Assignment 6

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11/21/2021

1. Consider the following activity-on-arc project network, where the 12 arcs (arrows) represent the 12 activities (tasks) that must be performed to complete the project and the network displays the order in which the activities need to be performed. The number next to each arc (arrow) is the time required for the corresponding activity. Consider the problem of finding the longest path (the largest total time) through this network from start (node 1) to finish (node 9), since the longest path is the critical path. Formulate

```
###Loading the packages and creating the object
library(lpSolveAPI)
a6 <- make.lp(nrow = 9, ncol = 12)
```

```
##Naming the arcs and nodes
A.names <- c("X12", "X13", "X24", "X25", "X35", "X46", "X47", "X57", "X58", "X69", "X79", "X89")
N.names <- c("N1", "N2", "N3", "N4", "N5", "N6", "N7", "N8", "N9")
```

```
rownames(a6) <- N.names
colnames(a6) <- A.names
```

```
##Setting the objective function and using -1 to get the min value
of <- c(5, 3, 4, 2, 3, 1, 4, 6, 2, 5, 4, 7)
set.objfn(a6, -1*of)
```

```
##Setting the constraints
set.row(a6, 1, c(1, 1), indices = c(1, 2))
set.row(a6, 2, c(1, -1, -1), indices = c(1, 3, 4))
set.row(a6, 3, c(1, -1), indices = c(2, 5))
set.row(a6, 4, c(1, -1, -1), indices = c(3, 6, 7))
set.row(a6, 5, c(1, 1, -1, -1), indices = c(4, 5, 8, 9))
set.row(a6, 6, c(1, -1), indices = c(6, 10))
set.row(a6, 7, c(1, 1, -1), indices = c(7, 8, 11))
set.row(a6, 8, c(1, -1), indices = c(9, 12))
set.row(a6, 9, c(1, 1, 1), indices = c(10, 11, 12))
set.constr.type(a6, rep("="), 9)
rhs <- c(1, rep(0, 7), 1)
set.rhs(a6, rhs)
set.type(a6, 1:12, "binary")
```

```
solve(a6)
```

```
## [1] 0
```

```
get.objective(a6)
```

```
## [1] -17
```

```
get.variables(a6)
```

```
##  [1] 1 0 0 1 0 0 0 1 0 0 1 0
```

As what we can see, the longest path is 17 (largest total time) which is from Node1>-Node2>-Node5>-Node7>-Node9

2. Selecting an Investment Portfolio An investment manager wants to determine an opti- mal portfolio for a wealthy client. The fund has $2.5 million to invest, and its objective is to maximize total dollar return from both growth and dividends over the course of the coming year. The client has researched eight high-tech companies and wants the portfolio to consist of shares in these firms only. Three of the firms (S1 – S3) are primarily software companies, three (H1–H3) are primarily hardware companies, and two (C1–C2) are internet consulting companies. The client has stipulated that no more than 40 percent of the investment be allocated to any one of these three sectors. To assure diversification, at least $100,000 must be invested in each of the eight stocks. Moreover, the number of shares invested in any stock must be a multiple of 1000. The table below gives estimates from the investment company's database relating to these stocks. These estimates include the price per share, the projected annual growth rate in the share price, and the anticipated annual dividend payment per share.

First lets prepare the decision variables, objective function and constraints:

Decision Variables:

Xi= the number of 1000's shares for each company X1=S1, X2=S2, X3=S3 X4=H1, X5=H2, X6=H3 X7=C1, X8=C2

Objective Function: To maximize total dollar return from both growth and dividends over the course of the coming year. Max Z= Σ Xi* [(price per share * growth rate) + dividend] Max Z= 4Xi + 6.5X2 + 5.9X3 + 5.4X4 + 5.15X5 + 10X6 + 8.4X7 + 6.25X8

Constraints:

Total Investment: 40Xi + 50X2 + 80X3 + 60X4 + 45X5 + 60X6 + 30X7 + 25X8 <= 2,500

Diversification: 40Xi + 50X2 + 80X3<=1,000 60X4 + 45X5 + 60X6<=1,000 30X7 + 25X8<=1,000

Min Investment: 40Xi>=100 50X2>=100 80X3>=100 60X4>=100 45X5>=100 60X6>=100 30X7>=100 25X8>=100

1. Determine the maximum return on the portfolio. What is the optimal number of shares to buy for each of the stocks? What is the corresponding dollar amount invested in each stock?

```
library(lpSolveAPI)
##We have 4 constraints and 8 decision variables
lprec <- make.lp(12,8)
```

```
##Set the coefficient of the objective function
set.objfn(lprec, c(4,6.5,5.9,5.4,5.15,10,8.4,6.25))
lp.control(lprec, sense = "max")
```

```
## $anti.degen
## [1] "fixedvars" "stalling"
##
## $basis.crash
## [1] "none"
##
## $bb.depthlimit
## [1] -50
##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy"       "dynamic"      "rcostfixing"
##
## $break.at.first
## [1] FALSE
##
## $break.at.value
## [1] 1e+30
##
## $epsilon
##       epsb       epsd      epsel     epsint epsperturb   epspivot
##      1e-10      1e-09      1e-12      1e-07      1e-05      2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
##
## $infinite
## [1] 1e+30
##
## $maxpivot
## [1] 250
##
## $mip.gap
## absolute relative
##    1e-11    1e-11
##
## $negrange
## [1] -1e+06
##
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"    "adaptive"
##
## $presolve
## [1] "none"
##
## $scalelimit
## [1] 5
##
## $scaling
## [1] "geometric"   "equilibrate" "integers"
##
## $sense
## [1] "maximize"
##
## $simplextype
## [1] "dual"   "primal"
##
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"
```

```
## i will use set.row to set the a column object

##Total Investment:
set.row(lprec, 1, c(40,50,80,60,45,60,30,25))

##Diversification
set.row(lprec, 2, c(40,50,80), indices = c(1,2,3))
set.row(lprec, 3, c(60,45,60), indices = c(4,5,6))
set.row(lprec, 4, c(30,25), indices = c(7,8))
##Min Investment:
set.row(lprec, 5, 40, indices = 1)
set.row(lprec, 6, 50, indices = 2)
set.row(lprec, 7, 80, indices = 3)
set.row(lprec, 8, 60, indices = 4)
set.row(lprec, 9, 45, indices = 5)
set.row(lprec, 10, 60, indices = 6)
set.row(lprec, 11, 30, indices = 7)
set.row(lprec, 12, 25, indices = 8)
```

```
##Set corresponding value for max investment and diversification
rhs <- c(2500,1000,1000,1000,100,100,100,100,100,100,100,100)### these numbers are multiply by 1
000
set.rhs(lprec, rhs)
```

```
set.constr.type(lprec, c("<=","<=","<=","<=",">=",">=",">=",">=",">=",">=",">=",">="))
```

```
##Setting the boundary
set.bounds(lprec, lower = rep(1, 8))
```

```
print(lprec)
```

```
## Model name:
##              C1    C2    C3    C4    C5    C6    C7    C8
## Maximize      4   6.5   5.9   5.4  5.15    10   8.4  6.25
## R1           40    50    80    60    45    60    30    25  <=  2500
## R2           40    50    80     0     0     0     0     0  <=  1000
## R3            0     0     0    60    45    60     0     0  <=  1000
## R4            0     0     0     0     0     0    30    25  <=  1000
## R5           40     0     0     0     0     0     0     0  >=   100
## R6            0    50     0     0     0     0     0     0  >=   100
## R7            0     0    80     0     0     0     0     0  >=   100
## R8            0     0     0    60     0     0     0     0  >=   100
## R9            0     0     0     0    45     0     0     0  >=   100
## R10           0     0     0     0     0    60     0     0  >=   100
## R11           0     0     0     0     0     0    30     0  >=   100
## R12           0     0     0     0     0     0     0    25  >=   100
## Kind        Std   Std   Std   Std   Std   Std   Std   Std
## Type       Real  Real  Real  Real  Real  Real  Real  Real
## Upper       Inf   Inf   Inf   Inf   Inf   Inf   Inf   Inf
## Lower         1     1     1     1     1     1     1     1
```

```
##Integer constrains
set.type(lprec, 1:8, "integer")
```

```
solve(lprec)
```

```
## [1] 0
```

```
get.objective(lprec)
```

```
## [1] 477.4
```

```
get.variables(lprec)
```

```
## [1]  3  5  2  2  3 12 29  5
```

Based on the objective function the max reuturn on protfolio with integer restriction is $477,400 The optimum number of shares to buy for each of the stock: S1=X1=3000 shares = $120,000 S2=X2=5000 shares = $250,000 S3=X3=2000 shares = $160,000 H1=X4=2000 shares = $120,000 H2=X5=3000 shares = $135,000 H3=X6=12000 shares = $720,000 C1=X7=29000 shares = $870,000 C2=X8=5000 shares = $125,000

2. Compare the solution in which there is no integer restriction on the number of shares invested. By how much (in percentage terms) do the integer restrictions alter the value of the optimal objective function? By how much (in percentage terms) do they alter the optimal investment quantities? So to detrmine this part, I have run the coding without the integer restriction and here is what I got:

Based on the objective function the max reuturn on protfolio with integer restriction is $487,152.8 The optimum number of shares to buy for each of the stock: S1=X1=2,500 shares = $100,000 S2=X2=6,000 shares = $300,000 S3=X3=1,250 shares = $100,000 H1=X4=1,666 shares = $99,960 H2=X5=2,222 shares = $99,990 H3=X6=13,333 shares = $799,980 C1=X7=30,000 shares = $900,000 C2=X8=4,000 shares = $100,000

Based on the obove output the integer restiction altered the value of the optimum function by 0.39%

Based on the obove output the integer restiction altered the optimum investment quantities by:

S1= 16.67% S2= 20% S3= 37.5% H1= 16.7% H2= 25.93% H3= 11.11% C1= 3.45% C2= 20%