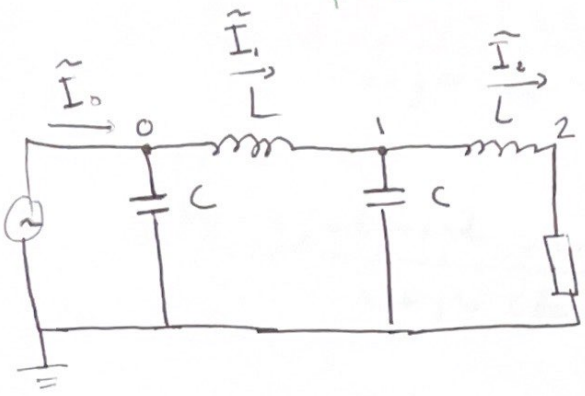
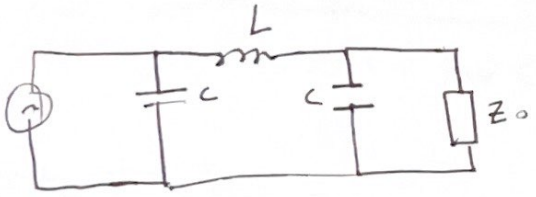
Nasser Alrasbi
Homework 10

[1]

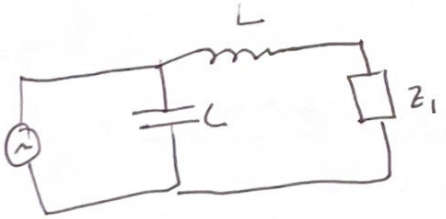


$$Z_c = \frac{1}{j\omega C}$$

$$Z_L = j\omega L$$



$$Z_a = Z_L + j\omega L$$



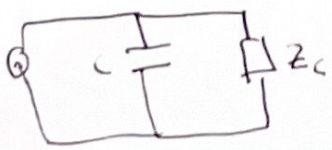
$$\frac{1}{Z_1} = \frac{1}{Z_a} + j\omega C = \frac{1}{Z_L + j\omega L} + j\omega C$$

$$\frac{1}{Z_1} = \frac{1 + j\omega C Z_L - \omega^2 LC}{Z_L + j\omega L}$$

$$\boxed{Z_1 = \frac{Z_L + j\omega L}{1 + j\omega C Z_L - \omega^2 LC}}$$

if $\omega = L = C = Z_L = 1$

$$Z_1 = \frac{1+j}{1+j-1} = \frac{1+j}{j}\left(\frac{j}{j}\right) = \frac{j-1}{-1} = \boxed{1-j = Z_1}$$



$$Z_c = j\omega L + Z_1$$

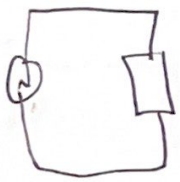
$$Z_c = \frac{Z_1 + j\omega L}{1 + j\omega C Z_c - \omega^2 LC} + j\omega L$$

[1]

$$Z_L = \frac{Z_L + jwL + jwL(1 + jwCZ_L - w^2LC)}{1 + jwCZ_L - w^2LC}$$

$$= \frac{Z_L + jwL + jwL - w^2LCZ_L - jw^3L^2C}{1 + jwCZ_L - w^2LC}$$

$$Z_L = \frac{Z_L + 2jwL - w^2LCZ_L - jw^3L^2C}{1 + jwCZ_L - w^2LC}$$



$$\frac{1}{Z_0} = jwC + \frac{1 + jwCZ_L - w^2LC}{Z_L + 2jwL - w^2LCZ_L - jw^3L^2C}$$

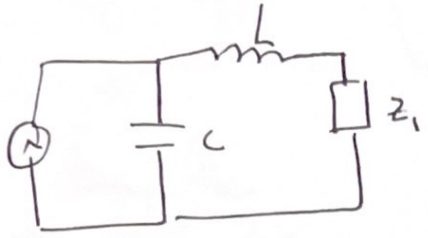
$$\frac{1}{Z_0} = \frac{jwCZ_L - 2w^2LC - jw^3LC^2Z_L + w^4L^2C^2 + jwCZ_L - w^2LC}{Z_L + 2jwL - w^2LCZ_L - jw^3L^2C}$$

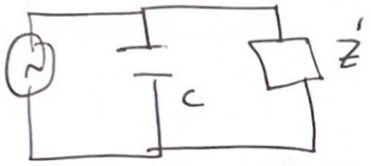
$$\boxed{Z_0 = \frac{Z_L + 2jwL - w^2LCZ_L - jw^3L^2C}{2jwCZ_L - 3w^2LC - jw^3LC^2Z_L + w^4L^2C^2 + 1}}$$

if $w = L = C = Z_L = 1$

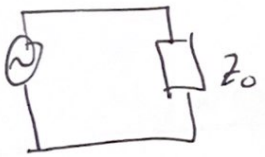
$$Z_0 = \frac{1 + 2j - 1 - j}{2j - 3 - j + 1 + 1} = \frac{j}{j-1}\left(\frac{j+1}{j+1}\right) = \frac{-1+j}{-1-1} = \boxed{\frac{1-j}{2} = Z_0}$$

- An alternative way to find Z_0 which we will use in parts-s





$$Z' = j\omega L + Z_1$$



$$\frac{1}{Z_0} = j\omega C + \frac{1}{j\omega L + Z_1}$$

$$\frac{1}{Z_0} = \frac{j\omega C(j\omega L + Z_1) + 1}{j\omega L + Z_1}$$

$$\boxed{Z_0 = \frac{j\omega L + Z_1}{1 - \omega^2 LC + j\omega C Z_1}}$$

Same as Z_1, but swapped Z_L for Z_1

$$\boxed{Z_{n+1} = \frac{j\omega L + Z_n}{1 - \omega^2 LC + j\omega C Z_n}}$$