

# ENPM673 PERCEPTION FOR AUTONOMOUS ROBOTS

## PROJECT 4

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### Implementation of Lucas Kanade Tracker

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## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Optical Flow</b>	<b>2</b>
<b>3</b>	<b>Warping the image for Affine transformation</b>	<b>2</b>
<b>4</b>	<b>Lucas Kanade Algorithm</b>	<b>3</b>
<b>5</b>	<b>Pipeline for Lucas Kanade</b>	<b>3</b>
<b>6</b>	<b>Compensating for Information Loss</b>	<b>4</b>
<b>7</b>	<b>Template Images</b>	<b>5</b>
<b>8</b>	<b>Results</b>	<b>6</b>
<b>9</b>	<b>Challenges Faced</b>	<b>8</b>
<b>10</b>	<b>References</b>	<b>8</b>

## List of Figures

1	Algorithm Flowchart[4]	4
2	Template image for Car	5
3	Template image for Baby vs. Dragon	5
4	Template image for Bolt	6
5	Output result-1 for Baby vs Dragon	6
6	Output result-1 for Baby vs Dragon	7
7	Output result-2 for Baby vs Dragon	7
8	Output result-1 for Bolt	7

# 1 Introduction

Lucas Kanade is a method to generate the optical flow of events in a frame. In this project we implement the Lucas Kanade Algorithm to track objects from the given video sequence. There are 3 video sequence for which the tracking has to be done.

To initialize the tracker we determine a template by drawing a bounding box around the object to be tracked. For each of the subsequent frames the tracker will update an affine transform that warps the current frame such that the template is completely aligned with the warped image. From this, we get the optical flow of the overall video sequence from which we keep a track on the object of interest.

The output videos can be found from the link:

<https://drive.google.com/drive/u/0/folders/1aovczAYYDBDMtwZcIV0nqoE4NFXeM0IV>

## 2 Optical Flow

Optical flow is the pattern of apparent motion of image object between two consecutive frames. This can be caused because of object or camera movement. Optical flow is 2D field vector where each vector represents the displacement between the two consecutive frames.

Also, there are some assumptions for Optical flow to work:

1. The pixel intensities of the object do not change between consecutive frames.
2. All the neighbouring pixels have similar motion.

## 3 Warping the image for Affine transformation

Here, the task is to find the transformation parameters which maximises the similarity between reference and subsequent frames of the tracked object.

Hence minimizing the function,

$$\sum_x [I(W(x; p)) - T(x, y)]^2$$

We take an initial assumption for the value of p for which the equation can be rewritten as,

$$\sum_x [I(W(x; p + \Delta p)) - T(x, y)]^2$$

Here,  $I(W(x; p))$  is the warped image for affine transformation which is given by,

$$W(x; p) = \begin{bmatrix} 1 + p_1 & p_3 & p_5 \\ p_2 & 1 + p_4 & p_6 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

and  $p_1, p_2, p_3, p_4, p_5, p_6$  represents the six parameters for affine transformation.

## 4 Lucas Kanade Algorithm

As per the assumption of Optical flow, all the neighbouring pixels have similar motion. Lucas Kanade takes in a  $3 \times 3$  patch around the point, so that, all the 9 elements of the matrix have similar motion. Now inorder to approximate the solution we use the least square criterion.

$$E(u, v) = \sum [I(x + u, y + v) - T(x, y)]^2$$

$$E(u, v) \approx \sum [I(x, y) + uI_x(x, y) + vI_y(x, y) - T(x, y)]^2$$

$$E(u, v) = \sum [uI_x(x, y) + vI_y(x, y) + D(x, y)]$$

where  $u$  and  $v$  are the location of the template in the current frame

Now taking the derivatives we get,

$$\frac{\partial E}{\partial u} = \sum [uI_x(x, y) + vI_y(x, y) + D(x, y)]I_x(x, y) = 0$$

$$\frac{\partial E}{\partial v} = \sum [uI_x(x, y) + vI_y(x, y) + D(x, y)]I_y(x, y) = 0$$

The above equation can be represented in matrix form as:

$$\begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_x D(x, y) \\ I_y D(x, y) \end{bmatrix}$$

The above matrix form contains 2 equations and 2 unknowns which can be easily solved for values of  $u$  and  $v$ .

## 5 Pipeline for Lucas Kanade

There are basic nine steps of applying the Lucas Kanade Algorithm. But there is a huge computational cost for it. Hence, we divide the approach into two parts i.e considering the pre-computed steps and iterative steps.

Pre-Computation Steps:

- Decide on a bounding box (ROI) manually.
- Based on this bounding box, find the template of the image frame.

Iterative Steps:

- Warp the Image  $I(x, y)$  with the  $W(x; p)$
- Compute the error in the image  $[T(x) - I(W(x; p))]$

- Compute the gradient  $\nabla I$
- Find out the jacobian for the warped image:  $\frac{\partial W}{\partial p}$
- Compute the steepest descent  $\nabla I \frac{\partial W}{\partial p}$
- Compute Hessian Matrix:  $H = \sum_x \left[ \nabla I \frac{\partial w}{\partial p} \right]^T \left[ \nabla I \frac{\partial w}{\partial p} \right]$
- Get the change in parameters  $\Delta p = H^{-1} \sum_x \left[ I \frac{\partial w}{\partial p} \right]^T [T(x) - I(W(x; p))]$
- Update the parameters i.e  $p = p + \Delta p$

These above steps are repeated until our algorithm converges. Below is the schematic for the algorithm.

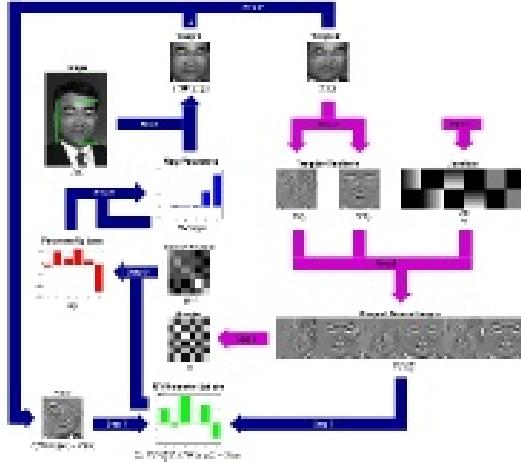


Figure 1: Algorithm Flowchart[4]

## 6 Compensating for Information Loss

As we know, Lucas Kanade works by comparing the features from the template frame with subsequent frames. The least square equation that we try to minimize gives equal amount of weights to all the n-pixels. But in practice, it is better to give more weight to the pixels that are closer to the central pixel. Hence, to implement this we use the weighted version of the least square equation.

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum_i w_i I_x^2 & \sum_i w_i I_x I_y \\ \sum_i w_i I_x I_y & \sum_i w_i I_y^2 \end{bmatrix}^{-1} \begin{bmatrix} -\sum_i w_i I_x D(x, y) \\ -\sum_i w_i I_y D(x, y) \end{bmatrix}$$

where  $w_i$  is the gaussian weight distribution which then forms the diagonal Weight matrix  $W$ .

## 7 Template Images

We need a reference frame to compare the features from. Here, we choose the first frame from every video to be our reference frame which are shown below.



Figure 2: Template image for Car



Figure 3: Template image for Baby vs. Dragon

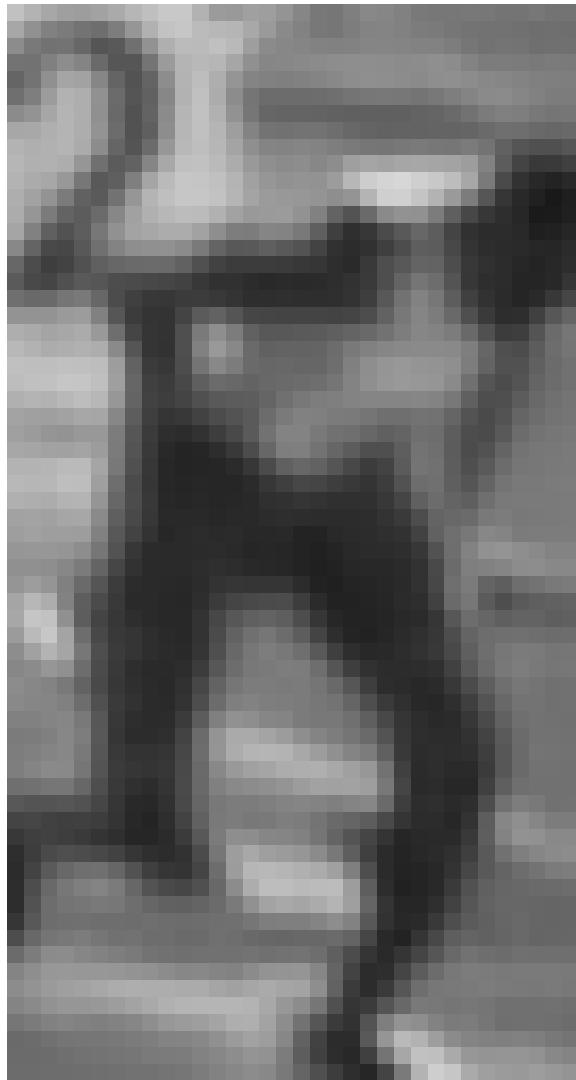


Figure 4: Template image for Bolt

## 8 Results



Figure 5: Output result-1 for Baby vs Dragon



Figure 6: Output result-1 for Baby vs Dragon

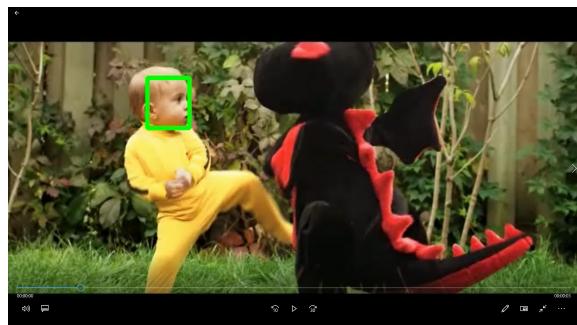


Figure 7: Output result-2 for Baby vs Dragon



Figure 8: Output result-1 for Bolt

## 9 Challenges Faced

- The algorithm breaks down when there is a change in the illumination. To avoid this, histogram equalization was used.
- The algorithm was tested for various threshold values.
- After applying the basic Lucas Kanade method, information loss was observed which was then rectified using the weighted least square equation.
- The algorithm isn't the most efficient when it comes tracking objects or features which are in continuous fast motion in the optical flowline.

## 10 References

- <https://www.youtube.com/watch?v=tzO245uWQxA>
- [https://en.wikipedia.org/wiki/Lucas-Kanade\\_method](https://en.wikipedia.org/wiki/Lucas-Kanade_method)
- <http://www.cse.psu.edu/~rtc12/CSE486/lecture30.pdf>
- Lucas-Kanade 20 Years On: A Unifying Framework: Part 1 by Simon Baker and Ian Matthews