Probability Fun

Conditional Probability

You toss an unbiased coin. The first two throws are heads. What is the probability that the third toss will also be a head?

Answer = 1/2

Conditional Probability

- Formal definition:

$$P(X = a|Y = b) = \frac{P(X = a, Y = b)}{P(Y = b)}$$

Joint probability

Suppose we know that a dice throw was odd, and want to know the probability of outcome of dice being "one".

Let X be the random variable of the dice throw, and Y be an indicator variable that takes on the value of 1 if the dice throw turns up odd,

$$P(X=1|Y=1) = \frac{P(X=1, Y=1)}{P(Y=1)} = \frac{1/6}{1/2} = 1/3$$

Bayes Rule

- Perhaps one of the most important equations for this class ☺

$$P(\text{hypothesis} \mid \text{evidence}) = \frac{P(\text{evidence} \mid \text{hypothesis}) \times P(\text{hypothesis})}{P(\text{evidence})}$$

If this seems too complicated, let's look at concrete examples:

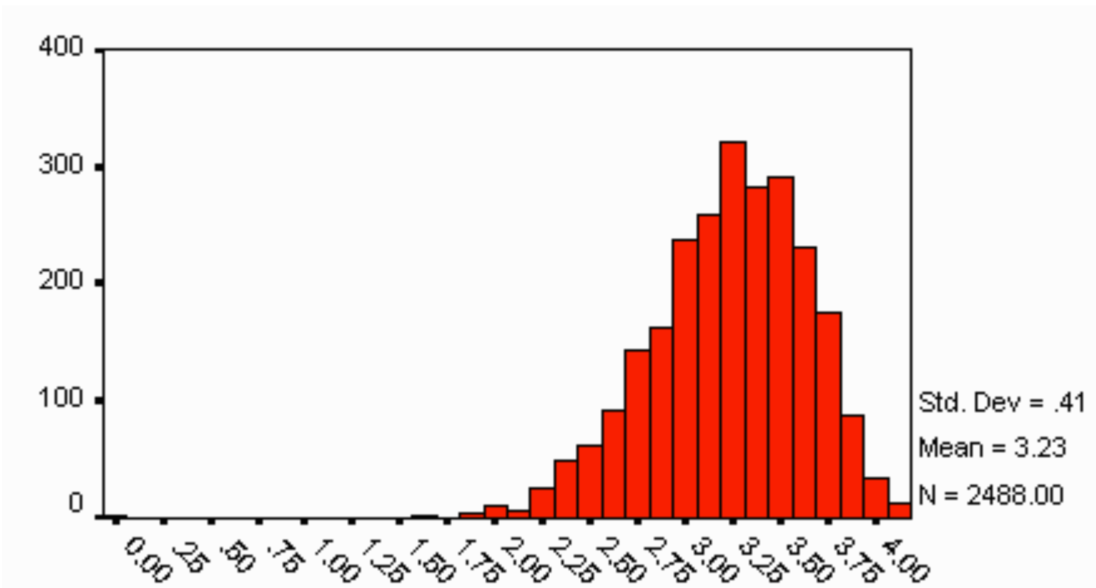
In a given population, the probability of getting flu is 1/40 and the probability of getting a headache is 1/10. If someone has the flu, the probability of headache goes up to 1/2.
A patient comes to the doctor complaining of headache, what is the probability that he has the flu.

Hypotheses = He has the flu

Evidence = He has headache

Probability Distribution

- How are variables distributed?
- What is the probability of a specific value?
- Example – how is GPA distributed?

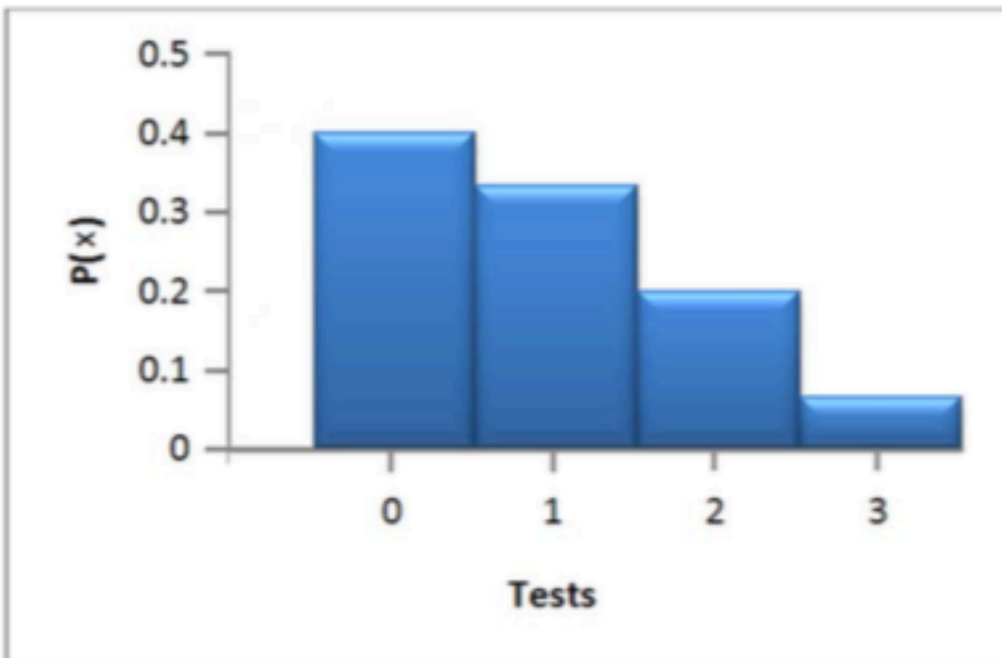


The probabilities that a patient will have 0, 1, 2 or 3 tests performed upon entering a hospital are $\frac{6}{15}$, $\frac{5}{15}$, $\frac{3}{15}$, and $\frac{1}{15}$ respectively.

The probability distribution is:

x	0	1	2	3
P(x)	$\frac{6}{15}$	$\frac{5}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

The graph of the distribution is:



Mean of probability distribution

- Discrete case:

$$x = \frac{\sum xf}{n} = \sum\left(x\frac{f}{n}\right) = \sum x\,P\,(x)$$

Also known as Expected Value

A box contains:

- Two \$1 bills
- Three \$5 bills
- One \$10 bill
- Three \$20 bills

Construct a probability distribution for the data. Then, use the formula to calculate the mean.

x	\$1	\$5	\$10	\$20
P(x)				

A random variable x has possible values {-1, 0, 1}. All values of x are equally likely (i.e. $P(-1) = P(0) = P(1) = 1/3$). Find the mean of x

a) 0.
b) 1/3.
c) 1.
d) I do not know.

Examples

For a daily lottery, a person selects a 3-digit number(i.e. numbers can be from 000 to 999). If the person plays for $1, she can win $500. Find the expectation.

Examples

For a daily lottery, a person selects a 3-digit number(i.e. numbers can be from 000 to 999). If the person plays for \$1, she can win \$500. Find the expectation of winning amount in \$ terms.

Two cases – win or loss

For win:

$x = 500 - 1 = 499$

$P(x) = 1/1000$

For loss:

$x = -1$

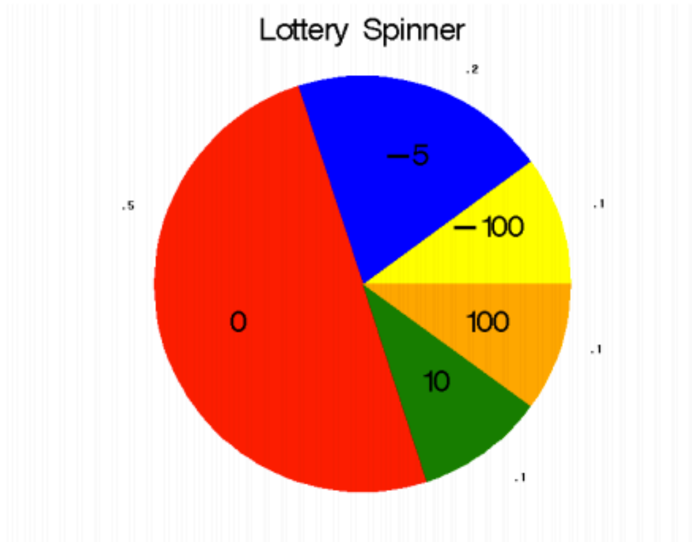
$P(x) = 999/1000$

$E(x) = 499 * (1/1000) - 1 * (999/1000) = -500/1000 = -0.5$

Question

- What is the expected value of the output for a fair dice?

Lottery Spinner



Color	Y	$P(Y)$
Yellow	-100	.10
Blue	-5	.20
Red	0	.50
Green	10	.10
Tan	100	.10

Calculate expected win or loss for the lottery spinner

Variance of probability distribution

- Variance:

$$\sum[x^2 \cdot P(x)] - \mu^2$$

Famous
Probability Distributions

Discrete Uniform

- Defined between two values (say 1 and n)
- $p(x) = 1/n$ for $1 <= x <= n$

What is $E(x)$ and $var(X)$?

Continuous Uniform

- Defined between two values (say a and b)
- $p(x) = 1/(b-a)$ for $a<= x <= b$

What is $E(x)$ and $var(X)$?
(Use integrals)

Bernoulli Distribution

- Has only two outcomes -> 0 and 1
- $p(1) = p$, so $p(0) = 1-p$
- Can be written as:

  $p(x) = p^x (1-p)^{(1-x)}$ where $x = \{0,1\}$

- Can you compute mean and variance?

Binomial

- You conduct n independent Bernoulli trials.

$$P(x) = \binom{n}{k} p^k (1-p)^{n-k}$$

- Calculate mean and variance