MATH 109: Homework 2

Professor Rabin

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Problem 2.2

(a) True.

(b) False, 33 is not a Fibonacci number.

(c) False, $22 \in A \implies 22 \in A \cup D$.

(d) True.

(e) False, $\phi \notin B$ and $\phi \notin D$.

(f) False, 53 is prime and $53 \neq 2$.

Problem 2.13

(a) The real number r is greater than $\sqrt{2}$.

(b) The absolute value of the real number a is greater than or equal to 3.

(c) At least two angles of the triangle are not 45°.

(d) The area of the circle is less than 9π.

(e) All sides of the triangle have different length.

(f) The point P in the plane is on or inside of the circle C.

Problem 2.20

P	Q	$P \implies Q$	$\sim P$	$(P \implies Q) \implies (\sim P)$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

Problem 2.22

(a) If $\sqrt{2}$ is rational and $\frac{2}{3}$ is rational, then $\sqrt{3}$ is rational.
Since $(P \wedge Q)$ is false and R is false, $(P \wedge Q) \implies R$ is **true**.

(b) If $\sqrt{2}$ is rational and $\frac{2}{3}$ is rational, then $\sqrt{3}$ is not rational.
Since $(P \wedge Q)$ is false and $\sim R$ is true, $(P \wedge Q) \implies \sim R$ is **true**.

(c) If $\sqrt{2}$ is not rational and $\frac{2}{3}$ is rational, then $\sqrt{3}$ is rational.
Since $((\sim P) \wedge Q)$ is true and R is false, $((\sim P) \wedge Q) \implies R$ is **false**.

(d) If $\sqrt{2}$ is rational or $\frac{2}{3}$ is rational, then $\sqrt{3}$ is not rational.
Since $(P \vee Q)$ is true and $\sim R$ is true, $(P \vee Q) \implies \sim R$ is **true**.

Problem 2.24

(a) False

(b) True

(c) False

(d) True

(e) False

(f) True

Problem 2.32

(a) Let us consider the truth value of $P(x)$.

$P(x)$ is only true for $x = 7$. However, $Q(7) : 7 \geq 8$ is false. Therefore, $P(7) \implies Q(7)$ is **false**.

For all other $x \neq 7$, $P(x)$ is false. Therefore, $P(x) \implies Q(x) \quad \forall x \in S - \{7\}$ is **true**.

Therefore, $P(x) \implies Q(x)$ is **true** $\forall x \in S - \{7\}$.

(b) Let us consider the truth value of $P(x)$.

For $P(x)$ to be true, $x^2 \geq 1 \implies x \geq 1$ or $x \leq -1$.

For the case $x \geq 1$, $P(x)$ is true and $Q(x) : x \geq 1$ is also true. Therefore, for $x \in [1, \infty)$, $P(x) \implies Q(x)$ is **true**.

Alternately, $x \leq -1$, in which case, $Q(x) : x \geq 1$ is false. Therefore, for $x \in (-\infty, -1]$, $P(x) \implies Q(x)$ is **false**.

For $x \in (-1, 1)$, $P(x)$ is false. Therefore, $P(x) \implies Q(x)$ is **true**.

Therefore, $P(x) \implies Q(x)$ is **true** $\forall x \in (-1, \infty)$.

(c) Note that $P(x)$ and $Q(x)$ are the same as (b), only $S = \mathbb{N}$ is different. Therefore, we only need to consider the case where $x \in \mathbb{N}$.

As before, for $x \geq 1$, $P(x)$ is true and $Q(x) : x \geq 1$ is also true. Therefore, for $x \in [1, \infty)$, $P(x) \implies Q(x)$ is **true**.

Therefore, $P(x) \implies Q(x)$ is **true** $\forall x \in S$.

(d) Consider the truth value of $P(x)$. Since $S = [-1, 1] \subseteq [-1, 2]$, $P(x) : x \in [-1, 2]$ is true $\forall x \in S$.

Now, consider the truth value of $Q(x)$. $Q(x) : x^2 \leq 2$ is true when $x \leq \sqrt{2}$ or $x \geq -\sqrt{2}$. Since $S \subseteq [-\sqrt{2}, \sqrt{2}]$, $Q(x) : x^2 \leq 2$ is also true $\forall x \in S$.

Therefore, $P(x) \implies Q(x)$ is **true** $\forall x \in S$.

Problem 2.42

Let $P(n)$: The integer $\frac{n \cdot (n-1)}{2}$ is odd.
Let $Q(n)$: $\frac{n \cdot (n+1)}{2}$ is even.
$P(n) \iff Q(n)$ when either both are **true** or both are **false**.

For $n = 2$, $P(2) : \frac{2\cdot(1)}{2} = 1$ is odd, which is true and $Q(2) : \frac{2\cdot(3)}{2} = 3$ is even, which is false. Therefore, $P(2) \iff Q(2)$ is **false**.
For $n = 3$, $P(3) : \frac{3\cdot(2)}{2} = 3$ is odd, which is true and $Q(3) : \frac{3\cdot(4)}{2} = 6$ is even, which is true. Therefore, $P(3) \iff Q(3)$ is **true**.
For $n = 4$, $P(4) : \frac{4\cdot(3)}{2} = 6$ is odd, which is false and $Q(4) : \frac{4\cdot(5)}{2} = 10$ is even, which is true. Therefore, $P(4) \iff Q(4)$ is **false**.

Problem 2.56

P	Q	$\sim P$	$\sim Q$	$P \wedge (\sim P)$	$(\sim Q) \implies (P \wedge (\sim P))$
T	T	F	F	F	T
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	F	F

Since the truth values of Q and $(\sim Q) \implies (P \wedge (\sim P))$ are equal for all possible truth values of P and Q, Q and $(\sim Q) \implies (P \wedge (\sim P))$ are logically equivalent.

Problem 2.62

(a) x and y are even only if xy is even.

(b) If xy is even, then x and y are even.

(c) Either x and y are not even, or xy is even.

(d) x and y are even and xy is not even.