Homework3

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3.1.1

$$\nabla^2 \Phi = \frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right) = 0$$

$$\nabla^2 \Phi = 2r \frac{\partial \Phi}{\partial r} + r^2 \frac{\partial^2 \Phi}{\partial r^2} = 0 \quad \Rightarrow (1)$$

$$\Phi(r+h) = \Phi(r) + h \frac{\partial \Phi}{\partial r} + \frac{h^2}{2} \frac{\partial^2 \Phi}{\partial r^2}$$

$$\Phi(r-h) = \Phi(r) - h \frac{\partial \Phi}{\partial r} + \frac{h^2}{2} \frac{\partial^2 \Phi}{\partial r^2}$$

$$\Rightarrow \boxed{\frac{\partial^2 \Phi}{\partial r^2}} \quad \Phi(r+h) + \Phi(r-h) = 2\Phi(r) + h^2 \frac{\partial^2 \Phi}{\partial r^2}$$

$$\frac{\partial^2 \Phi}{\partial r^2} = \frac{\Phi(r+h) - 2\Phi(r) + \Phi(r-h)}{h^2} \quad \Rightarrow (2)$$

$$\boxed{\frac{\partial \Phi}{\partial r}} \quad \Phi(r+h) - \Phi(r-h) = 2h \frac{\partial \Phi}{\partial r}$$

$$\frac{\partial \Phi}{\partial r} = \frac{\Phi(r+h) - \Phi(r-h)}{2h} \quad \Rightarrow 3$$

apply 2 and 3 in 1

$$\frac{r}{h}\left(\Phi(r+h) - \Phi(r-h)\right) + \frac{h^2}{h^2}\left(\Phi(r+h) - 2\Phi(r) + \Phi(r-h)\right) = 0$$

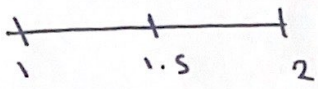
$$\Phi(r) = \left(\frac{1}{2} + \frac{h}{2r}\right)\Phi(r+h) + \left(\frac{1}{2} - \frac{h}{2r}\right)\Phi(r-h)$$

$$\boxed{\Phi(r) = \frac{1}{2}\left(\left(1 + \frac{h}{r}\right)\Phi(r+h) + \left(1 - \frac{h}{r}\right)\Phi(r-h)\right)}$$

3.1.2

$\Phi(2) = 1$

$\Phi(1) = 0$



$h = 0.5$

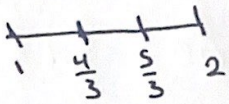
$$\Phi(1.5) = \left(\frac{1}{2} + \frac{0.5}{3}\right)\Phi(2) + \left(\frac{1}{2} - \frac{0.5}{3}\right)\Phi(1)$$

$$\boxed{\Phi(1.5) = \frac{2}{3}}$$

3.1.3

$\Phi(2) = 1$

$\Phi(1) = 0$



$h = \frac{1}{3}$

$\Phi_0(\frac{4}{3}) = \frac{1}{2} = \Phi_0(5/3)$

Iter 1

$$\Phi_1(4/3) = \left(\frac{1}{2} + \frac{1/3}{8/3}\right)\Phi(\tfrac{5}{3}) + 0 = \frac{5}{8}\frac{1}{2} = \frac{5}{16} = 0.3125$$

$$\Phi_1(5/3) = \left(\frac{1}{2} + \frac{1/3}{10/3}\right)\Phi(2) + \left(\frac{1}{2} - \frac{1/3}{10/3}\right)\Phi(4/3)$$

$$= \frac{3}{5} + \frac{2}{5}\cdot\frac{5}{16} = \frac{3}{5} + \frac{1}{8} = 0.725$$

Iter 2

$$\Phi_2(4/3) = (0.625)(0.725) = 0.453$$

$$\Phi_2(5/3) = 0.6 + (0.4)(0.453) = 0.787$$

Iter 3

$$\Phi_3(4/3) = (0.625)(0.787) = 0.488$$

$$\Phi_3(5/3) = 0.6 + (0.4)(0.488) = 0.795$$

→ approx