Wake Fields of Ring-Like Electron Clouds

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Abstract

Electron cloud interacting with passing relativistic bunches induces electric fields. These fields act back on the bunch leading to instabilities or energy loss. The distribution of the cloud plays a significant role in these phenomena. Recently it was shown that the cloud forms a thin sheath near the pipe wall. It was found that it modifies the stopping powers and affects the results of the Microwave Transmission Measurements. In this note the fields for the ring-like electron cloud are investigated analytically and in simulations.

1. Introduction

In Ref. [1] it was shown that the cloud forms a sheath near the pipe wall. Such electron cloud structure modifies the electron cloud wake fields and corresponding stopping powers. In this note a simplified case of an infinitely thin electron cloud in cylindrical geometry is analyzed analytically and in simulations.

2. Analytical Wake Field of a Ring-Like Electron Cloud

In Ref. [1] a kick approximation was used to obtain the stopping powers analytically. In this approximation the electron positions are assumed to be fixed during the interaction with the beam. In Ref. [2] it was shown that one can obtain the longitudinal wake fields in the kick approximation. Such wake fields agree with simulations only for very small bunch intensities i.e. when electrons approximately do not change their position during the interaction. The kick approximation can be slightly modified to yield a much better agreement with simulation results. For this study we neglect the self space charge forces of the cloud.

Let $\lambda_i(z)$ be a longitudinal line density of a beam. In the kick-approximation the electron will gain the following velocity:

$$v = \frac{1}{c}\int_{-\infty}^{z} \frac{e^2\lambda_i(z)}{2\pi\epsilon_0 R_0 m_e} dz, \tag{1}$$

where e is the electron charge, c is the velocity of light, ϵ_0 is the permittivity, m_e is the electron mass, R_0 is the initial electron position. We now assume additionally than an electron is actually moving with this velocity. Its distance from the pipe center is the given as follows

$$R(z) = R_0 - \frac{1}{c}\int_{-\infty}^{z} v dz. \tag{2}$$

The next step is to study the electric potential associated with the ring-like electron cloud:

$$\phi(z) = \frac{e\lambda_e}{2\pi\epsilon_0} \ln\left(\frac{R(z)}{R_p}\right), \tag{3}$$

where R_p is the pipe radius, λ_e is the cloud line density. The potential inside the ring like electron cloud is constant. Outside the cloud it changes logarithmically. If the density change along the pipe is slow compared to the transverse plane, then one can calculate a longitudinal field inside the ring-like cloud:

$$E_z(z) = -\frac{\partial\phi(z)}{\partial z} = -\frac{e\lambda_e}{2\pi\epsilon_0}\frac{R'(z)}{R(z)}, \tag{4}$$

This is the longitudinal wake field generated by the cloud in the extended kick approximation.

3. Analytical Wake Fields for Some Longitudinal Beam Profiles

3.1. Rectangular Bunch

One can obtain a wake field for a small rectangular bunch as a piecewise function. We assume that the bunch line density starts at $z = 0$ and ends at $z = d$ The velocity change during the bunch passage is

$$v(z) = \frac{1}{c}\frac{e^2\lambda_i z}{2\pi\epsilon_0 R_0 m_e}. \tag{5}$$

Distance to the pipe center during the bunch passage is

$$R = R_0 - \frac{1}{c^2}\frac{e^2\lambda_i z^2}{4\pi\epsilon_0 R_0 m_e} \tag{6}$$

After the bunch passage the distance is

$$R = R_0 - \frac{1}{c^2}\frac{e^2\lambda_i d^2}{4\pi\epsilon_0 R_0 m_e} - \frac{1}{c^2}\frac{e^2\lambda_i d\cdot(z-d)}{2\pi\epsilon_0 R_0 m_e}. \tag{7}$$

Now we can calculate the field. Before the bunch passage it is given as

$$E_z(z) = 0. \tag{8}$$

During the bunch passage the field is

$$E_z(z) = \frac{e\lambda_e}{\pi\epsilon_0}\frac{z}{\frac{2m_eR_0c^2}{eE_i^r(R_0)} - z^2}, \tag{9}$$

where $E_i^r(R_0)$ stands for the bunch electric field at distance R_0. Finally, after the bunch passage the field is

$$E_z(z) = \frac{e\lambda_e}{\pi\epsilon_0}\frac{z}{\frac{2m_eR_0c^2}{eE_i^r(R_0)} + d^2 - 2dz}. \tag{10}$$

3.2. Gaussian Bunch

We assume that the bunch line density has a maximum at $z = 0$, the bunch intensity is N_i and the r.m.s. length is σ_z. The velocity change during the bunch passage is then

$$v(z) = \frac{1}{c}\frac{e^2N_i}{4\pi\epsilon_0R_0m_e}(1 + erf\left[\frac{z}{\sqrt{2}\sigma_z}\right]), \tag{11}$$

where $erf(...)$ is the Gauss error function. The corresponding position change is

$$R(z) = R_0 - \frac{1}{c^2}\frac{e^2N_i}{4\pi\epsilon_0R_0m_e}\int_{-\infty}^{z}\left(1 + erf\left[\frac{s}{\sqrt{2}\sigma_z}\right]\right)ds \tag{12}$$

Using Eq. 12 one can directly obtain the corresponding wake field.

$$E_z(z) = \frac{e\lambda_e}{2\pi\epsilon_0}\frac{\left(1 + erf\left[\frac{z}{\sqrt{2}\sigma_z}\right]\right)}{\frac{m_eR_0c^2}{\sqrt{2}\sigma_z eE_{i,max}^r(R_0)} - \int_{-\infty}^{z}\left(1 + erf\left[\frac{s}{\sqrt{2}\sigma_z}\right]\right)ds}, \tag{13}$$

where $E_{i,max}^r(R_0)$ stands for the maximum bunch electric field at distance R_0.

4. Simulations

For simulations the infinitely thin ring-like electron distribution was formed near the wall. Parameters of the pipe and the cloud were taken from Fig. 9 in Ref. [1] and are listed in Table. 1. During the simulation, the interaction between the bunch and the beam is only transverse. The fields are calculated using the FFT Poisson solver. Longitudinal fields are obtained by means of the post-processing similar to Ref. [1].

Table 1: Main simulation parameters

Parameter	Value
N_i	10^{11}
Bunch length, σ_z / m	0.1
Bunch velocity, v / [m/s]	$2.99 \cdot 10^8$
Bunch radius / mm	2
Average cloud density / m^{-3}	10^{13}

Fig. 1 shows the pinch of the ring-like electron cloud. The corresponding longitudinal wake field is shown in Fig. 2. Analytical results are depicted as well.

One can see a perfect agreement between the wake fields for the studied case. Simulation results indicate a rapid drop of the electric field after the bunch passage. This is due to the loss of the electrons on the wall. In the analytical theory one just needs to assume zero electric field when the cloud size reaches the beam pipe size.

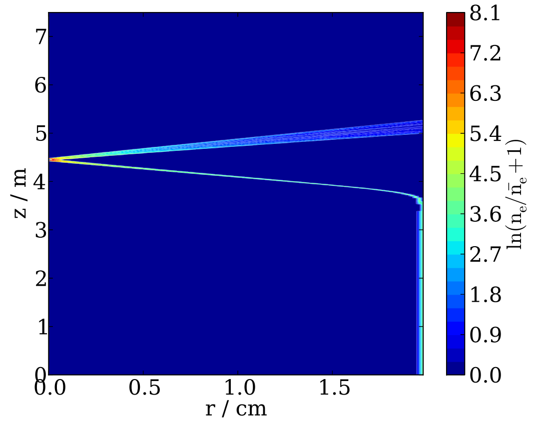

Figure 1: (Color) Pinch of the ring-like electron cloud under the influence of the passing proton bunch.

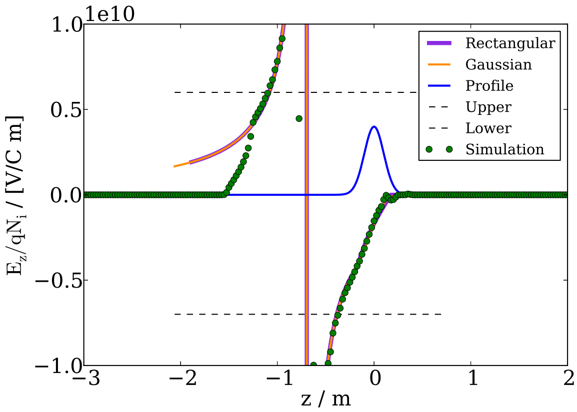

Figure 2: (Color) Comparison of the analytical and simulated wake fields. For the analytical results two profiles: Gaussian and rectangular were assumed. Simulations were performed with the Gaussian bunch. Dashed lines show the maximum and minimum fields from Fig.9 in Ref. [1].

However, if one reduces the initial electron cloud radius, the discrepancy arises. Fig. 3 shows the results of these simulations. The initial radius of the cloud is $R_0 = 3/4R_p$. This is approximately the distance where the electron density in Fig.9 from Ref. [1] disappears. One needs a higher order approximation in this case.

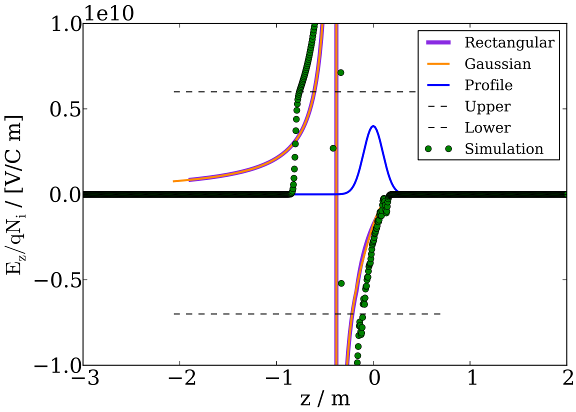

Figure 3: (Color) Comparison of the analytical and simulated wake fields for $R_0 = 3/4R_p$.

5. Possible Next Steps

One can see that the analytical theory gives a singularity for the infinitely thin cloud (simulation is close to this). This singularity can be removed and a more realistic wake field can be obtained if one integrates Eq. 4 with a suitable electron cloud sheath distribution.

References

[1] O. Boine-Frankenheim, Phys. Rev. ST Accel. Beams 15, 054402 (2012)

[2] F.Petrov, PhD Thesis, 2013