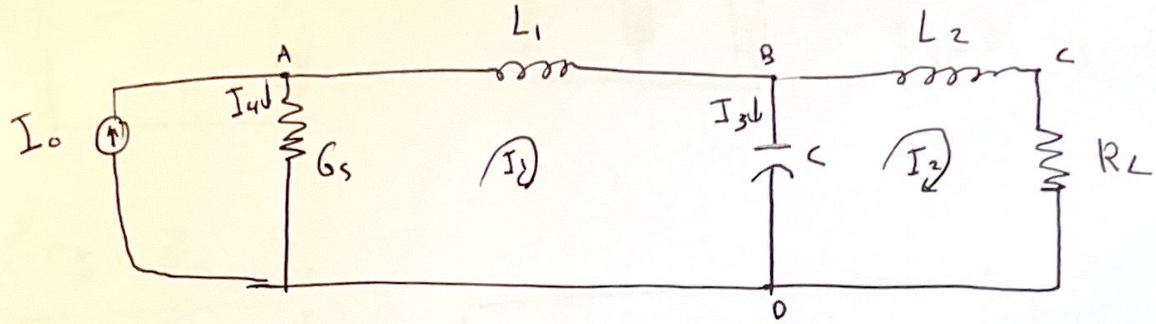
Nasser Alrasbi
Homework 7

4.3b



Loop equations

$I_3 + I_2 = I_1$ $\qquad$ $I_0 = I_1 + I_4$

$I_3 = I_1 - I_2$ $\qquad$ $I_4 = I_0 - I_1$

$$-L_1 \frac{dI_1(t)}{dt} - \frac{1}{C}\int (I_1(t) + I_2(t))\,dt + (I_0 - I_1)\frac{1}{G_s} = 0$$

$$-L_2 \frac{dI_2(t)}{dt} - I_2(t) R_L + \frac{1}{C}\int (I_1(t) - I_2(t))\,dt = 0$$

Node equations

$V_D = 0$

$$A: \quad I_0 + V_A G_s + \frac{1}{L_1}\int (V_A - V_B)\,dt = 0$$

$$B: \quad \frac{1}{L_1}\int (V_B - V_A)\,dt + C\frac{dV_B}{dt} + \frac{1}{L_2}\int (V_B - V_C)\,dt = 0$$

$$C: \quad \frac{1}{L_2}\int (V_C - V_B)\,dt + \frac{V_C}{R_L} = 0$$

Resistor: $V = IR \Rightarrow I = \frac{V}{R}$

Capacitor: $V = \frac{1}{C}\int I(t)\,dt$

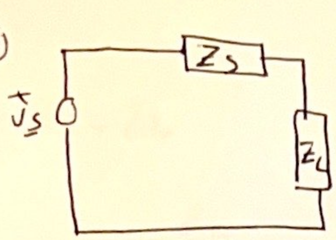
$I = C\frac{dv}{dt}$

Inductor: $V = L\frac{dI}{dt}$

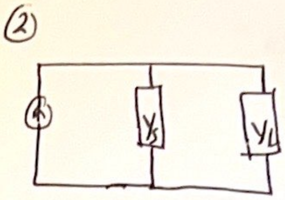
$I = \frac{1}{L}\int v\,dt$

4.3c

①



②



$Y_s = Z_s^{-1}$

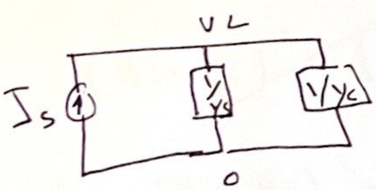
$I_s = V_s Y_s$

①

$$Z_{eq} = Z_s + Z_L$$

$$\boxed{I_L = \frac{V_s}{Z_s + Z_L}} \qquad \boxed{V_L = \frac{V_s}{Z_s + Z_L} Z_L}$$

② convert from admittance to impedance



$Z_s = Y_s^{-1}$

$Z_L = Y_L^{-1}$

$$\frac{1}{Z_{eq}} = \frac{1}{1/Y_s} + \frac{1}{1/Y_L} = Y_s + Y_L$$

$$Z_{eq} = \frac{1}{Y_s + Y_L}$$

Node equation

$$-I_s + \frac{V_L}{1/Y_s} + \frac{V_L}{1/Y_L} = 0$$

$$I_s = V_L\left(\frac{1}{Z_s} + \frac{1}{Z_L}\right)$$

$$I_s = \frac{V_s}{Z_s} \rightarrow V_L = \frac{I_s}{\frac{1}{Z_s} + \frac{1}{Z_L}} = \frac{V_s}{Z_s\left(\frac{1}{Z_s} + \frac{1}{Z_L}\right)}$$

$$V_L = \frac{V_s}{1 + Z_s/Z_L}$$

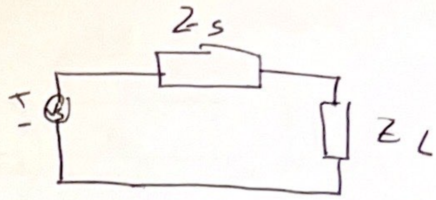
$$\boxed{V_L = \frac{V_s Z_L}{Z_L + Z_s}}$$

$$V_L = I_L Z_L$$

$$\boxed{I_L = \frac{V_s}{Z_L + Z_s}}$$

②

4.3 d



$$Z_{eq} = Z_s + Z_L$$

$$V_L = V_s \frac{Z_L}{Z_s + Z_L} \qquad I_L = \frac{V_s}{Z_s + Z_L} \qquad Z = R + iX$$

$$P = Re\left[V I^* \right]$$

$$P_L = Re\left[V_s \frac{Z_L}{Z_s + Z_L} \left(\frac{V_s}{Z_s^* + Z_L^*} \right) \right] = \frac{V_s^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

$$\frac{\partial P_L}{\partial R_L} = \frac{\partial}{\partial R_L} \left(V_s^2 R_L \left((R_s + R_L)^2 + (X_s + X_L)^2 \right)^{-1} \right) = 0$$

$$\left((R_s + R_L)^2 + (X_s + X_L)^2 \right)^{-1} + R_L(-1)2(R_s + R_L)\left((R_s + R_L)^2 + (X_s + X_L)^2 \right)^{\cancel{-2}\,-1} = 0$$

① $$(R_s + R_L)^2 + (X_s + X_L)^2 = 2R_L(R_s + R_L)$$

$$\frac{\partial P_L}{\partial X_L} = \frac{\partial}{\partial X_L} \frac{1}{(R_s + R_L)^2 + (X_s + X_L)^2} = 0$$

$$\frac{-2(X_s + X_L)}{(R_s + R_L)^2 + (X_s + X_L)^2} = 0$$

$$\boxed{X_s = -X_L}$$

$$(R_s + R_L)^{\cancel{2}} = 2R_L(\cancel{R_s + R_L})$$

$$R_s = R_L$$

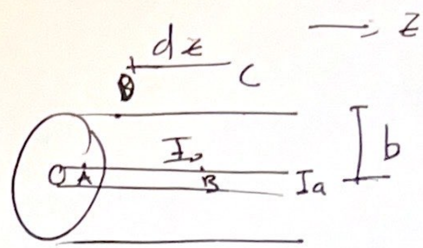
Power maximized when $R_L = R_s$ and $-X_L = X_s$

or

$$Z_L = Z_s^*$$

4.6e

eq. 1.9(4) $C = \dfrac{2\pi\epsilon}{\ln(b/a)}$



$$-\oint \vec{E}\cdot d\vec{\ell} = \frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{s}$$

High frequency

$$-\overset{①}{\int_A^B \vec{E}\cdot d\vec{\ell}} - \overset{②}{\int_B^C \vec{E}\cdot d\vec{\ell}} - \overset{③}{\int_C^D \vec{E}\cdot d\vec{\ell}} - \overset{④}{\int_D^A \vec{E}\cdot d\vec{\ell}} = \overset{⑤}{\frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{s}}$$

① $-\int_A^B \vec{E}\cdot d\vec{\ell} = -\dfrac{1}{\sigma_A}\int_A^B \vec{J}\cdot d\vec{\ell} = -\dfrac{1}{\sigma_A}\int_A^B \dfrac{I}{2\pi a\delta_a}\,d\ell = \dfrac{-1}{\sigma_A}\dfrac{I\,dz}{2\pi a\delta_A} = \dfrac{-IR_{sa}dz}{2\pi a}$

$\vec{J} = \sigma\vec{E}$ $\quad J = \dfrac{I}{2\pi\delta}$ $\quad R_{sa} = (\sigma_a\delta_a)^{-1}$

③ $-\int_C^D \vec{E}\cdot d\vec{\ell} = \dfrac{-1}{\sigma_b}\int_C^D \vec{J}\cdot d\vec{\ell} = -\dfrac{1}{\sigma_b}\int_C^D \dfrac{I}{2\pi b\delta_b}\,d\ell = -\dfrac{I\,dz}{\sigma_b 2\pi b\delta_b} = \dfrac{-IR_{sb}dz}{2\pi b}$

I'm still working on it :(