Homework4
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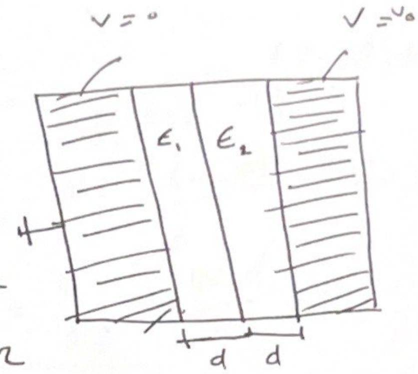
4.1.4



$V_1 = A_1 x + B_1$

$V_1(0) = 0$

$B_1 = 0$

$V_2 = A_2 x + B_2$

$\underline{V_2(2d) = V_0}$

$V_0 = 2dA_2 + B_2$

$B_2 = V_0 - 2dA_2$

$\therefore V_1 = A_1 x$

$V_2 = A_2(x-2d) + V_0$

$\underline{V_1(d) = V_2(d)}$

$A_1 d = A_2(-d) + V_0$

$$A_1 = \frac{V_0}{d} - A_2$$

$$V_1 = \left(\frac{V_0}{d} - A_2\right) x$$

$V_2 = A_2(x-2d) + V_0$

$\vec{D}_1(d) = \vec{D}_2(d)$

$\because \vec{D} = \epsilon \vec{E} = -\epsilon \vec{\nabla} V$

$$\therefore \vec{D}_1 = -\epsilon_1 \left(\frac{V_0}{d} - A_2\right) \hat{x}$$

$\vec{D}_2 = -\epsilon_2 A_2 \hat{x}$

$$-\epsilon_1\left(\frac{V_0}{d} - A_2\right) = -\epsilon_2 A_2$$

$$\frac{V_0}{d} - A_2 = \frac{\epsilon_2}{\epsilon_1} A_2$$

$$\frac{V_0}{d} = \left(\frac{\epsilon_2}{\epsilon_1} + 1\right) A_2$$

$$A_2 = \frac{V_0}{d\left(\frac{\epsilon_2}{\epsilon_1} + 1\right)}$$

$$V_1 = \frac{V_0}{d}\left(1 - \frac{1}{\epsilon_2/\epsilon_1 + 1}\right) x$$

$$V_2 = \frac{V_0}{d\left(\frac{\epsilon_2}{\epsilon_1} + 1\right)}(x-2d) + V_0$$

4.1.2

$\epsilon_1 = \epsilon_2$

$$V_1 = \frac{V_0}{d}\left(1 - \frac{1}{\epsilon_2/\epsilon_1 + 1}\right)x$$

$$\boxed{V_1 = \frac{V_0 x}{2d}}$$

$$V_2 = \frac{V_0}{d(\epsilon_2/\epsilon_1 + 1)}(x-2d) + V_0$$

$$V_2 = \frac{V_0(x-2d)}{2d} + V_0$$

$$V_2 = \frac{V_0 x}{2d} - \frac{V_0 2d}{2d} + V_0$$

$$\boxed{V_2 = \frac{V_0 x}{2d}}$$

$\therefore \quad V_1 = V_2$ when $\epsilon_1 = \epsilon_2$

4.1.3

$\chi_{e2} \to \infty$ then $\epsilon_2 = \epsilon_0(1 + \chi_{e2}) \to \infty$

$$V_1 = \frac{V_0}{d}\left(1 - \cancel{\frac{1}{\epsilon_2/\epsilon_1 + 1}}\right)x \qquad V_2 = \cancel{\frac{V_0}{d(\epsilon_2/\epsilon_1 + 1)}}(x-2d) + V_0$$

$$\boxed{V_1 = \frac{V_0 x}{d}} \qquad \boxed{V_2 = V_0}$$

4.1.4

$$\vec{D} = \epsilon \vec{E} = -\epsilon \vec{\nabla} V \qquad \sigma_b = \vec{P} \cdot \hat{n}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = -\epsilon_0 \vec{\nabla} V + \vec{P}$$

$$-\epsilon \vec{\nabla} V = -\epsilon_0 \vec{\nabla} V + \vec{P}$$

Dilelectric 1

$$-\epsilon_1 \vec{\nabla} V_1 = -\epsilon_0 \vec{\nabla} V_1 + \vec{P}_1$$

$$-\epsilon_1 \left(\frac{V_0}{d}\left(\frac{\epsilon_2/\epsilon_1 + 1 - 1}{\epsilon_2/\epsilon_1 + 1}\right)\right) = -\epsilon_0 \frac{V_0}{d}\left(\frac{\epsilon_2/\epsilon_1}{\epsilon_2/\epsilon_1 + 1}\right) + P_{1x}$$

$$-\epsilon_1 \frac{V_0}{d}\left(\frac{\epsilon_2}{\epsilon_2 + \epsilon_1}\right) = -\epsilon_0 \frac{V_0}{d}\left(\frac{\epsilon_2}{\epsilon_2 + \epsilon_1}\right) + P_{1x}$$

$$P_{1x} = \frac{-V_0}{d}\left(\frac{\epsilon_2}{\epsilon_2 + \epsilon_1}\right)(\epsilon_1 - \epsilon_0)$$

left : $\hat{n}_{1L} = -\hat{x} \longrightarrow \sigma_{b1L} = \vec{P} \cdot \hat{n}_{1L} = \boxed{\frac{V_0}{d}\left(\frac{\epsilon_2}{\epsilon_2 + \epsilon_1}\right)(\epsilon_1 - \epsilon_0)}$

right : $\hat{n}_{1r} = \hat{x} \rightarrow \sigma_{b1r} = \boxed{\frac{-V_0}{d}\left(\frac{\epsilon_2}{\epsilon_2 + \epsilon_1}\right)(\epsilon_1 - \epsilon_0)}$

Dielectric 2

$$-\epsilon_2 \vec{\nabla} V_2 = \epsilon_0 \vec{\nabla} V_2 + \vec{P_2}$$

$$-\epsilon_2 \frac{V_0}{d}\left(\frac{\epsilon_1}{\epsilon_2+\epsilon_1}\right) = -\epsilon_0 \frac{V_0}{d}\left(\frac{\epsilon_1}{\epsilon_2+\epsilon_1}\right) + P_{2x}$$

$$P_{2x} = \frac{-V_0}{d}\left(\frac{\epsilon_1}{\epsilon_2+\epsilon_1}\right)(\epsilon_2 - \epsilon_0)$$

left : $\hat{n}_{2L} = -\hat{x}$ $\rightarrow$ $\boxed{\sigma_{b2L} = \frac{V_0}{d}\left(\frac{\epsilon_1}{\epsilon_2+\epsilon_1}\right)(\epsilon_2 - \epsilon_0)}$

right : $\hat{n}_{2r} = \hat{x}$ $\longrightarrow$ $\boxed{\sigma_{b2r} = \frac{-V_0}{d}\left(\frac{\epsilon_1}{\epsilon_2+\epsilon_1}\right)(\epsilon_2 - \epsilon_0)}$

4.1.5

$$\oint E \cdot d\vec{a} = \frac{\sigma}{\epsilon_0}$$

$$E_{1x} - E_{2x} = \frac{1}{\epsilon_0}(\sigma_{b1r} + \sigma_{b2L})$$

$$-\frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial x} = \frac{1}{\epsilon_0}\left(\left(-\frac{V_0}{d}\left(\frac{\epsilon_2}{\epsilon_2+\epsilon_1}\right)(\epsilon_1 - \epsilon_0)\right) + \left(\frac{V_0}{d}\left(\frac{\epsilon_1}{\epsilon_2+\epsilon_1}\right)(\epsilon_2 - \epsilon_0)\right)\right)$$

$$-\frac{V_0}{d}\left(\frac{\epsilon_2}{\epsilon_1+\epsilon_2}\right) + \frac{V_0}{d}\left(\frac{\epsilon_1}{\epsilon_1+\epsilon_2}\right) = \frac{V_0}{\epsilon_0 d(\epsilon_1+\epsilon_2)}\left(-\cancel{\epsilon_2\epsilon_1} + \epsilon_2\epsilon_0 + \cancel{\epsilon_1\epsilon_2} - \epsilon_1\epsilon_0\right)$$

$$\boxed{\frac{V_0}{d(\epsilon_1+\epsilon_2)}(-\epsilon_2 + \epsilon_1) = \frac{V_0}{d(\epsilon_1+\epsilon_2)}(\epsilon_2 - \epsilon_1)}$$

I misplaced a minus sign somewhere, but I can't find it :(

4.1.6

$$\oint \vec{D} \cdot d\vec{a} = \sigma_{free} \qquad \sigma_{free} = 0$$

$$D_{1x} - D_{2x} = 0$$

$$\epsilon_1 E_{1x} - \epsilon_2 E_{2x} = 0$$

$$-\epsilon_1 \vec{\nabla}_x V_1 + \epsilon_2 \vec{\nabla}_x V_2 = 0$$

$$-\epsilon_1 \frac{V_0}{d}\left(\frac{\epsilon_2}{\epsilon_1+\epsilon_2}\right) + \epsilon_2 \frac{V_0}{d}\left(\frac{\epsilon_1}{\epsilon_1+\epsilon_2}\right) = 0$$

$$\boxed{\frac{V_0}{d}\left[\frac{-\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2} + \frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}\right] = 0}$$