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EN2570 - DIGITAL SIGNAL PROCESSING

FIR Filter Design Project - Bandstop Filter

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1 ABSTRACT

This report outlines the design process of a Non - Recursive (Finite Impulse Response) Bandstop Filter using the Kaiser Window Function. MATLAB 2014a software package was used as the programming environment for the design project. In the following sections the basic theoractical construct underlying te design process, results obtained and the conclusion would be discussed.
Keywords - Non Recursive (FIR) Bandstop Filter, Kaiser Window Function, MATLAB 2014a software package

2 INTRODUCTION

There are two classical methods for the design of nonrecursive filters.

- **Fourier series method (Window Method)**
  Here, the Fourier series is used in conjunction with a class of functions known as window functions.

- **weighted-Chebyshev method**
  This is a a multivariable optimization method

In this report the Fourier series method is used in conjuction with the window known as the Kaiser Window.

3 Basic Theory

A fundamental property of digital filters in general is that they have a periodic frequency response with period equal to the sampling frequency ω_s

$$H(e^{j(\omega+k\omega_s)T}) = H(e^{j(\omega})$$

Since the frequency response is periodic we can represent it as a Fourier Series as below.

$$H(e^{j\omega T}) = \sum_{n=-\infty}^{n=\infty} h(nT)e^{-j\omega nT}$$

where,

$$h(nT) = \frac{1}{\omega_s}\int_{-\omega_s/2}^{\omega_s/2} H(e^{j\omega T})e^{j\omega nT}d\omega$$

Define the impulse response of the Filter as $h(nT)$ and then by By substituting $z = e^{j\omega T}$ we obtain the Transfer function of the filter as follows.

$$H(z) = \sum_{n=-\infty}^{n=\infty} h(nT)z^{-n}$$

Since the Fourier series coefficients are defined over the range $-\infty < n < \infty$, there are two problems regarding the Fourier series method.

- The nonrecursive filter obtained is of infinite length.
- The filter is noncausal because the impulse response is nonzero for negative time.

A finite filter length can be achieved by truncating the impulse response in such way that, $h(nT) = 0$ for $|n| > M$ where $M = (N-1)/2$. And a causal filter can be obtained by delaying the impulse response by a period MT seconds or by M sampling periods.
Since delaying the impulse response by M sampling periods amounts to multiplying the transfer function by z^{-M}, the transfer function of the causal filter has the following form.

$$H'(z) = z^{-M}\sum_{n=-\infty}^{n=\infty} h(nT)z^{-n}$$

And we obtain the frequeny response as,

$$H(e^{j\omega T}) = e^{-j\omega MT}\sum_{n=-\infty}^{n=\infty} h(nT)e^{-j\omega nT}$$

The amplitude response of the filter does not change with the sample shift as $|e^{-j\omega MT}| = 1$. However, the phase response will change from that of the original.

However, we can observe that the amplitude response of the filter would have undesirable oscillations (Gibbs ocsillations) in both the stop band and the pass band. They are caused by the truncation of the Fourier series. And unfortunately we cannot reduce this phenomenon by increasing the truncation length.

As a solution this problem we use a window function ($w(nT)$) to truncate the infinite duration filter response $h(nT)$. This is done as follows.

$$h_w(nT) = h(nT)w(nT)$$

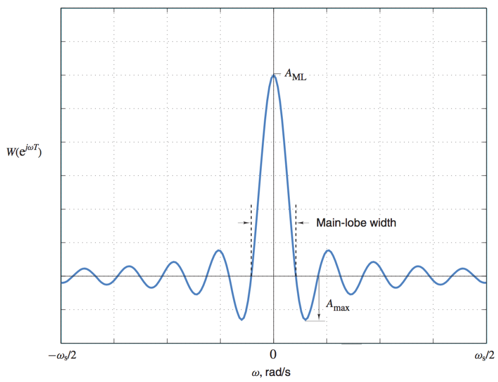
Then the Transfer Fucntion of the modified filter is given by,

$$H_w(z) = Z[h(nT)w(nT)]$$

Hence, the frequency response of the modified filter is given by the convolution integral.

$$H_w(e^{j\omega T}) = \frac{T}{2\pi}\int_0^{2\pi/T} H(e^{j\omega' T})W(e^{j(\omega-\omega')T})d\omega'$$

Figure 3.1: Frequency Spectrum of a usual window



Windows are characterized by their **main-lobe width**, B_{ML}, which is the bandwidth between the first negative and the first positive zero crossings, and by their **ripple ratio**, r, which is defined as,

$$R = 20log\frac{A_{max}}{A_{ML}}$$

where A_{max} and A_{ML} are the maximum side-lobe and main-lobe amplitudes, respectively. The main-lobe width and ripple ratio should be as low as possible. The spectral energy of the window should be concentrated as far as possible in the main lobe and the energy in the side lobes should be as low as possible.

These are some of the window functions that are used to design filters.

- Rectangular
- von Hann
- Hamming
- Blackman
- Dolph-Chebyshev
- Kaiser
- Ultraspherical window

In this design project the Kaiser window is used as the window function to arrive at a band stop filter according to the given specifications.

4 Kaiser Window

The Kaiser window function is given by,

$$w_K(nT) = \begin{cases} \frac{I_o(\beta)}{I_o(\alpha)}, & \text{for } |n| < (N-1)/2 \\ 0, & \text{otherwise} \end{cases} \tag{4.1}$$

here,

$$\beta = \alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2}$$

and

$$I_o(x) = 1 + \sum_{k=1}^{\infty}\left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2$$

α is an independent parameter, and $I_o(x)$ is a zeroth-order modified Bessel function of the first kind.

Figure 4.1: Ripple Ration vs α

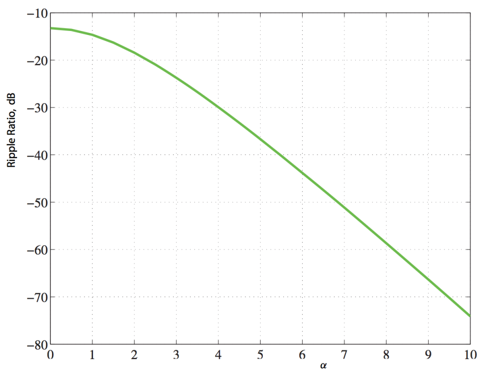
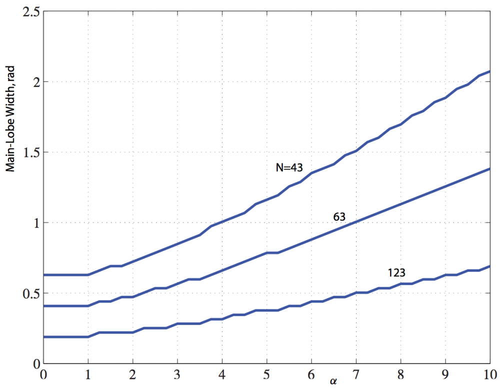


Figure 4.2: Main Lobe Width vs. α



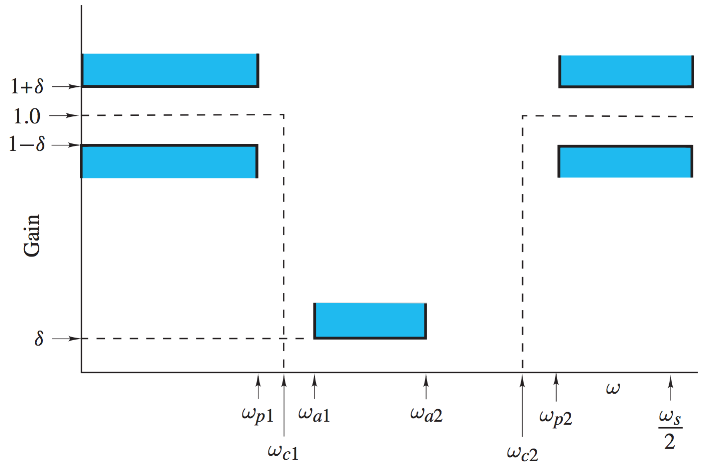
A filter-design method is available, also proposed by Kaiser, that can be used to design nonrecursive filters that would satisfy arbitrary prescribed specifications. This method can be used to design lowpass (LP), highpass (HP), bandpass (BP), and bandstop (BS) filters.

5 Design Procedure of a Non Recursive Band Stop Filter

Let us assume that the following are the design specifications of the desired bandpass filter.

- Passband Ripple $\leqslant \tilde{A}_p$
- Minimum Stop Band Attenuation $\geqslant \tilde{A}_a$
- Lower passband edge ω_{p1}
- Lower stopband edge ω_{a1}
- Upper stopband edge ω_{a2}
- Upper passband edge ω_{p2}
- Sampling frequency ω_s

Figure 5.1: Design Specifications Visualized



From the above specifications, the more critical transition width is,

$$B_t = min[(\omega_{a1} - \omega_{p1}), (\omega_{p2} - \omega_{a2})]$$

Hence, idealized frequency response for a Bandstop filter is given as,

$$H(e^{j\omega T}) = \begin{cases} 1, & \text{for } 0 \leqslant |\omega| \leqslant \omega_{c1} \\ 0, & \text{for } \omega_{c1} \leqslant |\omega| \leqslant \omega_{c2} \\ 1, & \text{for } \omega_{c1} \leqslant |\omega| \leqslant \omega_s/2 \end{cases} \tag{5.1}$$

where $\omega_{c1} = \omega_{p1} + B_t/2$ and $\omega_{c2} = \omega_{p2} + B_t/2$.

Using the Fourier series, the impulse response of the ideal bandstop filter can be obtained as,

$$h(nT) = \begin{cases} 1 + \frac{2(\omega_{c1} - \omega_{c2})}{\omega_s}, & \text{for } n = 0 \\ \frac{1}{n\pi}(sin(\omega_{c1} nT) - sin(\omega_{c2} nT), & \text{otherwise} \end{cases} \tag{5.2}$$

Now we can start the design procedure.

1. Choose a suitable values for δ using the below eqautions.

$$\tilde{\delta_p} = \frac{10^{0.05\tilde{A}_p} - 1}{10^{0.05\tilde{A}_p} + 1}$$

$$\tilde{\delta_a} = 10^{-0.05\tilde{A}_a}$$

$$\delta = min(\tilde{\delta_p}, \tilde{\delta_a})$$

2. Calculate the actual stop band attenuation A_a by

$$A_a = -20log(\delta)$$

3. Choose parameter α as,

$$\alpha = \begin{cases} 0, & \text{for } A_a \leqslant 21 \\ 0.5842(A_a - 21)^0.4 + 0.07886(A_a - 21), & \text{for } 21 < A_a \leqslant 50 \\ 0.1102(A_a - 8.7), & \text{for } A_a > 50 \end{cases} \tag{5.3}$$

4. Choose parameter D as,

$$D = \begin{cases} 0.9222, & \text{for } A_a \leqslant 21 \\ \frac{Aa-7.95}{14.26}, & \text{for } A_a > 21 \end{cases} \tag{5.4}$$

5. Then select the lowest odd value of N that would satisfy the inequality,

$$N \geqslant \frac{\omega_s D}{B_t} + 1 \text{ where } B_t = min[(\omega_{a1} - \omega_{p1}), (\omega_{p2} - \omega_{a2})]$$

6. Then using the definition of the Kaiser Window, find $w_K(nT)$:

$$w_K(nT) = \begin{cases} \frac{I_o(\beta)}{I_o(\alpha)}, & \text{for } |n| < (N-1)/2 \\ 0, & \text{otherwise} \end{cases} \tag{5.5}$$

here,

$$\beta = \alpha\sqrt{1 - \left(\frac{2n}{N-1}\right)^2}$$

and

$$I_o(x) = 1 + \sum_{k=1}^{\infty} \left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2$$

7. Finally form the modified transfer function of the filter.

$$H'_w(z) = z^{-(N-1)/2} H_w(z) \text{ where } H_w(z) = Z[h(nT)w_K(nT)]$$

6 RESULTS

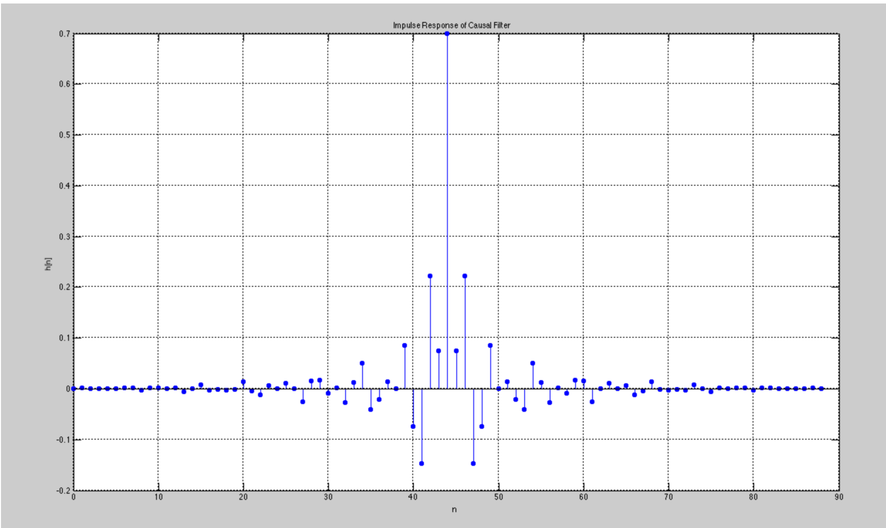
MATLAB 2014a Software package was used to design, simulate and validate the digital filter. The aforementioned procedure was used for the design process.

The following design specifications were used.

Parameter	Value
Passband Ripple $\tilde{A}_p$	0.06 dB
Minimum Stop Band Attenuation $\tilde{A}_a$	40 dB
Lower passband edge ω_{p1}	600 rad/s
Lower stopband edge ω_{a1}	700 rad/s
Upper stopband edge ω_{a2}	1000 rad/s
Upper passband edge ω_{p2}	1150 rad/s
Sampling frequency ω_s	3000 rad/s

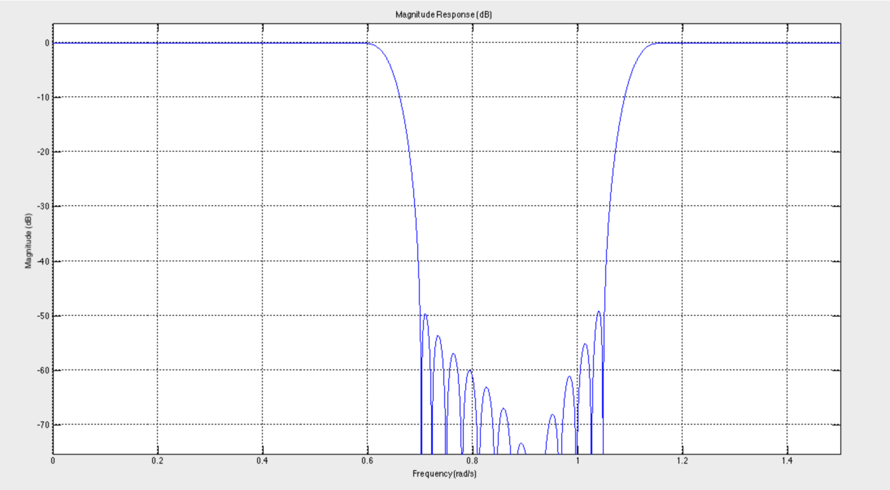
6.1 THE CAUSAL IMPULSE RESPONSE OF THE DESIGNED BANDSTOP FILTER

Figure 6.1: Causal Impulse Response



6.2 THE MAGNITUDE RESPONSE OF THE DESIGNED BANDSTOP FILTER

Figure 6.2: Magnitude Response



6.3 THE MAGNITUDE RESPONSE OF UPPER AND LOWER PASSBANDS

Figure 6.3: Lower Passband

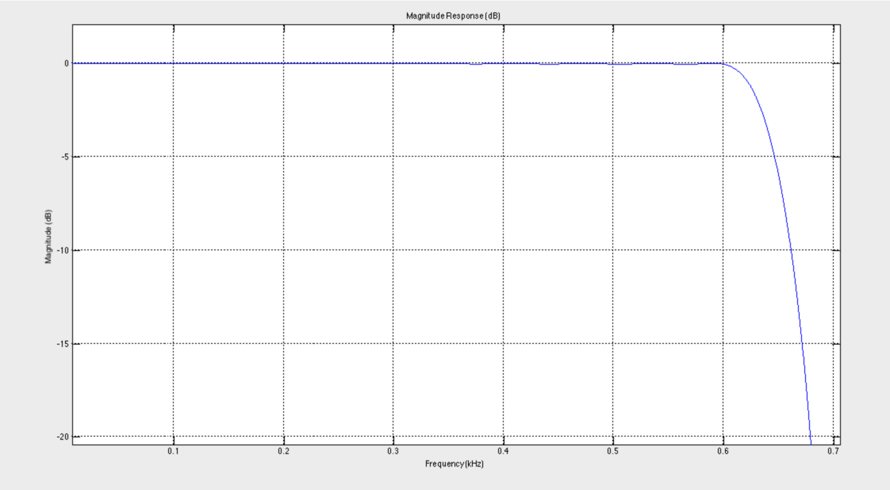
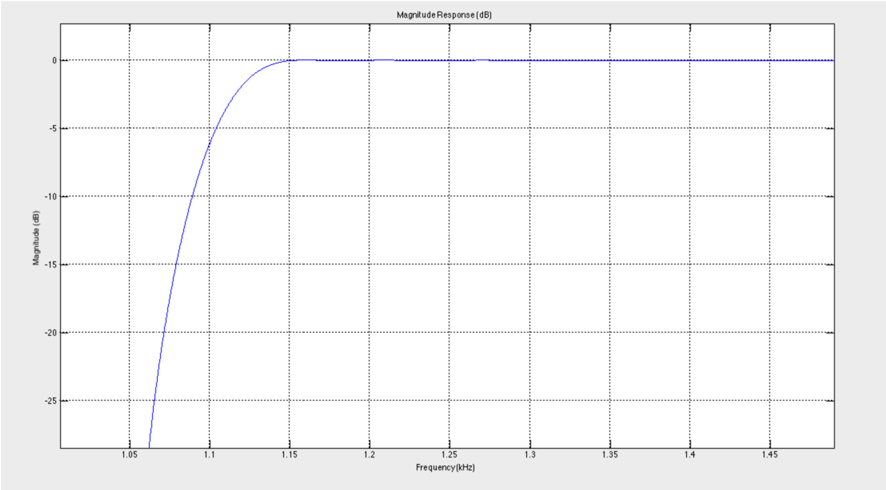


Figure 6.4: Upper Passband



7 VALIDATION OF THE DESIGNED BAND STOP FILTER

For the validation purposes, the following time domain excitation was used.

$$x(nT) = \sum_{i=1}^{3} sin(\Omega_i nT)$$

Since the cutoff frequencies of the designed bandstop filter was $\Omega_{c1} = 650 rads^{-1}$ and $\Omega_{c2} = 1100 rads^{-1}$,

$$\Omega_1 = 325 rads^{-1}$$

,

$$\Omega_2 = 850 rads^{-1}$$

,

$$\Omega_3 = 1275 rads^{-1}$$

was chosen as the frequencies of the excitation signal.

As it was observed, the filtered signall using the bandstop filter and the ideally filtered signal were almost the same.

Figure 7.1: Excitation in time domain

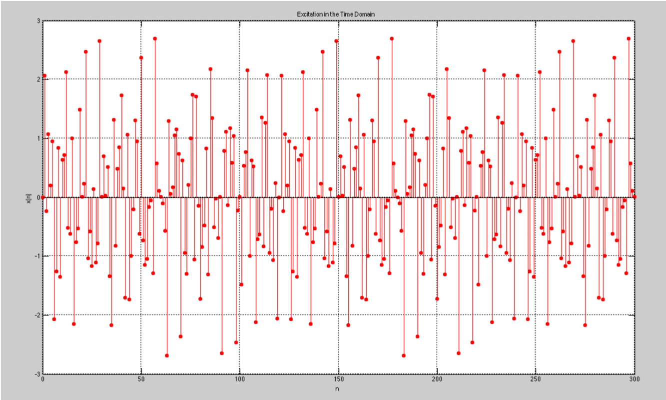


Figure 7.2: Filtered signal in the time domain

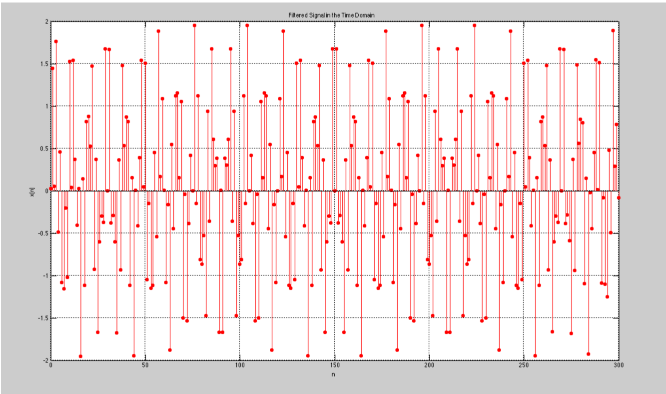
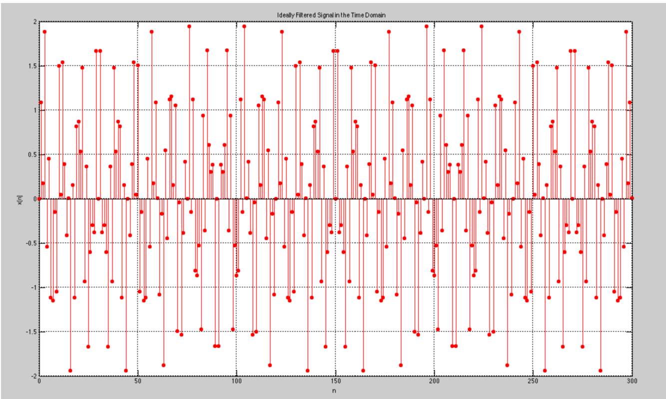


Figure 7.3: Ideally Filtered Signal in the Time Domain



As it can be observed, the filtered signall using the bandstop filter and the ideally filtered signal were almost the same.

When the frequency domain was considered, the following results were obtained.

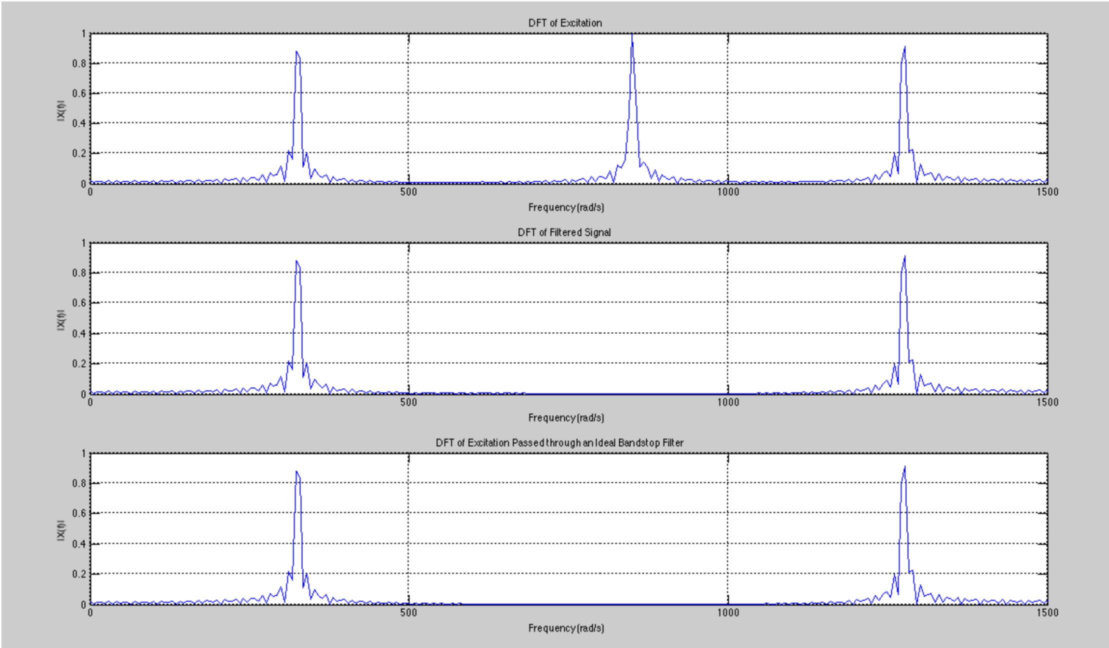


Figure 7.4: Frequency Domain Representation of the excitation, filtered excitation and the ideally filtered excitation

As the final step of the validation, the same filter was designed using the designfilt function of the MATLAB 2014a software package. The following magnitude response of the filter was obatained.

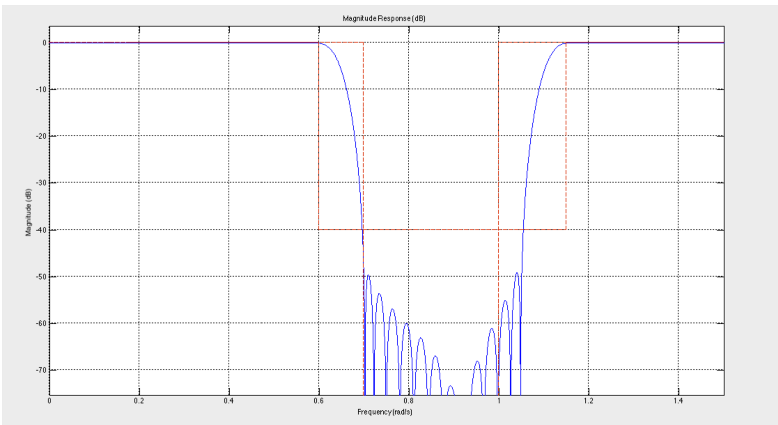


Figure 7.5: The magnitude response of the filter designed using the designfilt fucntion

8 Conclusion

The Kaiser Window Function based Bandstop filter for the above specifications was successfully implemented adhering to the aforementioned procedure.

9 Acknowledgements

I would like to extend my sincere gratitude towards Dr.Chamira Edussooriya for providing valuable support during this project.

10 References

- Digital Signal Processing: Signals, Systems, and Filters 1st Edition by Andreas Antoniou
- `https://en.wikipedia.org/wiki/Kaiser_window`
- `https://en.wikipedia.org/wiki/Kaiser_window`

11 APPENDIX

The following code was used to design, simulate and validate the band stop filter.

```
%EN2570 Digital Signal Processing - Project

clear all;
close all;

%K.G.G.L.A. de Silva
%150103P
%Last Updated : 2017 Nov 10

% Get A,B,C in 150ABC (index)

indexNo = str2double(inputdlg('Enter Index Number', 'Index Number',1));
C = mod(indexNo,10);
B = mod(floor(indexNo/10),10);
A = mod(floor(indexNo/100),10);

%A = 1; B = 0; C = 3;

%Calculate the paramteres

Ap = 0.05 + (0.01 * A);
Aa = 40 + B;
Omega_p1 = C*100 + 300;
Omega_p2 = C*100 + 850;
Omega_a1 = C*100 + 400;
Omega_a2 = C*100 + 700;
Omega_s  = 2*(C*100 + 1200);



%Compute the transition width, cut off freqeuncies and sampling period

Bt = min((Omega_a1 - Omega_p1),(Omega_p2 - Omega_a2));
Omega_c1 = Omega_p1 + Bt/2;
Omega_c2 = Omega_p2 - Bt/2;
T = 2*pi / Omega_s;

%Compute delta value

```

```
delta_p = (10^(0.05*Ap) - 1)/(10^(0.05*Ap) + 1); delta_a = 10^(-0.05*Aa);
delta = min(delta_p,delta_a);

%Compute actual stop band attenuation

Aaa = -20*log10(delta);

%Choose Parameter alpha

if Aaa <= 21
    alpha = 0;
elseif (21 < Aaa) && (Aaa <= 50)
    alpha = 0.5842*(Aaa-21)^0.4 + 0.07886*(Aaa-21);
else
    alpha = 0.1102*(Aaa-8.7);
end

%Choose parameter D

if Aaa <= 21
    D = 0.9222;
else
    D = (Aaa - 7.95)/14.36;
end

%Select N

if mod(ceil(Omega_s*D/Bt + 1),2) == 1
    N = ceil(Omega_s*D/Bt + 1);
else
    N = ceil(Omega_s*D/Bt + 1) + 1;
end

%Compute and plot the Window function

range2 = (N-1)/2;
n = -range2 : 1 : range2;
beta = alpha*(1 - (2*n/(N-1)).^2).^0.5;
I_beta = 0; I_alpha = 0;
for k = 1 : 100
    I_beta = I_beta + ((1/factorial(k))*(beta/2).^k).^2;
    I_alpha = I_alpha + ((1/factorial(k))*(alpha/2)^k)^2;
end
```

```
I_beta = I_beta + ones(1,numel(I_beta));
I_alpha = I_alpha + ones(1,numel(I_alpha));

w = I_beta./I_alpha;

figure;stem(n,w,'fill');
xlabel('n');ylabel('w[n]');title('Windowing Function');
grid on;

%Compute and plot h[n]

range1 = (N-1)/2;
n1 = -range1 : 1 : -1;
h1 = ((1/pi)./n1).*(sin(Omega_c1*T.*n1) - sin(Omega_c2*T.*n1));
h0 = 1 + 2*(Omega_c1 - Omega_c2)/Omega_s;
n2 = 1 : 1 : range1;
h2 = ((1/pi)./n2).*(sin(Omega_c1*T.*n2) - sin(Omega_c2*T.*n2));
h = [h1,h0,h2];
n = [n1,0,n2];
figure;stem(n,h,'fill');grid on;
xlabel('n');ylabel('h[n]');title('Impulse Response of Ideal Bandstop
    Filter');

%Compute the filter respone

h_filter = h.*w;

%subplot(1,2,1);
stem(n,h_filter);
xlabel('n');ylabel('h[n]');title('Impulse Response of Non Causal Filter'
    );grid on;
n_shifted = [0:1:N-1];
%subplot(1,2,2);
stem(n_shifted,h_filter,'fill');
xlabel('n');ylabel('h[n]');title('Impulse Response of Causal Filter');
    grid on;

%Magnitude Response of the filter

fvtool(h_filter);
freqz(h_filter);

%generate the excitation
```

```
n = 0 : 1 : 300;
L = numel(n);
x = sin(325*T*n) + sin(850*T*n) + sin(1275*T*n);

%Plot the DFT of the excitation

NFFT = 2^nextpow2(L); % Next power of 2 from length of y
Y = fft(x,NFFT)/L;
f = (Omega_s)/2*linspace(0,1,NFFT/2+1);
figure;
subplot(3,1,1);
plot(f,2*abs(Y(1:NFFT/2+1)));
title('DFT of Excitation')
xlabel('Frequency (rad/s)')
ylabel('|X(f)|');grid on;

%Plot the DFT of filtered signal

x_f = conv(x,h_filter,'same');
L = numel(x_f);
NFFT = 2^nextpow2(L); % Next power of 2 from length of y
Y = fft(x_f,NFFT)/L;
f = (Omega_s)/2*linspace(0,1,NFFT/2+1);
subplot(3,1,2);
plot(f,2*abs(Y(1:NFFT/2+1)));
title('DFT of Filtered Signal')
xlabel('Frequency (rad/s)')
ylabel('|X(f)|');grid on;


%Plot the DFT of the excitation passed through an ideal bandstop filter

x_i = sin(325*T*n) + sin(1275*T*n);
L = numel(n);
NFFT = 2^nextpow2(L); % Next power of 2 from length of y
Y = fft(x_i,NFFT)/L;
f = (Omega_s)/2*linspace(0,1,NFFT/2+1);
subplot(3,1,3);
plot(f,2*abs(Y(1:NFFT/2+1)));
title('DFT of Excitation Passed through an Ideal Bandstop Filter')
xlabel('Frequency (rad/s)')
ylabel('|X(f)|');grid on;

```

```matlab
%Plot the excitation in time domain

figure;
stem(n,x,'r','fill');
title('Excitation in the Time Domain')
xlabel('n')
ylabel('x[n]');grid on;

%Plot the Filtered signal in the time domain

nl = [0:1:numel(x_f)-1];

figure;
stem(nl,x_f,'r','fill');
title('Filtered Signal in the Time Domain')
xlabel('n')
ylabel('x[n]');grid on;

%Plot the Ideally Filtered signal in the time domain

figure;
stem(n,x_i,'r','fill');
title('Ideally Filtered Signal in the Time Domain')
xlabel('n')
ylabel('x[n]');grid on;

%Validation using the designfilt function

HpFilt = designfilt('bandstopfir','PassbandFrequency1',600, ...
               'StopbandFrequency1',700,'StopbandFrequency2',1000,'PassbandFrequency2',1150,'PassbandRipple1',0.06, ...
               'StopbandAttenuation',40,'PassbandRipple2',0.06,'DesignMethod','kaiserwin','SampleRate',3000);
        fvtool(HpFilt);
```