Simon Fraser University Surrey

Department of Mechatronic Systems Engineering

MSE 221 Project Report

Strength Analysis and Design of Bicycle Frame

Submitted to:

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Objective

To evaluate a bicycle frame and suggest a new model to improve its design

Introduction

The invention of the wheel is one of the greatest in the history of mankind. This led to the invention of bicycles, which are still used after its initial inauguration 327 years ago. Today, bicycles are not just used as a means of transportation, but also as a means of recreational sport. The bicycle has undergone vast improvements over time in terms of design, material, aerodynamics, strength and mechanics. The focus of this project is to analyze a bicycle and provide some improvement to it in terms of design and mechanics.

Theory

A bicycle has many components due to which it can fail. Generally, spokes and axles have a high chance of failure, but our main concern is the frame of the bicycle. A bicycle frame consists of a head tube, fork, top tube, down tube, seat tube, seat stay and chain stay. These can be seen in the Figure 1.

Figure 1: Bike Frame Used for Analysis

The frame is a major part of the bicycle; as such, it is important that the frame be strong. To simplify the analysis, the tubes of the bike will be assumed to be trusses and beams. As we look towards the criteria for the failure of the frame, we need to calculate fatigue stress, S_{FL}; this value depends directly on the loading cycle. We can calculate fatigue stress by:

$$S_{FL} = S_f{}'(NFL)^b \ (\mathbf{1})$$

where S_f' and b are the material's stress-cycle curve's intercept and slope respectively. *NFL* is the number of loading cycles. Because the bike frame is made of "Steel 4130 BHN=259", the values of S_f', b, and *NFL* will be taken as 1195 MPa, -0.077, and 10^6 respectively [2]. To understand where the frame will likely fail, the forces and stress in each of the bike's tubes must be calculated. To do so, we assume stress, σ, is equal to:

$$\sigma = \frac{F}{A} \ (\mathbf{2})$$

where F is the force in the supposed member, and A is the member's cross-sectional area. To be suitable for use, the safety factor, *FS*, in each beam should be above 3. To find the safety factor:

$$FS = \frac{\sigma_U}{\sigma_A} \ (\mathbf{3})$$

Where σ_U and σ_A and are the respective ultimate and allowable stress in the member. Due to the complexity of finding σ_U, S_{FL} will used instead. Therefore, Equation 3 can be rewritten as:

$$FS = \frac{S_{FL}}{\sigma_A} \ (\mathbf{4})$$

Initial Design

Table 1: Truss Analysis, All Tubes Present

Frame Part	Axial Force in Part, *F* (lb, Tension)	Axial Stress in Part, σ (psi, Tension)	Safety Factor
Head Tube	-135	-525	107
Fork	-135	-1020	55
Top tube	-115	-550	102
Down tube	105	408	138
Seat tube	-110	-430	131
Seat stays	-299	-7275	7.74
Chainstays	155	1682	33.5

Table 2: Truss and Beam Analysis, Top Tube Removed

Frame Part	Axial Stress in Part, σ (psi, Tension)	Shear Stress In Part, σ (psi, Tension)	Safety Factor	Bending Moment, *M* (in•lb)
Head tube	-	841	66.9	-2610
Fork	-	841	66.9	-5290
Down tube	-	-841	66.9	-5290
Seat tube	-	-841	66.9	-2610
Seat stays	-7280	-	7.74	-
Chain stays	1680	-	33.5	-

Table 3: Beam Analysis, All Tubes Present

Frame Part	Axial Force in Part, *F* (lb, Tension)	Axial Stress in Part, *σ* (psi, Tension)	Safety Factor	Bending Moment, *M* (in•lb)	Max Bending Stress, σ_b *(psi)*
Head Tube	-80	-310	37.7	429	10.4
Fork	-118	-714	83.6	678	32
Top Tube	-103	-494	121	50.0	1.64
Down Tube	131	507	118	249	6.05
Seat Tube	-126	-486	123	136	3.31
Seat Stays	-280	-6830	8.76	34	13.0
Chainstays	148	159	37.65	275	30.9

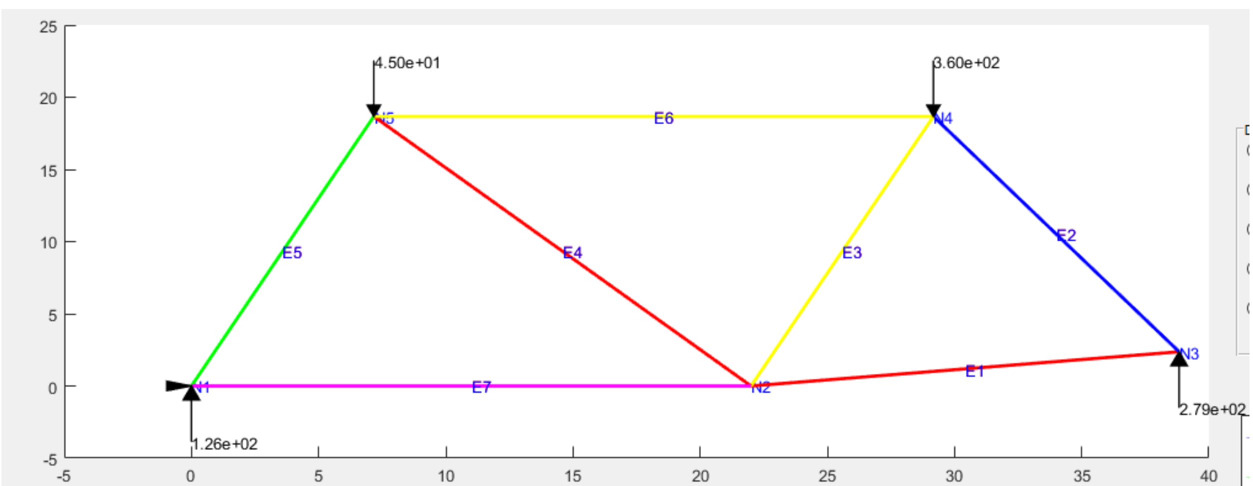


Figure 2: Axial Stress in Frame, FEA Analysis

Figure 2 shows the bicycle frame as ideal trusses, with one more truss connecting fork and downtube joints. The warmer colors show greater axial force compared to colder ones; it is clear that downtube and chain stays bear highest force.

Discussion

Comparing the data of Table 1 to Table 2, it can be seen that the frame is a lot safer in the first scenario than the second due to the presence of the top tube. The top tube stabilizes the frame, reducing the horizontal stress in the frame and decreasing the overall stress in the other trusses and beams. As the result, we can see that the safety factor increases when the top tube is present. Table 3 shows the most realistic analysis of the frame. The third scenario treats the whole structure as a beam structure, where forces can be applied along each part. Comparing the safety factor in the third scenario with the safety factors in other parts, it can be seen clearly that the beam analysis has the largest safety factor, thus proving that the structure is safe.

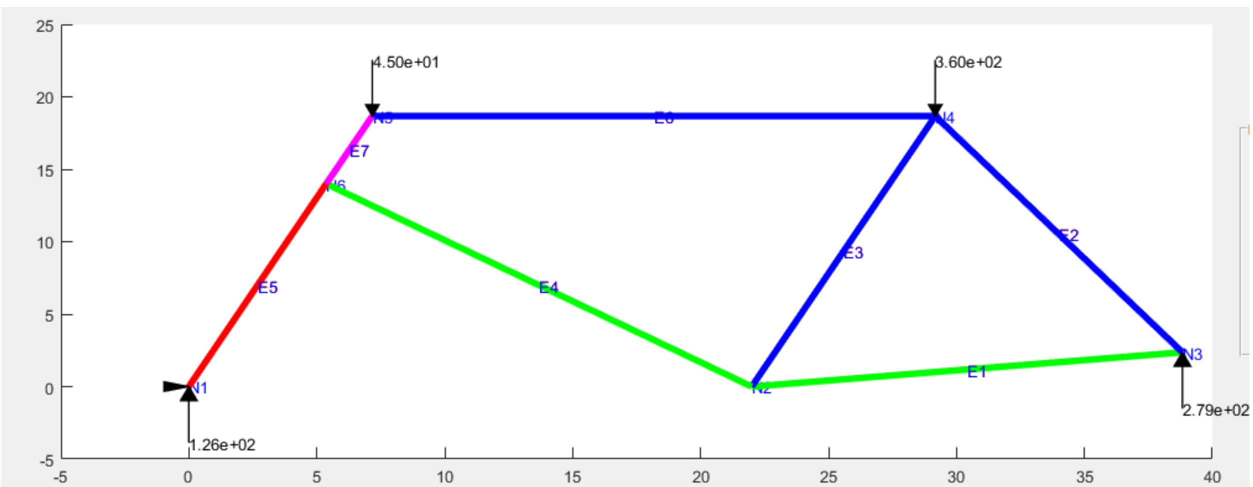


Figure 3: Bending Moment in Frame, FEA Analysis

Figure 3 ranks the bending moment ranking in each beam, with red indicating a high value and blue indicating a small one. After calculating the safety factor, the bars with lowest values are the chain stays, seat stays and forks. Specifically, the chain stays have a safety factor 3 to 4 times lower than that of the parts with values over 100, and it is only around 8.5 for the seat stays, resulting from sustaining most of the cycler's weight. The bending moment is highest in the fork due to the constraint at one end and the connections of the head and down tubes at the other end. Therefore, a new design that will lower the moment in this beam is desired.

If the weight of the frame is to be reduced, the cross-sectional area of the beams can be decreased within the margin of the safety factor. Changing the material with a high strength to weight ratio can also be a good way to reduce weight. Carbon fibre reinforced polymers [CFRP]) is an ideal candidate.

Redesign

Due to high bending moment in the fork (E5) of Figure 3, the main objective is to reduce the moment in this beam and divide stress more equally between beams. The angle between head tube-fork section and down tube can be decreased from 60° to 50° to reduce the horizontal force component in the headtube-fork, as well as the bending moment. Because bending moment and length have an inversely proportional relationship (as per Equation 5), the length of the beam can be increased from 15" to 18" to reduce maximum strain, resulting in decreasing maximum stress and bending moment.

$$M = -\frac{\delta EI}{yL} \ (\mathbf{5})$$

Locations of different joints are also adjusted to maintain dynamic structure. The new design and bending moment in each part is as follows:

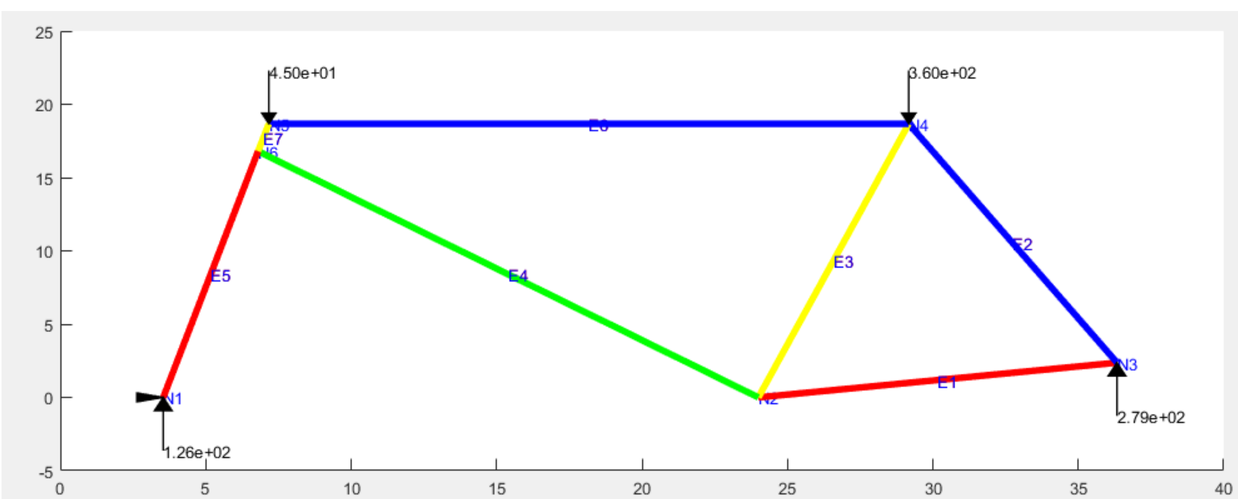


Figure 4: Bending Moment in Redesigned Frame

Table 4: Redesign Beam Analysis, All Tubes Present

Frame Part	Axial Force in Part, *F* (lb, Tension)	Axial Stress in Part, σ (psi, Tension)	Safety Factor	Bending Moment, *M* (lb.in)	Max Bending Stress, σ_b *(psi)*
Head Tube	-57.0	-221	255	207	5.04
Fork	-117	-706	79.8	387	18.6
Top Tube	-77.6	-371	152	78.1	2.56
Down Tube	109	421	134	180	4.38
Seat Tube	-134	-520.	108	194	470
Seat Stays	-254	-6190	9.10	60.9	23.0
Chainstays	114	1230	45.8	374	42

The new moment for the fork (E5) is now recorded at 386.7 lb.in, which shows a great improvement in safety from previous design. In the downtube (E4), the moment is decreased from 249 to 180.4 lb.in, and it is lessened by more than 50% in seat stays (E7). Nevertheless, the safety factors in other parts are still satisfied, as their moments are not much different from previous results.

If more changes are allowed, a different material is probably effective in lowering risk level. As elasticity modulus is increased, maximum stress and moment decrease; therefore a Young's modulus higher than 200 GPa is desired. Hexcel UHM carbon fiber (3000 filaments) [3] is a suitable candidate, as it has a modulus of 440 GPa.

Conclusion

This era of materials has revolutionized the design of bicycles. In the design of a bicycle, there are various factor to consider like aerodynamics, material, stress and toughness. The design of the bike will also be influenced by its purpose, whether it is to be used for commuting, racing, mountain biking, etc. We also see that the frame is a fairly safe part of the bike; other parts (i.e. gears, shifters, brake pads) have higher chances of failure.

References

[1] Beer, F. P., Johnston Jr., E. R., DeWolf, J. T., & Mazurek, D. F. (2015). *Mechanics of Materials*. New York: McGraw-Hill Education.

[2] Material Property Finder. (n.d.). Retrieved November 18, 2017, from https://www.efatigue.com/constantamplitude/stresslife/materials/#a

[3] MatWeb. (n.d.). Retrieved November 28, 2017, from http://www.matweb.com/index.aspx

Appendix

Table 5: Bike Dimensions

Frame Part	Length, L (± 1 in)	Outer Diameter, d_{out} (± 0.1 in)	Thickness, t (in)	Inner diameter, d_{in} (in)	Cross-sectional Area (in²)	Moment of Inertia, I (in⁴)
Head Tube	5	1.25	$0.13\overline{8}$	$1.\overline{1}$	0.258	1.44
Fork	15	1.25	$0.13\overline{8}$	$1.\overline{1}$	0.258	1.44
Top tube	22	1.125	0.125	1	0.209	0.945
Down tube	24	1.25	$0.13\overline{8}$	$1.\overline{1}$	0.258	1.44
Seat tube	20	1.25	$0.13\overline{8}$	$1.\overline{1}$	0.258	1.44
Seat stays	19	0.5	$0.0\overline{5}$	$0.\overline{4}$	0.0412	0.0369
Chainstays	17	0.75	$0.08\overline{3}$	0.6	0.0927	0.187

Sample Calculations

Fatigue Stress

$$S_{FL} = S_f{}'(NFL)^b$$

$$S_{FL} = (1195\times10^6)(10^6)^{-0.077}$$

$$S_{FL} = (388\ldots\times10^6\text{ Pa})\left(\frac{1.45038\times10^{-4}\,psi}{1\,Pa}\right)$$

$$S_{FL} \approx 56.3\text{ ksi}$$

Thickness

Thickness of the tubes was found using the standard dimension ratio (SDR), which is a means of rating a pipe's durability against pressure. The SDR values of all the tubes were assumed to be 9:

$$SDR = \frac{d_{out}}{t}$$

$$t = \frac{d_{out}}{SDR}$$

$$t = \frac{1.25}{9}$$

$$t = 0.13\bar{8}\ in$$

Inner Diameter

$$d_{in} = d_{out} - t$$

$$d_{in} = 1.25 - 0.13\bar{8}$$

$$d_{in} = 1.\bar{1}\ in$$

Cross Sectional Area

$$A = \frac{\pi}{4}\left({d_{out}}^2 - {d_{in}}^2\right)$$

$$A = \frac{\pi}{4}(1.25^2 - 1.\bar{1}^2)$$

$$A \approx 0.258\ in^2$$

Moment of Inertia

$$I = \frac{\pi}{2}\left({d_{out}}^4 - {d_{in}}^4\right)$$

$$I = \frac{\pi}{2}(1.25^4 - 1.\bar{1}^4)$$

$$I \approx 1.44\ in^4$$

Stress in Part

$$\sigma = \frac{F}{A}$$

$$\sigma = \frac{-126.5}{0.258}$$

$$\sigma \approx -491\ \text{psi, Tension}$$

Factor of Safety

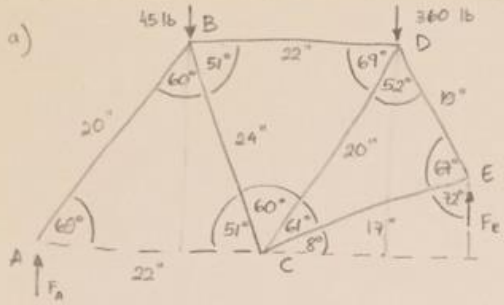
$$FS = \frac{S_{FL}}{\sigma_A}$$

$$FS = \left|\frac{56.3\times10^3}{-491}\right|$$

$$FS \approx 115$$

Part 3a Calculations

a)



Whole Structure

$\to \Sigma M_A = 0$

$-45 \cdot 20 \cdot \cos(69°) - 360 \cdot (22 + 20\cos(69°)) + F_E \cdot (22 + 17\cos(8°)) = 0$

$F_E = 278.6894 \text{ lb} \uparrow$

$\to \Sigma F_y = 0$

$F_A - 45 - 360 + F_E = 0$

$F_A = 126.3106 \text{ lb} \uparrow$

Joint A

$\to \Sigma F_y = 0$

$F_A + F_{AB} \sin(69°) = 0$

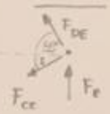
$F_{AB} = -135.2969 \text{ lb} \swarrow$

$\to \Sigma F_x = 0$

$F_{AC} + F_{AB}\cos(69°) = 0$

$F_{AC} = 48.4861 \text{ lb} \rightarrow$

Joint E



$\to \Sigma F_x = 0$

$-F_{DE}\cos(59°) - F_{CE}\cos(8°) = 0$

$F_{DE} = -\dfrac{F_{CE}\cos(8°)}{\cos(59°)} = -299.8107 \text{ lb} \searrow$

$\to \Sigma F_y = 0$

$F_E + F_{DE}\sin(59°) - F_{CE}\sin(8°) = 0$

$F_{CE} = 155.9314 \text{ lb} \swarrow$

Joint C

$\to \Sigma F_x = 0$

$-F_{AC} - F_{BC}\cos(51°) + F_{CD}\cos(69°) + F_{CE}\cos(8°) = 0$

$F_{BC} = \dfrac{1}{\cos(51°)}\left(F_{CD}\cos(69°) + 105.9278\right) = 105.2106 \text{ lb} \nwarrow$

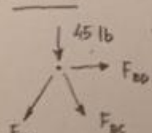
$\to \Sigma F_y = 0$

$F_{BC}\sin(51°) + F_{CD}\sin(69°) + F_{CE}\sin(8°) = 0$

$\dfrac{\sin(51°)}{\cos(51°)}\left(F_{CD}\cos(69°) + 105.9278\right) + F_{CD}\sin(69°) + 21.7015 = 0$

$F_{CD} = -110.8265 \text{ lb} \swarrow$

Joint B

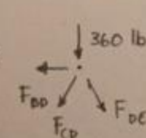


$\to \Sigma F_x = 0$

$F_{BD} - F_{AB}\cos(69°) + F_{BC}\cos(51°) = 0$

$F_{BD} = -114.6972 \text{ lb} \leftarrow$

Check! Joint D



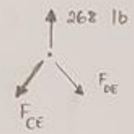
$\to \Sigma F_x = 0$

$F_{BD} + F_{CD}\cos(69°) = F_{DE}\cos(59°)$

$-154.4139 = -154.4139$ ✓

Part 3b Calculations

Joint E



$\Sigma F_x = 0$

$F_{DE} \cdot \sin(32°) - F_{CE} \cdot \sin(81°) = 0$

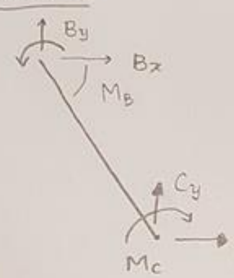
$\Sigma F_y = 0$

$-F_{DE} \cos(32°) - F_{CE} \cos(81°) + 268 = 0$

$F_{DE} = 287.56$ lb

$F_{CE} = 154.28$ lb

Beam BC



$\Sigma F_x = 0$

$B_x + C_x = 0$

$B_x = 24.261$ lb →

$\Sigma F_y = 0$

$B_y + C_y = 0$

$B_y = -248.072$ ↓

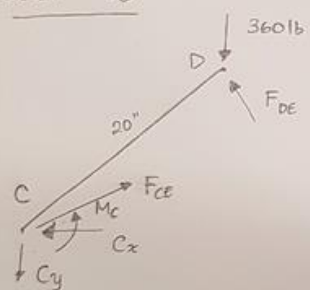
$\Sigma M_C = 0$

$M_C + B_x \cdot 24 \cdot \sin(51°) + B_y \cdot 24 \cdot \cos(51°)$

$= M_B$

$M_B = 6849.3693$ lb·in CCW

Beam CD



$\Sigma F_x = 0$

$-F_{DE} \cos(52°) - C_x + F_{CE} \cos(8°) = 0$

$C_x = -24.261$ lb ⟶

$\Sigma F_y = 0$

$F_{DE} \sin(52°) - C_y + F_{CE} \sin(8°) = 0$

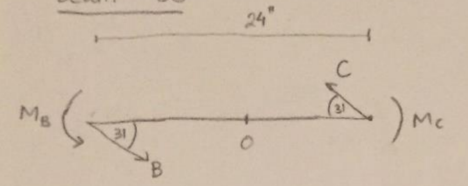
$C_y = 248.072$ lb ↓

$\Sigma M_C = 0$

$M_C + F_{DE} \cdot 20 \cdot \sin(60°) - 360 \cdot 20 \cdot \cos(69°) = 0$

$M_C = -2400.436$ lb in CW

Beam BC

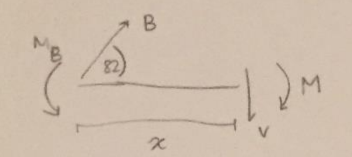


$B = 216.6632$ lb $M_B = -5290.4817$ lb in

$C = 216.6632$ lb $M_C = -2612.6244$ lb in

$\Sigma F_x = 0$

$B_x = C_x$ → cancel each other



$\boxed{1}$ $\Sigma F_y = 0$

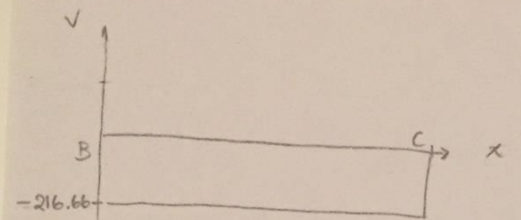
$B \sin(82°) = V$

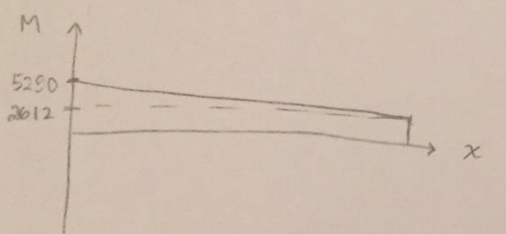
$V = 214.5546$ lb

$\Sigma M_o = 0$

$M_B + B \cdot x \sin(82°) = M$

@ $x = 24"$

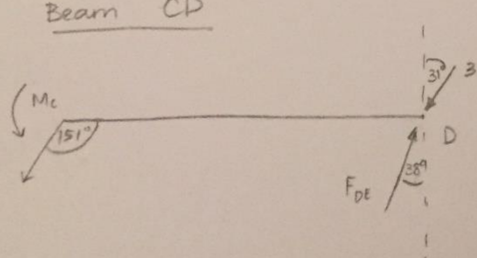


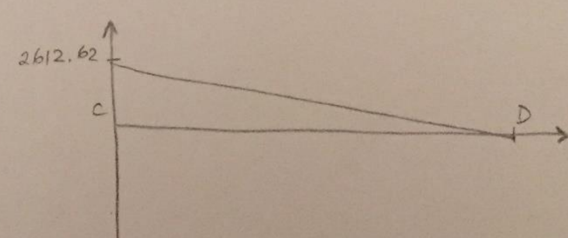


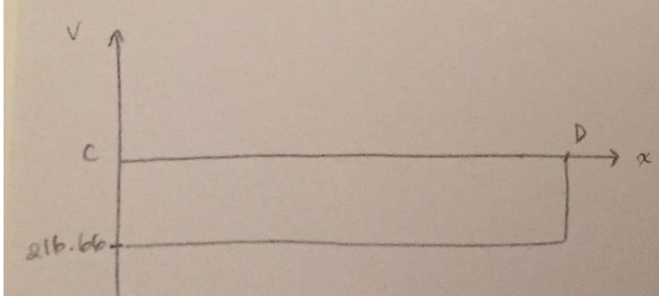
Beam CD

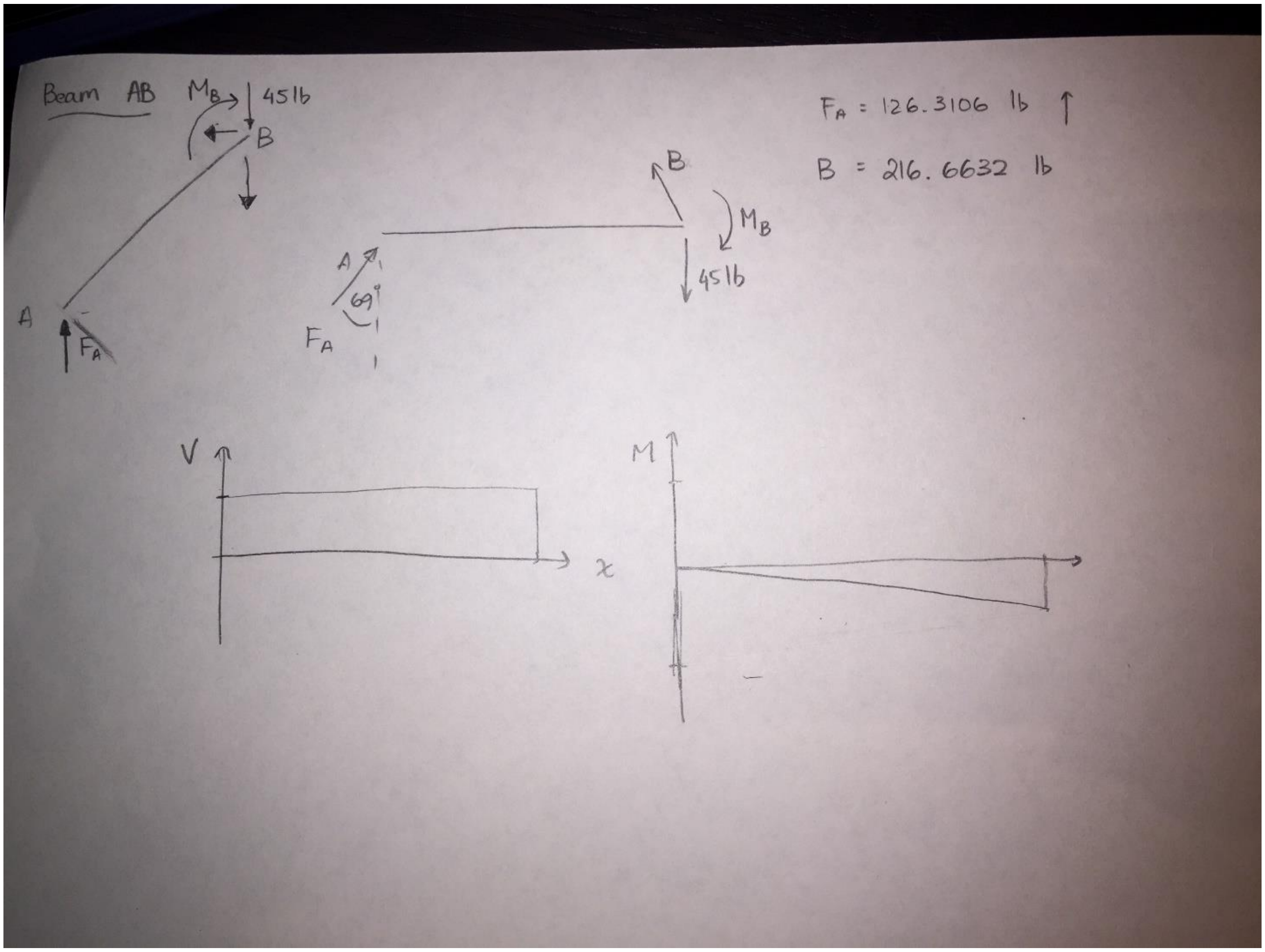
$C = 216.6632$ lb $M_c = -2612.6244$ lb in

$F_{DE} : 299.8107$







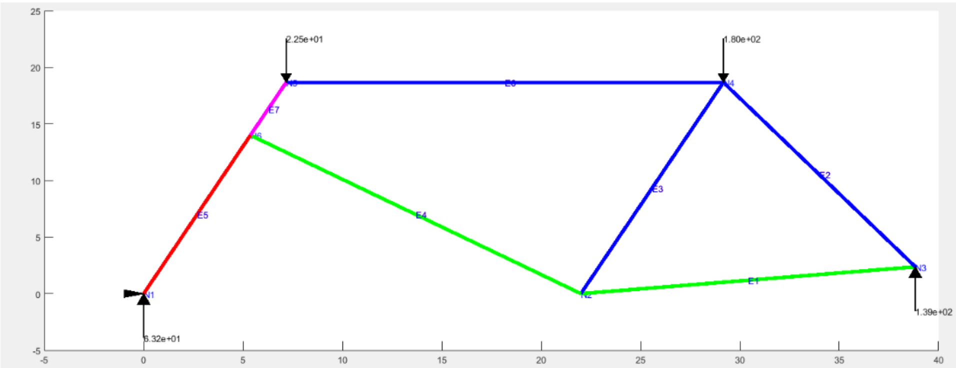


3a Extra Data

The color shows axial force

Elements	Axial Force (lb)	Shearing Force (lb)	Bending Moment (lb.in)	Axial Stress (psi)	Maximum Bending Stress (psi)
1	1.540e+02	0	0	1.656e+03	0
2	-2.991e+02	0	0	-7.294e+03	0
3	-1.100e+02	0	0	-4.265e+02	0
4	1.038e+02	0	0	4.022e+02	0
5	-1.352e+02	0	0	-5.242e+02	0
6	-1.130e+02	0	0	-5.408e+02	0
7	4.848e+01	0	0	2.320e+02	0

When cycler's weight is reduced by half (180 to 90 lb) color shows bending moment ranking



Elements	Axial Force (lb)	Shearing Force (lb)	Bending Moment (lb.in)	Axial Stress (psi)	Maximum Bending Stress (psi)
1	7.381e+01	9.098e+00	1.374e+02	1.656e+03	1.543e+01
2	1.399e+02	-5.677e-01	1.722e+01	-7.294e+03	6.516e+00
3	-6.274e+01	-2.484e+00	6.821e+01	-4.265e+02	1.657e+00
4	6.537e+01	2.544e+00	1.245e+02	4.022e+02	3.024e+00
5	-5.894e+01	-2.261e+01	3.390e+02	-5.242e+02	1.609e+01
6	-5.165e+01	-4.707e-01	2.499e+01	-5.408e+02	8.197e-01
7	-4.001e+01	3.993e+01	2.145e+02	2.320e+02	5.210e+00