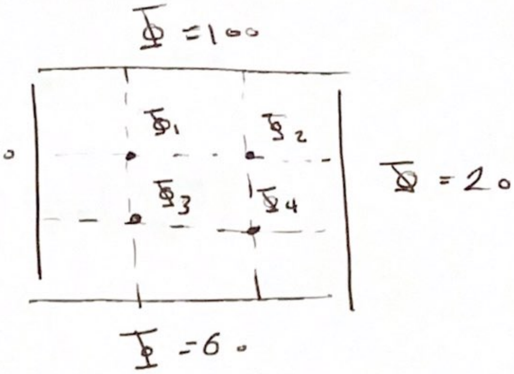
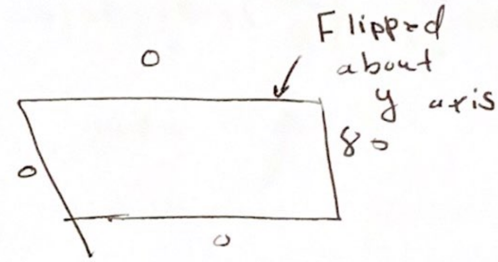
2.3.1

Take average

Step#	Φ_1	Φ_2	Φ_3	Φ_4
0	65	65	65	65
1.1	77.5	65	65	65
1.2	77.5	65.63	65	65
1.3	77.5	65.63	70.63	65
col 1	77.5	65.63	70.63	54.06





2.3.2

$\nabla^2 \Phi = 0 \qquad \Phi = X(x)\,Y(y)$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{d^2 Y}{\partial y^2} = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = \alpha^2$$

$$\frac{\partial^2 X}{\partial x^2} = \alpha^2 X$$

$X(x) = A\sinh \alpha x + B\cosh \alpha x$

$X = 0 \quad \Phi = 0$

$0 = A\sinh(0) + B\cosh(0)$

$B = 0$

$$\frac{\partial^2 Y}{\partial y^2} = -\alpha^2 Y$$

$Y(y) = C\sin \alpha y + D\cos \alpha y$

$Y = 0 \quad \Phi = 0$

$0 = C\sin(0) + D\cos(0)$

$\therefore D = 0$

$Y = a \quad \Phi = 0$

$0 = \sin \alpha a$

$$\alpha = \frac{n\pi}{a}$$

- combining the x and y solutions

$$\Phi(x,y) = \sum_{n=1}^{\infty} A_n \sinh\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)$$

$x = a \qquad \Phi = V_0 = 80V$

$$V_0 = \sum_{n=1}^{\infty} A_n \sinh(n\pi) \sin\left(\frac{n\pi y}{a}\right)$$

$$V_0 \int_0^a \sin\left(\frac{m\pi y}{a}\right) dy = \sum_{n=1}^{\infty} A_n \sinh(n\pi) \int_0^a \sin\left(\frac{m\pi y}{a}\right) \sin\left(\frac{n\pi y}{a}\right) dy$$

$$\int_0^{\pi} \sin mx \sin nx \, dx = \begin{cases} 0 & m \neq n \\ \frac{\pi}{2} & m = n \end{cases}$$

$$V_0 \int_0^a \sin \frac{m\pi y}{a} dy = A_m \sinh(n\pi) \int_0^a \sin^2 \frac{m\pi y}{a} dy$$

$$-V_0 \cos \frac{m\pi y}{a} \Big|_0^a \frac{a}{n\pi} = A_n \sinh(n\pi) \frac{1}{2} \int_0^a \left(1 - \cos\left(\frac{2\pi y}{a}\right)\right) dy$$

$$-V_0 (\cos n\pi - 1) \frac{a}{n\pi} = A_n \sinh(n\pi) \frac{1}{2} \left[a - \sin\left(\frac{2n\pi y}{a}\right) \Big|_0^a \frac{a}{2n\pi} \right]$$

$$-V_0 (\cos n\pi - 1) \frac{a}{n\pi} = A_n \sinh(n\pi) \frac{a}{2} \left[1 - (\sin 2n\pi - \sin 0) \frac{a}{2n\pi} \right]$$

$$A_n = \frac{2V_0 (1 - \cos n\pi)}{n\pi \sinh(n\pi)} \qquad A_n \begin{cases} \dfrac{4V_0}{n\pi \sinh(n\pi)} & n = \text{odd} \\ 0 & n = \text{even} \end{cases}$$

$$\boxed{\Phi = \sum_{n=\text{odd}}^{\infty} \frac{4V_0}{n\pi \sinh(n\pi)} \sinh\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)}$$