MATH 109: Homework 1

Professor Rabin

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Problem 1.8

(a) $A = \{-2, -3, 2, 3\}$

(b) $2.5, 2.6, 2.7$

(c) $C = \{2, \sqrt{2}\}$

(d) $D = \{2\}$

(e) $|A| = 4, |C| = 2, |D| = 1$

Problem 1.9

$B = \{5, 7, 8, 10, 13\}$
$C = \{5, 8\}$

Problem 1.12

$$\begin{aligned}
A = \{n \in \mathbb{Z} : |n| < 2\} &= \{0, 1, -1\} \\
B = \{n \in \mathbb{Z} : n^3 = n\} &= \{0, 1, -1\} \\
C = \{n \in \mathbb{Z} : n^2 \leq n\} &= \{0, 1\} \\
D = \{n \in \mathbb{Z} : n^2 \leq 1\} &= \{0, 1, -1\} \\
E &= \{-1, 0, 1\}
\end{aligned}$$

Therefore, $A = B = D = E$.

Problem 1.22

$U = \{1, 3, 5, \ldots, 15\}$
$A = \{1, 5, 9, 13\}$
$B = \{3, 9, 15\}$

(a) $A \cup B = \{1, 5, 9, 13, 3, 15\}$

(b) $A \cap B = \{9\}$

(c) $A - B = \{1, 5, 13\}$

(d) $B - A = \{3, 15\}$

(e) $\overline{A} = \{3, 7, 11, 15\}$

(f) $A \cap \overline{B} = \{1, 5, 13\}$

Problem 1.24

$A = \{1, 2\}$
$B = \{3, 1\}$
$C = \{3, 2\}$
Here, $B \neq C$, but $B - A = \{3\} = C - A$

Problem 1.30

$A = \{x \in \mathbb{R} : |x-1| \leq 2\} = \{x \in \mathbb{R} : -1 \leq x \leq 1 \text{ or } 1 \leq x \leq 3\}$
$B = \{x \in \mathbb{R} : |x| \geq 1\} = \{x \in \mathbb{R} : x \leq -1 \text{ or } x \geq 1\}$
$C = \{x \in \mathbb{R} : |x+2| \leq 3\} = \{x \in \mathbb{R} : -5 \leq x \leq -2 \text{ or } -2 \leq x \leq 1\}$

(a) $A = [-1, 3]$
$B = (-\infty, -1] \cup [1, \infty]$
$C = [-5, 1]$

(b) $A \cup B = (-\infty, \infty)$
$A \cap B = [-1, -1] \cup [1, 3] = \{-1\} \cup [1, 3]$
$B \cap C = [-5, -1] \cup [1, 1] = [-5, -1] \cup \{1\}$
$B - C = (-\infty, -5) \cup (1, \infty)$

Problem 1.38

For $r \in \mathbb{R} : A_r = \{r^2\}, B_r = [r-1, r+1], C_r = (r, \infty)$
$S = \{1, 2, 4\}$

(a) $A_1 = \{1\}, A_2 = \{4\}, A_4 = \{16\}$

$\bigcup_{\alpha \in S} A_\alpha = \{1, 4, 16\}$

$\bigcap_{\alpha \in S} A_\alpha = \phi$

(b) $B_1 = [0, 2], B_2 = [1, 3], B_4 = [3, 5]$

$\bigcup_{\alpha \in S} B_\alpha = [0, 5]$

$\bigcap_{\alpha \in S} B_\alpha = \phi$

(c) $C_1 = (1, \infty), C_2 = (2, \infty), C_4 = (4, \infty)$

$\bigcup_{\alpha \in S} C_\alpha = (1, \infty)$

$\bigcap_{\alpha \in S} C_\alpha = (4, \infty)$

Problem 1.43

For $r \in \mathbb{R}^+, A_r = \{x \in \mathbb{R} : |x| < r\}$

Since for all $n \in \mathbb{R}$, there exists $r \in \mathbb{R}^+$ such that $r > |n| \implies n \in A_r \implies \bigcup_{r \in \mathbb{R}^+} A_r = \mathbb{R}$

Since $\lim_{r \to 0} A_r = \{0\}, \bigcap_{r \in \mathbb{R}^+} A_r = \{0\}$

Problem 1.46

(a) For $n < m, \frac{1}{n} > \frac{1}{m}$
$\implies \frac{1}{1} > \frac{1}{k}, \quad \forall k > 1$

Similarly, $\frac{-1}{1} < \frac{-1}{k}, \quad \forall k > 1$

Therefore, $\bigcup_{n=1}^{\infty} (\frac{-1}{n}, \frac{1}{n}) = (-1, 1)$

Since $\lim_{n\to\infty} \frac{1}{n} = \lim_{n\to\infty} \frac{-1}{n} = 0$, $\lim_{n\to\infty} A_n = \{0\}$

Therefore, $\bigcap_{n=1}^{\infty} (\frac{-1}{n}, \frac{1}{n}) = \{0\}$

(b) For any $n, [\frac{n-1}{n}, \frac{n+1}{n}] = [1 - \frac{1}{n}, 1 + \frac{1}{n}]$
Note that this is symmetric to (a), except it is centred at 1 instead of 0.

$\bigcup_{n=1}^{\infty} [\frac{n-1}{n}, \frac{n+1}{n}] = [0, 2]$

$\bigcap_{n=1}^{\infty} [\frac{n-1}{n}, \frac{n+1}{n}] = \{1\}$