<u>Homework 1</u>

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1. From least to greatest

$f_1(n) = O(n^2 \log n)$ $\qquad f_6$

$f_2(n) = O(n^2)$ $\qquad f_2$

$f_3(n) = O(n^2)$ $\qquad f_3$

$f_4(n) = O(a^n)$ $\qquad f_1$

$f_5(n) = O(a^n)$ $\qquad f_4$

$f_6(n) = O(n \log n)$ $\qquad f_5$

2. $\sum_{i=1}^{n} i(i!) = (n+1)! - 1$

Base: $n=1$ $\quad 1 = 1$ ✓

Assume: $i(i!) = (n+1)! - 1$

Prove: $\sum_{i=1}^{n+1} i(i!) = (n+2)! - 1$

$$\underbrace{\sum_{i=1}^{n} i(i!)}_{=(n+1)!-1} + n+1((n+1)!) = (n+1)! - 1 + (n+1)!(n+1)$$

$$= (n+1)!\,(1 + n + 1) - 1$$

$$= (n+1)!\,(n+2) - 1$$

$$= (n+2)! - 1 \quad ✓$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Base: $n=1$ $\quad 1^3 = 1^2$ ✓

Assume: $i^3 = \left[\frac{n(n+1)}{2}\right]^2$

Prove: $\sum_{i=1}^{n+1} i^3 = \left[\frac{n+1(n+2)}{2}\right]^2 = \frac{n^2(n+1)^2}{4}$

$$\underbrace{\sum_{i=1}^{n} i^3}_{=\left[\frac{n(n+1)}{2}\right]^2} + (n+1)^3 = \left[\frac{n(n+1)}{2}\right]^2 + (n+1)^3$$

$$= \left[\frac{n^2(n+1)^2}{4}\right] + (n+1)^3$$

$$= (n+1)^2\left(\frac{n^2}{4} + (n+1)\right)$$

$$= \left[\frac{(n+1)^2(n^2-4n+4)}{4}\right]$$

$$= \left[\frac{(n+1)^2(n+2)^2}{4}\right]$$

$$= \left[\frac{(n+1)(n+2)}{2}\right] ✓$$

3. a) n^3 is the highest order term, and $n^3 \leq n^3$, so $\in O(n^3)$.

b) n^3 is the highest order term, and $n^3 \geq n^2$, so $\in \Omega(n^2)$.

c) $n!$ is the highest order term, and $n! \leq n^n$ (although seems to be debated and I could not find many examples of $O(n^n)$ functions) so $\in O(n^n)$

d) n^2 is the highest order function, and $n^2 \geq n$ clearly, so $n^2 \notin O(n)$.