Homework 11

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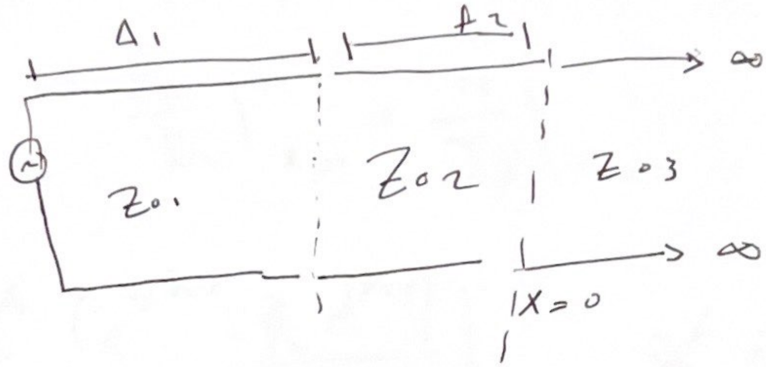

$Z_{02} = \frac{1}{2} Z_{01}$

$Z_{03} = \frac{1}{3} Z_{01}$

$\Delta_1 = \lambda_1$

$\Delta_2 = \frac{3\lambda_2}{4}$

$$\hat{V}_n(x) = \hat{V}_n^+ \left(e^{-j\beta_n x} + \hat{\rho}_n e^{j\beta_n x} \right)$$

$$\hat{I}_n(x) = \frac{\hat{V}_n^+}{Z_{0n}} \left(e^{-j\beta_n x} - \hat{\rho}_n e^{j\beta_n x} \right)$$

- Since region 3 goes to ∞, we assume that there is no wave traveling in the $-x$ direction

Region 3

$$\tilde{V}_3 = \tilde{V}_3^+ \left(e^{-j\beta_3 x} \right) \qquad \hat{I}_3 = \frac{\tilde{V}_3^+}{Z_{03}} \left(e^{-j\beta_3 x} \right)$$

Region 2

$$\tilde{V}_2 = \hat{V}_2^+ \left(e^{-j\beta_2 x} + \tilde{\rho}_2 e^{j\beta_2 x} \right)$$

$$\hat{I}_2 = \frac{\tilde{V}_2^+}{Z_{02}} \left(e^{-j\beta_2 x} - \hat{\rho}_2 e^{j\beta_2 x} \right)$$

Region 1

$$\hat{V}_1(x) = \hat{V}_1^+ \left(e^{-j\beta_1 x} + \hat{\rho}_1 e^{j\beta_1 x} \right)$$

$$\hat{I}_1(x) = \frac{\hat{V}_1^+}{Z_{01}} \left(e^{-j\beta_1 x} - \hat{\rho}_1 e^{j\beta_1 x} \right)$$

- Apply boundary conditions at the 2 and 3 interface

$V_2(x=0) = V_3(x=0) \qquad I_2(x=0) = I_3(x=0)$

$$\left.\frac{V_2}{I_2}\right|_{x=0} = \left.\frac{V_3}{I_3}\right|_{x=0}$$

$$\left.\frac{\cancel{\tilde{V}_2^+}\left(e^{-j\beta_2 x} + \hat{P}_2 e^{j\beta_2 x}\right)}{\frac{\cancel{\tilde{V}_2^+}}{Z_{02}}\left(e^{-j\beta_2 x} - \hat{P}_2 e^{j\beta_2 x}\right)}\right|_{x=0} = \left.\frac{\cancel{\tilde{V}_3^+}\,\cancel{e^{-j\beta_3 x}}}{\frac{\cancel{\tilde{V}_3^+}}{Z_{03}}\,\cancel{e^{-j\beta_3 x}}}\right|_{x=0}$$

$$\frac{Z_{02}(1+\hat{P}_2)}{1-\hat{P}_2} = Z_{03}$$

$$Z_{02}(1+\hat{P}_2) = Z_{03}(1-\hat{P}_2)$$

$$\hat{Z_0} + Z_{02}\hat{P}_2 = Z_{03} - Z_{03}\hat{P}_2$$

$$\hat{P}_2 + Z_{03}\hat{P}_2 = Z_{03} - Z_{02}$$

$$\boxed{\hat{P}_2 = \frac{Z_{03} - Z_{02}}{Z_{02} + Z_{03}}}$$

$$\tilde{P}_2 = \frac{\frac{1}{3}Z_{01} - \frac{1}{2}Z_{01}}{\frac{1}{2}Z_{01} + \frac{1}{3}Z_{01}} = \frac{\frac{1}{3} - \frac{1}{2}}{\frac{1}{2} + \frac{1}{3}} = \frac{2-3}{3+2} = -\frac{1}{5}$$

$$\boxed{\hat{P}_2 = \frac{-1}{5}}$$

- Boundary conditions at the interface of region 1 and 2

$x = -\Delta_2 = \frac{-3}{4}\lambda$

$$\left.\frac{\cancel{\tilde{V}_1^+}\left(e^{-jB_1x} + \tilde{\rho}_1 e^{jB_1x}\right)}{\frac{\cancel{\tilde{V}_1^+}}{Z_{01}}\left(e^{-jB_1x} - \tilde{\rho}_1 e^{jB_1x}\right)}\right|_{x=\Delta_2} = \left.\frac{\cancel{\tilde{V}_2^+}\left(e^{-jB_2x} + \hat{\rho}_2 e^{jB_2x}\right)}{\frac{\cancel{\tilde{V}_2^+}}{Z_{02}}\left(e^{-jB_2x} - \hat{\rho}_2 e^{jB_2x}\right)}\right|_{x=-\Delta_2}$$

$$\frac{Z_{01}\left(e^{+jB_1\Delta_2} + \tilde{p}_1 e^{-jB_1\Delta_2}\right)}{e^{+jB_1\Delta_2} - \tilde{\rho}_1 e^{-jB_1\Delta_2}} = \frac{Z_{02}\left(e^{+jB_2\Delta_2} + \tilde{\rho}_2 e^{-jB_2\Delta_2}\right)}{e^{+jB_2\Delta_2} - \hat{\rho}_2 e^{-jB_2\Delta_2}}$$

$$\left[Z_{02} = \frac{1}{2} Z_{01} \qquad \hat{\rho}_2 = \frac{-1}{5} \qquad \beta_2\Delta_2 = \frac{2\pi}{\cancel{\lambda_2}}\frac{3\cancel{\lambda_2}}{4} = \frac{3\pi}{4} \right.$$

$$\frac{\cancel{Z_{01}}\,\cancel{e^{+jB_1\Delta_2}}\left(1 + \tilde{p}_2 e^{-2jB_1\Delta_2}\right)}{\cancel{e^{+jB_1\Delta_2}}\left(1 - \tilde{p}_1 e^{-2jB_1\Delta_2}\right)} = \frac{\frac{1}{2}\cancel{Z_{01}}\left(-i + \left(\frac{-1}{5}\right)i\right)}{-i - \left(\frac{-1}{5}\right)i}$$

$$\frac{\left(1 + \tilde{p}_1 e^{-2jB_1\Delta_2}\right)}{\left(1 - \tilde{p}_1 e^{-2jB_1\Delta_2}\right)} = \frac{1}{2}\frac{i}{i}\left(\frac{-1 - \frac{1}{5}}{-1 + \frac{1}{5}}\right) = \frac{1}{2}\frac{(-5-1)}{(-5+1)} = \frac{1}{2}\left(\frac{-6}{4}\right) = \frac{3}{4}$$

$$1 + \tilde{p}_1 e^{-2jB_1\Delta_2} = \frac{3}{4} + \frac{3}{4}\tilde{p}_1 e^{-2jB_1\Delta_2}$$

$$\tilde{p}_1\left(e^{-2jB_1\Delta_2} + \frac{3}{4}e^{-2jB_1\Delta_2}\right) = \frac{3}{4} - 1 = \frac{-1}{4}$$

$$\tilde{p}_1 = \frac{-1}{4e^{-2jB_1\Delta_2}\left(1 + \frac{3}{4}\right)} = \frac{-1}{7}e^{2jB_1\Delta_2}$$

$$\tilde{P}_1 = \frac{-1}{7} e^{2jB_1 \Delta z}$$

$\lambda_1 = 10\,cm$, $\lambda_2 = 5\,cm$, $\frac{\lambda_1}{\lambda_2} = 2$

$$B_1 \Delta z = \frac{2\pi}{\lambda_1} \frac{3\lambda_2}{4} = \frac{3\pi}{2} \frac{\lambda_2}{\lambda_1} = \frac{3\pi}{4}$$

$$\tilde{P}_1 = \frac{-1}{7} e^{2j\frac{3\pi}{4}} = -\frac{1}{7} e^{j\frac{3\pi}{2}} = -\frac{1}{7}(-j) = \boxed{\frac{1}{7} j = \tilde{P}_1}$$

- Solve for amplitudes $\hat{V}_1^+, \hat{V}_2^+, \hat{V}_3^+$

$\boxed{\tilde{V}_1^+ = \hat{V}_s}$ The right region (1) has the amplitude of the source V

- at $x = -\Delta_2$ boundary

$$\hat{V}_1(x = -\Delta_2) = \hat{V}_2(x = -\Delta_2)$$

$$\tilde{V}_1^+ \left(e^{-j\beta_1 x} + \hat{\rho}_1 e^{j\beta_1 x} \right) \Big|_{x = -\Delta_2} = \hat{V}_2^+ \left(e^{-j\beta_2 x} + \tilde{\rho}_2 e^{j\beta_2 x} \right) \Big|_{x = \Delta_2}$$

$$\left[\tilde{V}_1^+ = \tilde{V}_s \qquad \tilde{\rho}_1 = \frac{j}{7} \qquad \hat{\rho}_2 = \frac{-1}{5} \right.$$

$$\tilde{V}_s \left(e^{j\beta_1 \Delta_2} + \frac{j}{7} e^{-j\beta_1 \Delta_2} \right) = \tilde{V}_2^+ \left(e^{j\beta_2 \Delta_2} - \frac{1}{5} e^{-j\beta_2 \Delta_2} \right)$$

$$\left\{ \beta_1 \Delta_2 = \frac{2\pi}{\lambda_1} \frac{3\lambda_2}{4} = \frac{3\pi}{2} \frac{5}{10} = \frac{3\pi}{4} \qquad \beta_2 \Delta_2 = \frac{2\pi}{\lambda_2} \frac{3\lambda_2}{4} = \frac{3\pi}{2} \right.$$

$$\tilde{V}_s \left(e^{j3\pi/4} + \frac{j}{7} e^{-j\frac{3\pi}{4}} \right) = \tilde{V}_2^+ \left(e^{j\frac{3\pi}{2}} - \frac{1}{5} e^{-j\frac{3\pi}{2}} \right)$$

$$\tilde{V}_s \left[\left(\frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}} \right) + \frac{j}{7} \left(\frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}} \right) \right] = \tilde{V}_2^+ \left(-j - \frac{1}{5} j \right)$$

$$\frac{\hat{V}_s}{\sqrt{2}} \left[-1 + j - \frac{j}{7} + \frac{1}{7} \right] = \frac{-6j}{5} \tilde{V}_2^+$$

$$\tilde{V}_2^+ = \left(\frac{-5}{6j}\right)\frac{\hat{V}_s}{\sqrt{2}}\left(\frac{6j}{7} - \frac{6}{7}\right) = \frac{-5}{7j\sqrt{2}}\hat{V}_s\,(j-1)\left(\frac{j}{j}\right)$$

$$\tilde{V}_2^+ = \frac{5}{7\sqrt{2}}\tilde{V}_s\,(-1-j)$$

$$\boxed{\hat{V}_2^+ = \frac{-5}{7\sqrt{2}}\tilde{V}_s\,(1+j)}$$

$\Rightarrow$ $x = 0$ Boundary

$$\hat{V}_2(x=0) = \hat{V}_3(x=0)$$

$$\tilde{V}_2^+\left(e^{-j\beta_2 x} - \frac{1}{5}e^{j\beta_2 x}\right) = \tilde{V}_3^+ e^{-j\beta_3 x}\Big|_{x=0}$$

$$\hat{V}_2^+\left(1 - \frac{1}{5}\right) = \tilde{V}_3^+$$

$$\tilde{V}_3^+ = \left(\frac{4}{\cancel{5}}\right)\left(\frac{-\cancel{5}}{7\sqrt{2}}\tilde{V}_s\,(1+j)\right)$$

$$\boxed{\tilde{V}_3^+ = \frac{-4}{7\sqrt{2}}\tilde{V}_s\,(1+j)}$$

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$$\tilde{\rho}_2 = \frac{\frac{Z_{03}}{Z_{02}} - 1}{\frac{Z_{03}}{Z_{02}} + 1} \Rightarrow \frac{Z_{03}}{Z_{02}} = \frac{1/3}{1/2} = \frac{2}{3} + 0j$$

from smith chart $\boxed{\tilde{\rho}_2 = \frac{-1}{5}}$

$$\tilde{\rho}_1 = \frac{\frac{Z_{02}}{Z_{01}} - 1}{\frac{Z_{02}}{Z_{01}} + 1} \Rightarrow \frac{Z_{02}}{Z_{01}} = \frac{1/2}{1} = \frac{1}{2} + 0j$$

from smith chart $\boxed{\tilde{\rho}_1 = \frac{1}{3}}$