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# LAB 02: SOLVING RECURRENCE RELATIONS

## 1. Solve the following recurrence relations with Master theorem

a. $C_N = C_{N/2} + 1000$

$a = 1$, $b = 2$, $n^k = 1 \Rightarrow k = 0$;

$\Rightarrow a = b^k$

$\Rightarrow C(n) \in \Theta(n^k logn)$

$\Rightarrow C(n) \in \Theta(logn)$

b. $C_N = 3C_{N/2} + N$

$a = 3$, $b = 2$, $N^k = N \Rightarrow k = 1$;

$\Rightarrow a > b^k$ $(3 > 2^1)$

$\Rightarrow C(n) \in \Theta(n^{\log_a b})$

$\Rightarrow C(n) \in \Theta(n^{\log_3 2})$

c. $C(N) = 2C(N/2) + 1$

$a = 2$, $b = 2$, $n^k = 1 \Rightarrow k = 0$;

$\Rightarrow a > b^k$ $(2>1)$

$\Rightarrow C(n) \in \Theta(n^{\log_a b})$

$\Rightarrow C(n) \in \Theta(n^{\log_2 2})$

$\Rightarrow C(n) \in \Theta(n)$

d. $C_N = 4c_{N/2} + N$

$a = 4$, $b = 2$, $N^k = N \Rightarrow k = 1$

$\Rightarrow a > b^k$ $(4 > 2)$

$\Rightarrow C(n) \in \Theta(n^{\log_a b})$

$\Rightarrow C(n) \in \Theta(n^{\log_4 2})$

$\Rightarrow C(n) \in \Theta(n^{1/2})$

e. $C(N) = 9C(N/3) + N$

$a = 4$, $b = 2$, $N^k = N \Rightarrow k = 1$

$\Rightarrow a > b^k$ $(9 > 3)$

$\Rightarrow C(n) \in \Theta(n^{\log_a b})$

$\Rightarrow C(n) \in \Theta(n^{\log_9 3})$

$\Rightarrow C(n) \in \Theta(n^{1/2})$

f. C (N) = C (2N/3) + 1
a = 1, b = 3/2, $N^k$ = 1 => k = 0
=> a = $b^k$
=> $C(n) \in \Theta\left(n^k logn\right)$
=> $C(n) \in \Theta(logn)$

g. C (N) = 3C (N/4) + $N^2$
a = 3, b = 4, $N^k = N^2$ => k = 2
=> a < $b^k$ (3 < 16)
=> $C(n) \in \Theta\left(n^k\right)$
=> $C(n) \in \ \Theta(n^2)$

h. $T(n) = T(n/3) + 2T(n/3) + \sqrt{n}$
=> $T(n) = (1 + 2)T(n/3) + \sqrt{n} = 3T(n/3) + \sqrt{n}$
a = 3, b = 3, $n^k = \sqrt{n}$ => k = ½
=> a > $b^k$ (3 > $\sqrt{3}$ )
=> $C(n) \in \Theta\left(n^{\log_a b}\right)$
=> $C(n) \in \ \Theta\left(n^{\log_3 3}\right)$
=> $C(n) \in \ \Theta(n)$

i. $T(n) \le T(n/3) + 2T(n/3) + n^{2.5}$

=> (Ve phai) $\left(\frac{n}{3}\right) + 2T\left(\frac{n}{3}\right) + n^{2.5} = 3T\left(\frac{n}{3}\right) + n^{2.5}$
=> $T(n) \le \ 3T\left(\frac{n}{3}\right) + n^{2.5}$
a = 3, b = 3, $n^k = n^{2.5}$ => k = 2.5
=> a < $b^k$ (3 < $3^{2.5}$ )
=> $C(n) \in O\left(n^k\right)$
=> $C(n) \in \ O\left(n^k\right)$
=> $C(n) \in \ O(n^{2.5})$

j. $T(n) \le T(n/3) + 2T(n/3) + n^3$
=> (Ve phai) $\left(\frac{n}{3}\right) + 2T\left(\frac{n}{3}\right) + n^3 = 3T\left(\frac{n}{3}\right) + n^3$
=> $T(n) \le \ 3T\left(\frac{n}{3}\right) + n^3$
a = 3, b = 3, $n^k = n^3$ => k = 3
=> a < $b^k$ (3 < $3^3$ )
=> $C(n) \in O\left(n^k\right)$
=> $C(n) \in \ O\left(n^k\right)$
=> $C(n) \in \ O(n^3)$

k. $T(n) \geq T(n/3) + 2T(n/3) + 5^3$

=> (Ve phai) $\left(\frac{n}{3}\right) + 2T\left(\frac{n}{3}\right) + 5^3 = 3T\left(\frac{n}{3}\right) + 5^3$

=> $T(n) \geq\ 3T\left(\frac{n}{3}\right) + 5^3$

a = 3, b = 3, $n^k = 1$ => k = 0

=> $a = b^k$

=> $C(n) \in \Omega(n^k \log n)$

=> $C(n) \in \Omega(\log n)$

k. $T(n) \geq T\left(\frac{n}{3}\right) + 2T\left(\frac{n}{3}\right) + n \times n$

=> (Ve phai) $\left(\frac{n}{3}\right) + 2T\left(\frac{n}{3}\right) + \text{nxn} = 3T\left(\frac{n}{3}\right) + n^2$

=> $T(n) \geq\ 3T\left(\frac{n}{3}\right) + n^2$

a = 3, b = 3, $n^k = n^2$ => k = 2

=> $a < b^k$ (3<9)

=> $C(n) \in \Omega(n^k)$

=> $C(n) \in \Omega(n^2)$

## 2. *Backward*

m. $C_N = C_{N/2} + 1000$ with N ε2, $C_1 = 0$

$C_{N/2} = C_{N/4} + 1000$

$C_{N/4} = C_{N/8} + 1000$

….

$C_2 = C_1 = 0$

=> C(N) + ~~C(N/2) + C(N/4)~~ + …. C(2) = ~~C(N/2)~~ + 1000 + ~~C(N/4)~~ + 1000 + … + 0

=> C(N) = 1000 + 1000 +…. + 1000 + 0

=> C(N) = $(\log_2(n) – 1)$*1000

=> C(N) ∈ $\Theta(\log(n))$

n. $C_N = 3C_{N/2} + N$ with nε 2, $C_1 = 0$

$C(N/2) = 3C(N/4) + N/2$

$C(N/4) = 3C(N/8) + N/4$

….

$C(2) = C(1) + 2 = 0 + 2$

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=> $3^0 C_N = 3C_{N/2} + 3^0 N$

=> $3^1 C(N/2) = 3^2 * C(N/4) + 3^1 * N/2$

=> $3^2 C(N/4) = 3^3 * C(N/8) + 3^2 * N/4$

….

$3^{\log2(N)-1} * C(2) = 0 + 3^{\log2(N)-1} *(N/2^{\log2(N)-1})$

=> $3^0 C_N + \cancel{3^1 C(N/2)} + \cancel{3^2 C(N/4)} + \ldots + \cancel{3^{\log2(N)-1} * C(2)}$

$= \cancel{3C_{N/2}} + 3^0 N + \cancel{3^2 * C(N/4)} + 3^1 * N/2 + \cancel{3^3 * C(N/8)} + 3^2 * N/4 + \ldots + 0 + 3^{\log2(N)-1} *(N/2^{\log2(N)-1}$
)

=> $3C(N) = (3/2)^0 N + (3/2)^1 N + \ldots + (3/2)^{\log2(N)-1} N$

=> $C(n) = N*((3/2)^0 + (3/2)^1 + \ldots + 3/2)^{\log2(N)-1})$

=> $C(n) \in \Theta(n^{log_2 n-1})$

o. C (N) = 2C (N/2) + 1. With N ε2, C (1) = 0

C (N/2) = 2C (N/4) + 1.

….

C(2) = 2C(1) + 1

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=> $2^0 C(N) = 2^1 C(N/2) + 2^0$.

=> $2^1 C(N) = 2^2 C(N/2) + 2^1$.

…..

=> $2^{log_2 n-1} C(N) = 2^{log_2 n} C(N/2) + 2^{log_2 n-1}$.

=> $C(N) = 2^0 + 2^1 + \ldots 2^{log_2 n-1} = \sum_{k=0}^{log_2 n-1}(2^k)$

=> => $C(n) \in \Theta(n^{log_2 n-1}) \sim \Theta(n^{\log(n)})$

p. $T(n) = \{5 \times T(n-1) + 3, \quad if\ n > 1\ 4, \qquad if\ n = 1$

T(n-1) = 5 x T(n-2) +3

….

T(2) = 5T(1) + 3

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$5^0$ x T(n) = $5^1$ x T(n-1) + $5^0$ x 3

$5^1$ x T(n-1) = $5^2$ x T(n-2) + $5^1$ x 3

…..

$5^{n-2}$ x T(2) = $5^{n-1}$ x T(1) + $5^{n-2}$ x 3 = $5^{n-1}$ x4 + $5^{n-2}$ x 3

=> T(n) = $5^0$ x 3 + $5^1$ x 3 …. + $5^{n-2}$ x 3 + $5^{n-1}$ x4

=> $T(n) = 3 \times (5^0 + 5^1 + \ldots 5^{n-2}) + 5^{n-1} \times 4$

=> $T(n) \in \Theta(5^{n-1}) = \Theta(2^n)$