4.2

$$D_1(d) = D_2(d)$$

$$\epsilon_1 E_1(d) = \epsilon_2 E_2(d)$$

$$\epsilon_1 \nabla V_1|_d = \epsilon_2 \nabla V_2|_d$$

$$\epsilon_1 \frac{\partial V_1}{\partial x} = \epsilon_2 \frac{\partial V_2}{\partial x}$$

$$V_1(x-h) \approx V_1(x) - h\frac{\partial V_1}{\partial x}$$

$$V_2(x+h) \approx V_2(x) + h\frac{\partial V_2}{\partial x}$$

$$\Downarrow$$

$$\frac{\partial V_1}{\partial x} = \frac{V_1(x) - V_1(x-h)}{h}$$

$$\frac{\partial V_2}{\partial x} = \frac{V_2(x+h) - V_2(x)}{h}$$

$$\frac{\epsilon_1}{h}\left(V_1(x) - V_1(x-h)\right) = \frac{\epsilon_2}{h}\left(V_2(x+h) - V_2(x)\right)$$

$$\epsilon_1 V_1(x) - \epsilon_1 V_1(x-h) = \epsilon_2 V_2(x+h) - \epsilon_2 V_2(x)$$

at $x = d \longrightarrow V_1(d) = V_2(d)$

$$V_1(d)(\epsilon_1 + \epsilon_2) = \epsilon_2 V_2(x+h) + \epsilon_1 V_1(x-h)$$

$$V(d) = \frac{\epsilon_2 V_2(x+h) + \epsilon_1 V_1(x-h)}{\epsilon_1 + \epsilon_2}$$

when $\epsilon_2 \to \infty$

$\epsilon_2 \longrightarrow \infty$

$$\frac{\epsilon_1}{\epsilon_1 + \epsilon_2} \Rightarrow 0 \qquad \frac{\epsilon_2}{\epsilon_1 + \epsilon_2} \to 1$$

$$V(d) \longrightarrow V_2(d+h)$$

the potential should be constant throughout a conductor

when $\epsilon_1 = \epsilon_2 = \epsilon$

$$V(d) = \frac{\epsilon\left(V(x+h) + V(x-h)\right)}{2\epsilon}$$

$$V(d) = \frac{V(x+h) + V(x-h)}{2}$$

averaging the potential on either side

Grid 1 : $x = 0, d, 2d$ $V(0) = 0$ $V(2d) = V_0$

$V(0) = 0$ (boundary condition)

$$V(d) = \frac{\epsilon_2}{\epsilon_1 + \epsilon_2} V_0 + \frac{\epsilon_1}{\epsilon_1 + \epsilon_2} 0 = \boxed{\frac{\epsilon_2}{\epsilon_1 + \epsilon_2} V_0}$$

$V(2d) = V_0$ (boundary condition)

Grid 2 : $x = 0, \frac{d}{2}, d, \frac{3d}{2}, 2d$

$V(0) = 0$
$V(2d) = V_0$
$V(d) = \frac{\epsilon_2}{\epsilon_1 + \epsilon_2} V_0$

$V(0) = 0$ (boundary condition)

$$V\left(\frac{d}{2}\right) = \frac{1}{2}\left(0 + \frac{\epsilon_2}{\epsilon_1 + \epsilon_2} V\right) = \boxed{\frac{V_0}{2}\left(\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}\right)}$$

$V(d) = \frac{\epsilon_2}{\epsilon_1 + \epsilon_2} V_0$ (boundery condition)

$$V\left(\frac{3d}{2}\right) = \frac{1}{2} V_0\left(\frac{\epsilon_2}{\epsilon_1 + \epsilon_2} + 1\right) = \boxed{\frac{V_0}{2}\left(1 + \frac{\epsilon_2}{\epsilon_1 + \epsilon_2}\right)}$$

$V(2d) = V_0$ (boundary condition)