Homework2

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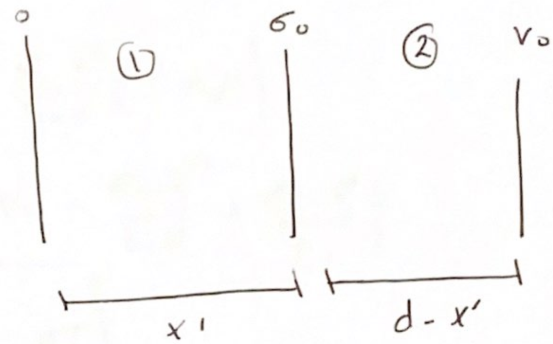
2.1.1



Region 1

$$\Phi_1 = A_1 x + B_1$$

$$\Phi_1(0) = 0 \Rightarrow B_1 = 0$$

$$\Phi_1 = A_1 x$$

Region 2

$$\Phi_2 = A_2 x + B_2$$

$$\Phi_2(d) = V_0 \Rightarrow V_0 = A_2 d + B_2 \Rightarrow B_2 = V_0 - A_2 d$$

$$\therefore \Phi_2 = A_2 x + V_0 - A_2 d$$

$$\Phi_2 = V_0 + A_2(x-d) \longrightarrow \quad -A_2 + A_1 = \frac{\sigma_0}{\varepsilon_0}$$

Coninuity

$$\Phi_1\big|_{x'} = \Phi_2\big|_{x'}$$

$$A_1 x' = V_0 + A_2(x'-d)$$

Smooth

$$E_2 - E_1 = \frac{\sigma_0}{\varepsilon_0}$$

$$-\frac{\partial V_2}{\partial x} + \frac{\partial V_1}{\partial x} = \frac{\sigma_0}{\varepsilon_0}$$

$$-A_2 + \frac{V_0 + A_2(x'-d)}{x'} = \frac{\sigma_0}{\varepsilon_0}$$

$$\frac{V_0 + A_2 x' - A_2 d - A_2 x'}{x'} = \frac{\sigma_0}{\varepsilon_0}$$

$$-A_2 d = \frac{\sigma_0 x'}{\varepsilon_0} - V_0 \Rightarrow A_2 = \frac{V_0}{d} - \frac{\sigma_0 x'}{\varepsilon_0 d}$$

$$\boxed{\begin{aligned} \Phi_1 &= \frac{V_0 + A_2(x'-d)}{x'}\, x \\ \Phi_2 &= V_0 + A_2(x-d) \\ A_2 &= \frac{V_0}{d} - \frac{\sigma_0 x'}{\varepsilon_0 d} \end{aligned}}$$

2) $\sigma_0 = \varepsilon_0 E_1 = -\varepsilon_0 \frac{\partial \Phi_1}{\partial x}\Big|_{x=0}$

$$\sigma_1 = \frac{-\varepsilon_0}{x'}\left(V_0 + \left(\frac{V_0}{d} - \frac{\sigma_0 x'}{\varepsilon_0 d}\right)(x'-d)\right)$$

$$\sigma_1 = \frac{-\varepsilon_0}{x'}\left(V_0 + \frac{V_0 x'}{d} - V_0 - \frac{\sigma_0 x'^2}{\varepsilon_0 d} + \frac{\sigma_0 x'}{\varepsilon_0}\right)$$

$$\boxed{\sigma_1 = \frac{\varepsilon_0 V_0}{d} + \frac{\sigma_0 x'}{d} - \sigma_0}$$

$$\sigma_2 = \varepsilon_0 E_2 = -\varepsilon_0 \frac{\partial \Phi_2}{dx}\Big|_{x=d}$$

$$\sigma_2 = -\varepsilon_0\left(\frac{V_0}{d} - \frac{\sigma_0 x'}{\varepsilon_0 d}\right)$$

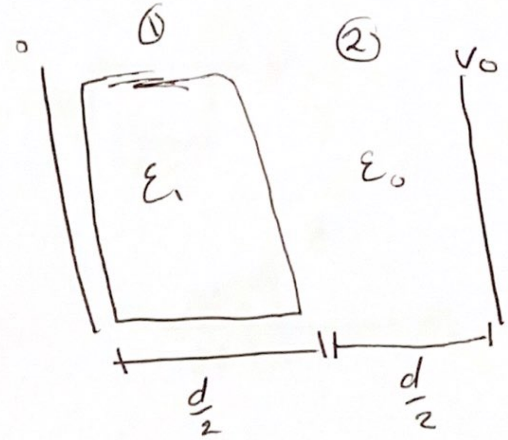
$$\boxed{\sigma_2 = \frac{-\varepsilon_0 V_0}{d} + \frac{\sigma_0 x'}{d}}$$

$$\sigma_{total} = \underbrace{\frac{-\varepsilon_0 V_0}{d} + \frac{\sigma_0 x'}{d} - \sigma_0}_{\sigma_1} \underbrace{- \frac{\varepsilon_0 V_0}{d} + \frac{\sigma_0 x'}{d}}_{\sigma_2} \underbrace{+ \sigma_0}_{\text{charge sheet}}$$

$$\sigma_T = \frac{2\sigma_0 x'}{d} - \frac{2\varepsilon_0 V_0}{d}$$

$$\sigma_{total} = \frac{2}{d}\left[\sigma_0 x' - \varepsilon_0 V_0\right]$$

2.1.3



$$\Phi_1 = A_1 x$$

$$\Phi_2 = V_0 + A_2(x-d)$$

$$D_2 - D_1 = \rho_s = 0$$

$$D_2 = D_1$$

$$\varepsilon_0 E_2 = \varepsilon_1 E_1 \qquad \because \; E = \frac{-\partial v}{\partial x}$$

$$\varepsilon_0 \frac{\partial \Phi_2}{\partial x} = \varepsilon_1 \frac{\partial \Phi_1}{\partial x}$$

$$\varepsilon_0 A_2 = \varepsilon_1 A_1$$

$$A_1 = \frac{\varepsilon_0}{\varepsilon_1} A_2 \longrightarrow \Phi_1 = \frac{\varepsilon_0}{\varepsilon_1} A_2 x, \quad \Phi_2 = V_0 + A_2(x-d)$$

$$\Phi_1 \Big|_{\frac{d}{2}} = \Phi_2 \Big|_{\frac{d}{2}}$$

$$\frac{A_1 d}{2} = V_0 + A_2\left(\frac{d}{2} - d\right)$$

$$\frac{\varepsilon_0}{\varepsilon_1} A_2 \frac{d}{2} = V_0 - A_2 \frac{d}{2}$$

$$A_2\left(\frac{\varepsilon_0}{\varepsilon_1} + 1\right)\frac{d}{2} = V_0$$

$$A_2 = \frac{V_0}{\frac{d}{2}\left(\frac{\varepsilon_0}{\varepsilon_1} + 1\right)}$$

$$\Phi_1 = \frac{\varepsilon_0}{\varepsilon_1}\left[\frac{V_0}{\frac{d}{2}\left(\frac{\varepsilon_0}{\varepsilon_1} + 1\right)}\right] x$$

$$\Phi_2 = V_0 + \left[\frac{V_0}{\frac{d}{2}\left(\frac{\varepsilon_0}{\varepsilon_1} + 1\right)}(x-d)\right]$$