$$= f(x) + u \cdot g(x) + \delta(x,t) \tag{4.17}$$

with

$$f(x) = \begin{bmatrix} v \\ -G \cdot m_{earth}\frac{p}{||p||^3} \end{bmatrix}, \quad g(x) = \begin{bmatrix} 0 \\ -\frac{1}{m_{sat}}||v||v \end{bmatrix}$$

and $\delta(x,t)$ reprensent the influence all the disturbances.

4.4 Relative dynamics

In order to analyse the distance between two satellites, the relative dynamics are analyzed. Furthermore, to simplify the system, satellites will be assumed to stay on the same plane. This assumption has also to be made due to the limitation of the direction of the input control (the drag force).

To compute the equations of the motion of one satellite compared to another, a new frame is used. The frame is illustrated in the *figure 4.4*, where the origin is the first satellite and the axis $\hat{\mathbf{x}}$ is defined by $\hat{\mathbf{x}} = \frac{\mathbf{R}}{||\mathbf{R}||}$, where $\mathbf{R}$ is the vector from the centre of the Earth to the first satellite. The axis $\hat{\mathbf{y}}$ is perpendicular to $\hat{\mathbf{x}}$ and in the plane of motion of the satellites and $\hat{\mathbf{z}}$ is defined by the right-hand law ($\hat{\mathbf{z}} = \hat{\mathbf{x}} \times \hat{\mathbf{y}}$).

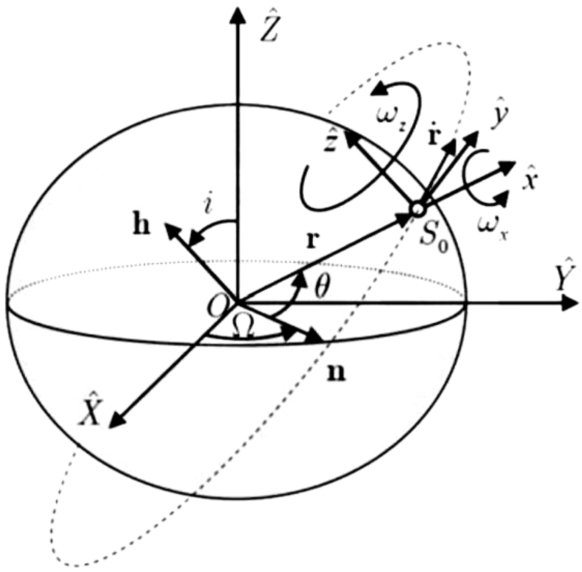

Figure 4.4: Frame for the relative dynamics [**SFF**]

Therefore, the vector position from the Earth to the fisrt satellite and the second satellite can be expressed in this frame:

$$\mathbf{p_1} = R\ \hat{\mathbf{x}} \tag{4.18}$$

$$\mathbf{p_2} = R\ \hat{\mathbf{x}} + x\ \hat{\mathbf{x}} + y\ \hat{\mathbf{y}} \tag{4.19}$$

The relative equation of motion can be writen as follows:

$$\begin{cases} \ddot{x} - 2\dot{y}w - (y+y^*)\dot{w} - (x+x^*)w^2 = \\ -(x+x^*)\dfrac{\mu}{R^3} + \dfrac{u_1}{m}||\dot{\mathbf{p}}_\mathbf{1}||\dot{R} - \dfrac{u_2}{m}||\dot{\mathbf{p}}_\mathbf{2}||(\dot{R} + \dot{x} - (y+y^*)w) + \dfrac{\Delta F_{dist,x}}{m} \\ \ddot{y} + 2\dot{x}w + (x+x^*)\dot{w} - (y+y^*)w^2 = \\ -(y+y^*)\dfrac{\mu}{R^3} + \dfrac{u_1}{m}||\dot{\mathbf{p}}_\mathbf{1}||wR - \dfrac{u_2}{m}||\dot{\mathbf{p}}_\mathbf{2}||(wR + (x+x^*)w + \dot{y}) + \dfrac{\Delta F_{dist,y}}{m} \end{cases} \tag{4.20}$$

The derivation of these equations can be found in *appendix B*.

Relative state space representation

Since the equations of relative motion that have been derived in *appendix B* are not linear, a linearization is made around the operating point, x^* and y^*, by introducing the states with a new variable as $s = [x\ \dot{x}\ y\ \dot{y}]^\mathsf{T}$. From the *equation (4.20)* and assuming that the radius is constant and the angular velocity equals to $w = \sqrt{\frac{\mu}{R^3}}$, a linearization of the system can be derived using some approximations. Moreover, the norm of the velocity of both satellite is assumed to be equal and constant as $||\dot{\mathbf{p}}_\mathbf{1}|| = ||\dot{\mathbf{p}}_\mathbf{2}|| = C$, where $C = \sqrt{\frac{\mu}{R}} = \omega\ R$. Therefore, the nominal system is given by:

$$\begin{cases} \dot{s}_1 = s_2 \\ \dot{s}_2 = 2ws_4 - u_2\dfrac{y^* wC}{m} \\ \dot{s}_3 = s_4 \\ \dot{s}_4 = -2ws_2 - \Big(u_2 - u_1\Big)\dfrac{wRC}{m} \end{cases} \tag{4.21}$$

using the approximation $\dot{x}$, $y \ll y^*$ and x, x^*, $\frac{\dot{y}}{w} \ll R$, therefore the system in state space can be writen as:

$$\dot{s}(t) = \underline{A}s(t) + \underline{B}u(t)$$

$$\underline{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2w \\ 0 & 0 & 0 & 1 \\ 0 & -2w & 0 & 0 \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -R^2\ w^2 \end{bmatrix}$$

With this assumptions and by using a control law $u = u_2 - u_1$ from the *equation (4.21)*, if u_2 and u_1 are chosen to be equal to u_{max} and u_{min} respectively, therefore, $u > 0$ and in this case y should decrease as seen in *figure 4.5*. From *figure 4.6* it can be seen that the theoretical and the practical of y are not the similar. This is due to the fact that when a satellite is subject to a drag force, the altitude of the satellite is decreasing and the angular velocity is increasing, which will be shown in the next section.

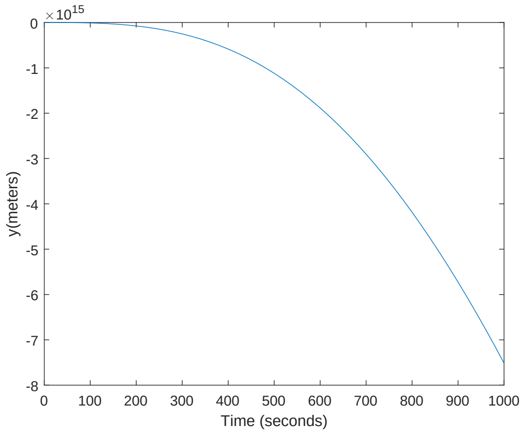

Figure 4.5: Approximation model

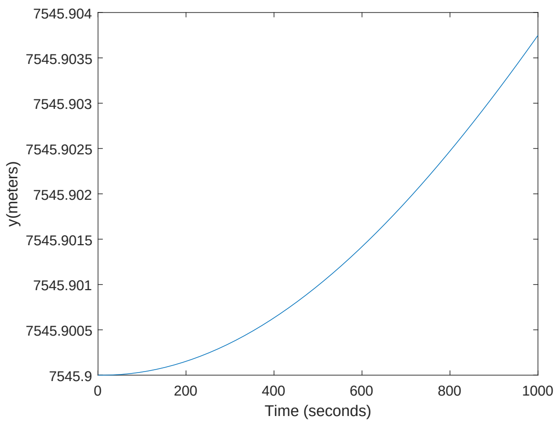

Figure 4.6: Real model

Therefore, when $u_2 > u_1$, then $w_2 > w_1$ this will lead to an increase in the angle θ between the satellites, and since $y = R\ sin\theta$, y will have an increasing attitude. The above mentioned issues are shown in *figure 4.6*.

In conclusion, the approximation that the time derivative of angular velocity is equal to 0 cannot be made for controlling the distance between the two satellites.

4.5 Modelling based on the angular velocity

As seen in the previous section, the angular velocity cannot be assumed constant. Therefore an equation is needed to estimate it as a function of the drag force. In *appendix C*, the time derivative of the angular velocity can be approximated as a linear function of the drag force.

$$\Delta\dot{\omega} = Cu \tag{4.22}$$

with $C = \frac{3\omega_0^2 R_0}{m}$. In order to check this equation and the coefficient, a simulation is performed using a constant drag force coefficient ($u = u_{min}$). The angular velocity as a function of time with constant drag force is shown in *figure 4.7*.

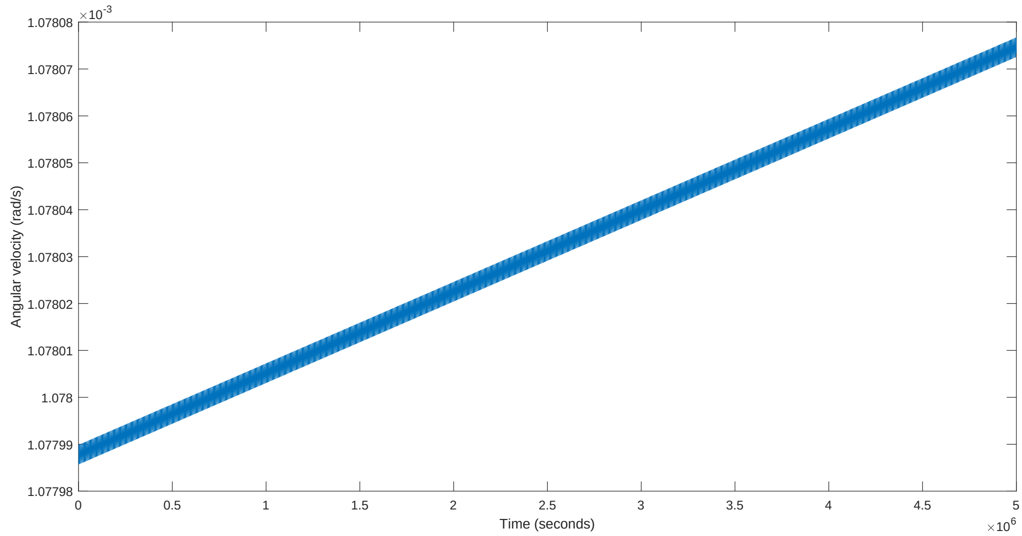

Figure 4.7: Angular velocity as function of time

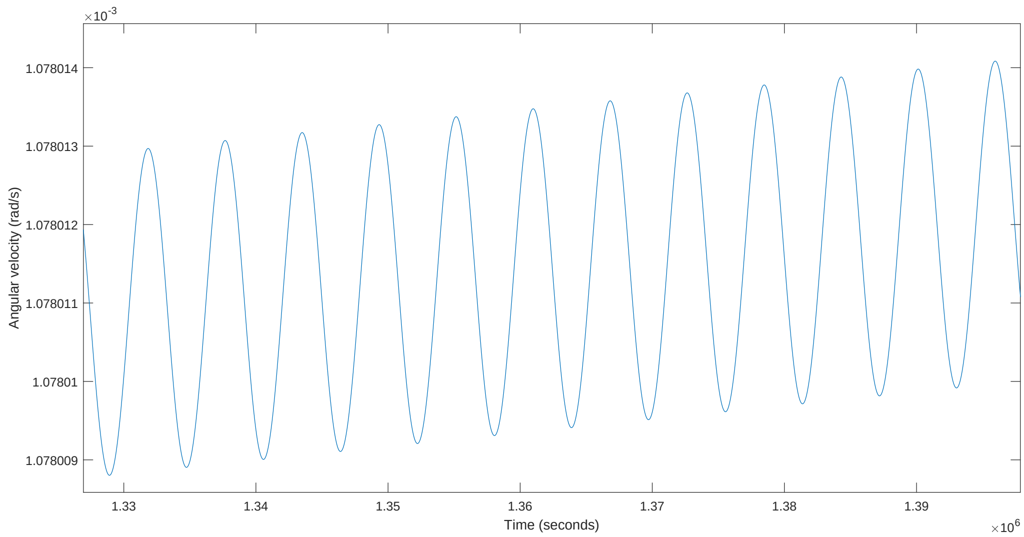

Figure 4.8

The coefficient found in the simulation is almost the same than in the theory. The oscillations can be observed in *figure 4.8* with a frequency equals to $f \approx \frac{2\pi}{\omega_0}$, where ω_0 is the angular velocity of the satellite in the beginning. This is due to the fact that the orbit is not exactly a circle but an ellipse. Thus, the angular velocity changes slightly during a turn around the Earth. In order to limit the influence of these variations on the controller, a second order low pass filter is added to reduce the amplitude of these oscillations. A state space representation can be derived with states θ and $\dot{\theta}$, where θ is the angle between two satellites. The time derritve of $\dot{\theta}$ is equal with $\dot{\omega}_2 - \dot{\omega}_1$ and using *equation (4.22)*, $\ddot{\theta} = Cu_2 - Cu_1 = Cu$.

$$s = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \tag{4.23}$$

$$\dot{s}(t) = \underline{A}s(t) + \underline{B}u(t) \tag{4.24}$$

where

$$\underline{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{4.25}$$

$$\underline{B} = \begin{bmatrix} 0 \\ \frac{3\omega_0^2 R_0}{m} \end{bmatrix} \tag{4.26}$$

4.6 Distance control design

A controller is designed to control the angle between two satellites. Due to the fact that the state representation is linear, Linear Quadratic Regulator(LQR) is chosen as control method with the following cost function:

$$\mathcal{I} = \int (\mathbf{x}^\mathsf{T}\ \underline{Q}\ \mathbf{x} + \mathbf{u}^\mathsf{T}\ \underline{R}\ \mathbf{u})dx \tag{4.27}$$

Due to the fact that no predefined cost is specified, the weighting matrices can be chosen as maximum acceptable values [optimality notes], thus, the $\underline{R}$ matrix is defined as $\underline{R} = [u_{delta}^{-2}]$ ($u_{delta} = u_{max} - u_{min}$). For the states weighting matrices no maximum value is defined. The weight for the second state is chosen to be equal to zero because the desired state to converge is θ and if the first state converge, the second state will also converge to zero. The weight for the first state (θ), is chosen by trial and error leading to the trade off between fast saturation and slow controller, since for big values of weights the controller will be in saturation mode faster and with low values the convergence will be slower. Therefore, the weighting matrices are given by :

$$\underline{Q} = \begin{bmatrix} (\frac{\pi}{7})^{-2} & 0 \\ 0 & 0 \end{bmatrix} \tag{4.28}$$

$$\underline{R} = \begin{bmatrix} u_{delta}^{-2} \end{bmatrix} \tag{4.29}$$

The vector of gains K is obtained by solving the Algebraic Riccati equation. The control input signal can be computed ($u = u_2 - u_1$) and therefore if u is bigger than zero, u_1 will be equal to u_{min} and u_2 will be equal to $u + u_{min}$. If u is smaller than zero, it will be the opposite. The control law used for designing the controller is: $u = -K(1)\ \theta\ - K(2)\ \dot{\theta}$.

Frequency Analysis

The loop transfer function can be computed $L(s) = P(s)\ C(s)\ LPF(s)\ H_{attittude}(s)$ where $P(s)$ is the transfer function of the state representation system, $C(s) = K_1 + K_2 s$ is the transfer function of the controller, $LPF(s)$ is the transfer function of the second order low pass filter and $H_{attitude}$ represent is the first order transfer function where the rise time is chosen to be equal to 10 minutes which is a estimation of time that the satellite take to converge into the good orientation. The bode diagram of L(s) is represented in the *figure 4.9*.

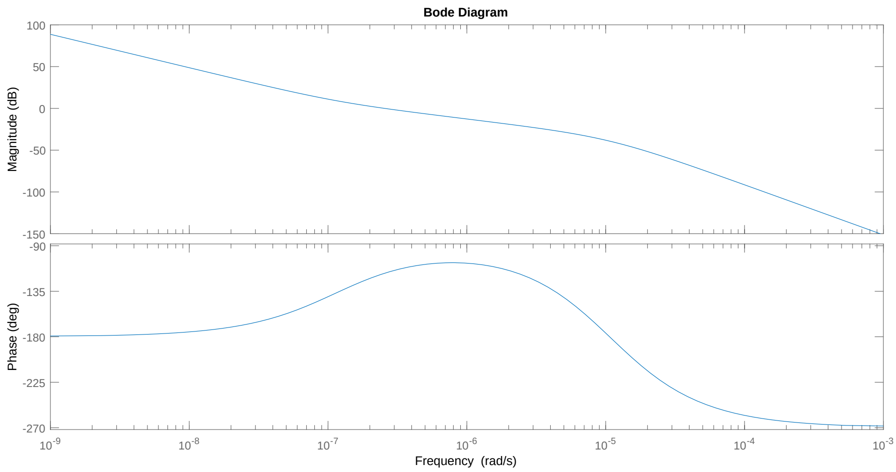

Figure 4.9: Bode diagram of the loop transfer function

The crossover frequency and the phase margin can be extracted from the bode diagram. The crossover frequency is equal to $2.6418 * 10^{-7} \frac{rad}{s}$ and the phase margin is equal to $62.7074°$ which is enough to accept a large delay due to the small crossover frequency.
.

Satellite Formation Control

Global algorithm

Since the formation consists of more than two satellites a second controller is designed to control n satellites around the Earth. The gain vector K computed above can be used to calculate the difference drag force between two neighbors satellites (called $u_a = u_2 - u_1$, $u_b = u_3 - u_2$, ...). Thus, a system of $n-1$ equations with n unknown variables is obtained. the last equation is chosen to set the minimum value of u_1, u_2,... equals to u_{min}. We have now a system of n equations with n unknown variables that can be solved to determine the drag force coefficient u_i for each satellite.

Distributed algorithm

In this case, u_1 is chosen to be equal with u_{medium}, therefore u_2 is computed using the control law found in *equation (4.6)*, which is a function of the angle between the first satellite and the second one. The drag force of satellite i can be determined by knowing the angle between the satellite i and $i-1$, the time derivative of this angle and the drag force of the satellite $i-1$.

Stability Analysis

In order to analyze the stability of the distributed formation controller, the global system is written in state space form as in *equation (4.24)*. The states are defined as

$$s = [\theta_n \ \dot{\theta_n}]^\mathsf{T}$$

where $\theta_n = [\theta_{12}\ \theta_{23}\ \theta_{34}\ \theta_{45}\ \theta_{56}\ \theta_{67}\ \theta_{78}]^\mathsf{T}$ are the angles between neighbour satellites and $\dot{\theta_n}$ is the time derivative of these angles. The system matrix will be a 14x14 matrix and for $i < 8$ satellites, $\dot{s}_i = s_{i+7}$. Therefore the $\underline{A}$ and $\underline{B}$ matrix can be written as:

$\underline{A} = \left(\dfrac{0_{(7\times7)} \,|\, I_{(7\times7)}}{0}\right), \underline{B} = \left(\dfrac{0_{(7\times7)}}{C\ I_{(7\times7)}}\right)$ with the constant $C = \frac{3\omega_0^2 R_0}{m}$ and $u = [u_2 - u_1; u_3 - u_2; ...; u_8 - u_7]$.

The controller gain that has been found in section 4.6, $K = [K_1\ K_2]$, now can be written as $\underline{K}_s = [K_1 I_{(7\times7)},\ K_2 I_{(7\times7)}]$ and by using a control law $u = -\underline{K}s$ a stability analysis can be made using a Lyapunov candidate function $V = s^T s$ where $V > 0$, $\forall s \neq 0$. Inserting $\underline{K}_s$, the state space equation becomes:

$$\dot{s} = (\underline{A} + \underline{BK}_s)s \tag{4.30}$$

From Lyapunov stability criterion, it has to be shown that $\dot{V} < 0, \forall s \neq 0$. The derivative of the candidate function can be written as

$$\dot{V} = \dot{s}^\mathsf{T} s + s^\mathsf{T} \dot{s} \tag{4.31}$$

and this is equal to

$$\dot{V} = s^\mathsf{T}(\underline{A} - \underline{BK})^\mathsf{T} s + s^\mathsf{T}(\underline{A} - \underline{BK})s \tag{4.32}$$

so the only thing that has to be shown in order the system to be stable is to show that the all the eigenvalues of $\underline{A} - \underline{BK} < 0$. The eigenvalues $\underline{A} - \underline{BK}$ were computed and the result was that all eigenvalues have negative real part.

4.7 Simulation and results

First, the simulation results of the control scheme between two satellites are shown. After the results of two satellites, the results for the whole satellite formation are shown with global and distributed algorithm. The reference angle is chosen to be 45^o for the whole formation. The inputs to the controller are $\Delta\theta$ and $\dot{\theta}$, where $\Delta\theta = \theta - \theta ref$ and $\dot{\theta} = w_2 - w_1$. In order to smooth the high frequency oscillations from θ, w_1 and w_2 to the controller a low pass filter is used. The control design is similar to PD controller. In *figure 4.10* is shown the response of the controller and in *figure 4.11* the angle between two satellites. The settling time is 2×10^7 which is equivalent to 230 days which is satisfactory, and also no saturation in the drag force appears.

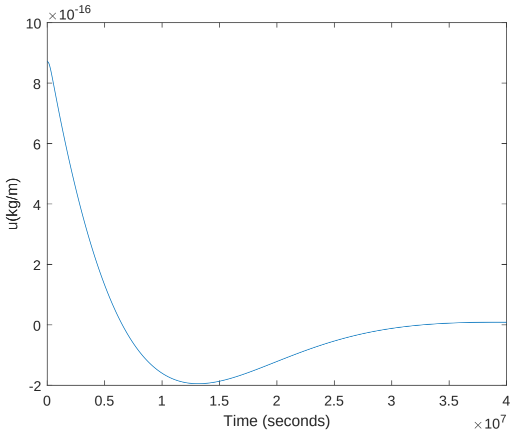

Figure 4.10: Distance control of two satellites

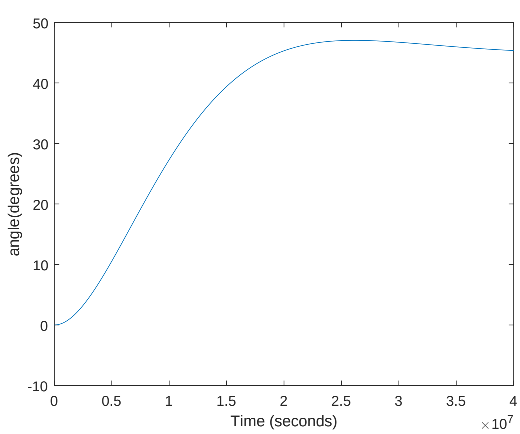

Figure 4.11: Angle between two satellites

The *figure 4.12* and *figure 4.13* show the input signal to the satellite 1 and satellite 2 respectively.

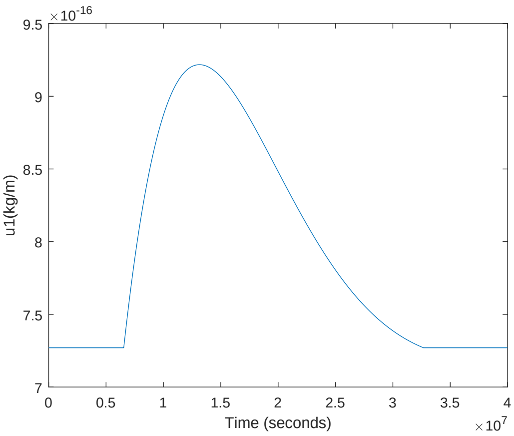

Figure 4.12: The applied input of satellite 1

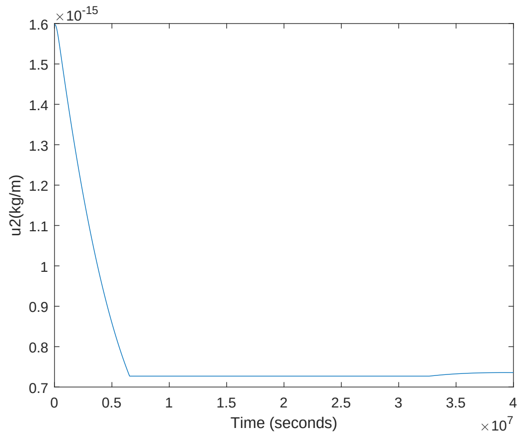

Figure 4.13: The applied input of satellite 2

Global algorithm results

The behaviour of the satellite formation using global algorithm was tested according to a hypothesis that the satellite formation starts at the same place. From *figure 4.14* it can be seen that in the beginning, some saturation appears, but finally, all the satellites converged to the desired angle of 45° .

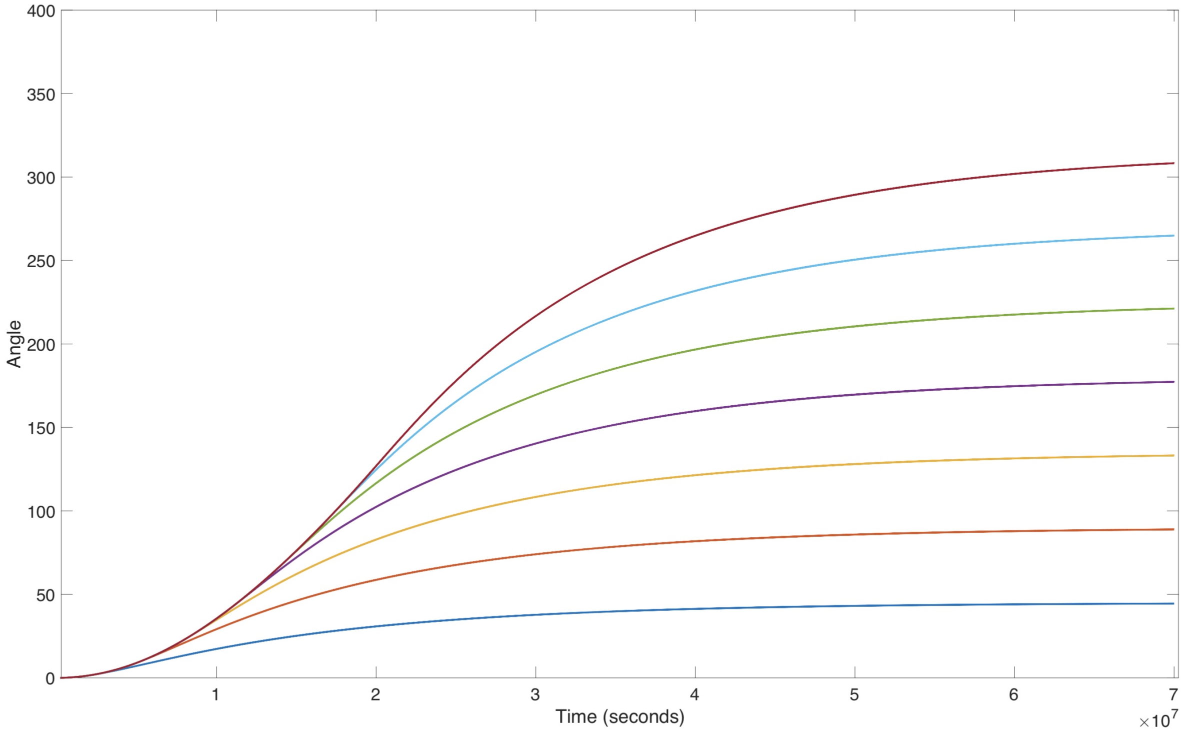

Figure 4.14: Eight satellites in flying formation on orbit

Distributed algorithm results

In the case of a distributed algorithm, the behaviour of the satellite formation is shown in *figure 4.15*, where it can be seen that the overshoot is big, therefore in order to reduce the overshoot, the derivative gain is increased by a factor of 2, which will correct the overshoot as seen in *figure 4.15*.

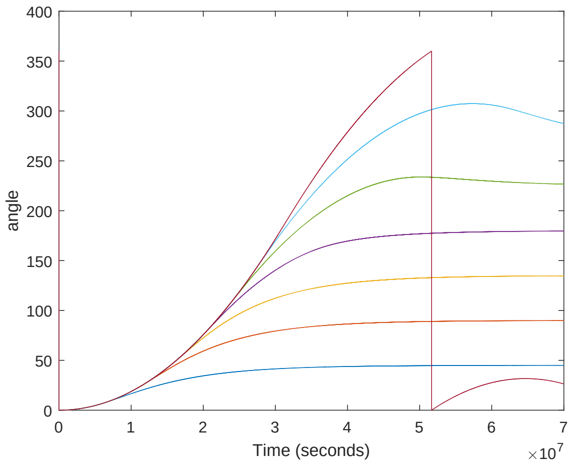

Figure 4.15: Distributed algorithm with overshoot

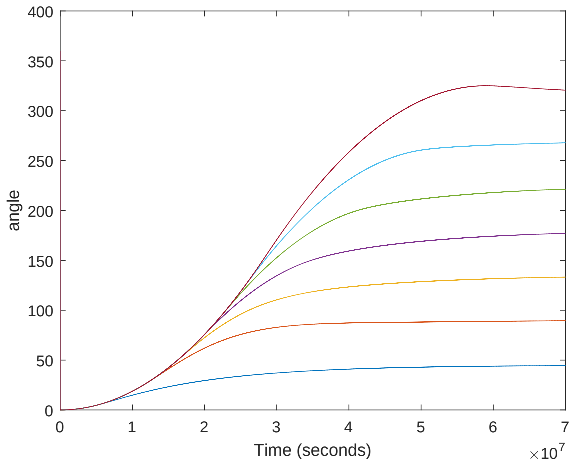

Figure 4.16: Distributed algorithm with corrected gain

The drawback of distributed algorithm is that the angle between satellites will converge slower compared with the global algorithm, moreover, all the satellites are converging to u_{min}, while in the case of distributed algorithm are converging to u_{medium}.

The difference between distributed and global algorithm, is that for distributed algorithm the drag force of one satellite is set to constant, that leads to less maneuverability, therefore more saturation

5 | Attitude control

5.1 Modelling

This section provides a description of the dynamic and kinematic equations of motion which constitute the basis for further analysis and description of the forces and/or disturbances, which may affect a rigid body within LEO. The model for the satellite is derived based on rigid body dynamics and kinematics.

Kinematics

This section will provide the orbit-attitude determination of the satellite using quaternion representation.

Quaternion parameterization is deemed useful for the kinematic analysis of the satellite. The orientation of the rigid body at time t is represented as $\mathbf{q}(t)$ and at time $\mathbf{q}(t+\Delta t)$ is the resulting quaternion at time $t+\Delta t$. The orientation of $\mathbf{q}(t+\Delta t)$ can be found by multiplying the quaternion at time t and the rotations of the satellite during the interval Δt, as follows [**SADC**]:

$$ {}^{\mathbf{s}}_{\mathbf{i}}\mathbf{q}(t+\Delta t) = \mathbf{q}(\Delta t) \otimes {}^{\mathbf{s}}_{\mathbf{i}}\mathbf{q}(t) \tag{5.1} $$

The axis $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ are the three axis of the satellite body frame at time t and $[e_x \; e_y \; e_z]$ are the components of the rotation axis unit vector along $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$. Therefore $q(\Delta t)$ can be expresed as [**SADC**]:

$$ q_1(\Delta t) = e_x \sin\frac{\Delta\Phi}{2} \tag{5.2} $$

$$ q_2(\Delta t) = e_y \sin\frac{\Delta\Phi}{2} \tag{5.3} $$

$$ q_3(\Delta t) = e_z \sin\frac{\Delta\Phi}{2} \tag{5.4} $$

$$ q_4(\Delta t) = \cos\frac{\Delta\Phi}{2} \tag{5.5} $$

where $\Delta\Phi$ is the rotation at time Δt

Combining the *equation (5.2)* - *equation (5.5)* with *equation (5.1)* results [**SADC**]:

$$ {}^{\mathbf{s}}_{\mathbf{i}}\mathbf{q}(t+\Delta t) = \left\{ \cos\frac{\Delta\Phi}{2}\underline{I}_{(4\times4)} + \sin\frac{\Delta\Phi}{2}\begin{bmatrix} 0 & e_z & -e_y & e_x \\ -e_z & 0 & e_x & e_y \\ e_y & -e_x & 0 & e_z \\ -e_x & e_y & -e_z & 0 \end{bmatrix} \right\} {}^{\mathbf{s}}_{\mathbf{i}}\mathbf{q}(t) \tag{5.6} $$

where $\underline{I}$ is the 4×4 identity matrix. Using the small angle approximation for infinitesimal $\Delta(t)$ and denoted $\boldsymbol{\omega}$ the instantaneous change in angular velocity it is obtained

$$ \mathbf{q(t+\Delta t)} = \left[1 + \frac{1}{2}\underline{\Omega}\Delta(t)\right]\mathbf{q(t)} \tag{5.7} $$

with $\underline{\Omega}$ be the skew symmetric matrix as [**SADC**]:

$$ \underline{\Omega} = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix} \tag{5.8} $$

the angle approximations where taken as $\cos\frac{\Delta\Phi}{2} \simeq 1$ and $\sin\frac{\Delta\Phi}{2} \simeq \frac{1}{2}\omega\Delta(t)$

Dynamic Model

In order to describe the behavior of the satellite a dynamic model based on reaction wheels and using Euler's equation of motion has been derived. Euler's equation of motion describing the rotation of a rigid body relates the time derivative of angular momentum to the applied torques and is given by [**SADC**]:

$$ \dot{\mathbf{L}} = \mathbf{N_{mw}} - \boldsymbol{\omega}\times\mathbf{L} \tag{5.9} $$

where $\mathbf{N_{tot}}$ represents all the external torques caused from the actuator and the disturbances, $\boldsymbol{\omega}$ is the angular velocity of the satellite and $\mathbf{L}$ is the total angular momentum of the satellite including reaction wheels, given by:

$$ \mathbf{L} = \underline{I}_s\boldsymbol{\omega} + \mathbf{h_{mw}} \tag{5.10} $$

where $\mathbf{h_{mw}}$ is the vector of the angular momentum of the wheels $[h_1\ h_2\ h_3]^T$ in the satellites coordinate system and $\underline{I}_s$ is the inertia matrix of the satellite. Inserting the *equation (5.10)* into *equation (5.9)* we obtain:

$$ \frac{d}{dt}(\underline{I}_s\boldsymbol{\omega}) + \dot{\mathbf{h}}_{\mathbf{mw}} = \mathbf{N_{mw}} - \boldsymbol{\omega}\times(\underline{I}_s\boldsymbol{\omega} + \mathbf{h_{mw}}) \tag{5.11} $$

The derivation of equation of motion can be found in the *appendix A*. For the ease of notation, the cross product can be written as matrix operation using the $\underline{S}()$ notation representing the skew symmetric matrix given by:

$$\underline{S}(\boldsymbol{\omega}) = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{5.12}$$

Solving for $\dot{\boldsymbol{\omega}}$ the dynamic equation can be written as

$$\dot{\boldsymbol{\omega}} = -\underline{I}_s^{-1}\underline{S}(\boldsymbol{\omega})\underline{I}_s\boldsymbol{\omega} - \underline{I}_s^{-1}\underline{S}(\boldsymbol{\omega})\mathbf{h}_{mw} - \underline{I}_s^{-1}\dot{\mathbf{h}}_{\mathbf{(mw)}} - \underline{I}_s^{-1}\mathbf{N_{mw}} \tag{5.13}$$

The rate of change of angular momentum $\dot{\mathbf{h}}_{\mathbf{mw}}$ is given by Newton's second law for rotation:

$$\dot{\mathbf{h}}_{\mathbf{mv}} = \mathbf{N_{mw}} \tag{5.14}$$

The total torque from external disturbances can be written as $\mathbf{N_{dis}}$. Therefore, *equation (5.13)* becomes:

$$\dot{\boldsymbol{\omega}} = -\underline{I}_s^{-1}\underline{S}(\boldsymbol{\omega})\underline{I}_s\boldsymbol{\omega} - \underline{I}_s^{-1}\underline{S}(\boldsymbol{\omega})\mathbf{h}_{mw} - \underline{I}_s^{-1}\mathbf{N}_{mw} + \underline{I}_s^{-1}\mathbf{N}_{dis} \tag{5.15}$$

which constitute the dynamics of the satellite with three reaction wheels.

Equation of motion

The behaviour of the satellite attitude is described by the dynamic and kinematic equations, which give a non-linear state space representations.

$$\begin{bmatrix} {}^{\mathbf{s}}_{\mathbf{i}}\dot{\mathbf{q}}(\mathbf{t}) \\ \dot{\boldsymbol{\omega}}(\mathbf{t}) \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\underline{\Omega}_{(4\times4)}{}^{\mathbf{s}}_{\mathbf{i}}\mathbf{q}(\mathbf{t}) \\ -\underline{I}_s^{-1}\underline{S}(\boldsymbol{\omega})\underline{I}_s\boldsymbol{\omega}(t) - \underline{I}_s^{-1}\underline{S}(\boldsymbol{\omega})\mathbf{h_{mw}} - \underline{I}_s^{-1}\mathbf{N_{mw}}(t) + \underline{I}_s^{-1}\mathbf{N_{dis}}(t) \end{bmatrix} \tag{5.16}$$

where,
${}^{\mathbf{s}}_{\mathbf{i}}\dot{\mathbf{q}}(\mathbf{t}) = [q_1\ q_2\ q_3\ q_4]^T$
$\dot{\boldsymbol{\omega}}(\mathbf{t}) = [\omega_1\ \omega_2\ \omega_3]^T$
$\underline{\Omega}(\omega)$ is the 4×4 skew symmetric matrix
$\underline{I}_s$ is the inertia matrix
$\underline{S}(\omega)$ is the 3×3 skew symmetric matrix
$\mathbf{N_{dis}}(t)$ is the disturbance torque
$\mathbf{N_{mw}}$ is the torque from momentum wheels

Linearized equation of motion

The kinematic and dynamic equations are linearized around the operating point for the purpose of designing a linear controller. The quaternion $\mathbf{q(t)}$ is split in the operating point ($\bar{\mathbf{q}}$) and the error quaternion ($\tilde{\mathbf{q}}$) and the angular velocity $\boldsymbol{\omega}$ is split in the nominal value $\tilde{\boldsymbol{\omega}}$ and the error $\tilde{\boldsymbol{\omega}}$.

$$ {}^{\mathbf{s}}_{\mathbf{i}}\mathbf{q} = {}^{\mathbf{s}}_{\mathbf{i}}\bar{\mathbf{q}} \otimes \tilde{\mathbf{q}} \tag{5.17} $$

$$ \tilde{\mathbf{q}} = {}^{\mathbf{s}}_{\mathbf{i}}\bar{\mathbf{q}}^{-1} \otimes {}^{\mathbf{s}}_{\mathbf{i}}\mathbf{q} \tag{5.18} $$

$$ \boldsymbol{\omega} = \bar{\boldsymbol{\omega}} + \tilde{\boldsymbol{\omega}} \tag{5.19} $$

Thus, the linearized equation of motion for the satellite are given by [**TH**]:

$$ \begin{bmatrix} \dot{\tilde{\mathbf{q}}}(\mathbf{t}) \\ \dot{\tilde{\boldsymbol{\omega}}}(\mathbf{t}) \end{bmatrix} = \begin{bmatrix} -\underline{S}(\bar{\boldsymbol{\omega}}) & \frac{1}{2}\underline{\mathbf{1}}_{(3\times3)} \\ \underline{0}_{(3\times3)} & \underline{I}_s^{-1}\underline{S}(\underline{I}_s\boldsymbol{\omega}(t)) - \underline{I}_s^{-1}\underline{S}(\boldsymbol{\omega})\underline{I}_s \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{q}}(\mathbf{t}) \\ \tilde{\omega}(t) \end{bmatrix} - \begin{bmatrix} \underline{\mathbf{0}}_{(3\times3)} \\ \underline{I}_s^{-1} \end{bmatrix} \tilde{\mathbf{N}}_{\mathbf{mw}} \tag{5.20} $$

5.2 Disturbance Models

Gravity-Gradient torque

An unbalanced satellite in orbit is subjected to a torque due to Earth's non-uniform gravitational field. Assumed that the Earth is a point mass and the satellite is a rigid body, the gravitational torque can be estimated. Each infinitesimal element of the satellite of mass dm_i is subjected to an infinitesimal force $\mathbf{d}F_i$ acting on the mass element located at position $\mathbf{R}i$ from the earth's geometric centre that can be calculated as [**SADC**]:

$$ d\mathbf{F_i} = -G\frac{m_{earth}}{\mathbf{R}_i^3}dm_i \cdot \mathbf{R_i} \tag{5.21} $$

where G is the gravitational constant, m_{earth} is the mass of the earth and $\mathbf{R_i}$ is the vector from the Earth's geometric centre to the infinitesimal element of the satellite.

The torque about the geometric centre of the satellite due to a force dF_i at a distance r_i from the geometric centre of the satellite is calculated as:

$$ d\mathbf{N_{gra}} = \mathbf{r}_i \times d\mathbf{F_i} \tag{5.22} $$

$\mathbf{r}_i$ can be written as the sum of two vectors as seen in the *figure 5.1*, $\mathbf{r}_{co}$ which is the vector from geometric centre of the satellite to the centre of mass, and $\mathbf{r}_{ci}$ which is the vector from the centre of mass to the infinitesimal element. Therefore, the expression of the gravity gradient torque is obtained by integrating *equation (5.22)* and thus obtaining:

$$ \mathbf{N}_{gra} = \int_{sat}(\mathbf{r}_{co} + \mathbf{r}_{ci}) \times d\mathbf{F_i} \tag{5.23} $$

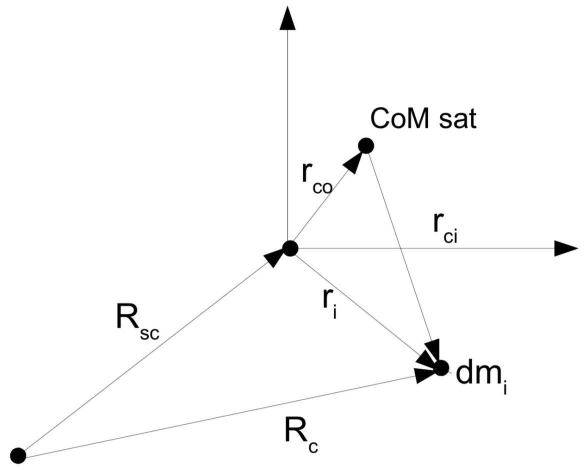

Figure 5.1: Coordinate system for the calculation of gravity gradient torque [**SADC**]

The vector $\mathbf{R_i}$ can be written as $\mathbf{R_i} = \mathbf{R_{sc}} + \mathbf{r_i}$. Finally assuming that the geometric centre of the satellite is aligned with its centre of mass, this is equal by setting $r_{co} = 0$, and by using the formula of inertia, and replacing the term dF_i by *equation (5.21)* the gravity gradient torque can be written as

$$\mathbf{N}_{gra} = \frac{3\mu}{\mathbf{R}_{sc}^3}[\mathbf{\hat{R}_{sc}} \times (\mathbf{I}\ \mathbf{\hat{R}_{sc}}] \tag{5.24}$$

where $\mathbf{\hat{R}_{sc}}$ is the unit vector from the earth's geometric center to the satellite's geometric center, and $\underline{I}$ is the diagonal inertia matrix of the satellite.

Solar radiation

The surface of the CubeSat will absorb or reflect the solar radiation, nevertheless, these two situations will alter the CubeSat, which will produce a torque about the satellite centre of mass (CoM). The torque around CoM is given by [**SADC**]:

$$\mathbf{N}_{rad} = \mathbf{F_{rad}} \times \mathbf{R}_{CoM} \tag{5.25}$$

where $\mathbf{F}_{rad}$ is the solar radiation and $\mathbf{R}_{CoM}$ is the vector from the centre of mass to the geometric centre of radiation

The solar radiation $\mathbf{F}_{rad}$ can be expressed as:

$$\mathbf{F}_{rad} = C_a P A\ \mathbf{\hat{R}_{sun,sat}} \tag{5.26}$$

where $\mathbf{\hat{R}_{sun,sat}}$ is the unit vector with the direction from the sun to the satellite, C_a is the surface's reflectance: 0 for a perfect absorber, 1 for a perfect reflector, while P is the solar flux and A is the radiated area

The solar flux can be computed as follows [**SADC**]:

$$P = \frac{F_s}{c} \tag{5.27}$$

where F_s is the mean solar energy and it is equal with 1358 W/m^2 and c is the speed of light

Atmospheric drag

For objects in LEO, friction with molecules can have a significant impact on the satellite surface by creating a force which acts upon the satellite surface. [ref: sat orbits and missions].

The equation of the atmospheric drag force is already computed as shown in *equation (4.3)*. Since the force has been computed also the torque can be computed by using the cross product between the drag force and the vector from the center of mass to the center of pressure of the satellite.

The aerodynamic torque acting on the satellite can be written as [**SADC**]:

$$\mathbf{N}_{drag} = \mathbf{F}_D \times \mathbf{r}_s \tag{5.28}$$

where:
$\mathbf{r}_s$ is the vector from the center of mass to the center of pressure, where the center of pressure is equal with the geometric center of the exposed area
$\mathbf{F}_D$ is the atmospheric drag force

5.3 Attitude control design

Relationship between drag force and quaternion

In order to apply the desired drag force, a function have to be found to compute the asociated quaternion. A method to find a bijective function between drag force and quaternion is to find two quaternions that give the minimum and the maximum drag force and analyse the drag force in the path between them.

A quaternion can be defined as a sequence of three rotations around the three axis.

$$\mathbf{q} = \mathbf{q}_x \otimes \mathbf{q}_y \otimes \mathbf{q}_z \tag{5.29}$$

with

$$\mathbf{q}_x = sin\left(\frac{\theta_x}{2}\right) * i + cos\left(\frac{\theta_x}{2}\right) \tag{5.30}$$

$$\mathbf{q}_y = sin\left(\frac{\theta_y}{2}\right) * j + cos\left(\frac{\theta_y}{2}\right) \tag{5.31}$$

$$\mathbf{q}_z = sin\left(\frac{\theta_z}{2}\right) * k + cos\left(\frac{\theta_z}{2}\right) \tag{5.32}$$

The drag force can be computed as the folowing:

$$u = \frac{u_{min}}{A_{min}} A_{\perp}({}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q}) \tag{5.33}$$

$$u = u_{min}\ f({}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q}) \tag{5.34}$$

where $A_{\perp}({}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q})$ is the perpendicular area of the satellite with a rotation ${}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q}$ and $f({}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q})$ is defined by $f = \frac{A_{\perp}}{A_{min}}$. Due to the fact that the velocity of the satellite is along the x axis in the orbit frame, a rotation around this axis doesn't change $A_{\perp}$. Therefore, f can be analysed using only rotation around y and z axes. The graph of f in function of θ_z and θ_y can be seen in the *figure 5.2*.

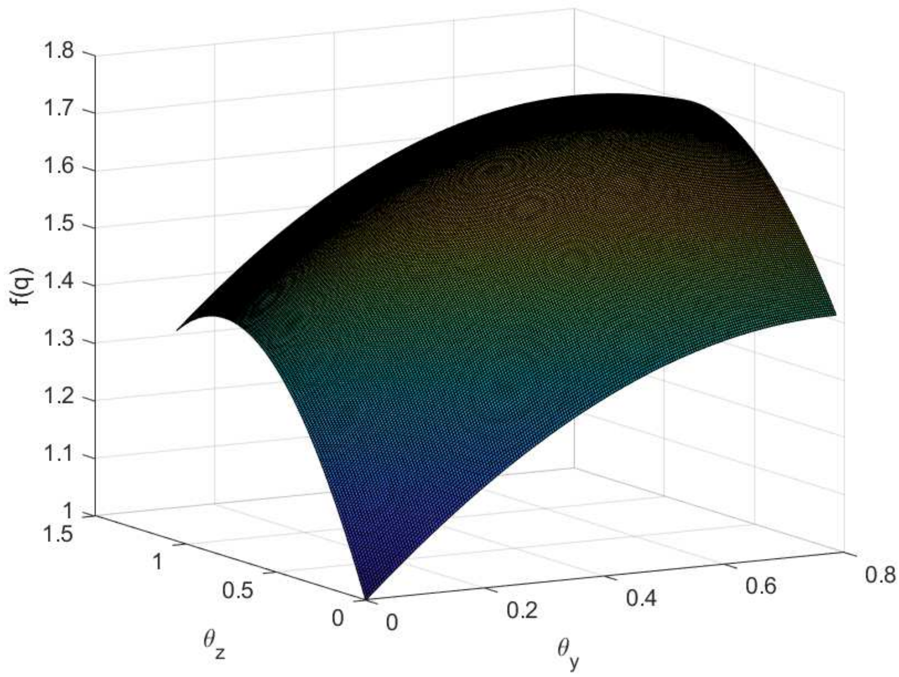

Figure 5.2: Perpendicular area in function of rotations around y and z axes

The quaternion for the minimum drag force is defined by the rotations $\theta_{y,min} = \theta_{z,min} = 0$ and the quaternion that give the maximum drag force is defined by the rotations $\theta_{y,max} = 0.61568$ rad and $\theta_{z,max} = \frac{\pi}{4}$.

The function $f({}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q})$ is computed in the pathm between the minimum and maximum drag force:

$${}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q} = {}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q_y} \otimes {}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q_z} \tag{5.35}$$

$$\tag{5.36}$$

with

$$\theta_y = \alpha\ \theta_{y,max} \tag{5.37}$$
$$\theta_z = \alpha\ \theta_{z,max} \tag{5.38}$$

and $\alpha \in [0, 1]$.

The *figure 5.3* shows $f({}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q})$ as a function of α:

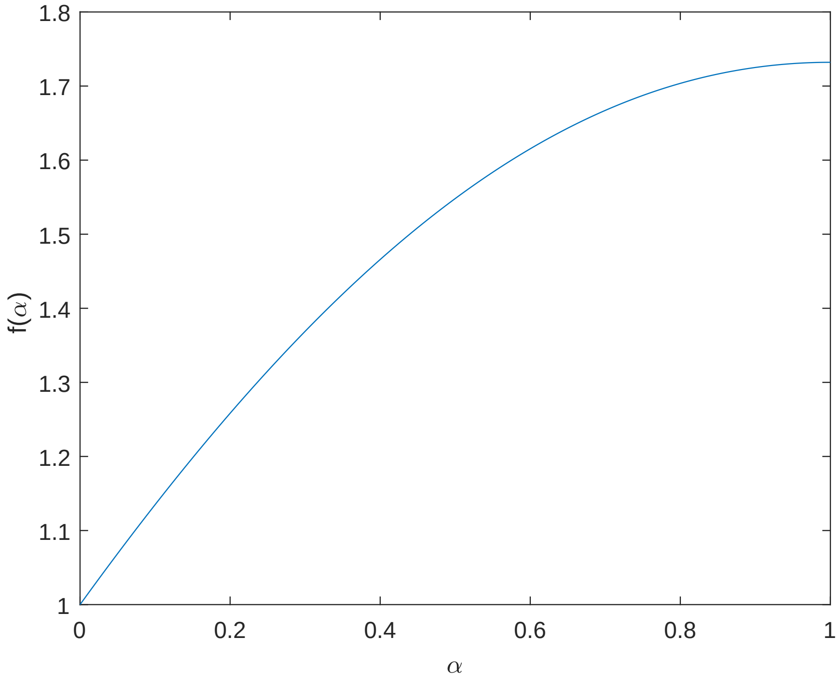

Figure 5.3: Perpendicular area in function of rotations around y and z axes

This function can be approximated by a polynomial of degree two. The relative error of this approximation represented in the *figure 5.4* is small and thus, the approximation by a polynomial of degree two can be used.

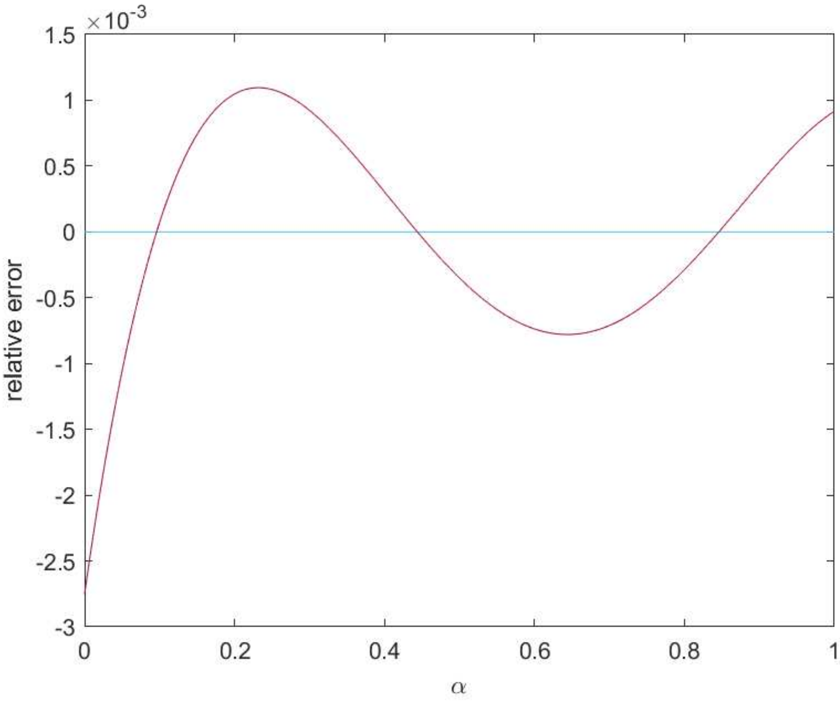

Figure 5.4: Relative error of the fitting approximation

Therefore, the quaternion that give the desired drag force in the orbit frame can be computed by following this algorithm:

$$\frac{u}{u_{min}} = f(\alpha) \approx p_1\alpha^2 + p_2\alpha + p_3 \tag{5.39}$$

$$\Rightarrow \alpha = \frac{-p_2 + \sqrt{p_2^2 - 4p_1\left(p_3 - \frac{u}{u_{min}}\right)}}{2p_1} \tag{5.40}$$

$$\Rightarrow \theta_y = \alpha\theta_{y,max} \tag{5.41}$$

$$\theta_z = \alpha\theta_{z,max} \tag{5.42}$$

$$\Rightarrow {}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q_y} = sin\left(\frac{\theta_y}{2}\right) * j + cos\left(\frac{\theta_y}{2}\right) \tag{5.43}$$

$${}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q} = sin\left(\frac{\theta_z}{2}\right) * k + cos\left(\frac{\theta_z}{2}\right) \tag{5.44}$$

$$\Rightarrow {}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q_{ref}} = {}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q_y} \otimes {}^{\mathbf{s}}_{\mathbf{o}}\mathbf{q_z} \tag{5.45}$$

Nota Bene: This is one solution of the problem, but there is an infinity of quaternions for one drag force. A better solution would be to choose the reference quaternion that give the desired drag force and that requires the smallest rotation relative to the current position of the satellite. However, this solution is very hard to implemented.

Linear Control Design - State Feedback

The linearized equation *equation (5.20)* of the attitude system allows to design a state feedback control. However, the nominal values of the quaternion and the angular velocity depends of the drag force desired and thus, they aren't constant all over the time. Therefore, the designed control have to stabilize the system for all the possibilities of $\bar{q}$ and $\bar{\omega}$.

The norm of $\bar{\boldsymbol{\omega}}$ is known and is equal to the angular velocity of the satellite around the Earth ($||\bar{\boldsymbol{\omega}}|| \approx 0.0011$). Due to the fact that this value is small compare to $\frac{1}{2}$, the A matrix can be approximated by:

$$\underline{A} \approx \begin{bmatrix} \underline{0}_{(3\times3)} & \frac{1}{2}\underline{1}_{(3\times3)} \\ \underline{0}_{(3\times3)} & \underline{0}_{(3\times3)} \end{bmatrix} \tag{5.46}$$

The system can be split in three subsystems defined by the matrix equation:

$$\begin{bmatrix} \dot{\tilde{q}}_i(t) \\ \dot{\tilde{\omega}}_i(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{q}_i(t) \\ \tilde{\omega}_i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ I_{i,s}^{-1} \end{bmatrix} N_{i,c}(t) \tag{5.47}$$

where i is equal to 1, 2, 3

The input control torque is defined by a state feedback law:

$$N_{i,mw} = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} \tilde{q}_i(t) \\ \tilde{\omega}_i(t) \end{bmatrix} \tag{5.48}$$

Therefore,

$$\underline{A} - \underline{BK} = \begin{bmatrix} 0 & \frac{1}{2} \\ -\frac{k_1}{I_{i,s}} & -\frac{k_2}{I_{i,s}} \end{bmatrix} \tag{5.49}$$

Pole Placement

$$det(s\underline{I} - (\underline{A} - \underline{BK})) = s^2 + \frac{k_2}{I_{i,s}}s + \frac{k_1}{2I_{i,s}} \tag{5.50}$$

By identification with a general second order equation $s^2 + 2\zeta\omega_n s + \omega_n^2$ where ζ is the damping factor and ω_n is the natural frequency, the gains are given by:

$$k_1 = -2\ I_{i,s}\ \omega_n^2 \tag{5.51}$$

$$k_2 = -2\ I_{i,s}\ \zeta\ \omega_n \tag{5.52}$$

The damping factor is chosen to be equal to 1 and the rise time is chosen to be equal to 60 and thus $\omega_n = \frac{2\pi}{T_n} = \frac{2\pi}{60/0.35}$. Therefore, all the eigenvalues are equal to -0.0367.

Stability

The stability of control designed in the previous section have to be checked for all $\bar{\boldsymbol{\omega}}$. Due to the fact that the matrix A is affine in $\bar{\boldsymbol{\omega}}$. The stability can be only checked on the vertices of the convex polyhedron (cube) that contains all the possibilities of $\bar{\boldsymbol{\omega}}$ as represented in the *figure* 5.5. [**NLCS**]

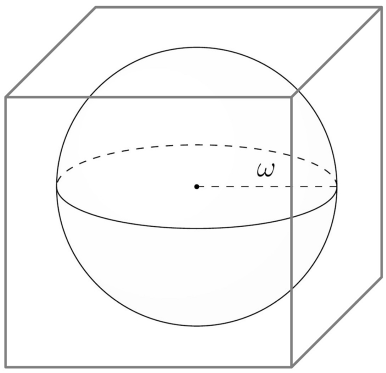

Figure 5.5: Convex Polyhedron

The maximum real part of the eigenvalue in the vertices of the cube is equal -0.0308 that is very close to the desired eigenvalue.

Non linear Control Design - Sliding Mode Control

In parallel to the linear feedback control design also a non-linear method has been designed.A sliding mode controller is implemented in order to compare the differences between the two methods. The sliding mode controller brings the system states to a designed manifold or a hyperplane s, such that when the states are on the manifold to converge to the desired reference. If the states are not on the manifold a control law that drives the states to the manifold is necessary. The outcome of this control law is the desired torque that will be fed back to the satellite. The satellite motion can be represented to the space of the sliding variable s. The sliding variable can be written in terms of the signal deviation as derived in *equation (5.19)*.The quaternion error can be written as [**WR**]:

$$\tilde{\mathbf{q}} = \bar{\mathbf{q}}^{-1} \otimes \mathbf{q} = \begin{bmatrix} q_{4r} & q_{3r} & -q_{2r} & q_{1r} \\ -q_{3r} & q_{4r} & q_{1r} & q_{2r} \\ q_{2r} & -q_{1r} & q_{4r} & q_{3r} \\ -q_{1r} & -q_{2r} & -q_{3r} & q_{4r} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \tag{5.53}$$

where $\bar{\mathbf{q}}$ represents the quaternion reference and $\mathbf{q}$ the measured quaternion. The time derivative of the error quaternion is given by[master thesis]:

$$\dot{\tilde{\mathbf{q}}} = \frac{1}{2}\Big(- \mathbf{q}_{\bar{\omega}} \otimes \tilde{\mathbf{q}} + \tilde{\mathbf{q}} \otimes \mathbf{q}_{\bar{\omega}} + \tilde{\mathbf{q}} \otimes \mathbf{q}_{\tilde{\omega}} \Big) \tag{5.54}$$

where $\mathbf{q}_{\bar{\omega}} = \bar{\omega_1} * i + \bar{\omega_2} * j + \bar{\omega_3} * k + 0$ and $\mathbf{q}_{\tilde{\omega}} = \tilde{\omega_1} * i + \tilde{\omega_2} * j + \tilde{\omega_3} * k + 0$. By choosing the sliding manifold variable to be equal:

$$s = F\tilde{\mathbf{q}} + \tilde{\boldsymbol{\omega}} \tag{5.55}$$

where $F = d * diag[1\ 1\ 1]$ is positive definite. The angular velocity error can be written as $\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega} - \bar{\boldsymbol{\omega}}$. Therefore, when s = $\mathbf{0}$, $\tilde{\boldsymbol{\omega}} = -d * \tilde{\mathbf{q}}_{\mathbf{1:3}}$ and thus the time derivative of the real part of the error quaternion is computed from the equation 5.54:

$$\dot{\tilde{q}}_4 = -\frac{1}{2}\tilde{\boldsymbol{\omega}} \cdot \tilde{\mathbf{q}}_{\mathbf{1:3}} \tag{5.56}$$

$$= \frac{d}{2}||\tilde{\mathbf{q}}_{1:3}||^2 \tag{5.57}$$

$$= \frac{d}{2}\Big(1 - \tilde{q}_4^2\Big) \tag{5.58}$$

According to equation 5.58, $\tilde{\mathbf{q}}$ will converge to $[0; 0; 0; 1]$ and d can be designed by trial and error to have the desired convergence. The behaviour of $\tilde{q}_4$ with d equal to 0.035 is represented in the *figure 5.6*.

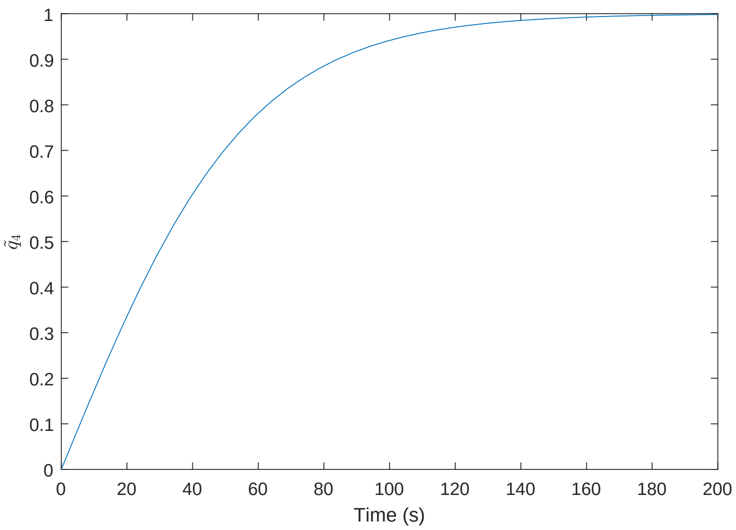

Figure 5.6: $\tilde{q}_4$ in function of time with d = 0.035

Assuming a lyapunov candidate function as

$$V = \frac{1}{2} s^T s \tag{5.59}$$

the time derivative of the lyapunov fuction can be written as

$$\dot{V} = \frac{1}{2}\left(\dot{s^T} s + s^T \dot{s}\right) \tag{5.60}$$

In order to prove stability it has to be shown that $\dot{V} < 0 \ \forall s \neq 0$ this is equal by showing

$$s^T \dot{s} < 0 \ \forall s \neq 0 \tag{5.61}$$

the *equation (5.55)* is substitute

$$\dot{V} = s^T (F\dot{\tilde{\mathbf{q}}} + \dot{\tilde{\boldsymbol{\omega}}}) \tag{5.62}$$

and using the *equation (5.16)* for $\dot{\tilde{\boldsymbol{\omega}}}$ the *equation (5.62)* becomes

$$\dot{V} = s^T \underline{I}_s^{-1}(-\underline{S}(\boldsymbol{\omega})\underline{I}_s\boldsymbol{\omega} - \underline{S}(\boldsymbol{\omega})\mathbf{h_{mw}} - \mathbf{N_{mw}} + \mathbf{N}_{dis} - I_s\dot{\bar{\omega}} + I_s F\dot{\tilde{\mathbf{q}}}) \tag{5.63}$$

where $\bar{\omega}$ is the desired angular velocity. Choosing o control law as

$$\mathbf{N_{mw}} = -\underline{S}(\boldsymbol{\omega})\underline{I}_s\boldsymbol{\omega} - \underline{S}(\boldsymbol{\omega})\mathbf{h}_{mw} - \underline{I}_s\dot{\omega} + I_s F\dot{\tilde{\mathbf{q}}} + I_s\lambda sign(s) \tag{5.64}$$

the *equation (5.63)* is written as

$$\dot{V} = -s^T(-\dot{\bar{\omega}} + \lambda sign(s) - \mathbf{N}_{dis}) \tag{5.65}$$

Therefore, with $\lambda > ||\dot{\omega}_{max}|| + ||\mathbf{N}_{dis}||$, the condition $\dot{V} < 0$ is satisfied. Thus, λ is chosen to be equal to $||\dot{\boldsymbol{\omega}}|| + c$ where c is designed by trial and error to have the desired behaviour of the sliding manifold s as represented in the figure 5.7

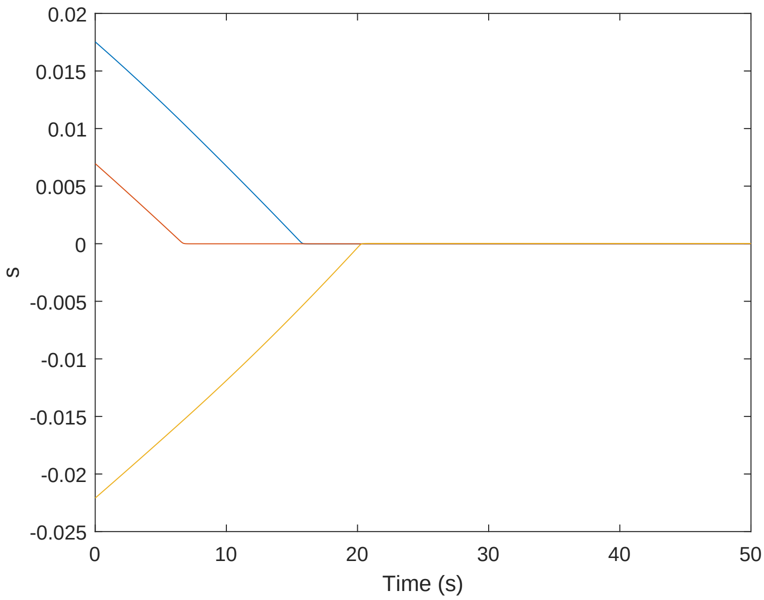

Figure 5.7: s in function of time with c = 10^{-3}

Sliding mode control suffers from chattering around the manifold. In order to reduce chattering the *sign* function is replaced with the hyperbolic tangent function $tanh(\frac{s}{\epsilon})$. The same result could be achieved with saturation function $sat(\frac{s}{\epsilon})$ but with saturation function the simulation of the system is very slow due to singularities. The difference between *sat* and *tanh* is seen in the *figure 5.8*

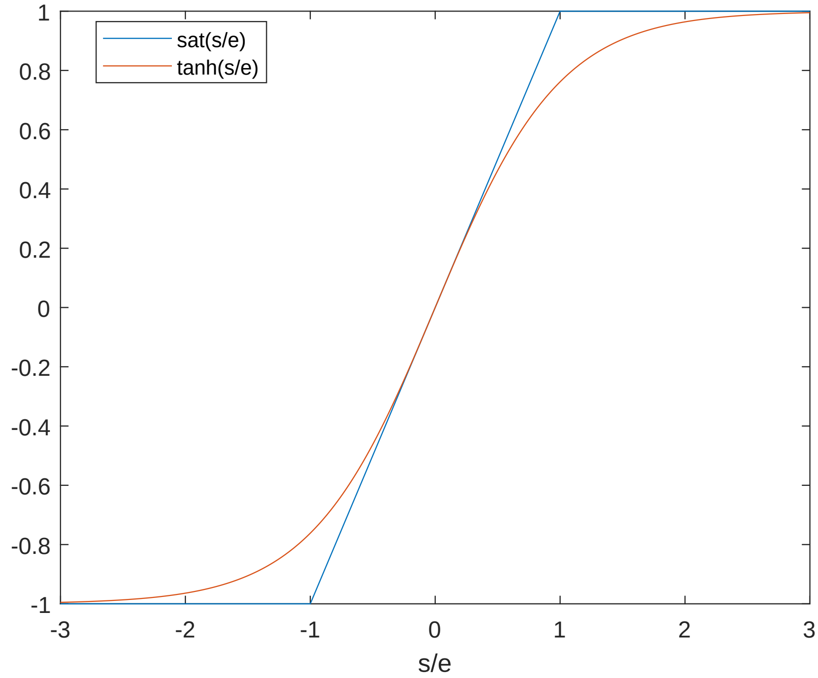

Figure 5.8: saturation and hyperbolic tangent functions

From the *figure 5.4*, it can be seen that the relative error due to the polynomial fitting is about 10^{-3}. Therefore, a choice of a smaller value for the maximum error quaternion will not influence the total error. Thus, the maximum acceptable error quaternion is chosen to be equal to 10^{-3}. Consequently, the maximum error on s is equal to $d * 10^{-3} = 3.5 * 10^{-5}$. ϵ is designed by trial and error to set the boundaries of s inside this maximum acceptable error for the sliding mode variable. The figure 5.9 shows s in function of time with $\epsilon = 2 * 10^{-3}$.

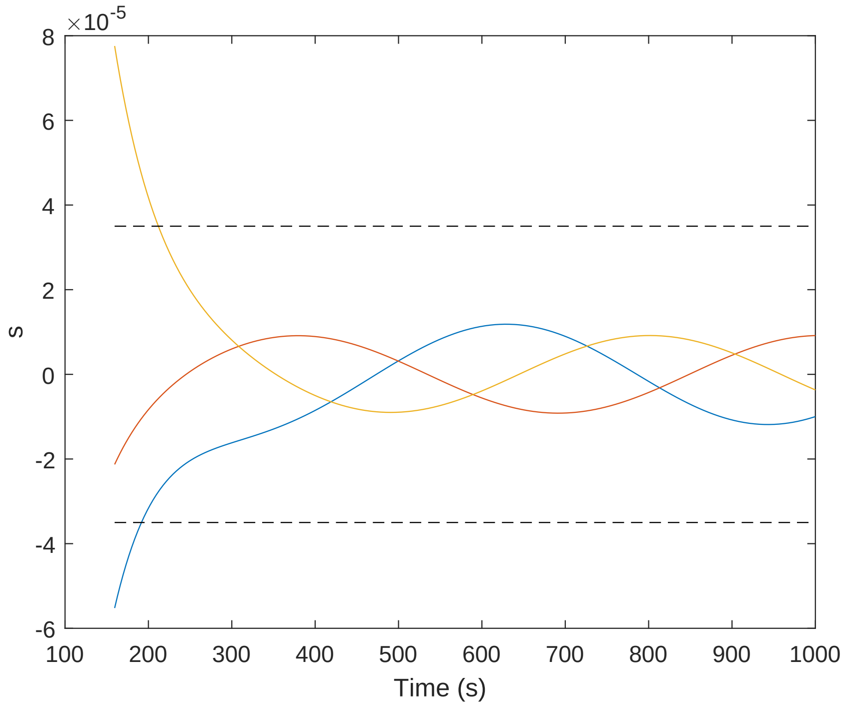

Figure 5.9: Design of ϵ

5.4 Simulation and results

The simulation of the state feedback control and sliding mode control are shown in the figure 5.10 and figure 5.11 respectively. At the beginning, the drag force reference is chosen to be equal to the minimum and after 500 seconds alters to the maximum.

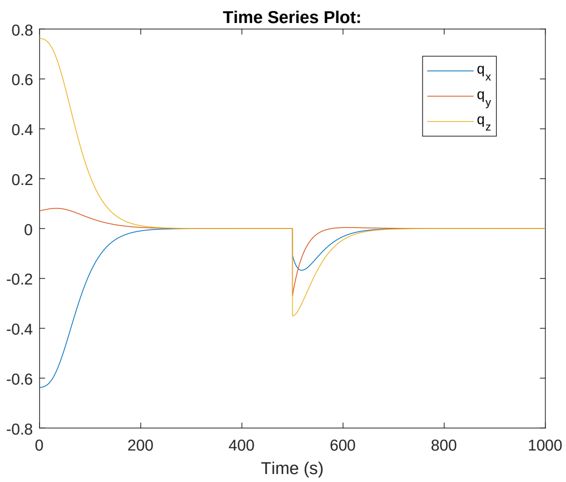

Figure 5.10: Simulation of the state feedback control

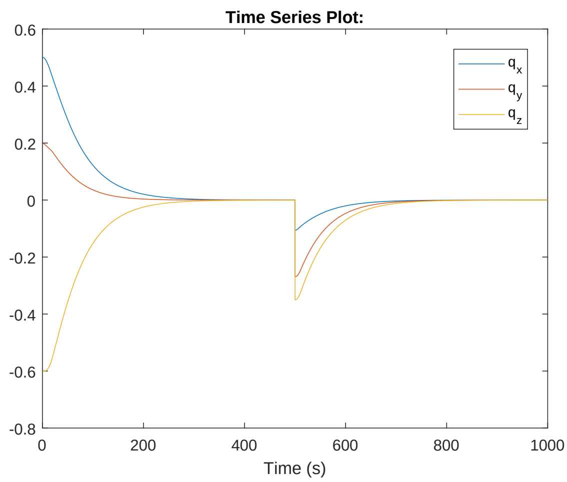

Figure 5.11: Simulation of the sliding mode control

It can be seen that, in the simulation of the state feedback control, the error increases sometimes due to the fact that the linear controller does not take into account the non-linearities of the system. However, in the simulation of sliding mode control, the error quaternion converges as expected.

6 | Conclusion

Future work

7 | Implementation and test

8 | Acceptance test

The system is tested to see if it fulfills the requirements put up (*chapter 3*).

A | Derivation of equation of motion

The general Euler's rotation equation with three reaction wheels aligned on the satellite body axis are derived as

$$I_1\dot{\omega}_1 = (I_2 - I_3)\omega_2\omega_3 + N_1 - \omega_2 h_3 + \omega_3 h_2 \tag{A.1}$$

$$I_2\dot{\omega}_2 = (I_3 - I_1)\omega_1\omega_3 + N_2 - \omega_3 h_1 + \omega_1 h_3 \tag{A.2}$$

$$I_3\dot{\omega}_3 = (I_1 - I_2)\omega_1\omega_2 + N_3 - \omega_1 h_2 + \omega_2 h_1 \tag{A.3}$$

The equation in compact form has been written as

$$\dot{\omega} = -\underline{I}_s^{-1}\underline{S}(\omega)\underline{I}_s^{-1}\boldsymbol{\omega} - \underline{I}_s^{-1}\underline{S}(\omega)\mathbf{h_{tot}} - \underline{I}_s^{-1}\dot{\mathbf{h}}_{\mathbf{mw}} + \underline{I}_s^{-1}\mathbf{N_{tot}} \tag{A.4}$$

where $\underline{S}(\omega)$ is the skew symmetric matrix given by

$$\underline{S}(\omega) = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{A.5}$$

and the angular momentum of the reaction wheels as $\mathbf{h}_{mw} = [h_1 \ h_2 \ h_3]^T$.

Inertia matrix

The inertia matrix for a solid cuboid of height z , width y, and depth x, amd mass m_i with respect the center of mass is given by

$$\underline{I}_i = \begin{bmatrix} \frac{1}{12}m_i(z^2+y^2) & 0 & 0 \\ 0 & \frac{1}{12}m_i(z^2+x^2) & 0 \\ 0 & 0 & \frac{1}{12}m_i(x^2+y^2 \end{bmatrix} \tag{A.6}$$

It is assumed that the Cube have a symmetric mass distribution around the axis of rotation to simplify the inertia matrix. With the mass distributed evenly and the axis of rotation being around one of the tree axis, the off diagonal term of the inertia matrix are equal to zero. These terms are also referred to as cross products of inertia.

B | Derivation of relative dynamics equations

The vector position from the centre of the Earth to the satellite 1 and the satellite 2 is given by

$$\mathbf{p_1} = R\hat{\mathbf{x}} \tag{B.1}$$
$$\mathbf{p_2} = R\hat{\mathbf{x}} + x\hat{\mathbf{x}} + y\hat{\mathbf{y}} \tag{B.2}$$

the first time derivative and second time relative of $\mathbf{p_1}$ and $\mathbf{p_2}$ is computed:

$$\dot{\mathbf{p_1}} = \dot{R}\hat{\mathbf{x}} + R(\mathbf{w} \times \hat{\mathbf{x}})$$

where $\mathbf{w}$ is the angular velocity vector and $\mathbf{w} = w\hat{\mathbf{z}}$ due to the fact the position of the satellites stay all over the time in the plan $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$. Therefore, the first time derivative and the second time derivative are given by:

$$\begin{aligned}
\dot{\mathbf{p_1}} &= \dot{R}\hat{\mathbf{x}} + wR\hat{\mathbf{y}} \\
\ddot{\mathbf{p_1}} &= \ddot{R}\hat{\mathbf{x}} + w\dot{R}\hat{\mathbf{y}} + \dot{w}R\hat{\mathbf{y}} + w\dot{R}\hat{\mathbf{y}} + wR(\mathbf{w} \times \hat{\mathbf{y}}) \\
&= \ddot{R}\hat{\mathbf{x}} + 2w\dot{R}\hat{\mathbf{y}} - w^2R\hat{\mathbf{x}} \\
\dot{\mathbf{p_2}} &= \dot{\mathbf{p_1}} + \dot{x}\hat{\mathbf{x}} + xw\hat{\mathbf{y}} + \dot{y}\hat{\mathbf{y}} - yw\hat{\mathbf{x}} \\
&= \dot{\mathbf{p_1}} + (\dot{x} - yw)\hat{\mathbf{x}} + (xw + \dot{y})\hat{\mathbf{y}} \\
\ddot{\mathbf{p_2}} &= \ddot{\mathbf{p_1}} + (\ddot{x} - \dot{y}w - y\dot{w})\hat{\mathbf{x}} + (\dot{x} - yw)w\hat{\mathbf{y}} + (\dot{x}w + x\dot{w} + \ddot{y})\hat{\mathbf{y}} - (xw + \dot{y})w\hat{\mathbf{x}} \\
&= \ddot{\mathbf{p_1}} + (\ddot{x} - 2\dot{y}w - y\dot{w} - xw^2)\hat{\mathbf{x}} + (\ddot{y} + 2\dot{x}w + x\dot{w} - yw^2)\hat{\mathbf{y}}
\end{aligned}$$

Furthermore, The Newton law gives:

$$m\ddot{\mathbf{p_1}} = \mathbf{F_{grav,1}} + \mathbf{F_{drag,1}} + \mathbf{F_{dist,1}} \tag{B.3}$$
$$m\ddot{\mathbf{p_2}} = \mathbf{F_{grav,2}} + \mathbf{F_{drag,2}} + \mathbf{F_{dist,2}} \tag{B.4}$$
$$\Rightarrow \ddot{\mathbf{p_2}} - \ddot{\mathbf{p_1}} = \frac{1}{m}(\Delta\mathbf{F_{grav}} + \Delta\mathbf{F_{drag}} + \Delta\mathbf{F}_{dist}) \tag{B.5}$$

with m is the mass of both satellites. The gravity is given by the universal law of gravitation:

$$\begin{aligned}
\frac{\mathbf{F_{grav,1}}}{m} &= -G\frac{m_{earth}}{||\mathbf{R}||^3}\mathbf{R} \\
\frac{\mathbf{F_{grav,2}}}{m} &= -G\frac{m_{earth}}{||\mathbf{R}+\mathbf{r}||^3}(\mathbf{R}+\mathbf{r})
\end{aligned}$$

where $\mathbf{r} = (x, y)$ is the vector from the satellite 1 to the satellite 2. The denominateur can be approximated using:

$$||\mathbf{R}+\mathbf{r}||^{-3} = ||\mathbf{r}||$$

and thus, the difference between the gravity force on satellite 2 and the the gravity force on 1 is:

$$\mathbf{F_{grav,2}} - \mathbf{F_{grav,1}} \approx -\frac{\mu}{R^3}\mathbf{r}$$

with $\mu = Gm_{earth}$, The drag force can be modelling be using *equation (4.4)*:

$$\begin{aligned}
\mathbf{F_{drag,1}} &= -u_1||\dot{\mathbf{p_1}}||\dot{\mathbf{p_1}} \\
&= -u_1||\dot{\mathbf{p_1}}||(\dot{R}\hat{\mathbf{x}} + wR\hat{\mathbf{y}}) \\
\mathbf{F_{drag,2}} &= -u_2||\dot{\mathbf{p_2}}||\dot{\mathbf{p_2}} \\
&= -u_2||\dot{\mathbf{p_2}}||((\dot{R} + \dot{x} - yw)\hat{\mathbf{x}} + (wR + xw + \dot{y})\hat{\mathbf{y}})
\end{aligned}$$

Therefore, the *equation (B.3)* becomes:

$$\begin{cases} \ddot{R} - w^2R = -\frac{\mu}{R^2} - \frac{u_1}{m}||\dot{\mathbf{p_1}}||\dot{R} + \frac{F_{dist,1,x}}{m} \\ 2w\dot{R} + \dot{w}R = -\frac{u_1}{m}||\dot{\mathbf{p_1}}||wR + \frac{F_{dist,1,y}}{m} \end{cases} \tag{B.6}$$

and the *equation (B.5)* gives:

$$\begin{cases} \ddot{x} - 2\dot{y}w - y\dot{w} - xw^2 = -x\frac{\mu}{R^3} + \frac{u_1}{m}||\dot{\mathbf{p_1}}||\dot{R} - \frac{u_2}{m}||\dot{\mathbf{p_2}}||(\dot{R} + \dot{x} - yw) + \frac{\Delta F_{dist,x}}{m} \\ \ddot{y} + 2\dot{x}w + x\dot{w} - yw^2 = -y\frac{\mu}{R^3} + \frac{u_1}{m}||\dot{\mathbf{p_1}}||wR - \frac{u_2}{m}||\dot{\mathbf{p_2}}||(wR + xw + \dot{y}) + \frac{\Delta F_{dist,y}}{m} \end{cases} \tag{B.7}$$

The operating point is the position (x^*, y^*) of the satellite 2 in the frame of satellite. x^* and y^* can be computed from *figure B.1*.

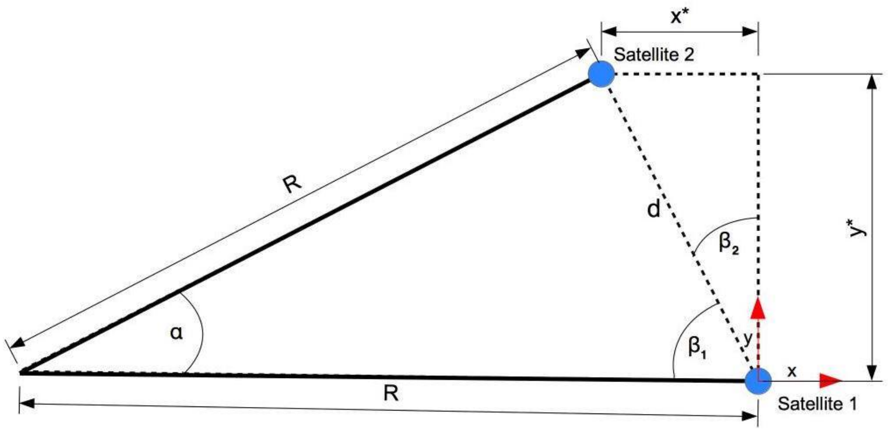

Figure B.1: Operating point

Using trigonometry relations:

$$\begin{aligned}
d &= 2Rsin(\frac{\alpha}{2}) \\
x^* &= -dsin(\beta_2) \\
&= -dsin(\frac{\alpha}{2}) \\
&= -2Rsin(\frac{\alpha}{2})^2 \\
y^* &= dcos(\frac{\alpha}{2}) \\
&= 2Rsin(\frac{\alpha}{2})cos(\frac{\alpha}{2}) \\
&= Rsin(\alpha)
\end{aligned}$$

with α is the desired angle between satellite and so $\beta_2 = 90° - \beta_1 = 90° - (90° - \frac{\alpha}{2}) = \frac{\alpha}{2}$. Therefore we change the coordinate reference as following:

$$\begin{aligned}
x &\Leftarrow x - x^* \\
y &\Leftarrow y - y^*
\end{aligned}$$

Thus, the equations *equation (B.7)* become:

$$\begin{cases}
\ddot{x} - 2\dot{y}w - (y + y^*)\dot{w} - (x + x^*)w^2 = \\
-(x + x^*)\dfrac{\mu}{R^3} + \dfrac{u_1}{m}||\dot{\mathbf{p_1}}||\dot{R} - \dfrac{u_2}{m}||\dot{\mathbf{p_2}}||(\dot{R} + \dot{x} - (y + y^*)w) + \dfrac{\Delta F_{dist,x}}{m} \\
\ddot{y} + 2\dot{x}w + (x + x^*)\dot{w} - (y + y^*)w^2 = \\
-(y + y^*)\dfrac{\mu}{R^3} + \dfrac{u_1}{m}||\dot{\mathbf{p_1}}||wR - \dfrac{u_2}{m}||\dot{\mathbf{p_2}}||(wR + (x + x^*)w + \dot{y}) + \dfrac{\Delta F_{dist,y}}{m}
\end{cases} \tag{B.8}$$

C | Angular velocity equations

The differential equations for R and ω for a satellite is given by the equation (ref B6). In order to find an expression of the angular velocity in function of the drag force, the little perturbation approximation is used for R and ω.

$$R = R_0 - \Delta R \tag{C.1}$$

$$\omega = \omega_0 - \Delta\omega \tag{C.2}$$

where R_0 and ω_0 are the initial value and $\omega_0 = \sqrt{\frac{\mu}{R_0^3}}$. Therefore, the system of equations B6 become:

$$\Delta \ddot{R} - (\omega_0 + \Delta\omega)^2(R_0 + \Delta R) = -\frac{\mu}{(R_0 + \Delta R)^2} - u\frac{v}{m}\Delta \dot{R} \tag{C.3}$$

$$2(\omega_0 + \Delta\omega)\Delta \dot{R} + \Delta\dot{\omega}(R_0 + \Delta R) = -u\frac{v}{m}(\omega_0 + \Delta\omega)(R_0 + \Delta R) \tag{C.4}$$

The *equation (C.3)* can be simplified using approximations that the speed of the satellites can be assumed constant ($||\dot{\mathbf{p}} = v_0 = \omega_0 R_0$), the second derivative of ΔR is neglectable and by deleting all the second order term. Thus the equation gives:

$$-\omega_0^2 R_0 - 2\omega_0 R_0 \Delta\omega - w_0^2 \Delta R = -\frac{\mu}{R_0^2}(1 + \frac{\Delta R}{R_0})^{-2} - u\frac{v_0}{m}\Delta \dot{R} \tag{C.5}$$

$$\Rightarrow -\omega_0^2 R_0 - 2\omega_0 R_0 \Delta\omega - \omega_0^2 \Delta R = -\omega_0^2 R_0 + 2\omega_0^2 \Delta R - u\frac{v_0}{m}\Delta \dot{R} \tag{C.6}$$

$$\Rightarrow \Delta R = -\frac{2R_0}{3\omega_0}\Delta\omega + u\frac{R_0}{3\omega_0 m}\Delta \dot{R} \tag{C.7}$$

$$\Rightarrow \Delta R \approx -\frac{2R_0}{3\omega_0}\Delta\omega \tag{C.8}$$

because $\frac{u}{m} << 1$. The second equation D.4 can be simplified by neglected the second order term:

$$2\omega_0 \Delta \dot{R} + \Delta\dot{\omega} R_0 = -\frac{u}{m}\omega_0^2 R_0^2 \tag{C.9}$$

$$\Rightarrow -\frac{4}{3}R_0 \Delta\dot{\omega} + \Delta\dot{\omega} R_0 == -\frac{u}{m}\omega_0^2 R_0^2 \tag{C.10}$$

$$\Rightarrow \Delta\dot{\omega} = \frac{3\omega_0^2 R_0}{m}u \tag{C.11}$$

Thanks to this equation, the angular velocity can be computed in function of the drag force on the satellite.