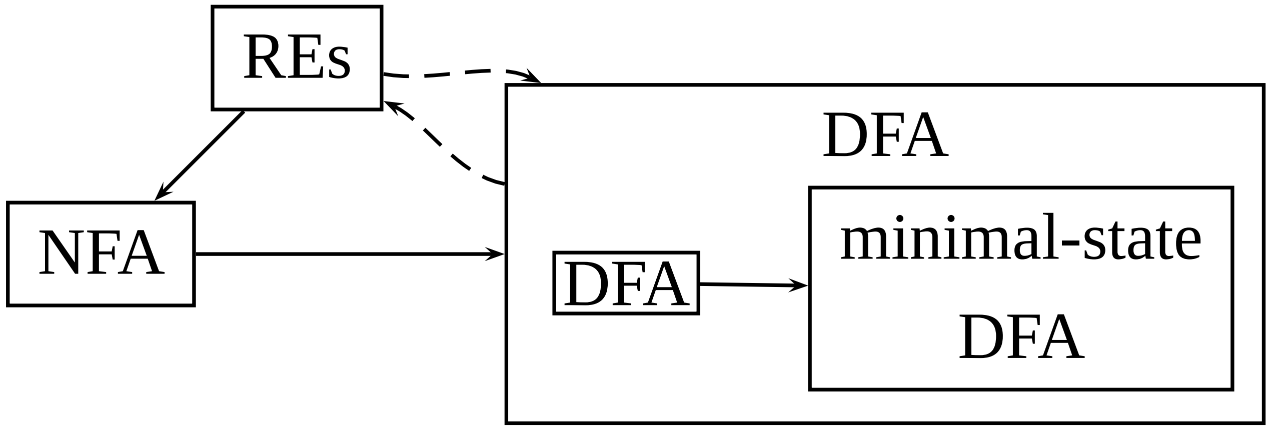
Regular Expressions, Finite State Automata, and Lexical Analysis

Nguyen Phuong Thai
Faculty of Information Technology
The University of Engineering and Technology
VNU Hanoi

The Big Picture



The two conversions in dashed arrows are not covered:

- REs $\longrightarrow$ DFA (pages 135 – 141, Red Dragon/§3.7, Purple Dragon)
- DFA $\longrightarrow$ REs: Chapter 3, J. Hopcroft, R. Motwani and J. Ullman, *Introduction to Automata Theory, Languages, and Computation*, Addison-Wesley, 2nd Edition, 2001. See `www-db.stanford.edu/~ullman/ullman-books.html`.
- DFA $\longrightarrow$ minimal-state DFA (pages 141 – 144, Red Dragon/§3.9.6, Purple Dragon)
- Tools: `http://www.jflap.org/`

Week 2: Regular Expressions, DFA and NFA

1. Definitions of REs, DFA and NFA
2. REs $\Longrightarrow$ NFA (*Thompson's construction, Algorithm 3.3, Red Dragon/Algorithm 3.23, Purple Dragon*)
3. NFA $\Longrightarrow$ DFA (*subset construction, Algorithm 3.2, Red Dragon/Algorithm 3.20, Purple Dragon*)
4. DFA $\Longrightarrow$ minimal-state DFA (*state minimisation, Algorithm 3.6, Red Dragon/Algorithm 3.39, Purple Dragon*)
5. Scanner generators
   - How to use them (straightforward)
   - How to write them (the most techniques introduced today)

Applications of Regular Expressions

- Anywhere when patterns of text need to be specified
- Unix system, database and networking administration:

  grep, fgrep, egrep, sed, awk
- HTML documents: Javascript and VBScript
- Perl:

  J. Friedl, Mastering Regular Expressions, O'reilly, 1997
- Token Specs for scanner generators (lex, Jflex, etc.)
- `http://www.robotwisdom.com/net/regexres.html`

Applications of Finite Automata (i.e., Finite State Machines)

- Hardware design (minimising states $\Longrightarrow$ minimising cost)
- Language theory
- Computational complexity
- Scanner generators (lex and Jflex)
- Automata tools:

  `http://members.fortunecity.com/boroday/Automatatools.html`

Alphabet, Strings and Languages

- Alphabet denoted Σ: any finite set of symbols
  - The binary alphabet $\{0,1\}$ (for machine languages)
  - The ASCII alphabet (for high-level languages)
- String: a finite sequence of symbols drawn from Σ:
  - Length $|s|$ of a string s: the number of symbols in s
  - ϵ: the empty string ($|\epsilon| = 0$)
- Language: any set of strings over Σ; its two special cases:
  - $\emptyset$: the empty set
  - $\{\epsilon\}$

Examples of Languages

- $\Sigma = \{0, 1\}$ – a string is an instruction
  - The set of M68K instructions
  - The set of Pentium instructions
  - The set of MIPS instructions
- $\Sigma =$ the ASCII set – a string is a program
  - the set of Haskell programs
  - the set of C programs
  - the set of VC programs

Terms for Parts of a String (Figure 3.7 of Text)

TERM	DEFINITION
prefix of s	a string obtained by removing 0 or more trailing symbols of s
suffix of s	a string obtained by removing 0 or more leading symbols of s
substring of s	a string obtained by deleting a prefix and a suffix from s
proper prefix suffix, substring of s	Any nonempty string x that is, respectively, a prefix, suffix or substring of s such that $s \neq x$

String Concatenation

- If x and y are strings, xy is the string formed by appending y to x
- Examples:

x	y	xy
key	word	keyword
java	script	javascript

- ϵ is the identity: $\epsilon x = x\epsilon = x$

Operations on Languages (Figure 3.8 of Text)

OPERATION	DEFINITION
union: $L \cup M$	$L \cup M = \{s \mid s \in L \text{ or } s \in M\}$
concatenation: LM	$LM = \{st \mid s \in L \text{ and } t \in M\}$
Kleene Closure: L^*	$L^* = \cup_{i=0}^{\infty} L^i = L^0 \cup L \cup LL \cup LLL \ldots$ where $L^0 = \{\epsilon\}$ (0 or more concatenations of L)
Positive Closure: L^+	$L^+ = \cup_{i=1}^{\infty} L^i = L \cup LL \cup LLL \ldots$ (1 or more concatenations of L)

Examples: Operations on Languages

- $L = \{a, \ldots, z, A, \ldots, Z, \_\}$
- $D = \{0, \ldots, 9\}$

EXAMPLE	LANGUAGE (THE SET OF)
$L \cup D$	
L^3	
LD	
L^*	
$L(L \cup D)^*$	
D^+	

Examples: Operations on Languages

- $L = \{a, \ldots, z, A, \ldots, Z, \_\}$
- $D = \{0, \ldots, 9\}$

Example	Language
$L \cup D$	letters and digits
L^3	all 3-letter strings
LD	strings consisting of a letter followed by a digit
L^*	all strings of letters, including the empty string ϵ
$L(L \cup D)^*$	all strings of letters and digits beginning with a letter
D^+	all strings of one or more digits

Regular Expressions (REs) Over Alphabet Σ

- Inductive Base:
  1. ϵ is a RE, denoting the RL $\{\epsilon\}$
  2. $a \in \Sigma$ is a RE, denoting the RL $\{a\}$
- Inductive Step: Suppose r and s are REs, denoting the RLs $L(r)$ and $L(s)$. Then (next slide):
  1. $(r)|(s)$ is a RE, denoting the RL $L(r) \cup L(s)$
  2. $(r)(s)$ is a RE, denoting the RL $L(r)L(s)$
  3. $(r)^*$ is a RE, denoting the RL $L(r)^*$
  4. (r) is a RE, denoting the RL $L(r)$

REs define regular languages (RL) or regular sets

Precedence and Associativity of “Regular” Operators

- Precedence:
  - “$*$” has the highest precedence
  - “Concatenation” has the second highest precedence
  - “$|$” has the lowest precedence
- Associativity: — all are left-associative
- Example:

$$(a)|((b)^*(c)) \equiv a|b^*c$$

Unnecessary parentheses can be avoided!

An Example (Following the Definition of REs)

- Alphabet: $\Sigma = \{0, 1\}$
- RE: $0(0|1)^*$
- Question: What is the language defined by the RE?
- Answer:

$$
\begin{array}{rcl}
L(0(0|1)^*) & = & L(0)L((0|1)^*) \\
& = & \{0\}L(0|1)^* \\
& = & \{0\}(L(0) \cup L(1))^* \\
& = & \{0\}(\{0\} \cup \{1\})^* \\
& = & \{0\}\{0, 1\}^* \\
& = & \{0\}\{\epsilon, 0, 1, 00, 01, 10, 11, \ldots\} \\
& = & \{0, 00, 01, 000, 001, 010, 011, \ldots\}
\end{array}
$$

The RE describes the set of strings of 0’s and 1’s beginning with a 0.

More Example Regular Expressions: $\Sigma = \{0, 1\}$

RE	LANGUAGE
1	$\{1\}$
$0\|1$	$\{0, 1\}$
1^*	$\{\epsilon, 1, 11, 111, \ldots\}$
1^*1	$\{1, 11, 111, \ldots\}$
$0\|0^*1$	the set containing 0 and all strings consisting of zero or more 0's followed by a 1.

Notational Shorthands

- One or more instances $^+$: $r^+ = rr^*$
  - denotes the language $(L(r))^+$
  - has the same precedence and associativity as *
- Zero or one instance ?: $r? = r|\epsilon$
  - denotes the language $L(r) \cup \{\epsilon\}$
  - written as $(r)?$ to indicate grouping (e.g., (12)?)
- Character classes:

$$[A - Za - z\_][A - Za - z0 - 9\_]^*$$

Regular Expressions for VC (or C)

TOKEN	RE
Identifiers	$\textbf{letter}(\textbf{letter}\mid\textbf{digit})^*$
Integers	$\textbf{digit}^+$
Reals	A bit long but can be obtained from the following page by substitutions

- In the VC spec, **letter** includes "_"
- In Java, letters and digits may be drawn from the entire Unicode character set. Examples of identifiers are:

abc $\alpha\beta\gamma$ 中文

Regular Grammars for Integers and Reals in VC

- Integers:

```
       digit: 0|1|2|...|9
  intLiteral: digit+
```

- Reals:

```
       digit: 0|1|2|...|9
    fraction: .digit+
    exponent: (E|e)(+|-)?digit+
floatLiteral: digit* fraction exponent?
            | digit+.
            | digit+.?exponent
```

Regular grammars are a special case of CFGs (Week 3).

Finite Automata (or Finite State Machines)

A finite automaton consists of a 5-tuple:

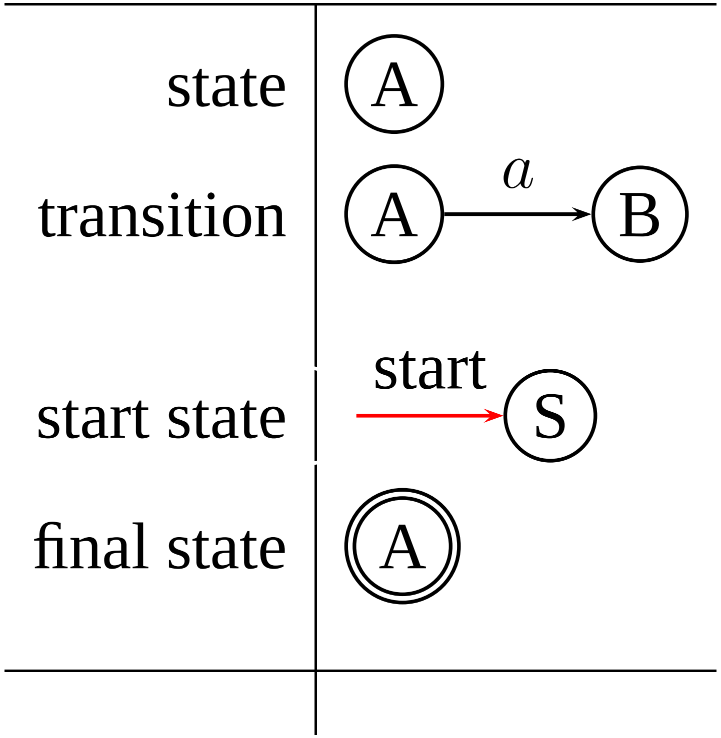
$$(\Sigma, S, T, F, I)$$

where

- Σ is an alphabet
- S is a finite set of states
- T is a state transition function: $T : S \times \Sigma \to S$
- F is a finite set of final or accepting states
- I is the start state: $I \in S$.

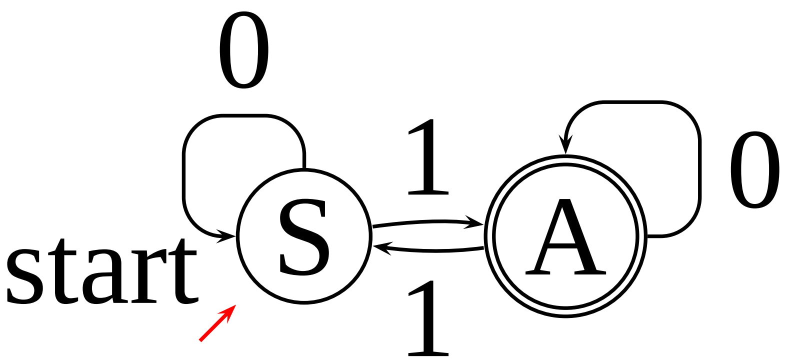
Representation and Acceptance

- Transition graph:



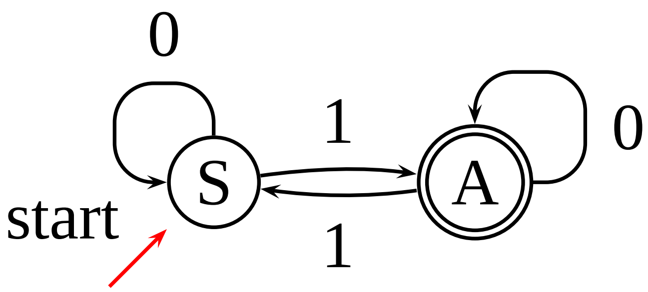
- Acceptance: A FA accepts an input string x iff there is some path in the transition graph from the start state to some accepting state such that the edge labels spell out x.

What Language does this FA accept?



Example 1

- The language: strings of 0 and 1 with an odd number of 1 (ϵ not included)



S: even number of 1's seen
A: odd number of 1's seen

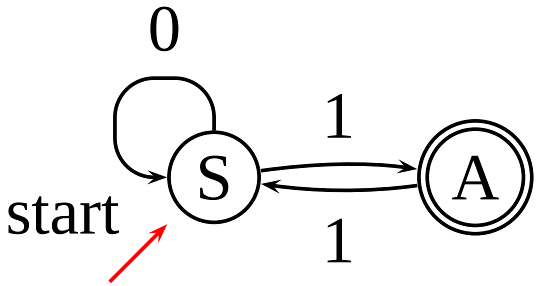
Σ	$\{0, 1\}$
S	$\{S, A\}$
T	$T(S, 0) = S, T(S, 1) = A, T(A, 0) = A, T(A, 1) = S$
F	$\{A\}$
I	S

- 01011 is recognised because

$$S \xrightarrow{0} S \xrightarrow{1} A \xrightarrow{0} A \xrightarrow{1} S \xrightarrow{1} A$$

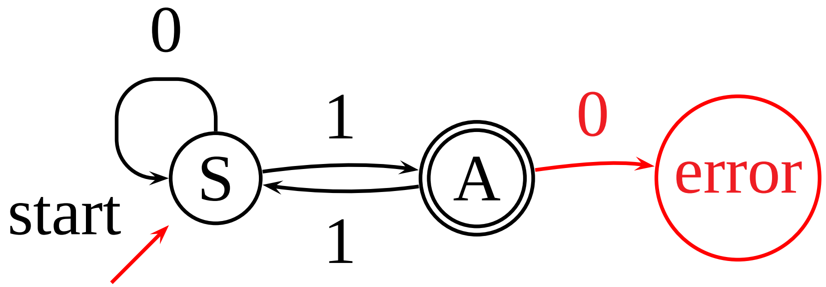
Implicit Error State

- By definition, T is a function from $S \times \Sigma$ to S, but ...



- If $T(s, a)$ is undefined at the state s on input a, then

$$T(s, a) = \text{error}$$



- The error state and transitions to it aren't drawn (by convention)

Deterministic FA (DFA) and Nondeterministic FA (NFA)

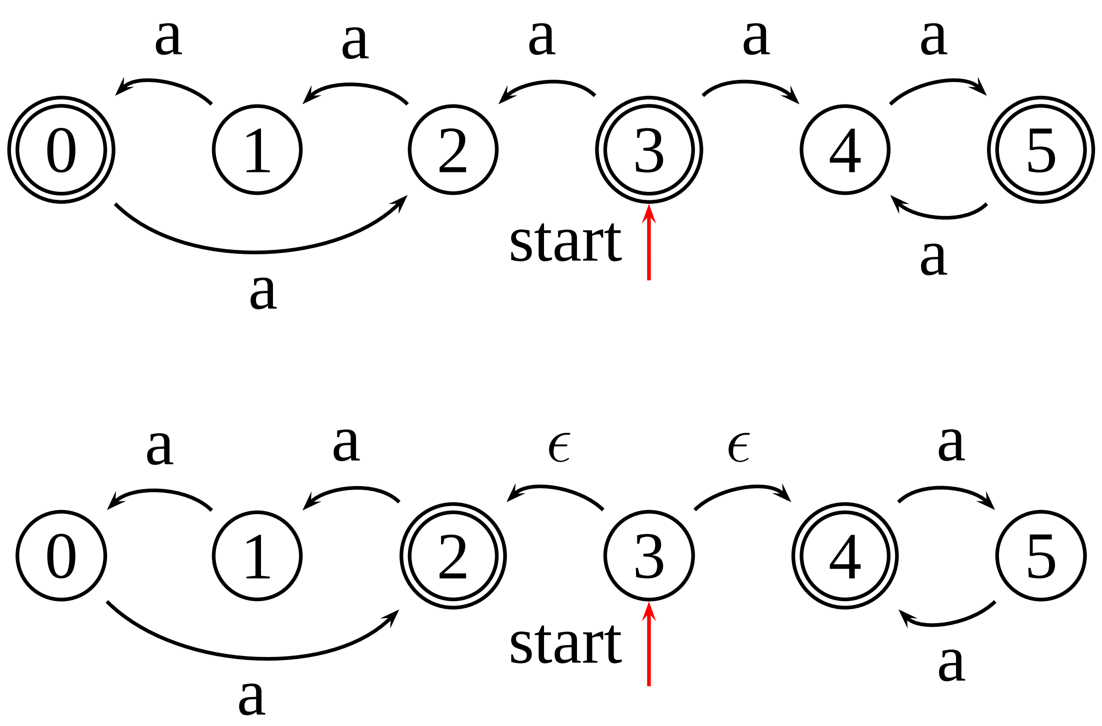
A FA is a DFA if

- no state has an ϵ-transition, i.e., an transition on input ϵ, and
- for each state s and input symbol a, there is at most one edge labeled a leaving s

A FA is an NFA if it is not a DFA:

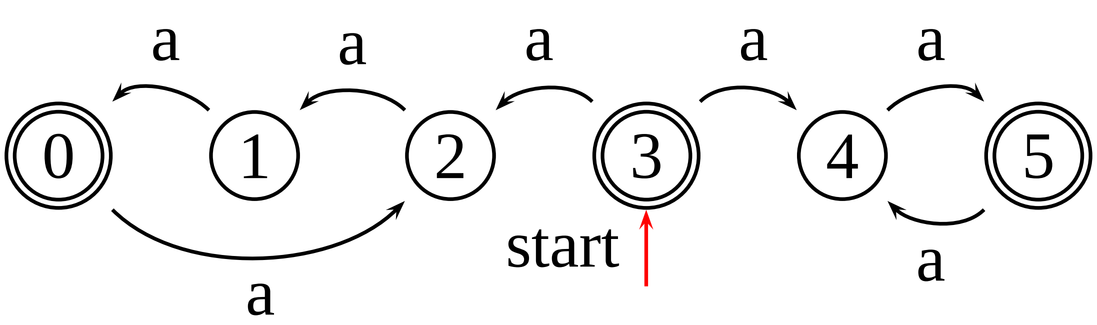
- Nondeterministic: can make several parallel transitions on a given input
- Acceptance: the existence of some path as per Slide 83

DFA or NFA? What are the Languages Recognised?

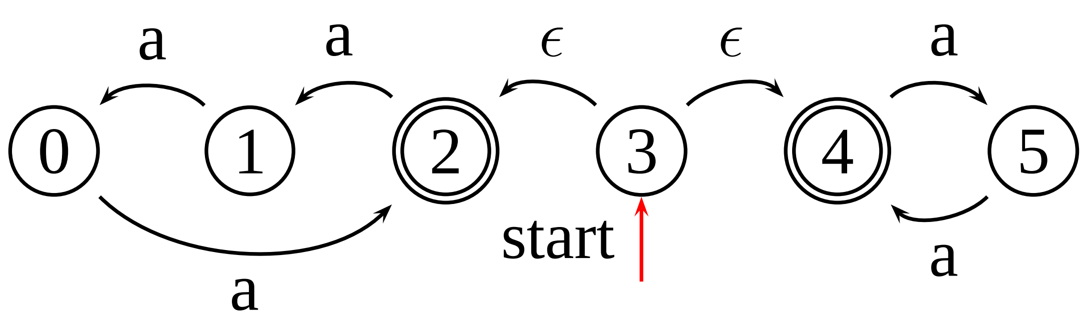


Two Examples

- NFA 1:



- NFA 2:



- The same language:
  the set of all strings of a's such that the length of each of these strings is a multiple of 2 or 3 (ϵ included)

Week 2: Regular Expressions, DFA and NFA

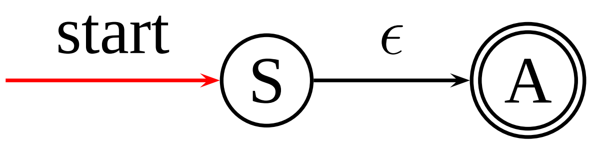
1. Definitions of REs, DFA and NFA $\surd$
2. REs $\Longrightarrow$ NFA (*Thompson's construction, Algorithm 3.3, Red Dragon/Algorithm 3.23, Purple Dragon*)
3. NFA $\Longrightarrow$ DFA (*subset construction, Algorithm 3.2, Red Dragon/Algorithm 3.20, Purple Dragon*)
4. DFA $\Longrightarrow$ minimal-state DFA (*state minimisation, Algorithm 3.6, Red Dragon/Algorithm 3.39, Purple Dragon*)
5. Scanner generators
   - How to use them (straightforward)
   - How to write them (the most techniques introduced today)

Thompson's Construction of NFA from REs

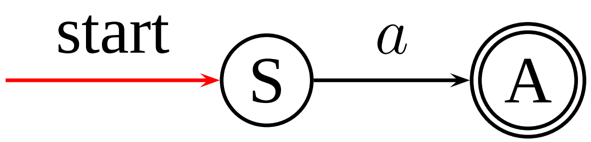
- Syntax-driven
- Inductive: The cases in the construction of the NFA follow the cases in the definition of REs
- Important: if a symbol a occurs several times in a RE r, a separate NFA is constructed for each occurrence
- Thompson's method is one of many available

Thompson's Construction

- Inductive Base:
  1. For ϵ, construct the NFA



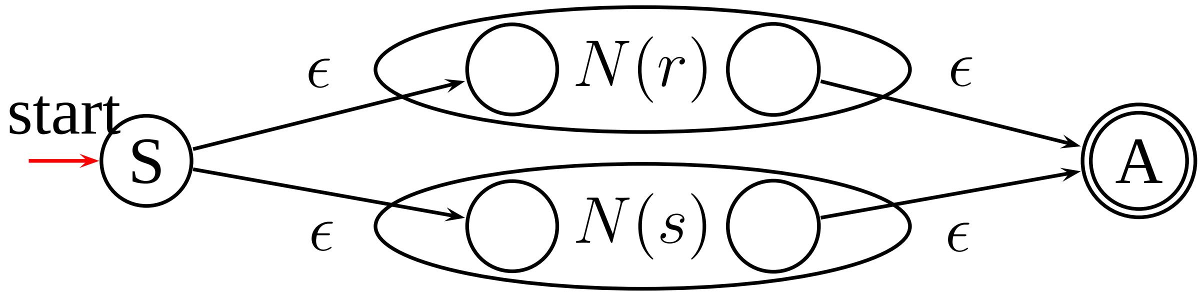
  2. For $a \in \Sigma$, construct the NFA



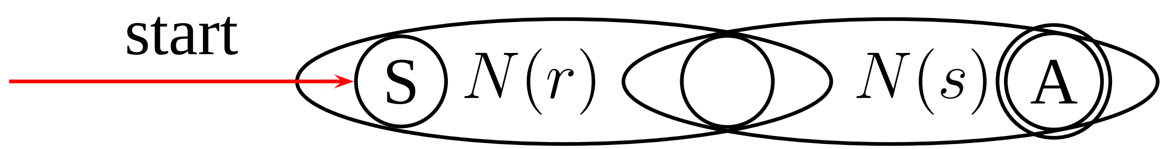
- Inductive step: suppose $N(r)$ and $N(s)$ are NFAs for REs r and s. Then

Thompson's Construction (Cont'd)

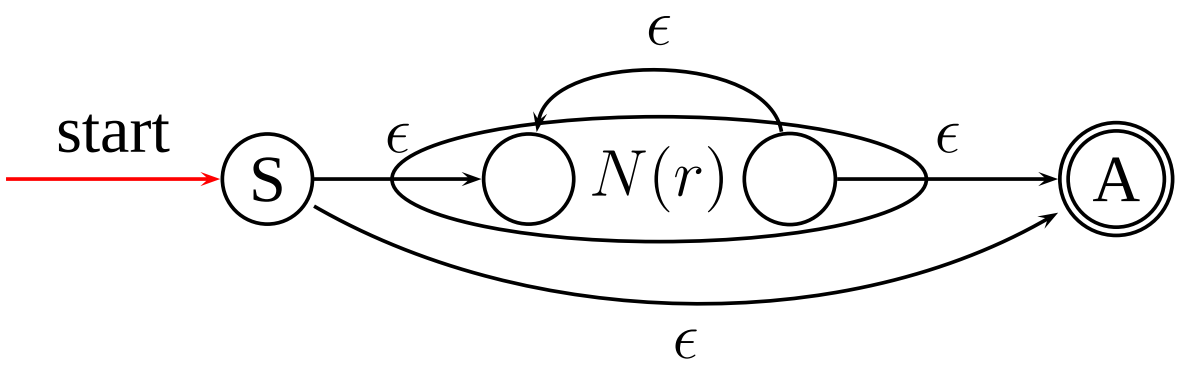
RE $r|s$:



RE rs :



RE r^* :



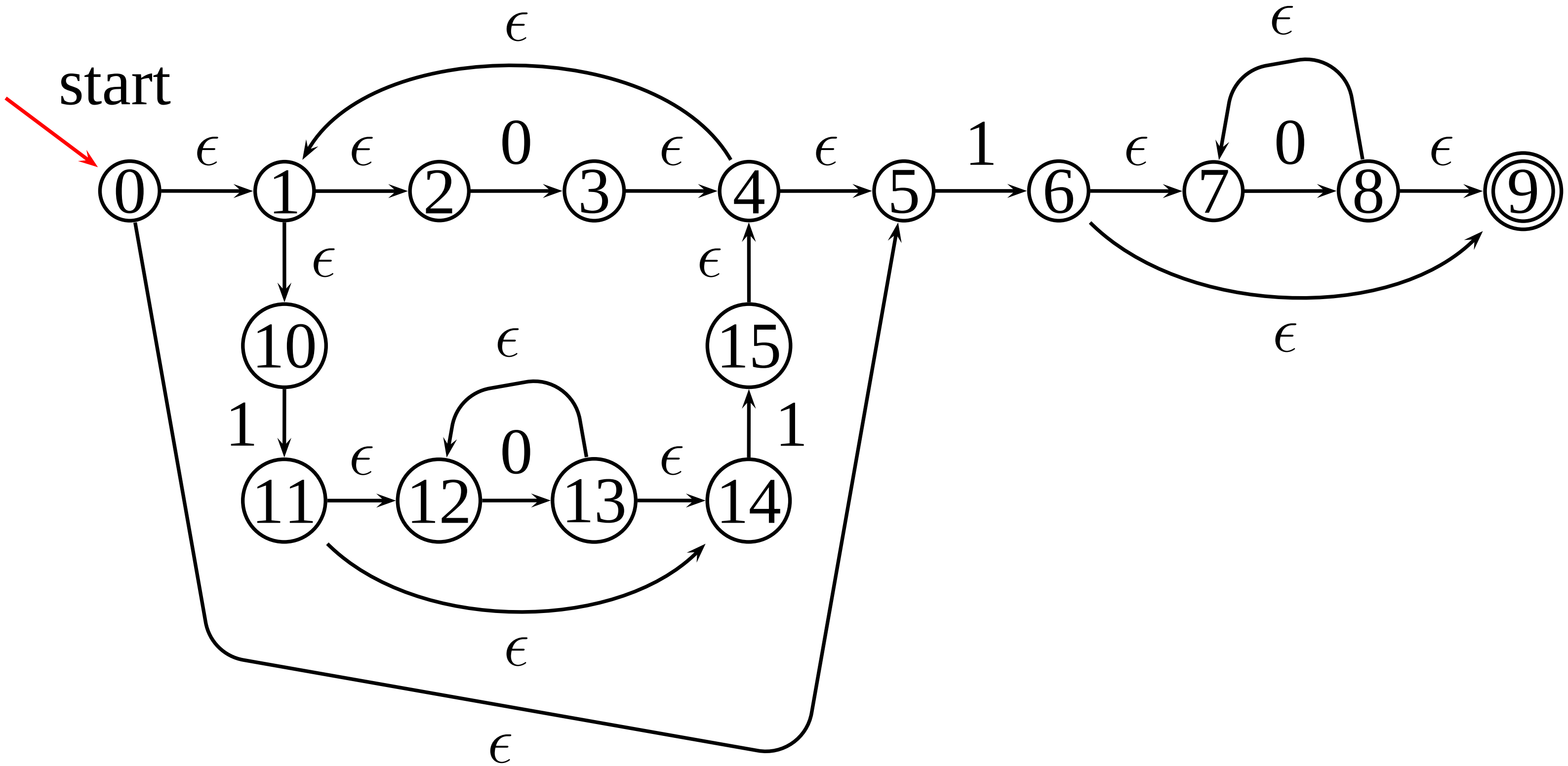
RE (r) : $N((r))$ is the same as $N(r)$

Example: RE $\Longrightarrow$ NFA

Converting $(0|10^*1)^*10^*$ to an NFA

Example: RE $\Longrightarrow$ NFA

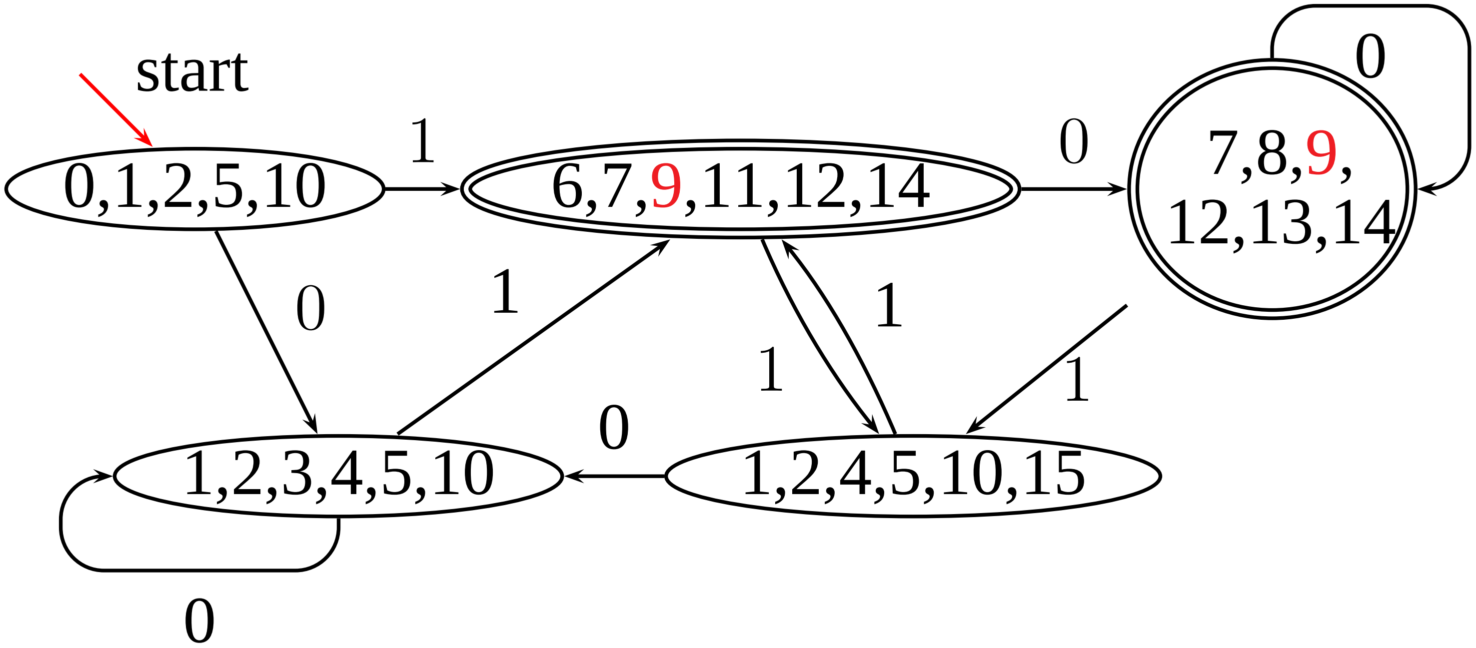
- Regular expression: $(0|10^*1)^*10^*$
- NFA:



Week 2: Regular Expressions, DFA and NFA

1. Definitions of REs, DFA and NFA $\surd$
2. REs $\Longrightarrow$ NFA (*Thompson's construction, Algorithm 3.3, Red Dragon/Algorithm 3.23, Purple Dragon*) $\surd$
3. NFA $\Longrightarrow$ DFA (*subset construction, Algorithm 3.2, Red Dragon/Algorithm 3.20, Purple Dragon*)
4. DFA $\Longrightarrow$ minimal-state DFA (*state minimisation, Algorithm 3.6, Red Dragon/Algorithm 3.39, Purple Dragon*)
5. Scanner generators
   - How to use them (straightforward)
   - How to write them (the most techniques introduced today)

Example: DFA (Cont'd)



- The algorithm used is known as the subset construction, because a DFA state corresponds to a subset of NFA states
- There are at most 2^n DFA states, where n is the total number of the NFA states

Subset Construction: The Operations Used

OPERATION	DESCRIPTION
ϵ-closure(s)	Set of NFA states readable from NFA state s on ϵ-transitions
ϵ-closure(T)	Set of NFA states readable from some state s in T on ϵ-transitions
move(T, a)	Set of NFA states to which there is a transition on input a from some state s in T

- s: a NFA state
- T: a set of NFA states

Subset Construction: The Algorithm

```
Let s_0 be the start state of the NFA;
DFAstates contains the only unmarked state ε-closure(s_0);
while there is an unmarked state T in DFAstates do begin
    mark T
    for each input symbol a do begin
        U := ε-closure(move(T, a));
        if U is not in DFAstates then
            Add U as an unmarked state in DFAstates;
        DFATrans[T, a] := U;
    end;
end;
```

Subset Construction: The Definition of the DFA

Let (Σ, S, T, F, s_0) be the original NFA. The DFA is:

- The alphabet: Σ
- The states: all states in DFAstates
- The start state: ϵ-closure(s_0)
- The accepting states: all states in DFAstates containing at least one accepting state in F of the NFA
- The transitions: DFATrans

Week 2: Regular Expressions, DFA and NFA

1. Definitions of REs, DFA and NFA $\surd$
2. REs $\Longrightarrow$ NFA (*Thompson's construction, Algorithm 3.3, Red Dragon/Algorithm 3.23, Purple Dragon*) $\surd$
3. NFA $\Longrightarrow$ DFA (*subset construction, Algorithm 3.2, Red Dragon/Algorithm 3.20, Purple Dragon*) $\surd$
4. DFA $\Longrightarrow$ minimal-state DFA (*state minimisation, Algorithm 3.6, Red Dragon/Algorithm 3.39, Purple Dragon*)
5. Scanner generators
   - How to use them (straightforward)
   - How to write them (the most techniques introduced today)

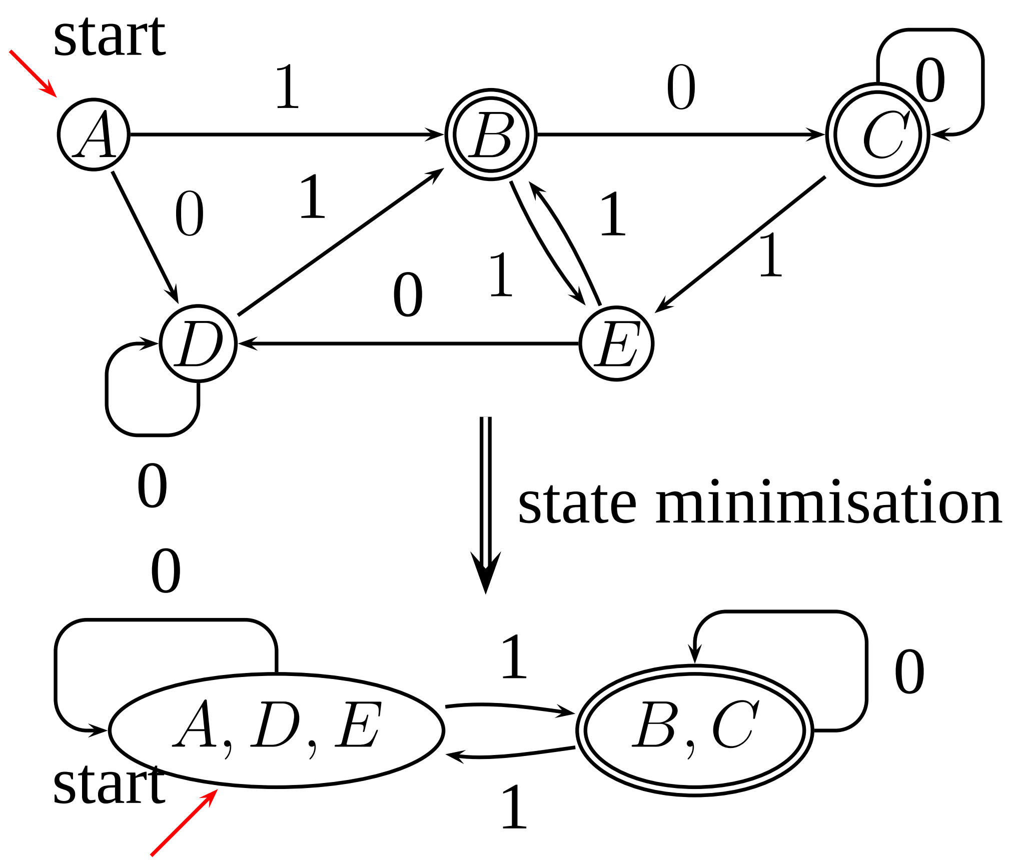
An Algorithm to Mimimise DFA Statements

```
Initially, let Π be the partition with the two groups:
  (1) one is the set of all final states
  (2) the other is the set of all non-final states

Let Π_new = Π
for (each group G in Π_new) {
   partition G into subgroups such that two states s and t
        are in the same subgroup iff for all input symbols
        a, states s and t have transitions on a to
        states in the same group of Π_new
   replace G in Π_new by the set of subgroups formed
}
```

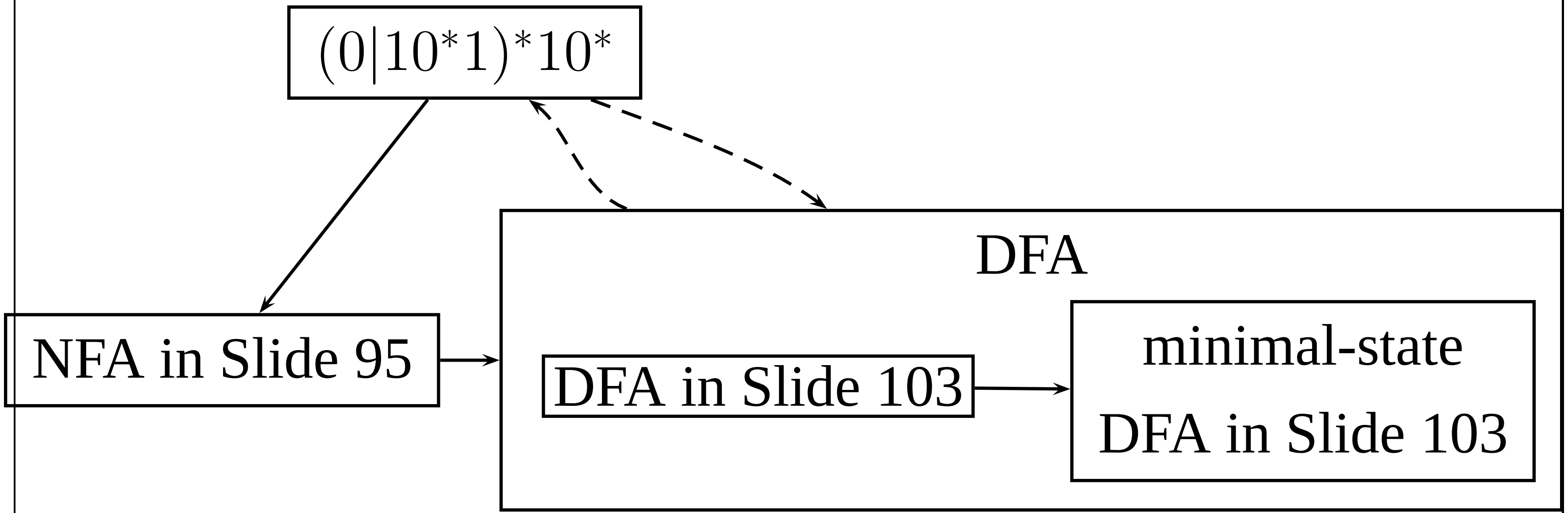
- Begins with the most optimistic assumption
- Also used in global value numbering (COMP4133)

Example (Cont'd): States Re-Labeled



Theoretical Result: every regular language can be recognised by a minimal-state DFA that is unique up to state names

Various Conversions for an Example



However, the conversions in dashed arrows are not covered.

Week 2: Regular Expressions, DFA and NFA

1. Definitions of REs, DFA and NFA $\surd$
2. REs $\Longrightarrow$ NFA (*Thompson's construction, Algorithm 3.3, Red Dragon/Algorithm 3.23, Purple Dragon*) $\surd$
3. NFA $\Longrightarrow$ DFA (*subset construction, Algorithm 3.2, Red Dragon/Algorithm 3.20, Purple Dragon*) $\surd$
4. DFA $\Longrightarrow$ minimal-state DFA (*state minimisation, Algorithm 3.6, Red Dragon/Algorithm 3.39, Purple Dragon*) $\surd$
5. Scanner generators
   - How to use them (straightforward)
   - How to write them (the most techniques introduced today)

Scanner Generators

- Scanners generated in C
  - lex (UNIX)
  - flex – GNU's fast lex (UNIX)
  - mks lex (MS-DOS and OS/2)
- Scanners generated in Java
  - Jflex
  - JavaCC (SUN Microsystems)

The Scanner Spec in Jflex

```
user code -- copied verbatim to the scanner file
%%
Jflex directives
%%
regular expression rules
```

How a Scanner Generator Works

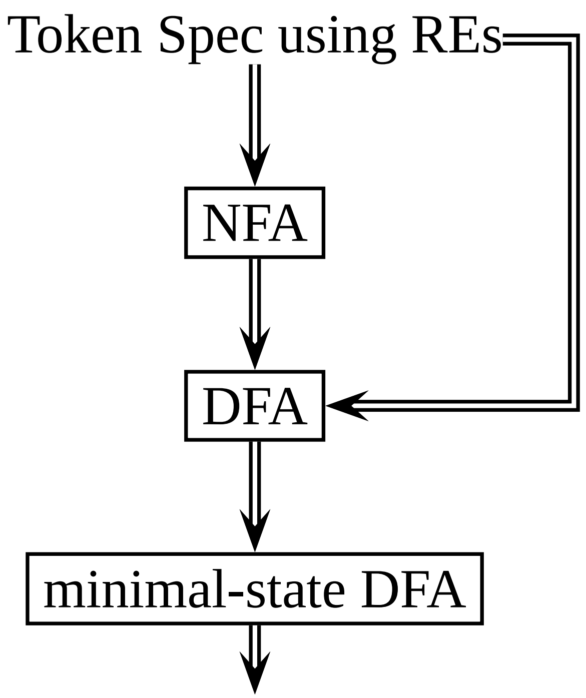


table-driven code (Jflex) – simulating a DFA on an input

or hard-wired code (§3.4 of either Dragon Book)

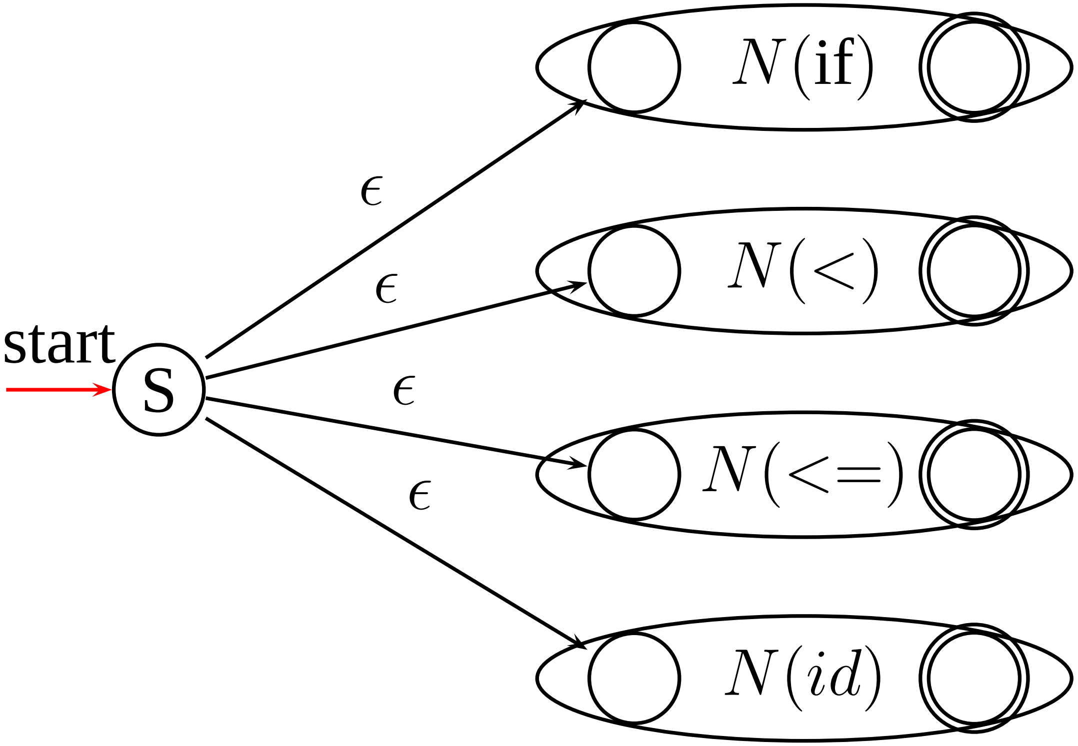
An Example: Spec

```
some user code
%%
LETTER=[A-Za-z_]
DIGIT=[0-9]
%%
"if"  { return new Token(Token.IF, "if", pos); }
"<"   { return new Token(Token.LT, "<", pos); }
"<="  { return new Token(Token.LT, "<=", pos); }
{LETTER}({LETTER}|{DIGIT})*
      { return new Token(Token.ID, "itsSpelling", pos); }
```

Two rules:

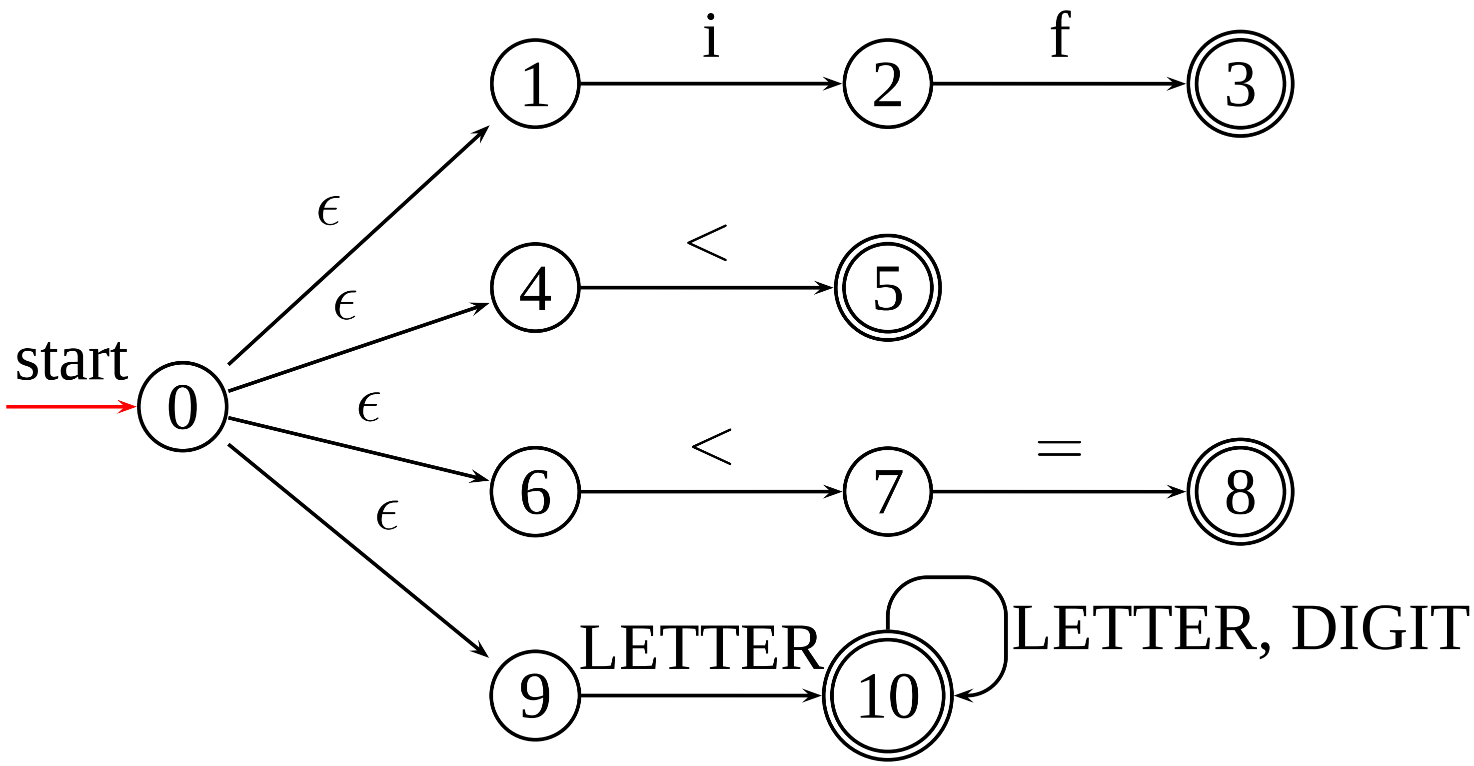
- The first pattern used when more than one are matched – “if” as a keyword not as an id
- The longest prefix of the input is always matched – ”<= as one token

An Example: NFA

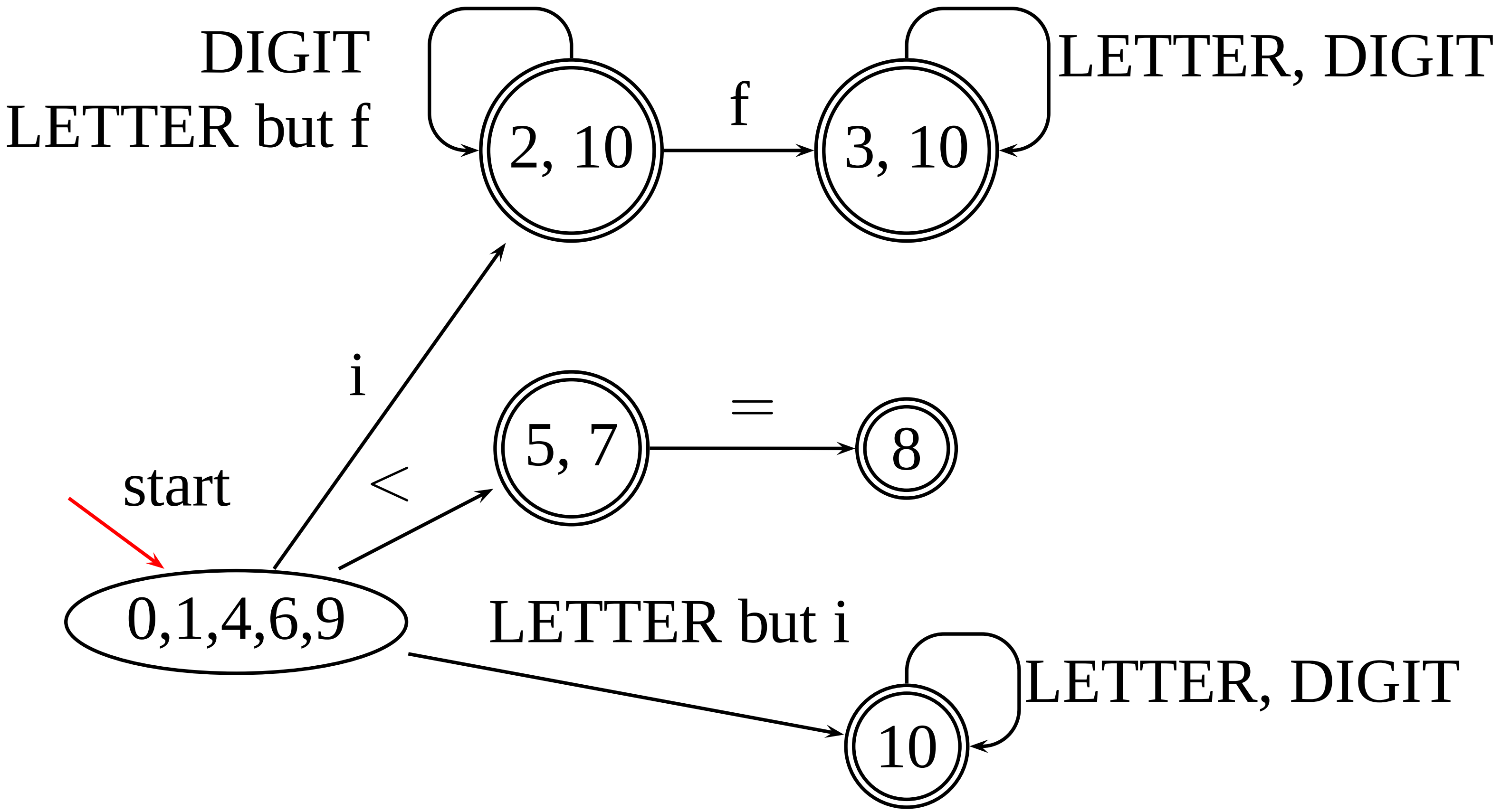


A DFA can also be used for each pattern.

An Example: NFA



An Example: DFA



Already a minimal-state DFA!

A DFA Represented as a Transition Table

State	Character						
	$<$	$=$	i	f	**LETTER** but i	**LETTER** but f	**DIGIT**
(0,1,4,6,9)	(5,7)		(2,10)		(10)		
(2,10)				(3,10)		(2,10)	
(5,7)		(8)					
(8)							
(10)			(10)		(10)		(10)
(3,10)			(3,10)		(3,10)		(10)

- **Letter** = $\{i\} \cup$ "**letter** but i"
- Character classes reduce the table size
- The blank entries are errors
- The tables are usually sparse (pages 146 – 177 of text for compression techniques)

The Scanner Driver for Simulating a DFA

```
state = initial_state
while (TRUE) {
  next_state = T[state][current_char];
  if (next_state == ERROR)  // cannot move any further
    break;
  state = next_state;
  if (current_char == EOF)  // input exhausted
    break;
  current_char = getchar(); // fetch the next char
}
Backtrack to the most recent accepting state
if (such a state exists)
  /* return the corresponding token
     reset current_char to the first after the token
  */
else
  lexical_error(state);
```

- There should be a column in the transition table for EOF
- Need to backtrack

The Output of Running Jflex on a Sample Scanner Spec

- Scanner.l: the spec for the scanner generator Jflex

```
[jxue@daniel lec2] java Jflex.Main Scanner.l

Processing first section -- user code.
Processing second section -- Jflex declarations.
Processing third section -- lexical rules.
Creating NFA machine representation.
NFA comprised of 315 states.
Working on character classes.::.::::.:..::..::.:...:.:.......
NFA has 46 distinct character classes.
Creating DFA transition table.
Working on DFA states.......................................
Minimizing DFA transition table.
109 states after removal of redundant states.
Outputting lexical analyzer code.
```

- Scanner.l.java: the scanner generated

```
javac Scanner.l.java
```

- java Scanner < test.vc

Limitations of Regular Expressions (or FAs)

- Cannot "count"
- Cannot recognise palindromes (e.g., racecar & rotator)
- The language of the balanced parentheses

$$\{(^n)^n \mid n \geqslant 1\}$$

  is not a regular language

  – cannot build a FA to recognise the language for any n (can trivially build a FA for n=3, for example)

  – but can be specified by a CFG (Week 3):

$$P \rightarrow (P) \mid ()$$

Chomsky's Hierarchy

Depending on the form of production

$$\alpha \longrightarrow \beta$$

four types of grammars (and accordingly, languages) are distinguished:

GRAMMAR	KNOWN AS	DEFINITION	LANGUAGE	MACHINE
Type 0	unrestricted grammar	$\alpha \neq \epsilon$	Type 0	Turing machine
Type 1	context-sensitive grammar CSGs	$\|\alpha\| \leq \|\beta\|$	Type 1	linear bounded automaton
Type 2	context-free grammar CFGs	$A \longrightarrow \alpha$	Type 2	stack automaton
Type 3	Regular grammars	$A \longrightarrow w \mid Bw$	Type 3	finite state automaton