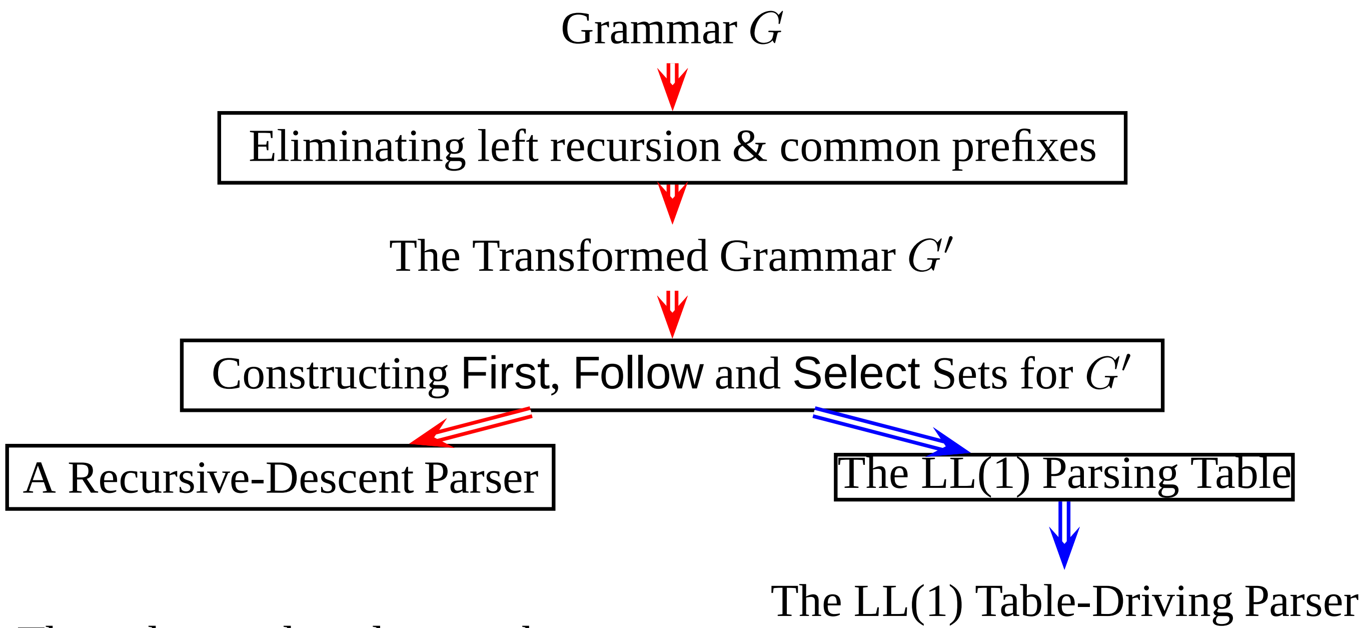
Lecture 5: Top-Down Parsing: Table-Driven

1. LL(1) table-driven parsing
2. Parser generators
3. Recursive-descent parsing revisited
4. Error recovery



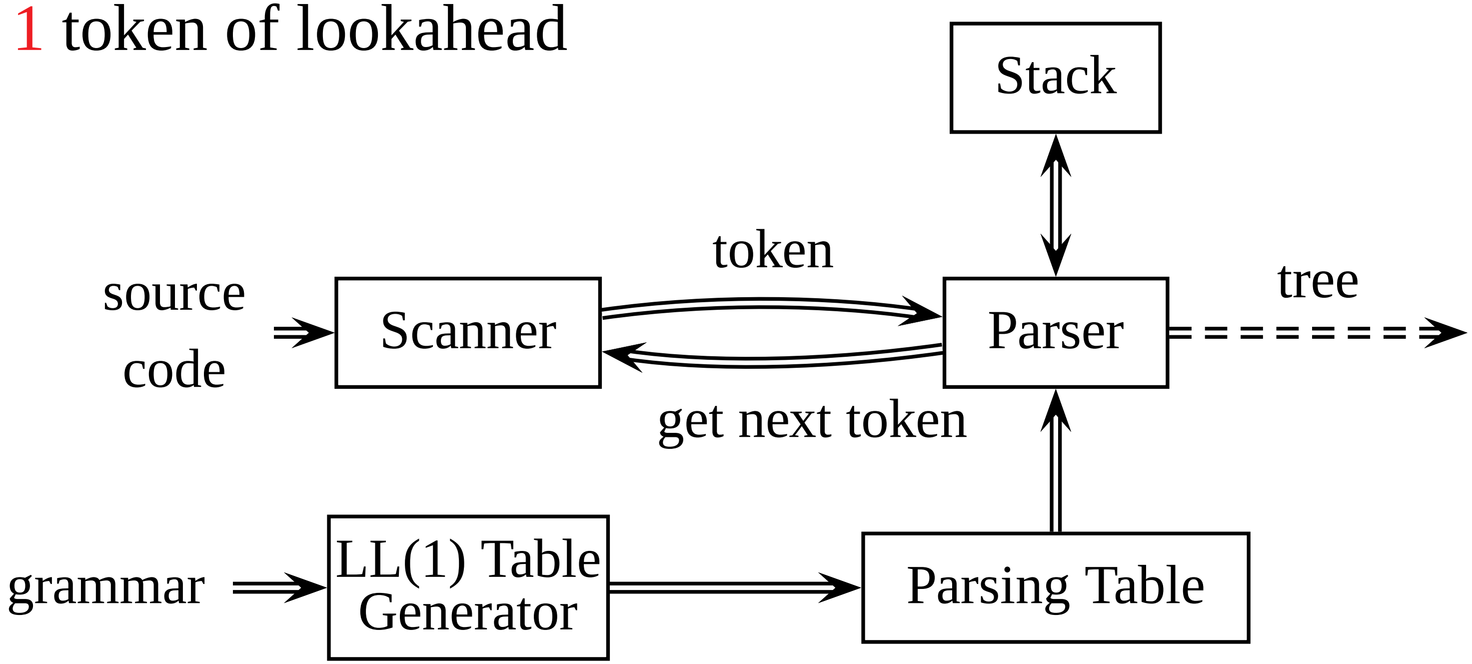
- The red parts done last week
- The the blue parts today

Predictive Non-Recursive Top-Down Parsers

- Recursion = Iteration + Stack
- Recursive calls in a recursive-descent parser can be implemented using
  - an explicit stack, and
  - a parsing table
- Understanding one helps your understanding the other

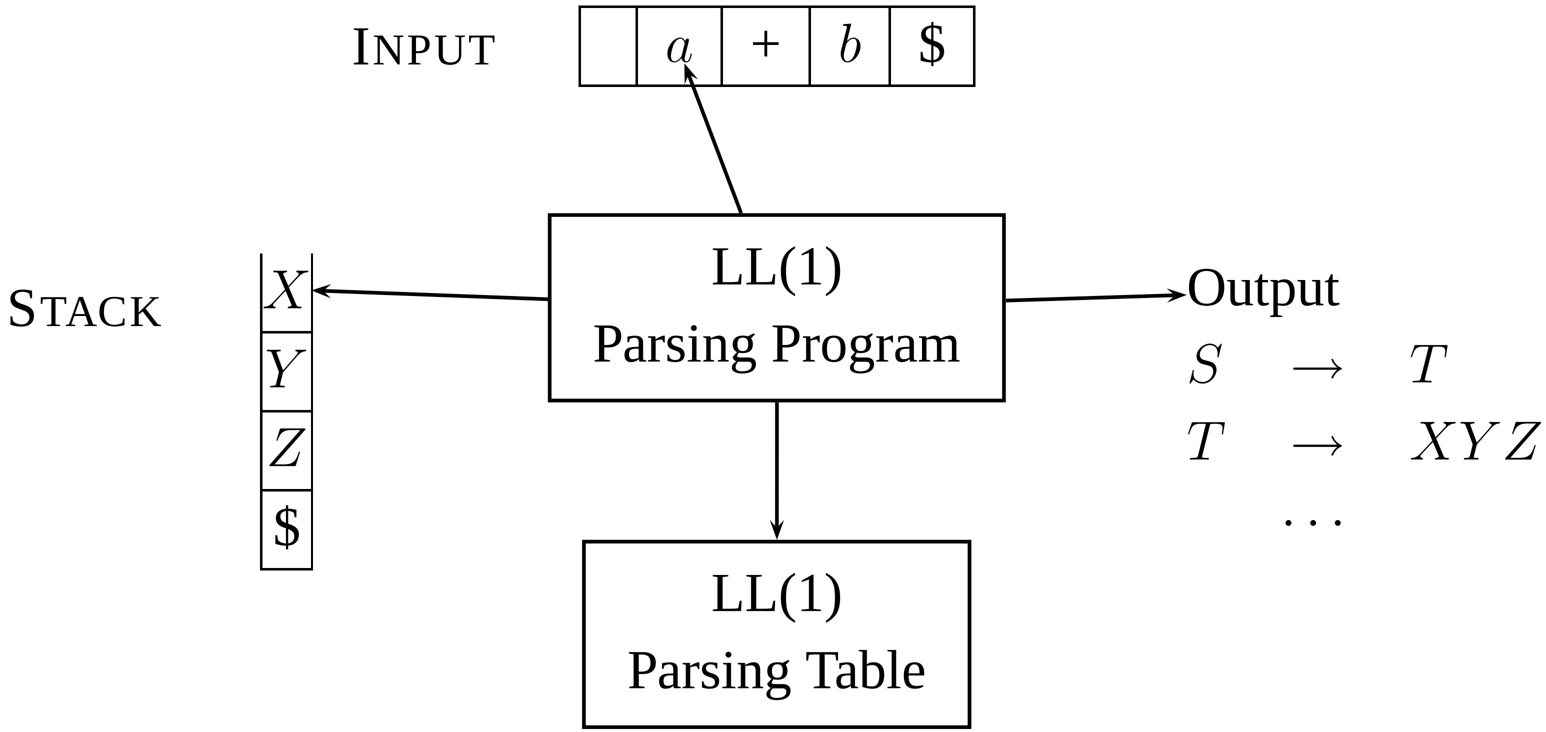
The Structure of a Table-Driven LL(1) Parser

- Input parsed from left to right
- Leftmost derivation
- 1 token of lookahead



- LR(1) parsers (almost always table-driven) also built this way

The Model of an LL(1) Table-Driven Parser



Output:

- The productions used (representing the leftmost derivation), or
- An AST (Lecture 6)

The LL(1) Parsing Program

Push \$ onto the stack

Push the start symbol onto the stack

WHILE (stack not empty) **DO**

  BEGIN

    Let X be the top stack symbol and a be the lookahead symbol in the input

    IF X is a terminal **THEN**

      IF $X = a$ then pop X and get the next token /* match */

      ELSE error

    ELSE /$*$ X is a nonterminal $*$/

      IF $Table[X, a]$ nonblank **THEN**

        Pop X

        Push $Table[X, a]$ onto stack in the reverse order

      ELSE error

  END

The parsing is successful when the stack is empty and no errors reported

Building a Table-Driving Parser from a Grammar

Grammar G

$\Downarrow$

$\boxed{\text{Eliminating left recursion \& common prefixes (Slide 247)}}$

$\Downarrow$

The Transformed Grammar G'

$\Downarrow$

$\boxed{\text{Constructing First, Follow and Select Sets for } G'}$

$\Downarrow$

$\boxed{\text{Constructing the LL(1) Parsing Table}}$

$\Downarrow$

The LL(1) Table-Driving Parser

- Follow Slide 247 to eliminate left recursion to get a BNF grammar
- Can build a table-driven parser for EBNF as well (but not considered)

The Expression Grammar

- The grammar with left recursion:

$$\text{Grammar 1:}\quad \begin{aligned} E &\rightarrow E + T \mid E - T \mid T \\ T &\rightarrow T * F \mid T/F \mid F \\ F &\rightarrow \textbf{INT} \mid (E) \end{aligned}$$

- The transformed grammar without left recursion:

$$\text{Grammar 2:}\quad \begin{aligned} E &\rightarrow TQ \\ Q &\rightarrow +TQ \mid -TQ \mid \epsilon \\ T &\rightarrow FR \\ R &\rightarrow *FR \mid /FR \mid \epsilon \\ F &\rightarrow \textbf{INT} \mid (E) \end{aligned}$$

First and Follow Sets for Grammar 2

- First sets:

$$
\begin{array}{rcl}
\text{First}(TQ) = \text{First}(FR) & = & \{(, \mathbf{i}\} \\
\text{First}(Q) & = & \{(+, -, \epsilon\} \\
\text{First}(R) & = & \{(*, /, \epsilon\} \\
\text{First}(+TQ) & = & \{+\} \\
\text{First}(-TQ) & = & \{-\} \\
\text{First}(*FR) & = & \{*\} \\
\text{First}(/FR) & = & \{/\} \\
\text{First}((E)) & = & \{(\} \\
\text{First}(\mathbf{i}) & = & \{\mathbf{i}\}
\end{array}
$$

- Follow sets:

$$
\begin{array}{rcl}
\text{Follow}(E) & = & \{\$,)\} \\
\text{Follow}(Q) & = & \{\$,)\} \\
\text{Follow}(T) & = & \{+, -, \$,)\} \\
\text{Follow}(R) & = & \{+, -, \$,)\} \\
\text{Follow}(F) & = & \{+, -, *, /, \$,)\}
\end{array}
$$

Select Sets for Grammar 2

$$
\begin{array}{lll}
\text{Select}(E \rightarrow TQ) & = \text{First}(TQ) & = \{(, \textbf{INT}\} \\
\text{Select}(Q \rightarrow +TQ) & = \text{First}(+TQ) & = \{+\} \\
\text{Select}(Q \rightarrow -TQ) & = \text{First}(-TQ) & = \{-\} \\
\text{Select}(Q \rightarrow \epsilon) & = (\text{First}(\epsilon) - \{\epsilon\}) \cup \text{Follow}(Q) & = \{), \$\} \\
\text{Select}(T \rightarrow FR) & = \text{First}(FR) & = \{(, \textbf{INT}\} \\
\text{Select}(R \rightarrow *FR) & = \text{First}(+FR) & = \{*\} \\
\text{Select}(R \rightarrow /FR) & = \text{First}(/FR) & = \{/\} \\
\text{Select}(R \rightarrow \epsilon) & = (\text{First}(\epsilon) - \{\epsilon\}) \cup \text{Follow}(T) & = \{+, -,), \$\} \\
\text{Select}(F \rightarrow \textbf{INT}) & = \text{First}(\textbf{INT}) & = \{\textbf{INT}\} \\
\text{Select}(F \rightarrow (E)) & = \text{First}((E)) & = \{(\}
\end{array}
$$

The Rules for Constructing an LL(1) Parsing Table

For every production of the $A \longrightarrow \alpha$ in the grammar, do:

for all a in Select$(A \longrightarrow \alpha)$, set $Table[A, a] = \alpha$

LL(1) Parsing Table for Grammar 2

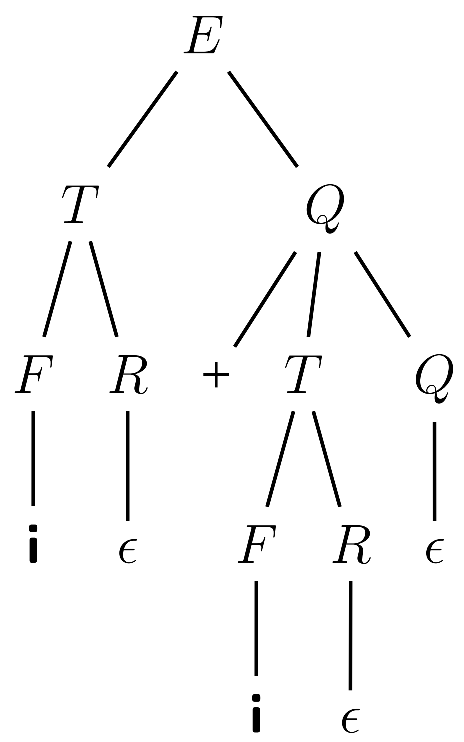
	INT	$+$	$-$	$*$	$/$	$($	$)$	$\$$
E	TQ					TQ		
Q		$+TQ$	$-TQ$				ϵ	ϵ
T	FR					FR		
R		ϵ	ϵ	$*FR$	$/FR$		ϵ	ϵ
F	**INT**					(E)		

The blanks are errors.

An LL(1) Parse on Input **i+i**: **INT** $\Longleftrightarrow$ **i**

STACK	INPUT	PRODUCTION	DERIVATION
$\$E$	**i+i$**	$E \rightarrow TQ$	$E \Longrightarrow_{\rm lm} TQ$
$\$QT$	**i+i$**	$T \rightarrow FR$	$\Longrightarrow_{\rm lm} FRQ$
$\$QRF$	**i+i$**	$F \rightarrow \mathbf{i}$	$\Longrightarrow_{\rm lm} \mathbf{i}RQ$
$\$QR\mathbf{i}$	**i+i$**	pop and go to next token	
$\$QR$	**+i$**	$R \rightarrow \epsilon$	$\Longrightarrow_{\rm lm} \mathbf{i}Q$
$\$Q$	**+i$**	$Q \rightarrow +TQ$	$\Longrightarrow_{\rm lm} \mathbf{i} + TQ$
$\$QT+$	**+i$**	pop and go to next token	
$\$QT$	**i$**	$T \rightarrow FR$	$\Longrightarrow_{\rm lm} \mathbf{i} + FRQ$
$\$QRF$	**i$**	$F \rightarrow \mathbf{i}$	$\Longrightarrow_{\rm lm} \mathbf{i} + \mathbf{i}RQ$
$\$QR\mathbf{i}$	**i$**	pop and go to next token	
$\$QR$	$	$R \rightarrow \epsilon$	$\Longrightarrow_{\rm lm} \mathbf{i} + \mathbf{i}RQ$
$\$Q$	$	$Q \rightarrow \epsilon$	$\Longrightarrow_{\rm lm} \mathbf{i} + \mathbf{i}Q$
$	$		

PARSE TREE



An LL(1) Parse on an Erroneous Input ”()”

STACK	INPUT	PRODUCTION	DERIVATION
$\$E$	()$	$E \rightarrow TQ$	$E \Longrightarrow_{\mathrm{lm}} TQ$
$\$QT$	()$	$T \rightarrow FR$	$E \Longrightarrow_{\mathrm{lm}} FRQ$
$\$QRF$	()$	$F \rightarrow (E)$	$E \Longrightarrow_{\mathrm{lm}} (E)RQ$
$\$QR)E($	()$	pop and go to next token	
$\$QR)E$	)$	$*\,*\,*$ Error: no table entry for $[E,)]$	

A better error message: expression missing inside ( )

LL(1) Grammars and Table-Driven LL(1) Parsers

- Like recursive descent, table-driven LL(1) parsers can only parse LL(1) grammars. Conversely, only LL(1) grammars can be parsed by the table-driven LL(1) parsers.
- Definition of LL(1) grammar given in Slide 241
- Definition of LL(1) grammar – using the parsing table:

  A grammar is LL(1) if every table entry contains at most one production.

Why Table-Driven LL(1) Parsers Cannot Handle Left Recursions?

- A grammar with left recursion:

$$\langle\text{expr}\rangle \quad \rightarrow \quad \langle\text{expr}\rangle + \textbf{id} \;\mid\; \textbf{id}$$

- Select Sets:

$$\begin{array}{lcl} \text{Select}(\langle\text{expr}\rangle+\textbf{id}) & = & \{\textbf{id}\} \\ \text{Select}(\textbf{id}) & = & \{\textbf{id}\} \end{array}$$

- The parsing table:

	id	$
⟨expr⟩	⟨expr⟩ + **id** **id**	

$Table[\langle\text{expr}\rangle, \textbf{id}]$ contains two entries!

- Any grammar with left recursions is not LL(1)

Why Table-Driven LL(1) Parsers Cannot Handle Left Recursions (Cont'd)?

- Eliminating the left recursion yields an LL(1) grammar:

$$\begin{array}{lcl} \langle \text{expr} \rangle & \longrightarrow & \textbf{id}\ \langle \text{expr-tail} \rangle \\ \langle \text{expr-tail} \rangle & \longrightarrow & \epsilon \mid +\ \textbf{id}\ \langle \text{expr-tail} \rangle \end{array}$$

- Select Sets:

$$\begin{array}{lcl} \text{Select}\langle \text{expr} \rangle \longrightarrow \textbf{id}\ \langle \text{expr-tail} \rangle) & = & \{\textbf{id}\} \\ \text{Select}(\langle \text{expr-tail} \rangle \longrightarrow \langle \text{expr} \rangle) = & = & \{\$\} \\ \text{Select}(\langle \text{expr-tail} \rangle \longrightarrow +\textbf{id}\ \langle \text{expr-tail} \rangle) & = & \{+\} \end{array}$$

- The parsing table for the transformed grammar:

	id	+	$
⟨expr⟩	**id** ⟨expr-tail⟩		
⟨expr-tail⟩		+ **id** ⟨expr-tail⟩	ϵ

Why LL(1) Table-Driven Parsers Cannot Handle Common Prefixes?

- A grammar with a common prefix:

$$\begin{array}{lcl} S & \rightarrow & \textbf{if}\ (E)\ S \mid \textbf{if}\ (E)\ S\ \textbf{else}\ S \mid \textbf{s} \\ E & \rightarrow & e \end{array}$$

- Select sets:

$$\begin{array}{lcl} \text{Select}(S\rightarrow\textbf{if}\ (E)\ \ S) & = & \{\textbf{if}\} \\ \text{Select}(S\rightarrow\textbf{if}\ (E)\ \ S\ \textbf{else}\ S) & = & \{\textbf{if}\} \end{array}$$

- Any grammar with common prefixes is not LL(1)
- Eliminating the common prefix does not yield an LL(1) grammar:

$$\begin{array}{lcl} S & \rightarrow & \textbf{if}\ (E)\ SQ \mid s \\ Q & \rightarrow & \textbf{else}\ S \mid \epsilon \\ E & \rightarrow & e \end{array}$$

Why LL(1) Table-Driven Parsers Cannot Handle Common Prefixes (Cont'd)?

- Select sets:

$$
\begin{array}{lcl}
\text{Select}(S \rightarrow \textbf{if}(E)SQ) & = & \{\textbf{if}\} \\
\text{Select}(S \rightarrow s) & = & \{s\} \\
\text{Select}(Q \rightarrow \textbf{else}S) & = & \{\textbf{else}\} \\
\text{Select}(Q \rightarrow \epsilon) & = & \{\textbf{else}, \epsilon\} \\
\text{Select}(E \rightarrow e) & = & \{e\}
\end{array}
$$

- The parsing table:

	if	(	e	)	s	**else**	\$
S	**if** E **then** SQ				s		
E			e				
Q						**else** S ϵ	ϵ

- This modified grammar, although having no common prefixes, is still ambiguous. You are referred to Week 6 Tutorial. To resolve the ambiguity in the grammar, we make the convention to select **else** S as the table entry. This effectively implements the following rule:

  Match an **else** to the most recent unmatched **then**

Recognise Palindromes Easily

- Grammar:

$$S \quad \rightarrow \quad (S) \mid \epsilon$$

- Parsing Table:

	(	)	\$
S	(S)	ϵ	ϵ

- Try to parse the following three inputs:

```
a. (())
b. (()
c. ()))
```

- Cannot design a DFA/NFA to recognise the language $L(S)$

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The Expression Grammar

- The grammar with left recursion:

$$\text{Grammar 1:} \quad \begin{aligned} E &\rightarrow E + T \mid E - T \mid T \\ T &\rightarrow T * F \mid T/F \mid F \\ F &\rightarrow \textbf{INT} \mid (E) \end{aligned}$$

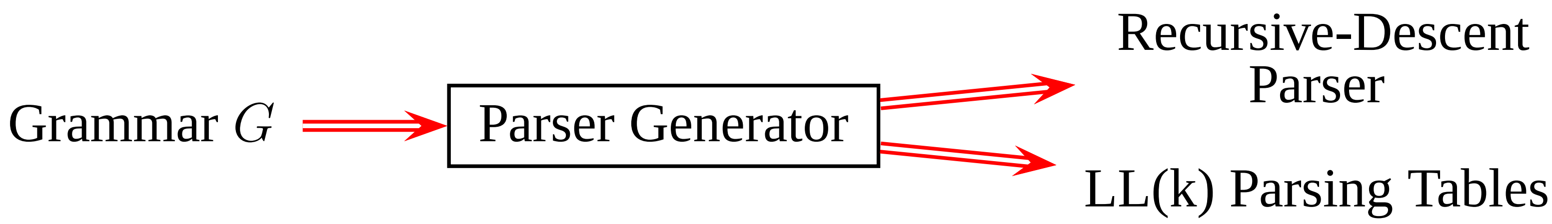
- Eliminating left recursion using the Kleene Closure

$$\text{Grammar 3:} \quad \begin{aligned} E &\rightarrow T\ (\text{"+"}\ T \mid \text{"-"}\ T)^* \\ T &\rightarrow F\ (\text{"*"}\ F \mid \text{"/"}\ F)^* \\ F &\rightarrow \textbf{INT} \mid \text{"("}\ E\ \text{")"} \end{aligned}$$

  All tokens are enclosed in double quotes to distinguish them for the regular operators: (,) and *

- Compare with Slide 269

Parser Generators (Generating Top-Down Parsers)



Tool	Grammar Accepted	Parsers and Their Implementation Languages
JavaCC	EBNF	Recursive-Descent LL(1) (with some LL(k) portions) in Java
COCO/R	EBNF	Recursive-Descent LL(1) in Pascal, C, C++,, Java, etc.
ANTLR	Predicated LL(k)	Recursive-Descent LL(k) in C, C++,, Java

- These and other tools can be found on the internet
- Predicated: a conditional evaluated by the parser at run time to determine which of the two conflicting productions to use

$$Q \quad \rightarrow \boxed{\text{if (lookahead is "else")}}\ \textbf{else}\ S \mid \epsilon$$

where the condition inside the box resolves the dangling-else problem.

Parser Generators (Generating Bottom-Up Parsers)

Grammar G $\Longrightarrow$ $\boxed{\text{Parser Generator}}$ $\Longrightarrow$ LALR(1) Parsing Tables

Tool	Grammar Accepted	Parsers and Their Implementation Languages
Yacc	BNF	LALR(1) table-driven in C
JavaCUP	BNF	LALR(1) table-driven in Java

- These and other tools can be found on the internet
- Likely to deal with LR parsing at the end of the semester