Homework 9

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II

$\vec{\tilde{B}} = \frac{1}{v}\hat{k} \times \vec{\tilde{E}}$

$\vec{\tilde{E}} = \tilde{E}_y$

$\vec{\tilde{B}} = \sqrt{\mu\epsilon}\ \hat{k}_x \tilde{E}_y \hat{z}$

Incident: $\hat{k} = \frac{k_1 \hat{x}}{k_1} = \hat{x}$

Reflected: $\hat{k} = \frac{-k_1 \hat{x}}{k_1} = -\hat{x}$

Transmitted: $\hat{k} = \frac{k_2 \hat{x}}{k_2} = \hat{x}$

$\frac{1}{v} = \sqrt{\mu\epsilon}$

$$\boxed{\begin{aligned} &\tilde{B}_{oI} = \sqrt{\mu_1\epsilon_1}\ \tilde{E}_{oI} \qquad \tilde{B}_{oT} = \sqrt{\mu_2\epsilon_2}\ \tilde{E}_{oT} \\ &\tilde{B}_{oR} = -\sqrt{\mu_1\epsilon_1}\ \tilde{E}_{oR} \end{aligned}}$$

$\tilde{E}_y^1(0,t) = \tilde{E}_y^2(0,t)$

$\left(\tilde{E}_{oI} e^{ik_1x} \cancel{e^{-i\omega t}} + \tilde{E}_{oR} e^{-ik_1x} \cancel{e^{-i\omega t}} = \tilde{E}_{oT} e^{ik_2x} \cancel{e^{-i\omega t}}\right)\Big|_{x=0}$

(1) $\tilde{E}_{oI} + \tilde{E}_{oR} = \tilde{E}_{oT}$

$\frac{1}{\mu_1} \tilde{B}_z^1(0,t) = \frac{1}{\mu_2} \tilde{B}_z^2(0,t)$

$\frac{1}{\mu_1}\left(\tilde{B}_{oI} + \tilde{B}_{oR}\right) = \frac{1}{\mu_2} \tilde{B}_{oT}$

$\frac{\sqrt{\mu_1\epsilon_1}}{\mu_1}\left(\tilde{E}_{oI} - \tilde{E}_{oR}\right) = \frac{\sqrt{\mu_2\epsilon_2}}{\mu_2} \tilde{E}_{oT}$

$\frac{\sqrt{\epsilon_1/\mu_1}}{\sqrt{\epsilon_2/\mu_2}}\left(\tilde{E}_{oI} - \tilde{E}_{oR}\right) = \tilde{E}_{oT}$

$\beta^{-1}\left(\tilde{E}_{oI} - \tilde{E}_{oR}\right) = \tilde{E}_{oT}$

$\tilde{E}_{oI} - \tilde{E}_{oR} = \beta \tilde{E}_{oT}$

(2)

- by combing equations 1 and 2

$$\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}$$
$$\tilde{E}_{0I} - \tilde{E}_{0R} = \beta \tilde{E}_{0T}$$

$$2\tilde{E}_{0I} = (1+\beta)\tilde{E}_{0T}$$

$$\boxed{\tilde{E}_{0T} = \frac{2}{1+\beta}\tilde{E}_{0I} = \tau \tilde{E}_{0I}}$$

$$\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}$$
$$\tilde{E}_{0I} + \tilde{E}_{0R} = \frac{2}{1+\beta}\tilde{E}_{0I}$$
$$\frac{1+\beta}{1+\beta}\tilde{E}_{0I} - \frac{2}{1+\beta}\tilde{E}_{0I} = -\tilde{E}_{0R}$$

$$-\frac{1+\beta}{1+\beta}\tilde{E}_{0I} = -\tilde{E}_{0R}$$

$$\boxed{\tilde{E}_{0R} = \frac{1-\beta}{1+\beta}\tilde{E}_{0I}} = \rho\,\tilde{E}_{0I}$$

Finally

$$\boxed{\begin{aligned} \tilde{E}_y^1 &= \tilde{E}_{0I}\left(e^{ik_1x} + \rho e^{-ik_1x}\right)e^{-i\omega t} & \tilde{E}_y^2 &= \tau \tilde{E}_{0I}\, e^{ik_2x} e^{-i\omega t} \\ \tilde{B}_z^1 &= \tilde{E}_{0I}\sqrt{\mu_1\epsilon_1}\left(e^{ik_1x} - \rho e^{-ik_1x}\right)e^{-i\omega t} & \tilde{B}_z^2 &= \tau \tilde{E}_{0I}\sqrt{\mu_2\epsilon_2}\, e^{ik_2x} e^{-i\omega t} \end{aligned}}$$