

Homework 4

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Q1

- a) Merge Sort is a stable sorting algorithm. When splitting up all the elements, duplicate elements will maintain the order they were in. Two '4's placed in an array, when sorted, will be in order from left to right as they were found.
- b) Quick Sort is not a stable sorting algorithm. It compares values to the pivot, disregarding the adjacent values (or, their order, I should say). Merge Sort assumes $\text{Left} \leq \text{Right}$, while Quick Sort does not. So, those two '4's may not be in the same order.
- c) Arrays with all equal elements are worst-case scenarios because the algorithm has to traverse the entire array to make comparisons, even if it doesn't swap elements.
- d) Descending elements are also a worst case scenario, as things are definitely out of order. The entire array must be traversed to compare all elements AND swap them.

Q2/

See problem2.cpp

Q3/

$$I\{A\} \begin{cases} 1: \text{correct bag} \\ 0: \text{incorrect bag} \end{cases}$$

$$X_A \rightarrow E[X_A] = \Pr\{A\}$$

$$E[X_A] = 1 \cdot \Pr(\text{correct}) + 0 \cdot \Pr(\text{incorrect})$$

$$\text{1st cust. } 1 \cdot \frac{1}{n} + 0 \cdot \frac{n-1}{n} = \frac{1}{n}$$

$$\text{2nd cust. } + (1 \cdot \frac{1}{n} + 0 \cdot \frac{n-1}{n}) = \frac{1}{n}$$

\vdots

$$\text{nth cust. } + (1 \cdot \frac{1}{n} + 0 \cdot \frac{n-1}{n}) = \frac{1}{n}$$

Sum all of these up for n customers.

$$\frac{1}{n} + \frac{1}{n} + \dots + \frac{1}{n} \rightarrow n\left(\frac{1}{n}\right) = 1$$

This means the expected number of visitors getting their bag back is one.

Q4/

$$I\{P\} \begin{cases} \$3: \text{working} \\ -\$80: \text{defective} \end{cases}$$

$$E[X_P] = 3 \cdot \Pr(W) - 80 \cdot \Pr(D)$$

$$3 \cdot \frac{49}{50} - 80 \cdot \frac{1}{50}$$

$$2.94 - 1.60 = \$1.34 \text{ profit}$$

(for any given toy)