8.2

1 $\vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{\partial t}$ $\quad\quad$ $\vec{\nabla}\times\vec{B} = \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$

$$\left(\cancel{\frac{\partial E_z}{\partial y}} - \cancel{\frac{\partial E_y}{\partial z}}\right)\hat{x} + \left(\cancel{\frac{\partial E_x}{\partial z}} - \frac{\partial E_z}{\partial x}\right)\hat{y} + \left(\frac{\partial E_y}{\partial x} - \cancel{\frac{\partial E_x}{\partial y}}\right)\hat{z} = \frac{-\partial\vec{B}}{\partial t}$$

$$\left(\cancel{\frac{\partial B_z}{\partial y}} - \cancel{\frac{\partial B_x}{\partial z}}\right)\hat{x} + \left(\cancel{\frac{\partial B_x}{\partial z}} - \frac{\partial B_z}{\partial x}\right)\hat{y} + \left(\frac{\partial B_y}{\partial x} - \cancel{\frac{\partial B_x}{\partial y}}\right)\hat{z} = \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$$

A $\left[\frac{-\partial E_z}{\partial x}\hat{y} + \frac{\partial E_y}{\partial x}\hat{z} = \frac{-\partial\vec{B}}{\partial t}\right.$ $\quad\quad$ $\frac{-\partial B_z}{\partial x}\hat{y} + \frac{\partial B_y}{\partial x}\hat{z} = \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$

$$\frac{-\partial E_z}{\partial x} = \frac{-\partial B_y}{\partial t} \quad\quad -\frac{\partial B_z}{\partial x} = \frac{1}{c^2}\frac{\partial E_y}{\partial t}$$

$$\frac{\partial E_y}{\partial x} = \frac{-\partial B_z}{\partial t} \quad\quad \frac{\partial B_y}{\partial x} = \frac{1}{c^2}\frac{\partial E_z}{\partial t}$$

Showing one Example

$$\frac{\partial}{\partial x}\left(-\frac{\partial E_z}{\partial x} = \frac{-\partial B_x}{\partial t}\right) \quad\quad \frac{\partial}{\partial t}\left(\frac{\partial B_y}{\partial x} = \frac{1}{c^2}\frac{\partial E_z}{\partial t}\right)$$

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{\partial^2 B_y}{\partial x\,\partial t} \quad\quad \frac{\partial^2 B_y}{\partial t\,\partial x} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2}$$

$$\boxed{\frac{\partial^2 E_z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2}}$$

The same can be done for E_y, B_y, B_z

2] using the equations marked A in the previous part (part 8.2.1)

$\hat{x}$: $0 = \frac{-\partial B_x}{\partial t}$ $\quad 0 = \frac{1}{c^2}\frac{\partial E_x}{\partial t}$

$\cancel{E_x = E}$ $\boxed{E_x = B_x = 0}$ (or constant)

3] part 1 uses a specific and nongeneral field for $\vec{E}$ and $\vec{B}$. They only rely on x. we can use the same method to derive the wave equations from Maxwell's equations.

Maxwell's eq :

$$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla} \times \left(\frac{-\partial \vec{B}}{\partial t}\right)$$

Gauss' Law $\rightarrow$ $\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = \frac{-\partial}{\partial t}(\vec{\nabla} \times \vec{B})$

$$-\nabla^2 \vec{E} = \frac{-\partial}{\partial t}\left(\frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}\right)$$

$$\boxed{\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \frac{1}{c^2}\nabla \times \frac{\partial \vec{E}}{\partial t}$$

$$\nabla(\vec{\nabla} \cdot \vec{B}) - \nabla^2 B = \frac{1}{c^2}\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{E})$$

$$-\nabla^2 \vec{B} = \frac{1}{c^2}\frac{\partial}{\partial t}\left(\frac{-\partial \vec{B}}{\partial t}\right)$$

$$\nabla^2 \vec{B} = \frac{+}{c^2}\frac{\partial \vec{B}}{\partial t}$$

$$\boxed{\nabla^2 \vec{B} = \frac{1}{c^2}\frac{\partial \vec{B}}{\partial t}}$$