Homework 8
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8.1

$$E_x = A_1\cos(\omega t - kx + \delta_1) + A_2\cos(\omega t - kx + \delta_2)$$

using $\cos(x+\delta) = \cos(x)\cos(\delta) - \sin(x)\sin(\delta)$ $\quad x = \omega t - kx$

$$E_x = A_1(\cos x\cos\delta_1 - \sin x\sin\delta_1) + A_2(\cos x\cos\delta_2 - \sin x\sin\delta_2)$$

$$E_x = (A_1\cos\delta_1 + A_2\cos\delta_2)\cos x - (A_1\sin\delta_1 + A_2\sin\delta_2)\sin x$$

$$E_x = \alpha_1\cos x + \alpha_2\sin x$$

$$\alpha_1 = A_1\cos\delta_1 + A_2\cos\delta_2$$
$$\alpha_2 = -A_1\sin\delta_1 - A_2\sin\delta_2$$

using $\quad a\cos x + b\sin x = c\cos(x+\phi)$

$$c = \operatorname{sgn}(a)\sqrt{a^2+b^2}$$
$$\phi = \arctan(-b/a)$$

$$c = \sqrt{(A_1\cos\delta_1 + A_2\cos\delta_2)^2 + (A_1\sin\delta_1 + A_2\sin\delta_2)^2}$$

$$= (A_1^2\cos^2\delta_1 + A_2^2\cos^2\delta_2 + 2A_1A_2\cos\delta_1\cos\delta_2 + A_1^2\sin^2\delta_1 + A_2^2\sin\delta_2 + 2A_1A_2\sin\delta_1\sin\delta_2)^{\frac{1}{2}}$$

$$= (A_1^2 + A_2^2 + 2A_1A_2(\cos\delta_1\cos\delta_2 + \sin\delta_1\sin\delta_2))^{\frac{1}{2}}$$

$$= (A_1^2 + A_2^2 + A_1A_2(\cos(\delta_1-\delta_2) + \cos(\delta_1+\delta_2) + \cos(\delta_1-\delta_2) + \cos(\delta_1+\delta_2)))^{\frac{1}{2}}$$

$$= \left[A_2^2 + A_1^2 + 2A_1A_2\cos(\delta_1-\delta_2)\right]^{\frac{1}{2}}$$

$$\phi = \arctan(-b/a)$$

$$= \arctan\left[\frac{A_1 \sin\delta_1 + A_2 \sin\delta_2}{A_1 \cos\delta_1 + A_2 \cos\delta_2}\right]$$

$$E_x = \left[A_1^2 + A_2^2 + 2A_1A_2\cos(\delta_1 - \delta_2)\right]^{\frac{1}{2}} \cos\left(\omega t - kx + \arctan\left[\frac{A_1 \sin\delta_1 + A_2 \sin\delta_2}{A_1 \cos\delta_1 + A_2 \cos\delta_2}\right]\right)$$