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Lab 01 homework

1. Identify the efficiency

a. Solution:

- $n \rightarrow T(n)$
- $Basic: A[i] == A[j]$
- $Worst\ case: Have\ no\ couple\ items\ same$
- $T(n) = (n-1) + (n-2) + \cdots + 1 = \frac{(n-1)n}{2}$
- $T(n) \in \Theta(n^2)$

a. Solution:

- $n \rightarrow T(n)$
- $Basic: A[i] > max$
- $Worst\ case: No$
- $T(n) = (n-1) + (n-2) + \cdots + 1 = \frac{(n-1)n}{2}$
- $T(n) \in \Theta(n^2)$

b. Solution:

- $n \rightarrow T(n)$
- $Basic: i \geq 1$
- $Worst\ case: No$
- $Loop\ \log_3 n\ times\ in\ while, each\ of\ that, loop\ n\ times\ in\ for => T(n) = \log_3 n\,(n)$
- $T(n) \in \Theta(nlog(n))$

c. Solution:

- $n \rightarrow T(n)$
- $Basic: A[i] == A[j]^2$
- $Worst\ case: There\ is\ no\ items\ is\ square\ of\ another$
- $T(n) = (n-1) + (n-2) + \cdots + 1 = \frac{(n-1)n}{2}$
- $T(n) \in \Theta(n^2)$

d. ***Solution:***

- $k \to T(k)$
- $Basic: i < k$
- $Worst\ case: No$
- $T(k) = 1 + 1 + \cdots + 1 = k + 1$
- $T(k) \in \Theta(k)$

e. ***Solution:***

- $k \to T(k)$
- $Basic: k = 1$
- $Worst\ case: No$
- $T(k) = T(k-1) + 2$
- $\mathrm{T(k-1) = T(k-1-1) + 2}$
- …
- $\mathrm{T(1) = T(0) + 2 = 2}$
- $\mathrm{T(k) = 2k}$
- $T(k) \in \Theta(k)$

h. ***Solution:***

- $k \to T(k)$
- $Basic: k = 0$
- $Worst\ case: No$
- $T(k) = 2T(k-1)$
- $\mathrm{T(k-1) = 2T(k-1-1)}$
- …
- $\mathrm{T(1) = 2T(0) = 2}$

…

k. ***Solution:***

- $k \to T(k)$
- $Basic: i < k$
- $Worst\ case: No$
- $T(k) = 1 + 1 + \cdots + 1 = k + 1$
- $T(k) \in \Theta(k)$

l. ***Solution:***

- $n \to T(n)$
- $Basic: i < n\ and\ A[i]\ != k$
- $Worst\ case: There\ is\ no\ items\ equal\ k$
- $T(n) = n$
- $T(n) \in \Theta(n)$

n. Solution:

- $n \rightarrow T(n)$
- $Basic: all\ is\ swapped$
- $Worst\ case: Array\ is\ gradualy\ reduce$
- $T(n) = n * n$
- $T(n) \in \Theta(n^2)$

o. Solution:

- $n \rightarrow T(n)$
- $Basic: A[i] == A[j]$
- $Worst\ case: There\ is\ no\ same\ 2\ items$
- $T(n) = (n-1) + (n-2) + \cdots + 1 = \frac{(n-1)n}{2}$
- $T(n) \in \Theta(n^2)$

p. Solution:

- $n \rightarrow T(n)$
- $Basic: j > 0\ and\ A[j] > x$
- $Worst\ case: Array\ is\ gradualy\ reduce$
- $T(n) = 1 - 1 + 2 + 3 + \cdots + n = \frac{(n-1)(n+2)}{2} = \frac{n^2+n}{2} - 1$
- $T(n) \in \Theta(n^2)$

2. Define statement

   a. $32n^2 + 17n + 32 \in O(n)$: False

   b. $32n^2 + 17n + 32 \in O(n^3)$: False

   c. $32n^2 + 17n + 32 \in \Omega(n^3)$: False

   d. $32n^2 + 17n + 32 \in \Omega(n)$: True

   - Prove:

   Because of that $32n^2 + 17n + 32 > c * n$ with all $n > n_0$ (c is constant)

   e. $2^{n+1} \in O(2^n)$: True

f. $2^{2n} = 2^n 2^n \in O(2^n)$: False

g. If $f(n) \in O(g(n))$ and $g(n) \in O(f(n))$ then $f(n) = g(n)$