8.3

1) $\vec{E} = E_{0x}\cos(kz - \omega t + \delta_x)\,\hat{x}$

$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

$$-\cancel{E_{0x}}\,k^2 \cancel{\cos(kz-\omega t+\delta_x)} = \frac{1}{c^2}(-1)\,\omega^2 \cancel{E_{0x}}\cancel{\cos(kz-\omega t+\delta_x)}$$

$$-k^2 = -\frac{\omega^2}{c^2} \Rightarrow \boxed{k_z^2 = \frac{\omega^2}{c^2}}$$

$$\vec{B} = B_{0x}\cos(kz-\omega t+\delta_x')\,\hat{x} + B_{0y}\cos(kz-\omega t+\delta_y')\,\hat{y}$$

$$\nabla^2 \vec{B} = \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}$$

$\hat{x}$: $-\cancel{B_{0x}}\,k^2\cancel{\cos(kz-\omega t+\delta_x')} = \frac{-1}{c^2}\cancel{B_{0x}}\,\omega^2\cancel{\cos(kz-\omega t+\delta_x')}$

$$\boxed{k_z^2 = \frac{\omega^2}{c^2}}$$

$\hat{y}$: $-\cancel{B_{0y}}\,k^2\cancel{\cos(kz-\omega t+\delta_y')} = \frac{-1}{c^2}\cancel{B_{0y}}\,\omega^2\cancel{\cos(kz-\omega t+\delta_y')}$

$$\boxed{k_z^2 = \frac{\omega^2}{c^2}}$$

2)

$$\vec{E} = E_{0x}\cos(kz-\omega t+\delta_x)\,\hat{x}$$

$$\vec{B} = B_{0x}\cos(kz-\omega t+\delta_x')\,\hat{x} + B_{0y}\cos(kz-\omega t+\delta_y')\,\hat{y}$$

① $\vec{\nabla}\cdot\vec{E} = 0$

② $\vec{\nabla}\cdot\vec{B} = 0$

③ $\vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{\partial t}$

④ $\vec{\nabla}\times\vec{B} = \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$

① $\vec{\nabla}\times\vec{E} = 0$

$$\frac{\partial E_x}{\partial x} = 0$$

$$0 = 0$$

② $\vec{\nabla}\cdot\vec{B} = 0$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0$$

$$0 + 0 = 0$$

③ $\vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{\partial t} \Rightarrow \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{x} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{y} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{z} = \frac{-\partial\vec{B}}{\partial t}$

$$\frac{\partial E_x}{\partial z}\hat{y} = \frac{-\partial\vec{B}}{\partial t}$$

$\hat{x}$: $\quad 0 = +B_{0x}\sin(kz-\omega t+\delta_x')(-\omega)$

$$\boxed{B_{0x} = 0}$$

$\hat{y}$: $\quad +E_{0x}k\sin(kz-\omega t+\delta_x) = +B_{0y}(+\omega)(+1)\sin(kz-\omega t+\delta_y')$

$$\boxed{\delta_x = \delta_y'}$$

$$E_{0x}k = B_{0y}\omega$$

$$\boxed{E_{0x} = cB_{0y}}$$

④ $\vec{\nabla}\times\vec{B} = \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \Rightarrow \frac{\partial B_y}{\partial z}\hat{x} = \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$

$\hat{x} = +B_{0y}k(+1)\sin(kz-\omega t+\delta_x) = \frac{1}{c^2}E_{0x}(+\omega)(+1)\sin(kz-\omega t+\delta_x)$

$$\boxed{k_z = \frac{\omega}{c}}$$

3)

$$\vec{E} = E_{0y} \cos(kz - \omega t + \delta_y)\hat{y}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$\hat{x}$: $\boxed{\begin{array}{l} -E_{0y} = cB_{0x} \\ \delta_y = \delta_x' \end{array}}$ $\hat{y}$: $\boxed{B_{0y} = 0}$

4) $\vec{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}$

$$\vec{B} = \frac{1}{c} \hat{k} \times \vec{E}$$

$$\vec{B} = \frac{1}{c}\left[(\hat{k}_y E_z - \hat{k}_z E_y)\hat{x} + (\hat{k}_z E_x - \hat{k}_x E_z)\hat{y} + (\hat{k}_x E_y - \hat{k}_y E_x)\hat{z}\right]$$

⇒ Testing part 2

$\hat{y}$: $B_y = \frac{1}{c}\hat{k}_z E_x$ $k = k_z \quad (k_x = k_y = 0)$

$$\cancel{B_0}_y \cancel{\cos}(k_z z - \omega t + \delta_x) = \frac{1}{\cancel{c}} \frac{k_z}{k} \cancel{c} \cancel{B_0}_y \cos(k_z z - \omega t + \delta_x)$$

$\boxed{\checkmark}$

⇒ Testing part 3

$\hat{x}$: $B_x = -\frac{1}{c}\hat{k}_z E_y$

$$\cancel{B_0}_x \cancel{\cos}(k_z z - \omega t + \delta_y) = +\frac{1}{\cancel{c}} \frac{\cancel{k_z}}{\cancel{k}} (+\cancel{c} \cancel{B_0}_x) \cos(\cancel{k_z} z - \omega t + \delta_y)$$

$\boxed{\checkmark}$