12.1 : final 1

Nasser Alrasbi

Let $P_1(x) = \frac{1}{\sqrt{2\pi}\,\sigma_0} e^{-\frac{(x-\mu_1)^2}{2\sigma_0^2}}$ with mean $\mu_1$ + standar deviatio $\sigma_0$

$P_2(x) = \frac{1}{\sqrt{2\pi}\,\sigma_0} e^{-\frac{(x-\mu_2)^2}{2\sigma_0^2}}$ with mean $\mu_2$ + std $\sigma_0$

$P_3(x) = \frac{1}{\sqrt{2\pi}\,\sigma_0} e^{-\frac{(x-\mu_3)^2}{2\sigma_0^2}}$ with mean $\mu_3$ + std $\sigma_0$

$$\therefore P_{123}(x) = \frac{1}{\sqrt{2\pi}\,\sigma_0} e^{-\frac{(x-\mu_1)^2}{2\sigma_0^2}} \frac{1}{\sqrt{2\pi}\,\sigma_0} e^{-\frac{(x-\mu_2)^2}{2\sigma_0^2}} \frac{1}{\sqrt{2\pi}\,\sigma_0} e^{-\frac{(x-\mu_3)^2}{2\sigma_0^2}}$$

$$P_{123}(x) = \left(\frac{1}{\sqrt{2\pi}\,\sigma_0}\right)^3 e^{-\frac{[(x-\mu_1)^2+(x-\mu_2)^2+(x-\mu_3)^2]}{2\sigma_0^2}}$$

$$[x-\mu_1)^2 + (x-\mu_2)^2 + (x-\mu_3)^2 = x^2 - 2x\mu_1 + \mu_1^2 + x^2 - 2x\mu_2 + \mu_2^2 + x^2 - 2x\mu_3 + \mu_3^2$$

$$= x^2 - 2x\mu_1 + \mu_1^2 + x^2 - 2x\mu_2 + \mu_2^2 - 2x\mu_3 + \mu_3^2$$

$$= 3x^2 - 2x(\mu_1 + \mu_2 + \mu_3) + \mu_1^2 + \mu_2^2 + \mu_3^2 \Rightarrow (1)$$

From Quadratic equation

$$ax^2 + bx + c = a(x+d)^2 + e$$

where $d = \frac{b}{2}$, $e = c - \frac{b^2}{4a}$

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$$\therefore d = \frac{b}{2} = \frac{-2(u_1+u_2+u_3)}{2} = -(u_1+u_2+u_3) \Rightarrow (2)$$

$$e = c - \frac{b^2}{4a} = (u_1^2+u_2^2+u_3^2) - \frac{4(u_1+u_2+u_3)^2}{12} \Rightarrow (3)$$

~~From 1~~ will apply $d$ and $e$ in the $a(x+d)^2+e$ form1

$$a(x+d)^2+e = 3\left[x+(-(u_1+u_2+u_3))\right]^2 + (u_1^2+u_2^2+u_3^2) - \frac{4}{12}(u_1+u_2+u_3)^2$$

$\because e$ is constant with no $x$ dependence

$$\therefore 3\left[x-(u_1+u_2+u_3)\right]^2 + \text{constant}$$

$$\therefore P_{123}(x) = \left(\frac{1}{\sqrt{2\pi}\,\sigma_0}\right)^3 e^{\frac{-3\left[x-(u_1+u_2+u_3)\right]^2}{2\sigma_0^2} + \text{constant}}$$

$$P_{123}(x) = \left(\frac{1}{\sqrt{2\pi}\,\sigma_0}\right)^3 e^{\frac{-3\left[x-(u_1+u_2+u_3)\right]^2}{2\sigma_0^2}} \; e^{\frac{\text{constant}}{2\sigma_0^2}}$$

$$P_{123}(x) = \left(\frac{1}{\sqrt{2\pi}\,\sigma_0}\right)^3 e^{\frac{\text{constant}}{2\sigma_0^2}} \; e^{-\frac{3\left[x-(u_1+u_2+u_3)\right]^2}{2\sigma_0^2}}$$

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$$P_{123}(x) = \left(\frac{1}{\sqrt{2\pi}\,\sigma_0}\right)^3 e^{\frac{\text{constant}}{2\sigma_0^2}} e^{\frac{-(x-(\mu_1+\mu_2+\mu_3))^2}{2\sigma_0^2/3}}$$

$\mu = \mu_1 + \mu_2 + \mu_3$ and $\sigma = \sigma_0$

$$\therefore P_{123}(x) = \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^3 e^{\frac{\text{constant}}{2\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2/3}}$$

$$P_{123}(x) = \underbrace{\text{constant}}_{\text{we can Normalized it}} e^{-\frac{(x-\mu)^2}{2\sigma^2/3}} \qquad \therefore P_{123}(x) = \underbrace{C}_{\text{constant}} e^{-\frac{(x-\mu)^2}{2\sigma^2/3}}$$

we can generlize it

$$P_n(x) = \text{constan}\; e^{\frac{(x-\mu)^2}{2\sigma^2/n}}$$

whene $n$ is the values of $\mu$

$\therefore$ ~~$P_{123}(x)$~~ $=$