2.2 Bayes' Theorem

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$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B|A_j)\,P(A_j)}{\sum_{i=1}^{k} P(B|A_i)\cdot P(A_i)} \quad j = 1, \ldots k$$

so when we apply Bayes' Theorem to our case.

$$P(\text{cancer}|\text{postive}) = \frac{P(\text{postive}|\text{cancer})\,P(\text{cancer})}{P(\text{postive})}$$

$P(\text{postive}|\text{cancer}) = 0.8$ people with colon cancer will test postive

$P(\text{cancer}) = 0.02$ people of age 50-60 have colon cancer

$P(\text{postive}) = P(\text{postive}|\text{cancer})\,P(\text{cancer}) \cancel{+ P(\text{postive}}$

$\ldots + P(\text{postive}|\text{no cancer})\,P(\text{no cancer})$

$P(\text{postive}) = (0.8)(0.02) + (0.096)*(1-0.02)$

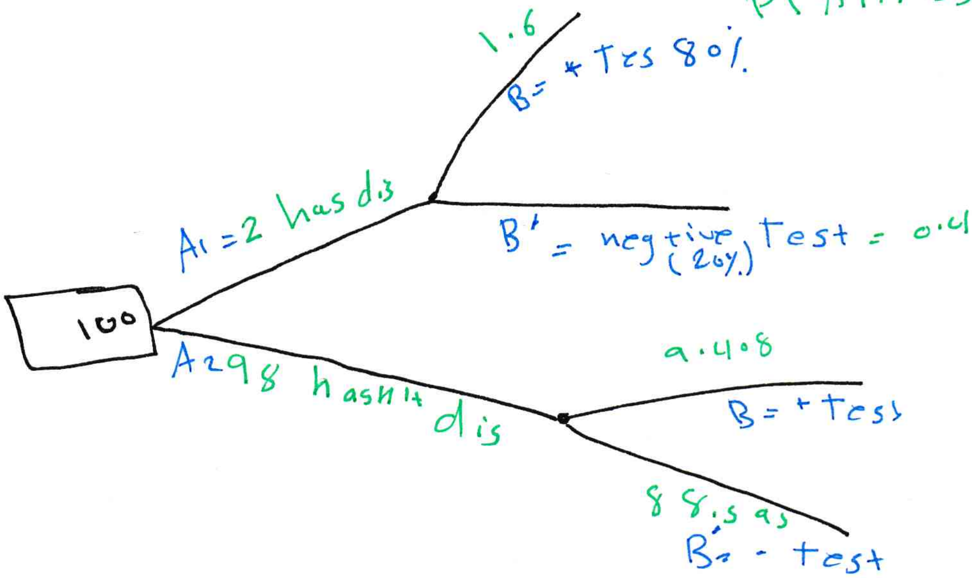
$= 0.016 + 0.094 = 0.11$

$$\boxed{P(\text{cancer}|\text{postive}) = \frac{(0.8)(0.02)}{(0.11)} = 0.145}$$

The probabity that actually have colon cancer is 0.145

Diagram for 100 people from ages 50-60

who participate in routing screen



$P(A_1 \cap B) = \frac{1.6}{100} = 0.016$

$P(A_2 \cap B) = \frac{9.408}{100}$
$= 0.09408$

$\therefore P(\text{postive}) = P(A_1 \cap B) + P(A_2 \cap B) = 0.016 + 0.09408 = 0.11$

$$P(\text{cancer} \mid \text{postive}) = \frac{P(\text{postive} \cap \text{cancer})}{P(\text{postive})} = \frac{0.016}{0.11}$$

$$\boxed{= 0.145}$$

$\Downarrow$

Same with Bayes' Theo