# MATH 4334 Numerical Analysis


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- Office hours: TBD + appt.

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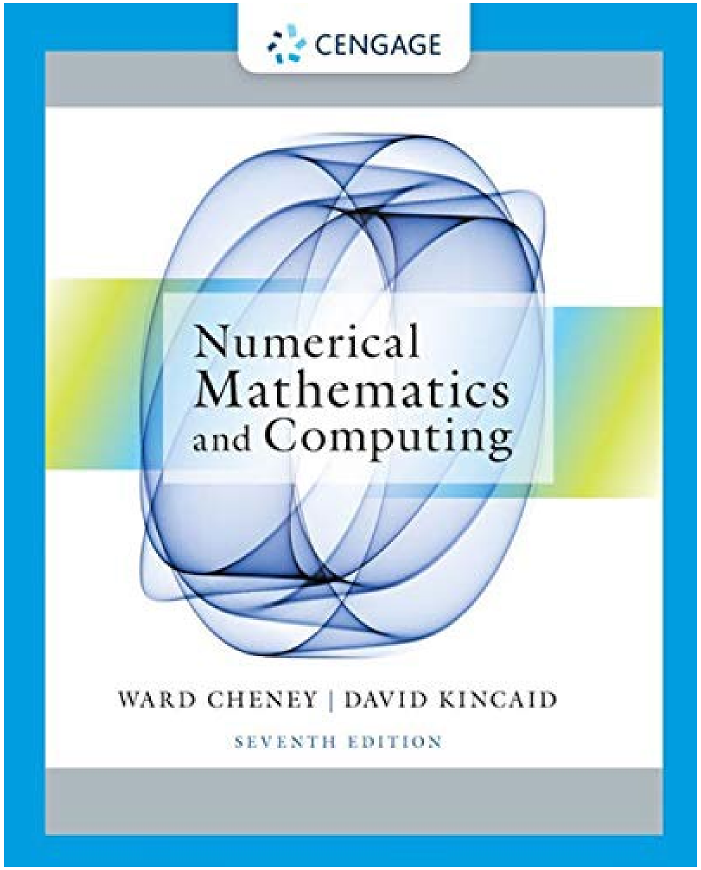
# About me

- I work at the intersection of computational math and data science.
- I have two thesis advisors.
- I took courses in CS/EE/Stat Departments.
- RA support in the summer is available if you impress me with your course work☺

# Prerequisites

- Multi-variable calculus
- Linear algebra
- Programming (Matlab)




## Textbook (ebook available)

*Numerical Mathematics and Computing*, 7th Edition, by Cheney and Kincaid.
Publisher: Brooks/Cole, 2013.

# Learning objectives

- Help students in understanding methods for solving scientific problems using computers.
- Understand the source, propagation, magnitude, and rate of growth of errors introduced by numerical computations.
- Learn not only how algorithms work, but also how they can fail.
- We consider problems after they have been cast into certain standard math forms.

# Overview

- Preliminaries (Ch 1)
- Solving nonlinear equation (Ch 3)
- Interpolation (Ch 4)
- Integration (Ch 5)

Midterm

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- Solving linear system (Ch 2+8)
- Initial value problem (Ch 7)

Final

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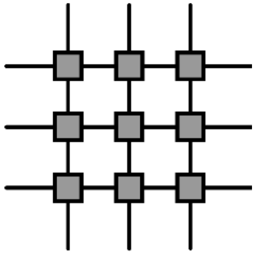
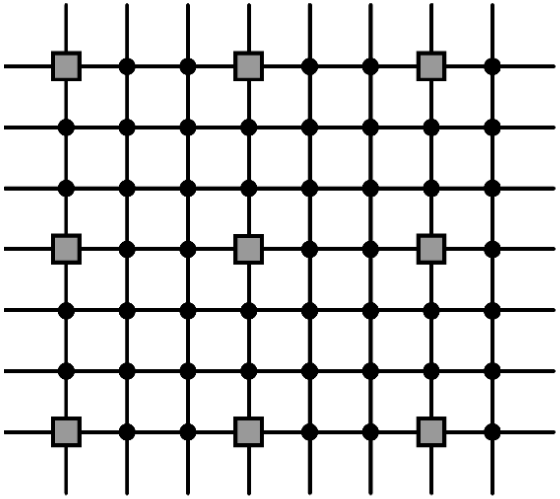
# Nonlinear equations

- Root-finding problem: find x s.t. f(x)=0
- Methods to be studied
  - Bisection method (3.1)
  - Newton's method (3.2)
  - Secant method (3.3)
- Convergence analysis
- Comparison and discussions

# Interpolation

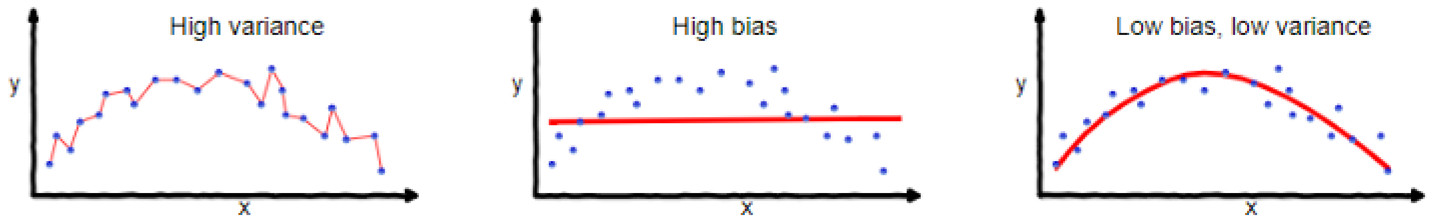
A special case: super-resolution





# Interpolation (cont'd)

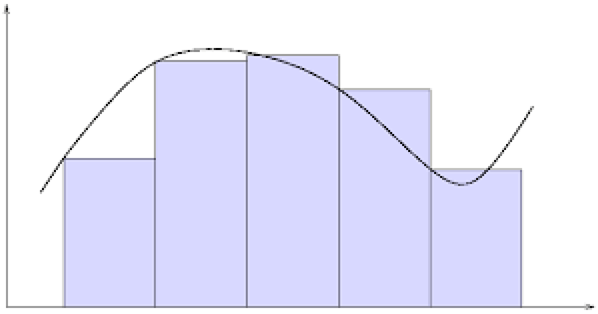
- Polynomial interpolation
- Error analysis
- Data fitting → machine learning



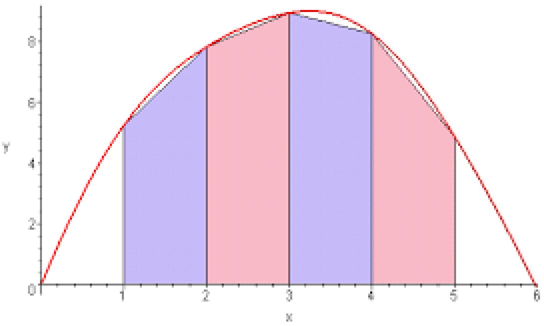

overfitting underfitting Good balance

# Integration

- Riemann sum



- Methods to be studied
  - Trapezoid method (5.1)
  - Simpson's rules (5.3)
  - Gaussian Quadrature formula (5.4)



- Error analysis

# Solving Ax = b

- Gaussian elimination (2.1)
- Pivoting (2.2)
- Tridiagonal and banded system (2.3)

- Matrix factorization (8.1)
- Singular Value Decomposition (8.2)
- Power method (8.3)
- Iterative method (8.4)

# Initial value problem

- Problem

$$\begin{cases} x' = f(x,t) \\ x(0) = given \end{cases}$$

Do you have ODE background?

- Methods
  - Taylor series method (7.1)
  - Runge-Kutta method (7.2)

# Tentative schedule

| Wk | Mon | Lecture | Wed | Lecture | Notes |
|---|---|---|---|---|---|
| 1 | 8/22 | Overview | 8/24 | Taylor (1.2) | |
| 2 | 8/29 | Number rep. (1.3) | 8/31 | Significance (1.4) | |
| 3 | 9/5 | **Holiday** | 9/7 | Bisection (3.1) | **HW1** |
| 4 | 9/12 | Newton (3.2) | 9/14 | Secant (3.3) | |
| 5 | 9/19 | Polynomial (4.1) | 9/21 | Polynomial (4.1) | **HW2** |
| 6 | 9/26 | Errors (4.2) | 9/28 | Errors (4.2) | |
| 7 | 10/3 | Trapezoid (5.1) | 10/5 | Simpson (5.2) | **HW3** |
| 8 | 10/10 | Gaussian (5.4) | 10/12 | Gaussian (5.4) | |
| 9 | 10/17 | Midterm review | 10/19 | **Midterm** | **HW4** |
| 10 | 10/24 | Gaussian elimination (2.1) | 10/26 | Structured system (2.3) | |
| 11 | 10/31 | Pivoting (2.2) | 11/2 | Pivoting (2.2) | |
| 12 | 11/7 | Factorization (8.1) | 11/9 | Eigenvalue (8.2) | **HW5** |
| 13 | 11/14 | Power method (8.3) | 11/16 | Iterative (8.4) | |
| 14 | 11/21 | **Fall break** | 11/23 | **Fall break** | |
| 15 | 11/28 | Taylor (7.1) | 11/30 | RK method (7.2) | |
| 16 | 12/5 | variants (7.3) | 12/8 | Review | **HW6**<br>**Final TBD** |

# Grading

- 6HWs: 36 % (once every 2-3 weeks)
- Midterm exam: 24% (in class Oct 19)
- Final exam: 40% (TBD)

**No extra credit**
Attendance may be taken for borderline scores

| | | |
|---|---|---|
| [96.6,100]…A+ | [93.3,96.6)…..A | [90,93.3)…..A- |
| [86.6,90)…..B+ | [83.3,86.6)…..B | [80,83.3)…..B- |
| [76.6,80)…..C+ | [73.3,76.6)…..C | [70,73.3),….C- |
| [66.6,70)…..D+ | [63.3,66.6)…..D | [60,63.3)…..D- |
| [0 ,60)……F | | |

# Homework

- HW will be posted on EL every 2-3 weeks.
- Each homework is due on Wednesday. You can turn in either in class or upload on EL.
- Only 1 file for electronic submission.
- No late homework will be accepted except for emergency (proof is required).
- Highly recommend to work on it in advance.
- You are encouraged to work together but must turn in your own work (plagiarism is not allowed).

# Major exams

- One midterm exam is scheduled on Oct 19 in class (mark your calendar).
- Final exam (TBD) is comprehensive.
- Both are closed-book, closed-note, in person exams.
- Accommodations for exams can be made; contact me as early as possible with valid reasons.

# Any questions?

Chapter 1.1

# MATHEMATICAL PRELIMINARIES

# Significant digits of precision

- Non-zero digits within given measurements are significant.
- Zeros to the right of the last non-zero digit are significant if within the measurement.
- Zeros to the left of the first nonzero digit are NOT significant.
- An exact number has an infinite number of significant digits (or figures).

# Example

Let's concentrate on solving for the variable $y$ in this **linear system of equations** in two variables

$$\begin{cases} 0.1036\,x + 0.2122\,y = 0.7381 \\ 0.2081\,x + 0.4247\,y = 0.9327 \end{cases} \qquad (1)$$

First, carry only three significant digits of precision in the calculations. Second, repeat with four significant digits throughout. Finally, use ten significant digits.

Data thought to be accurate should be carried with full precision and not be rounded prior to each of the calculation.

# Errors

$$\textbf{Absolute Error} = |\text{Exact Value} - \text{Approximate Value}|$$

$$\textbf{Relative Error} = \frac{|\text{Exact Value} - \text{Approximate Value}|}{|\text{Exact Value}|}$$

- In computing absolute error, the roles of these two values are the same, whereas in relative errors it is essential to distinguish the correct one.
- In practice, relative error is more meaningful. Sometimes, percentage error is used.
- The exact value may be the true value or the best known value.

# Rounding and chopping

- Rounding reduces the number of significant digits in a number.
- The result of rounding is a number similar in magnitude that is a shorter number having fewer nonzero digits.
- The round-to-even rule tends to reduce the total rounding error with (on average) an equal portion of numbers rounding up and down.
- Compared to chopping, rounding is preferrable.

# Nested multiplication

To evaluate the polynomial,

$$p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1} + a_n x^n$$

We group the terms in a nested multiplication

$$p(x) = a_0 + x(a_1 + x(a_2 + \cdots + x(a_{n-1} + x(a_n)) \cdots ))$$

Why and how?

# Pseudo-code

$$p(x) = a_0 + x(a_1 + x(a_2 + \cdots + x(a_{n-1} + x(a_n))\cdots))$$

**integer** $i, n$; **real** $p, x$
**real array** $(a_i)_{0:n}$
$p \leftarrow a_n$
**for** $i = n - 1$ **to** $0$
    $p \leftarrow a_i + xp$
**end for**