Final

Nasser Alrasbi

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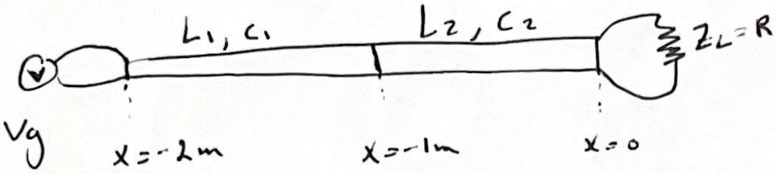

$V_g = 1V$

$L_1 = 0.1\,mH$

$C_1 = 1\,\mu f$

$L_2 = 0.4\,mH$

$C_2 = 1\,\mu F$

$R = 40\,\Omega$

$\ell = 1m$

$\omega = 1\times10^6\ 1/s$

$$Z_{01} = \sqrt{\frac{L_1}{C_1}} = 10\,\Omega \qquad Z_{02} = \sqrt{\frac{L_2}{C_2}} = 20\,\Omega$$

$$\tilde{V}_1(x) = \tilde{V}_1^+\left(e^{-j\beta_1 x} + \tilde{\rho}_1 e^{j\beta_1 x}\right) \qquad \tilde{V}_2(x) = \tilde{V}_2^+\left(e^{-j\beta_2 x} + \tilde{\rho}_2 e^{j\beta_2 x}\right)$$

$$I_1(x) = \frac{\tilde{V}_1^+}{Z_{01}}\left(e^{-j\beta_1 x} - \tilde{\rho}_1 e^{j\beta_1 x}\right) \qquad \tilde{I}_2(x) = \frac{\tilde{V}_2^+}{Z_{02}}\left(e^{-j\beta_2 x} - \tilde{\rho}_2 e^{j\beta_2 x}\right)$$

$$\tilde{\rho}_2 = \frac{Z_L - Z_{02}}{Z_L + Z_{02}} = \frac{40-20}{40+20} = \frac{1}{3}$$

$$\beta = \omega\sqrt{LC} \longrightarrow \beta_1 = 10\ 1/m \qquad \beta_2 = 20\ 1/m$$

$$\frac{V_1(x=-\ell)}{I_1(x=-\ell)} = \frac{V_2(x=-\ell)}{I_2(x=-\ell)}$$

$$\left.\frac{\cancel{\tilde{V}_1^+}\left(e^{-j\beta_1 x} + \tilde{\rho}_1 e^{j\beta_1 x}\right)}{\frac{\cancel{\tilde{V}_1^+}}{Z_{01}}\left(e^{-j\beta_1 x} - \tilde{\rho}_2 e^{j\beta_1 x}\right)} = \frac{\cancel{\tilde{V}_2^+}\left(e^{-j\beta_2 x} + \tilde{\rho}_2 e^{j\beta_2 x}\right)}{\frac{\cancel{\tilde{V}_2^+}}{Z_{02}}\left(e^{-j\beta_2 x} - \hat{\rho}_2 e^{j\beta_2 x}\right)}\right|_{x=-\ell}$$

$$\left.\frac{Z_{01}\left(1 + \tilde{\rho}_1 e^{2j\beta_1 x}\right)}{1 - \tilde{\rho}_1 e^{2j\beta_1 x}} = \frac{Z_{02}\left(1 + \tilde{\rho}_2 e^{2j\beta_2 x}\right)}{1 - \tilde{\rho}_2 e^{2j\beta_2 x}}\right|_{x=-\ell}$$

$x = -\ell$, $\hat{\rho}_2 = \frac{1}{3}$, $Z_{02} = 2Z_{01}$, $\beta_1 = 10\,\frac{1}{m}$, $\beta_2 = 20\,\frac{1}{m}$, $\ell = 1m$

$$\left(1 + \tilde{\rho}_1 e^{-2j\beta_1 \ell}\right)\left(1 - \frac{1}{3} e^{-2j\beta_2 \ell}\right) = \left(2 + \frac{2}{3} e^{-2j\beta_2 \ell}\right)\left(1 - \tilde{\rho}_1 e^{-2j\beta_1 \ell}\right)$$

$$1 - \frac{1}{3} e^{-2j\beta_2 \ell} + \tilde{\rho}_1 e^{-2j\beta_1 \ell} - \frac{1}{3}\tilde{\rho}_1 e^{-2j(\beta_1+\beta_2)\ell} = 2 - 2\tilde{\rho}_1 e^{-2j\beta_1 \ell} + \frac{2}{3} e^{-2j\beta_2 \ell} - \frac{2}{3}\tilde{\rho}_1 e^{-2j(\beta_1+\beta_2)\ell}$$

$$\tilde{\rho}_1\left(e^{-2j\beta_1 \ell} - \frac{1}{3} e^{-2j(\beta_1+\beta_2)\ell} + e^{-2j\beta_1 \ell} + \frac{2}{3} e^{-2j(\beta_1+\beta_2)\ell}\right) = 1 + e^{-2j\beta_2 \ell}$$

$$\boxed{\tilde{\rho}_1 = \frac{1 + e^{-2j\beta_2 \ell}}{3e^{-2j\beta_1 \ell} + \frac{1}{3} e^{-2j(\beta_1+\beta_2)\ell}}} \approx 0.29 + 0.026j$$

· Solve for amplitudes

$$V_g = V_1(x = -2\ell = -2m)$$

$$V_g = \tilde{V}_1^+ \left(e^{-j\beta_1 x} + \tilde{\rho}_1 e^{j\beta_1 x} \right) \Big|_{x=-2\ell}$$

$$\tilde{V}_1^+ = \frac{V_g}{e^{-j\beta_1 x} + \tilde{\rho}_1 e^{j\beta_1 x}} \Bigg|_{x=-2\ell}$$

$$\boxed{\tilde{V}_1^+ = \frac{V_g}{e^{2j\beta_1 \ell} + \tilde{\rho}_1 e^{-2j\beta_1 \ell}}}$$

$$V_1(x=-\ell) = V_2(x=-\ell)$$

$$\tilde{V}_1^+ \left(e^{-j\beta_1 x} + \tilde{\rho}_1 e^{j\beta_1 x} \right) = \tilde{V}_2^+ \left(e^{-j\beta_2 x} + \tilde{\rho}_2 e^{j\beta_2 x} \right) \Big|_{x=-\ell}$$

$$\tilde{V}_2^+ = \frac{\tilde{V}_1^+ \left(e^{-j\beta_1 x} + \tilde{\rho}_1 e^{j\beta_1 x} \right)}{e^{-j\beta_2 x} + \tilde{\rho}_2 e^{j\beta_2 x}} \Bigg|_{x=-\ell}$$

$$\boxed{\tilde{V}_2^+ = \frac{\tilde{V}_1^+ \left(e^{j\beta_1 \ell} + \tilde{\rho}_1 e^{-j\beta_1 \ell} \right)}{e^{+j\beta_2 \ell} + \tilde{\rho}_2 e^{-j\beta_2 \ell}}}$$