

3.3.1

$$P(\theta|D) = \underbrace{P(D|\theta)}_{\text{likelihood}} \frac{P(\theta)}{P(D)} \quad \begin{array}{l} \text{prior} \\ \text{normalization} \end{array}$$

$$P(k) = \binom{N}{k} (1-\theta)^{N-k} \theta^k \Rightarrow D = [H]$$

$$P(k=1 | N=1, \theta) = \frac{N!}{1! (N-1)!} (1-\theta)^0 \theta^1 = \theta \Rightarrow \text{likelihood}$$

$$\therefore P(D|\theta) = \theta \Rightarrow \text{likelihood}$$

$$3.3.2 \quad D = [H] \Rightarrow P(\theta) = ? = \text{constant} = C$$

$$P(D) = \int_0^1 P(\theta) P(D|\theta) d\theta$$

$$P(D) = \int_0^1 C P(D|\theta) d\theta = C \int_0^1 P(D|\theta) d\theta$$

$$= \int_0^1 C \theta d\theta = \left[ \frac{C\theta^2}{2} \right]_0^1 = \frac{C}{2} \approx \text{constant } C$$

3.3.3

$$P(\theta|D) = P(D|\theta) \frac{P(\theta)}{P(D)}$$

$$P(\theta|D) = \frac{\theta C}{[C/2]} = 2\theta$$

$$\therefore P(\theta|D) = 2\theta \Rightarrow \text{we will plot this vs } \theta$$

□

3.3.4

Nasser Alves

$$p(\theta|D) = p(D|\theta) \frac{p(\theta)}{p(D)}$$

$$D = [H, T], \quad p(\theta) = c$$

$$P(k=1, n=2, \theta) = \frac{n!}{k! (n-k)!} (1-\theta)^{n-k} \theta^k$$

$$p(D|\theta) = 2(1-\theta)\theta = 2\theta(1-\theta)$$

$$\therefore p(D|\theta) = 2\theta(1-\theta) = 2(\theta - \theta^2)$$

$$p(D) = \int_0^1 p(\theta) p(D|\theta) d\theta$$

$$p(D) = \int_0^1 c [2(\theta - \theta^2)] d\theta = 2c \left[ \frac{\theta^2}{2} - \frac{\theta^3}{3} \right]_0^1$$

$$p(D) = 2c \left[ \frac{1}{2} - \frac{1}{3} \right] = 2c \left( \frac{1}{6} \right) = \frac{c}{3}$$

$$\therefore p(\theta|D) = p(D|\theta) \frac{p(\theta)}{p(D)}$$

$$p(\theta|D) = 2(\theta - \theta^2) \times \frac{c}{c/3}$$

$$p(\theta|D) = 6(\theta - \theta^2) = 6\theta(1-\theta)$$

$$\therefore p(\theta|D) = 6\theta(1-\theta)$$

$\therefore$  we will plot  $p(\theta|D)$  vs  $\theta$   
 $6\theta(1-\theta)$  vs  $\theta$

3.3.5

Nasser Alrasbi

$$p(\theta) \propto e^{-(\theta - 0.5)^2 / -0.1}$$

→ Gaussian

$$p(\theta|D) \quad ?$$

$$D = [H, T]$$

$$p(\theta|D) = p(D|\theta) \frac{p(\theta)}{p(D)}$$

$$p(D|\theta) = 2\theta(1-\theta) \quad \text{From 3.3.4}$$

$$p(\theta) \propto e^{\frac{(\theta - 0.5)^2}{-0.1}}$$

$$p(D) = \text{constant}$$

$\therefore p(D)$  is constant  
So it will not  
effect equation

$$p(\theta|D) = 2\theta(1-\theta) \frac{e^{-\frac{(\theta - 0.5)^2}{0.1}}}{C}$$

$$\therefore p(\theta|D) = \frac{2}{C} \theta(1-\theta) e^{-\frac{(\theta - 0.5)^2}{0.1}}$$

we will plot  $p(\theta|D)$  vs  $\theta$  for  $D = [H, T]$