6.3.1

$$\vec{E} = E_{0x} \cos(k_z z - \omega t)\, \hat{x}$$

$$\vec{\nabla} \times \vec{E} = \frac{-\partial B}{\partial t}$$

$$\frac{\partial \vec{B}}{\partial t} = \frac{-\partial E_x}{\partial z}\, \hat{y} = + E_{0x} \sin(k_z z - \omega t)\, k\, \hat{y}$$

$$\int \partial B_y = \int dt\, E_{0x}\, k \sin(k_z z - \omega t)$$

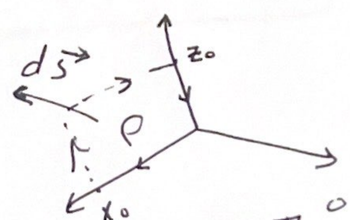
Let $u = kz - \omega t$
$du = -\omega dt$
$dt = -\frac{1}{\omega} du$

$$B_y = \frac{-E_{0x}\, k}{\omega} \int du \sin u = + \frac{E_{0x} k_z}{\omega} \cos u$$

$$\boxed{\vec{B} = \frac{E_{0x} k_z}{\omega} \cos(k_z z - \omega t)\, \hat{y}}$$

6.3.2

$$\oint \vec{E} \cdot d\vec{\ell} = \frac{-\partial \Phi_B}{\partial t}$$



LHS

$$\int_0^{x_0} E_x dx \Big|_{z=0} + \int_0^{z_0} \cancel{E_z dz}^{\,0} \Big|_{x=x_0} + \int_{x_0}^0 E_x dx \Big|_{z=z_0} + \int_{z_0}^0 \cancel{E_z dz}^{\,0} \Big|_{x=0}$$

$$= E_{0x} \int_0^{x_0} \cos(-\omega t)\, dx + E_{0x} \int_{x_0}^0 \cos(k z_0 - \omega t)\, dx$$

$$\boxed{LHS = E_{0x}\, x_0 \left(\cos(-\omega t) - \cos(k z_0 - \omega t)\right)}$$

RHs $\Phi_B = \int \vec{B} \cdot d\vec{s} \xrightarrow{} -\hat{y}\,ds = \frac{-E_{0x} k}{\omega} \int_0^{x_0} \int_0^{z_0} \cos(kz - \omega t)\,dx\,dz$

$$= \frac{-E_{0x} k x_0}{\omega} \int_0^{z_0} \cos(kz - \omega t)\,dz$$

$$= \frac{-E_{0x} \cancel{k} x_0}{\omega} \left. \frac{\sin(kz - \omega t)}{\cancel{k}} \right|_0^{z_0}$$

$$= \frac{-E_{0x} x_0}{\omega} \left(\sin(kz_0 - \omega t) - \sin(-\omega t) \right)$$

$$RHS = -\frac{\partial \Phi_B}{\partial t} = \frac{E_{0x} x_0}{\cancel{\omega}} \left((-\cancel{\omega}) \cos(kz_0 - \omega t) - (-\cancel{\omega}) \cos(-\omega t) \right)$$

$$\boxed{RHS = E_{0x} x_0 \left(\cos(-\omega t) - \cos(kz_0 - \omega t) \right)}$$

$\therefore$ LHs = RHs

$$\oint \vec{E} \cdot d\ell = \frac{-\partial \Phi_B}{\partial t}$$

6.3.3 $\oint \vec{B} \cdot d\vec{\ell} = \frac{1}{c^2} \frac{\partial \Phi_E}{\partial t}$

$$LHs = \int_0^{y_0} B_y\,dy \Big|_{z=0} + \int_0^{z_0} B_z\,dz \Big|_{y=y_0} + \int_{y_0}^{0} B_y\,dy \Big|_{z=z_0} + \int_{z_0}^{0} B_z\,dz \Big|_{y=0}$$

$$\boxed{LHs = \frac{E_{0x} k}{\omega} y_0 \left(\cos(-\omega t) - \cos(kz_0 - \omega t) \right)}$$

RHS

$$\Phi_E = \int \vec{E} \cdot d\vec{s} = E_{0x} \int \cos(kz - \omega t)\, ds$$

$\hat{x}\, ds$

$$= E_{0x} \int_0^{x_0} \int_0^{z_0} \cos(kz - \omega t)\, dy\, dz = E_{0x} y_0 \int_0^{z_0} \cos(kz - \omega t)\, dz$$

$$= \frac{E_0 y_0}{k} \sin(kz - \omega t) \Big|_0^{z_0}$$

$$= \frac{E_0 y_0}{k} \left(\sin(kz_0 - \omega t) - \sin(-\omega t) \right)$$

$$\frac{\partial \Phi_E}{\partial t} = \frac{E_0 y_0}{k} (-\omega) \left(\cos(kz_0 - \omega t) - \cos(-\omega t) \right)$$

$$= E_0 y_0 \frac{\omega}{k} \left(\cos(-\omega t) - \cos(kz_0 - \omega t) \right)$$

$$RHS = \frac{1}{c^2} \frac{\partial \Phi_E}{\partial t}$$

$$c = \lambda f, \quad k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f$$

$$c = \left(\frac{2\pi}{k}\right)\left(\frac{\omega}{2\pi}\right) = \frac{\omega}{k}$$

$$RHS = \frac{k^{\cancel{2}}}{\omega^{\cancel{2}}} E_0 y_0 \frac{\cancel{\omega}}{\cancel{k}} \left(\cos(-\omega t) - \cos(kz_0 - \omega t) \right)$$

$$\boxed{RHS = \frac{E_0 y_0 k}{\omega} \left(\cos(-\omega t) - \cos(kz_0 - \omega t) \right)}$$

$$\therefore \quad LHS = RHS$$