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6. from chapter 2 - Econometrics - simpleliner regression analysis I found the appropairate tes statistic to perform the test on.

⇒ Testing and confdience Interval for slope (b): and σ^2 is unknown

100(1-α)% Conidence interval b is

$$\left[b \pm t_{n-2,\alpha/2} \sqrt{\frac{SS_{res}}{(n-2)}} \right]$$

SS_{res} is the residual sum of squares

$$SS_{res} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

⇒ Testing and confdience intervel for intercept term (a)

100(1-α)% CI a is

$$\left[a \pm t_{n-2,\alpha/2} \sqrt{\frac{SS_{res}}{n-2} \left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}} \right)} \right.$$

$$S_{xx} = \sum_{i=1}^{n} (x - \bar{x})^2$$

6.7 $b = \dfrac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}} = \dfrac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$

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we need to show $E(b) = \beta$

$$b = \frac{\sum_{i=1}^{n}(x_i-\bar{x})y_i - \sum_{i=1}^{n}(x_i-\bar{x})\bar{y}}{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})y_i - \bar{y}\sum_{i=1}^{n}(x_i-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

$$b = \frac{\sum_{i=1}^{n}(x_i-\bar{x})y_i - \bar{y}\left[\sum_{i=1}^{n} x_i - \sum_{i=1}^{n}\bar{x}\right]}{\sum_{i=1}^{n}(x_i-\bar{x})^2} \Rightarrow 1$$

$\because \bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n} \quad \therefore n\bar{x} = \sum_{i=1}^{n} x_i \Rightarrow 2 \quad \therefore \sum_{i=1}^{n}\bar{x} = n\bar{x} \Rightarrow 3$

apply 2 and 3 in 1

$$b = \frac{\sum_{i=1}^{n}(x_i-\bar{x})y_i - [n\bar{x} - n\bar{x}]}{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})y_i - 0}{\sum_{i=1}^{n}(x_i-\bar{x})^2} \Rightarrow 4$$

$\because y_i = \beta x_i + \alpha + \epsilon_i$

$$\therefore b = \frac{\sum_{i=1}^{n}(x_i-\bar{x})\cdot(\beta x_i + \alpha + \epsilon_i)}{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{\sum_{i=1}^{n}\left[\beta(x_i-\bar{x})x_i + \alpha(x_i-\bar{x}) + \epsilon_i(x_i-\bar{x})\right]}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

$$b = \frac{\beta\sum_{i=1}^{n}(x_i-\bar{x})\cdot x_i + \alpha\sum_{i=1}^{n}(x_i-\bar{x}) + \sum_{i=1}^{n}\epsilon_i(x_i-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2} \Rightarrow 5$$

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(6) (7) As we know and proof from (4) that $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$

we will apply it in 5

$$b = \frac{\beta \sum_{i=1}^{n}(x_i - \bar{x}) \cdot x_i + \alpha[0] + \sum_{i=1}^{n} \epsilon_i (x_i - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$b = \frac{\beta \sum_{i=1}^{n}(x_i - \bar{x}) x_i + \sum_{i=1}^{n} \epsilon_i (x_i - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$b = \beta \frac{\sum_{i=1}^{n}(x_i - \bar{x}) x_i}{\sum_{i=1}^{n}(x_i - \bar{x})^2} + \frac{\sum_{i=1}^{n} \epsilon_i (x_i - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \Rightarrow (6)$$

$$\because \sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n}(x_i - \bar{x})(x_i - \bar{x}) = \sum_{i=1}^{n}(x_i^2 - x_i\bar{x} - \bar{x}x_i + \bar{x}^2)$$

$$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \bar{x}^2$$

$$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}(n\bar{x}) + n\bar{x}^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}\cdot\bar{x}$$

$$= \sum_{i=1}^{n} x_i^2 - n\bar{x}\frac{\sum_{i=1}^{n} x_i}{n} = \sum_{i=1}^{n} x_i^2 - \bar{x}\sum_{i=1}^{n} x_i = \sum_{i=1}^{n}(x_i^2 - \bar{x}x_i)$$

$$= \sum_{i=1}^{n}(x_i - \bar{x}) x_i \Rightarrow (7)$$

apply 7 in 6

$$b = \beta \frac{\sum_{i=1}^{n}(x_i - \bar{x}) x_i}{\sum_{i=1}^{n}(x_i - \bar{x}) x_i} + \frac{\sum_{i=1}^{n}(x_i - \bar{x}) \cdot \epsilon_i}{\sum_{i=1}^{n}(x_i + \bar{x})^2}$$

Complete ~~6~~ 7

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$$b = \beta + \frac{\sum_{i=1}^{n} (x_i - \bar{x}) . \epsilon_i}{\sum_{i=1}^{n} (x_i + \bar{x})^2}$$

Not sure what you mean by iterated expectations; should give a reference. Here you can just state that E[sum(z)] = sum(E[z]) - the proof is trivial and does not need to be shown.

$$E(b) = E\left[\beta + \frac{\sum_{i=1}^{n} (x_i - \bar{x}).\epsilon_i}{\sum_{i=1}^{n} (x_i - \bar{x})^2}\right]$$

$$E(b) = E(B) + E\left[\frac{\sum_{i=1}^{n} (x_i - \bar{x})\epsilon_i}{\sum_{i=1}^{n} (x_i - \bar{x})^2}\right]$$

From the law of Iterated expectations

$E(b) = E[E[b|x]]$ which com ~~E(x) = E~~ $E(x) = E[E(x|y)]$

$$E(b) = \beta + E\left[E\left[\frac{\sum_{i=1}^{n} (x_i - \bar{x})\epsilon_i}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \middle| x\right]\right]$$

$$E(b) = \beta + E\left[\frac{E[\sum_{i=1}^{n} (x_i - \bar{x})\epsilon_i | x]}{\sum_{i=1}^{n} (x_i - \bar{x})^2}\right]$$

$$E(b) = \beta + E\left[\frac{\sum_{i=1}^{n} (x_i - \bar{x}) . E[\epsilon_i | x]}{\sum_{i=1}^{n} (x_i - \bar{x})^2}\right]$$

$\because E[\epsilon_i | x] = 0$ The expected value of the error ~~is~~ term terms conditional on $x = 0$

$$\therefore E[b] = \beta + E\left(\frac{\sum_{i=1}^{n} (x_i - \bar{x})(0)}{\sum_{i} (x_i - \bar{x})}\right)$$

$$E[b] = \beta + E\left[\frac{0}{\sum_{i}^{n} (x_i - \bar{x})^2}\right]$$

$E[b] = \beta$ $\quad \therefore \boxed{E[b] = \beta}$

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Another way

$$b = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n}(x_i)^2 - n\bar{x}^2} = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sum_{i=1}^{n}(x_i)^2 - n\bar{x}^2}$$

$$\sum x_i = \frac{\bar{x}}{n}$$

$$\sum y_i = \frac{\bar{y}}{n}$$

$$b = \frac{\sum_{i=1}^{n} x_i y_i - \bar{x}\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n}(x_i)^2 - n\bar{x}^2} = \frac{\sum_{i=1}^{n}(x_i y_i - \bar{x} y_i)}{\sum_{i=1}^{n}(x_i)^2 - n\bar{x}^2}$$

$$b = \frac{\sum_{i=1}^{n}(x_i - \bar{x}) y_i}{\sum_{i=1}^{n}(x_i)^2 - n\bar{x}^2}$$

$$\cancel{b} = E[b|x_i] = \frac{\sum_{i=1}^{n}(x_i - \bar{x}) E[y_i|x_i]}{\sum_{i=1}^{n}(x_i)^2 - n\bar{x}^2}$$

$$E[b|x_i] = \frac{\sum_{i=1}^{n}(x_i - \bar{x}) E[\beta x_i + \alpha + \epsilon_i|x_i]}{\sum_{i=1}^{n}(x_i)^2 - n\bar{x}^2}$$

$$E[b|x_i] = \frac{\sum_{i=1}^{n}(x_i - \bar{x})\left[E[\beta x_i|x_i] + E[\alpha|x_i] + E[\epsilon_i|x_i]\right]}{\sum_{i=1}^{n}(x_i)^2 - n\bar{x}^2}$$

$$E[b|x_i] = \frac{\sum_{i=1}^{n}(x_i - \bar{x})[\beta x_i + \alpha + 0]}{\sum_{i=1}^{n}(x_i)^2 - n\bar{x}^2}$$

$$E[b|x_i] = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(\beta x_i + \alpha)}{\sum_{i=1}^{n}(x_i)^2 - n\bar{x}^2}$$

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$$E[b|x_i] = \frac{\sum_{i=1}^{n} (\beta x_i^2 + \alpha x_i - \beta x_i \bar{x} - \alpha \bar{x})}{\sum_{i=1}^{n} (x_i)^2 - n\bar{x}^2}$$

$$E[b|x_i] = \frac{\sum_{i=1}^{n} \alpha x_i - \sum_{i=1}^{n} \alpha \bar{x} + \beta\left[\sum_{i=1}^{n} (x_i^2 - x_i \bar{x})\right]}{\sum_{i=1}^{n} (x_i)^2 - n\bar{x}^2}$$

$$\because \frac{\sum_{i=1}^{n} x_i}{n} = \bar{X}$$

$$\therefore E[b|x_i] = \frac{\alpha\left[\sum_{i=1}^{n} x_i - n\bar{x}\right] + \beta\left(\sum_{i=1}^{n} x_i^2 - \bar{x}\sum_{i=1}^{n} x_i\right]}{\sum_{i=1}^{n} (x_i)^2 - n\bar{x}^2}$$

$$E[b|x_i] = \frac{\alpha\left[\sum_{i=1}^{n} x_i - n\left(\frac{\sum x_i}{n}\right)\right] + \beta\left[\sum x_i^2 - n\bar{x}^2\right]}{\sum_{i=1}^{n} (x_i)^2 - n\bar{x}^2}$$

$$E[b|x_i] = \frac{\alpha\left[\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i\right] + \beta\left[\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right]}{\sum_{i=1}^{n} (x_i)^2 - n\bar{x}^2}$$

$$E[b|x_i] = \frac{\alpha[0] + \beta\left[\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right]}{\sum_{i=1}^{n} (x_i)^2 - n\bar{x}^2}$$

$$E[b|x_i] = \frac{\beta \sum_{i=1}^{n} (x_i)^2 - n\bar{x}^2}{\sum_{i=1}^{n} (x_i)^2 - n\bar{x}^2}$$

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$E[b|x_i] = \beta$

$\because E[b|x_i] = E[b]$

$\therefore E[b] = \beta$