Artificial Intelligence

CS4365 --- Fall 2022

Knowledge Representation and Reasoning

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Summary of Propositional Logic

- Syntax
    - literals
    - Connectives
    - Clauses
    - CNF
- Semantics
    - Model
- Inference
    - Semantically: Model Checking
    - Syntactically: Inference rules and axioms
        - Modus Ponens
        - Resolution

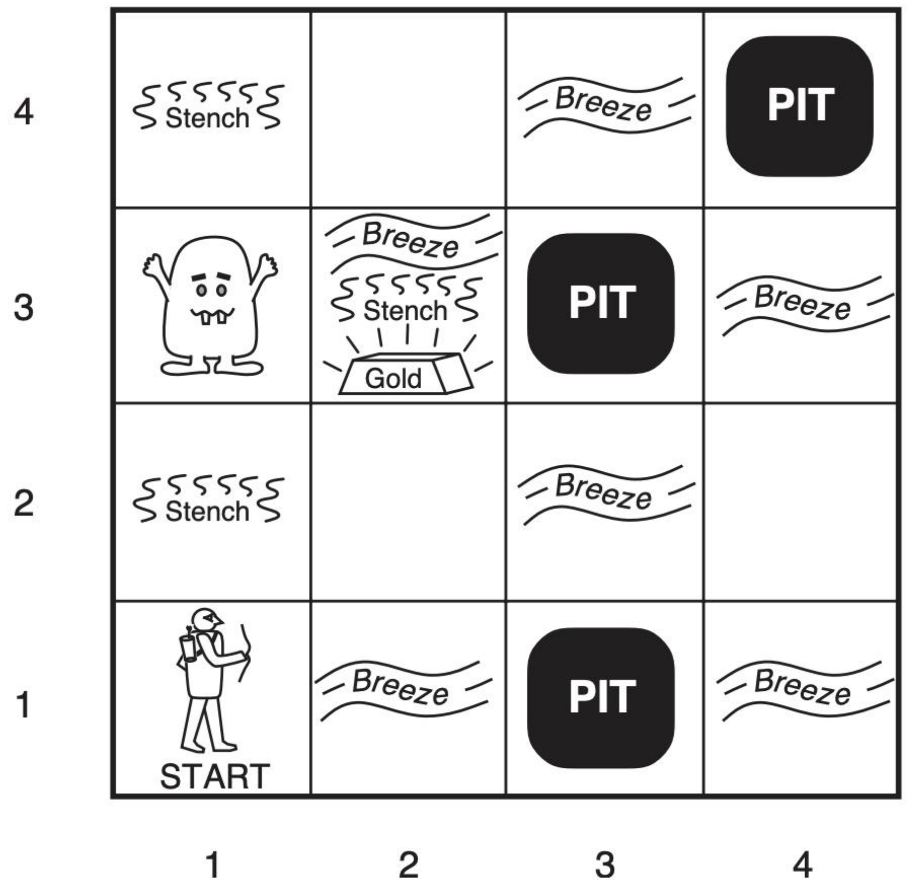
Agent Based on Propositional Logic

- Construct wumpus world agents that use propositional logic.

- Enable the agent to deduce, to the extent possible, the state of the world given its percept history

The Wumpus World

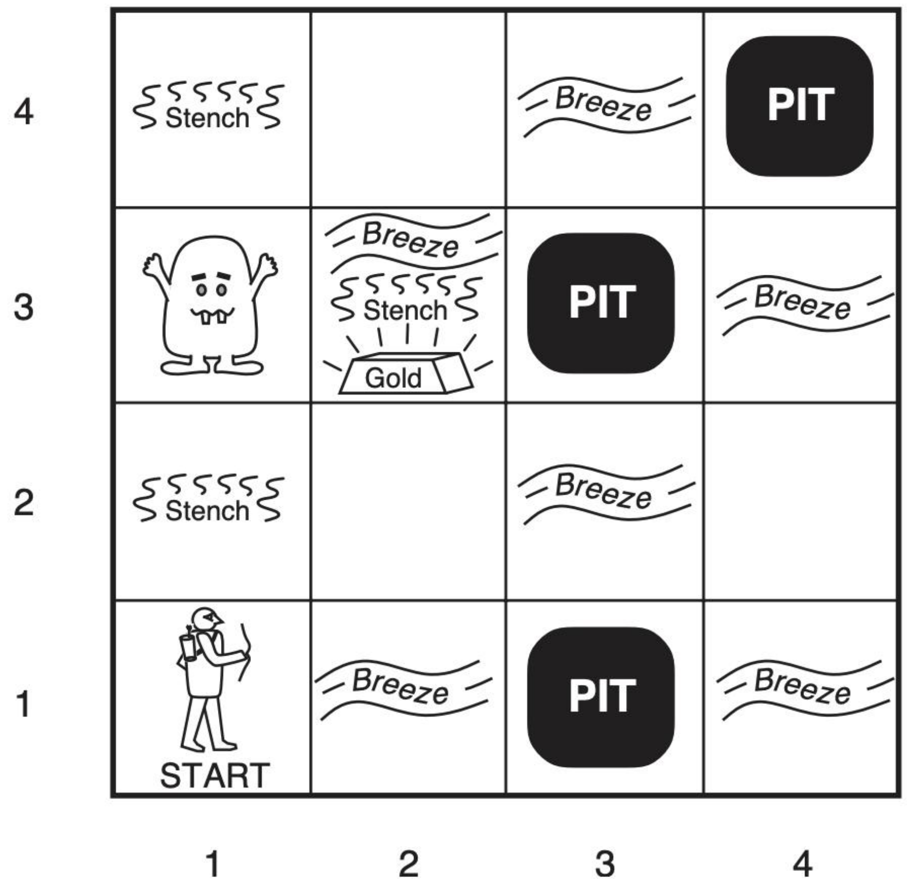
- Performance meaure:
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow



The Wumpus World

- Environment
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square

- Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot
- Sensors: Stench, Breeze, Glitter, Bump, Scream (shot Wumpus)



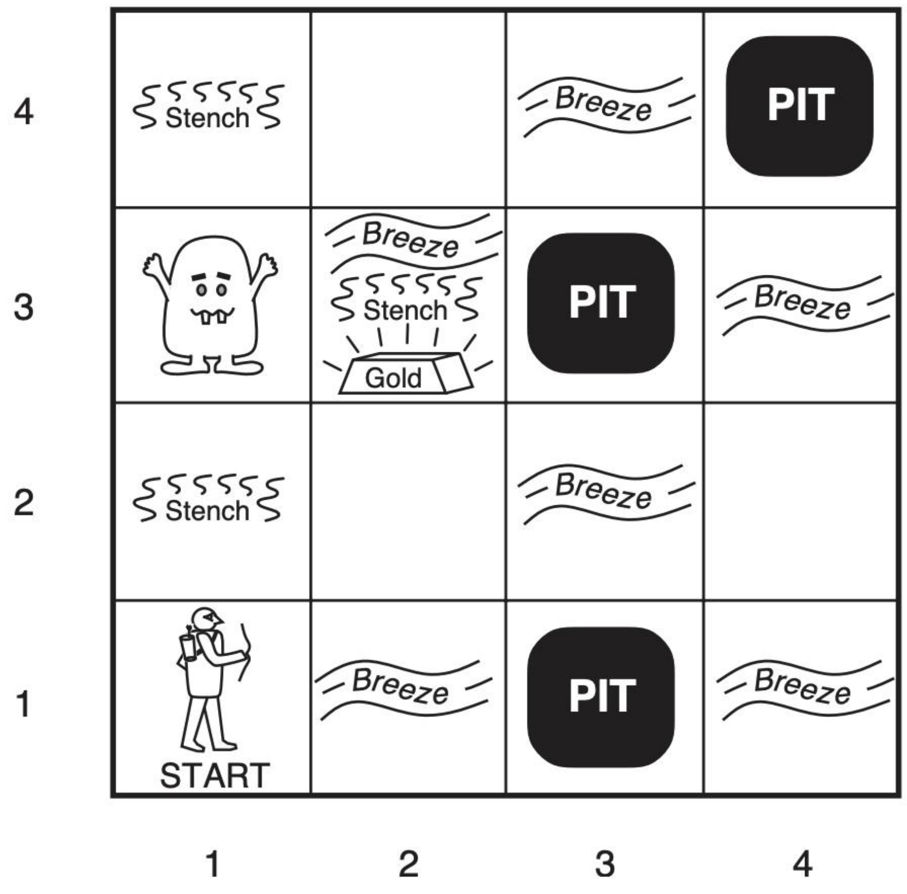
Wumpus world characterization

- Fully Observable?
  - No – only local perception
- Deterministic?
  - Yes – outcomes exactly specified
- Episodic?
  - No – sequential at the level of actions
- Static?
  - Yes – Wumpus and Pits do not move
- Discrete?
  - Yes
- Single-agent?
  - Yes – Wumpus is essentially a natural feature

The Wumpus World

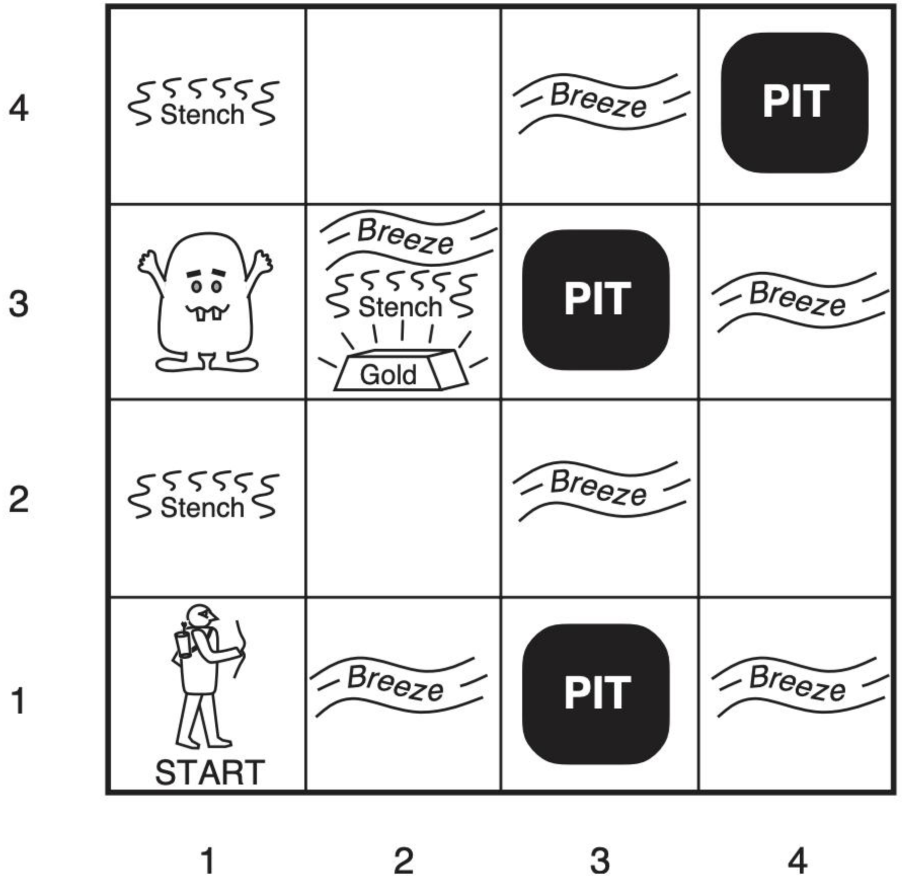
- A decision-maker needs to represent knowledge of the world and reason with it in order to safely explore this world.

- Principle Difficulty: agent is initially ignorant of the configuration of the environment – going to have to reason to figure out where the gold is without getting killed!



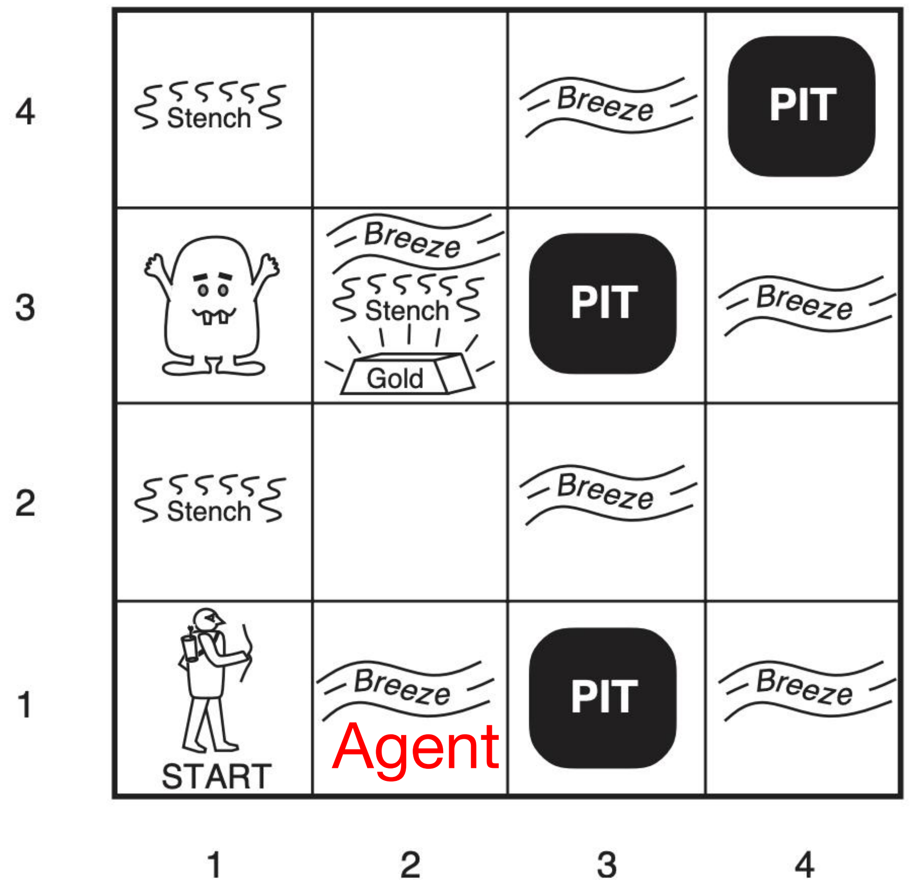
The Wumpus World

- Define the propositional symbols:
  - $P_{x,y}$: true if there is a pit in [x,y]
  - $W_{x,y}$: true if there is a wumpus in [x, y], dead or alive
  - $B_{x,y}$: true if the agent perceives a breeze in [x ,y]
  - $S_{x,y}$: true if the agent perceives a stench in [x,y]



Knowledge Base

- Build the knowledge base:
  - There is no pit in [1,1]:
    - R1: $\neg P_{1,1}$
  - For each square, it knows that the square is breezy if and only if a neighboring square has a pit;
    - R2: $B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$
    - R3: $B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$
  - Percepts:
    - R4: $\neg B_{1,1}$
    - R5: $B_{2,1}$

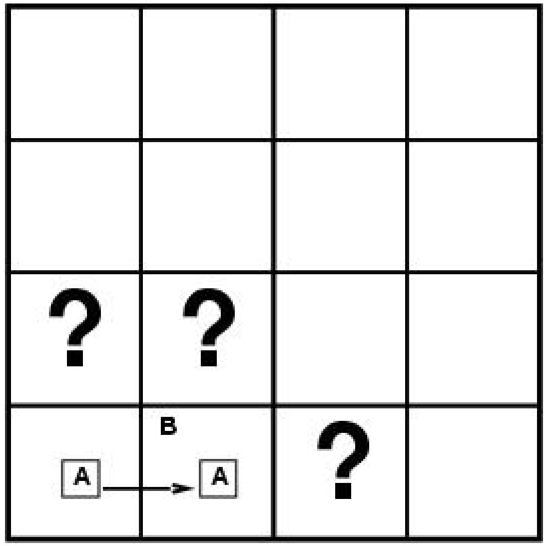


Entailment in the Wumpus World

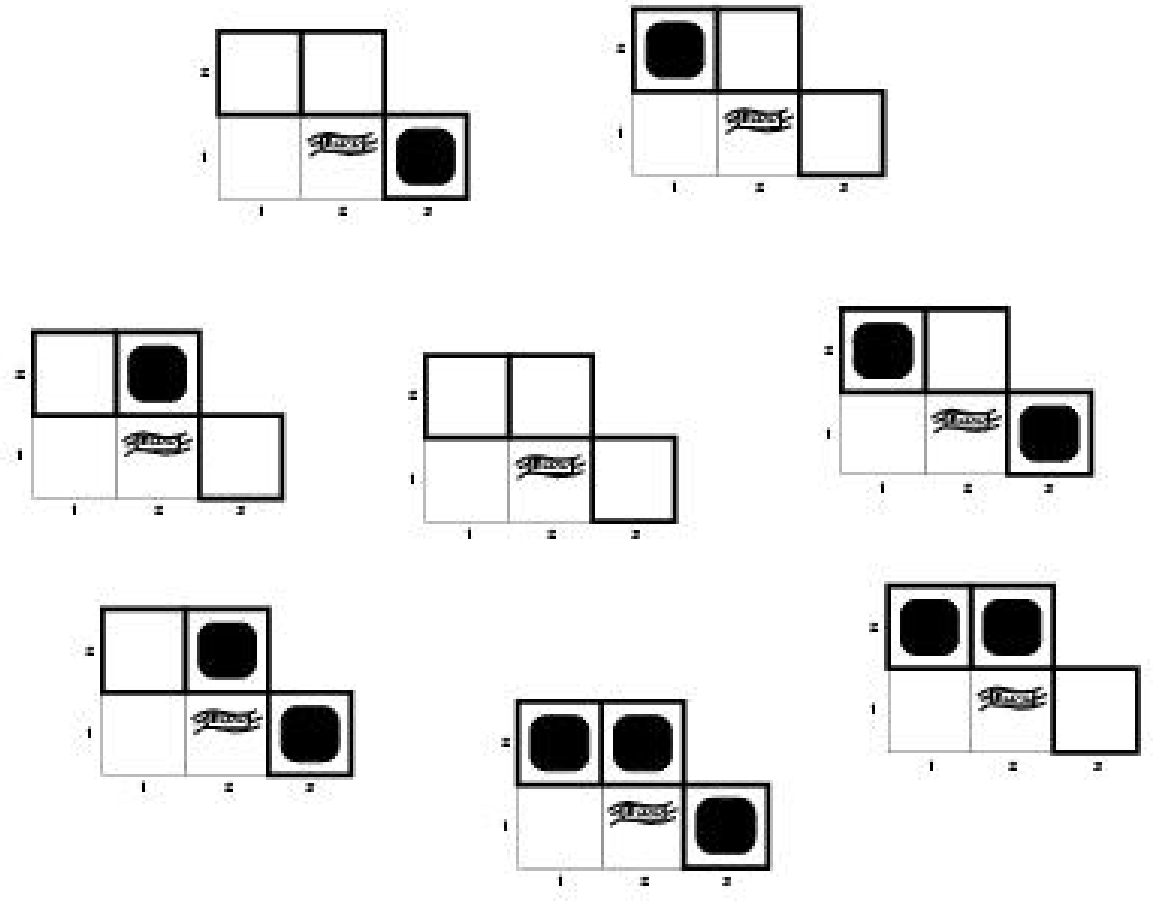
- Situation after detecting nothing in [1,1], moving right breeze in [2,1]

- Consider possible models for KB assuming only pits

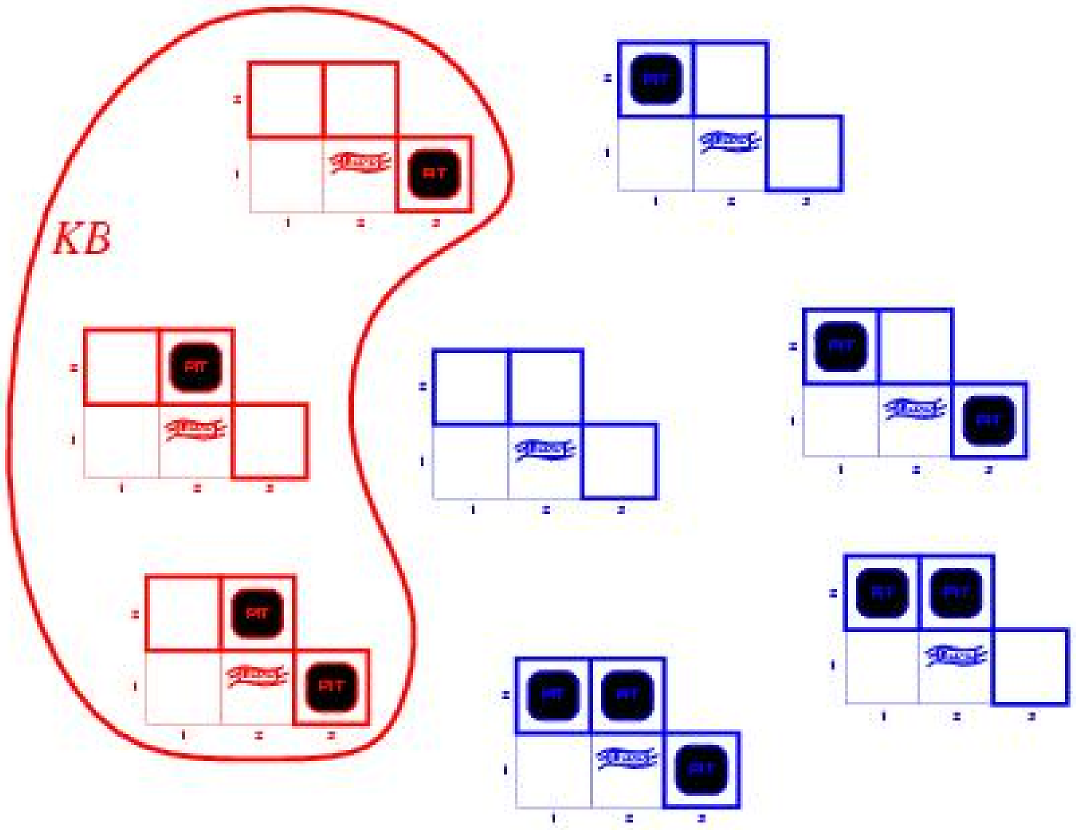
- 3 Boolean choices $\Rightarrow$ 8 possible models



Wumpus possible models

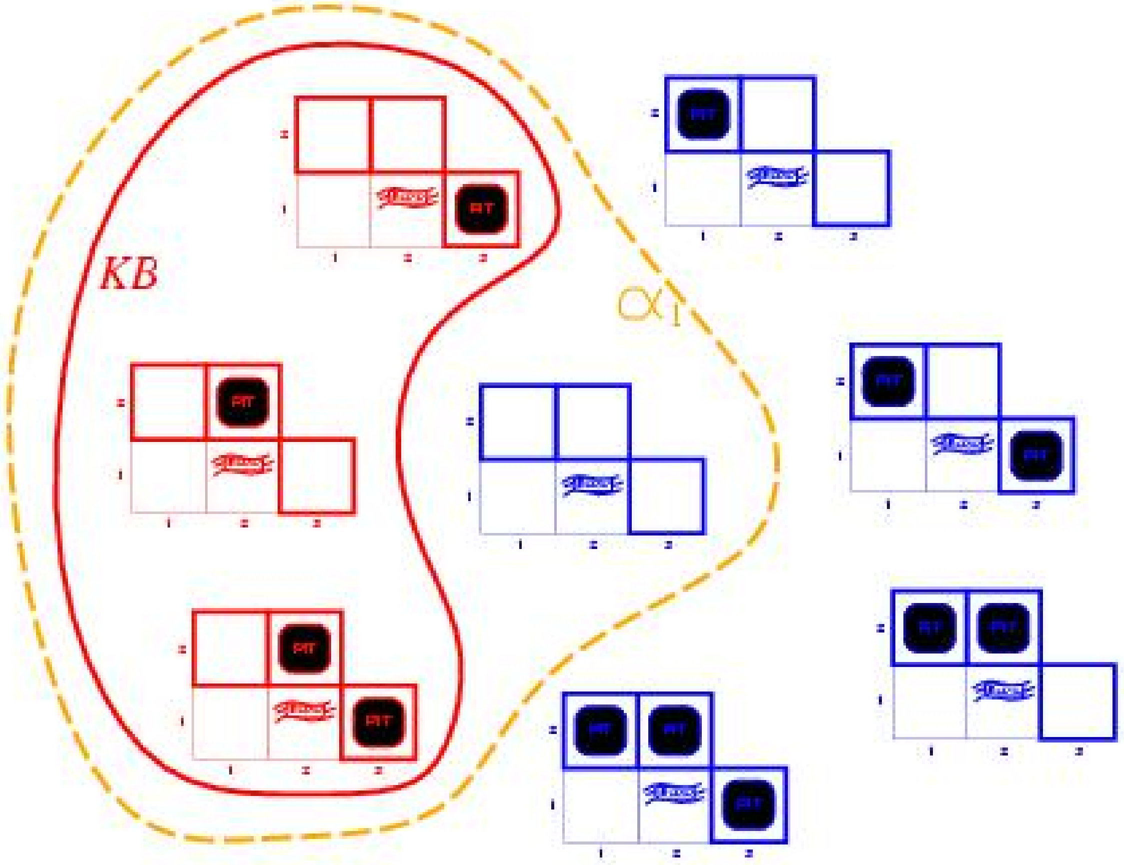


Wumpus models



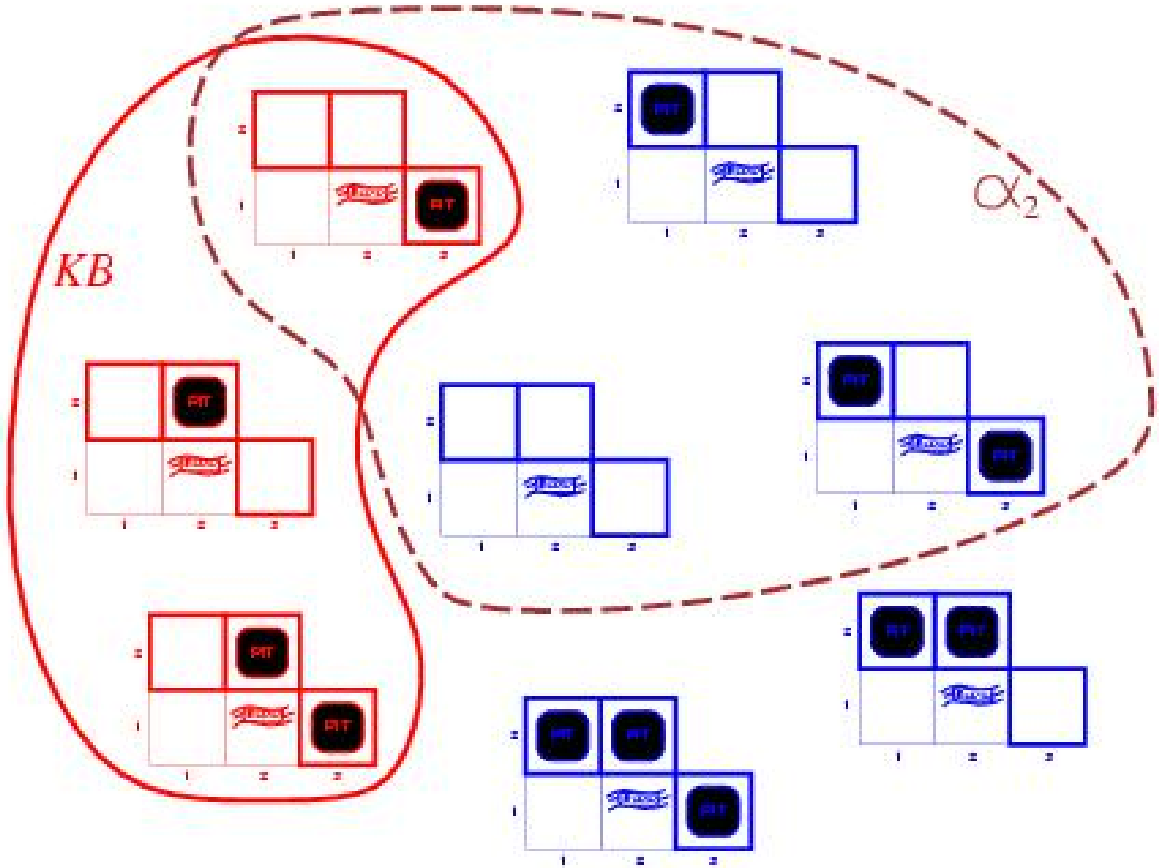
KB = wumpus-world rules + observations

Wumpus models



- KB = wumpus-world rules + observations
- α_1 = “there is no pit in [1,2]", KB $\models \alpha_1$, proved by model checking

Wumpus models



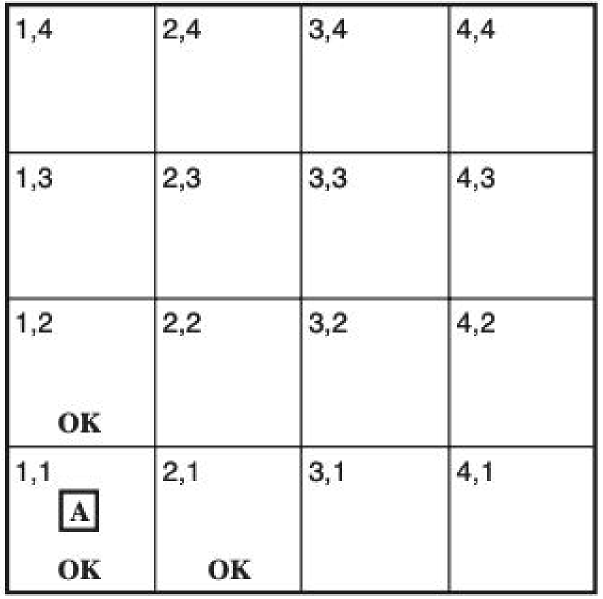
- KB = wumpus-world rules + observations
- α_2 = “there is no pit in [2,2]"

Inference

- Proof $\neg P1,2$ and $\neg P2,1$:
- R2: $B1,1 \Leftrightarrow (P1,2 \vee P2,1)$
- R6: $(B1,1 \Rightarrow (P1,2 \vee P2,1)) \wedge ((P1,2 \vee P2,1) \Rightarrow B1,1)$

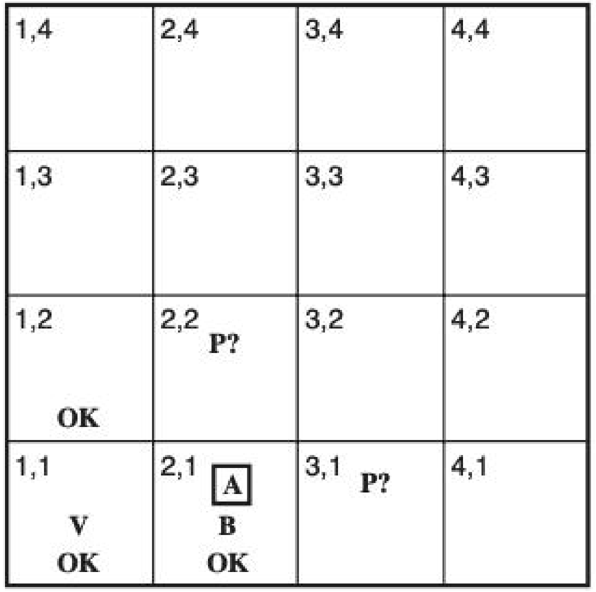
- R7: $(P1,2 \vee P2,1) \Rightarrow B1,1$ — And elimination

- R8: $\neg B1,1 \Rightarrow \neg(P1,2 \vee P2,1)$ — Contrapositive
- R9: $\neg(P1,2 \vee P2,1)$ — Modus Ponens
- R10: $\neg P1,2 \wedge \neg P2,1$ — De Morgan



Action and Percept

- Move to [2, 1]
- B2,1 is True
- There is a pit in [2,2] or [3,1]



Inference

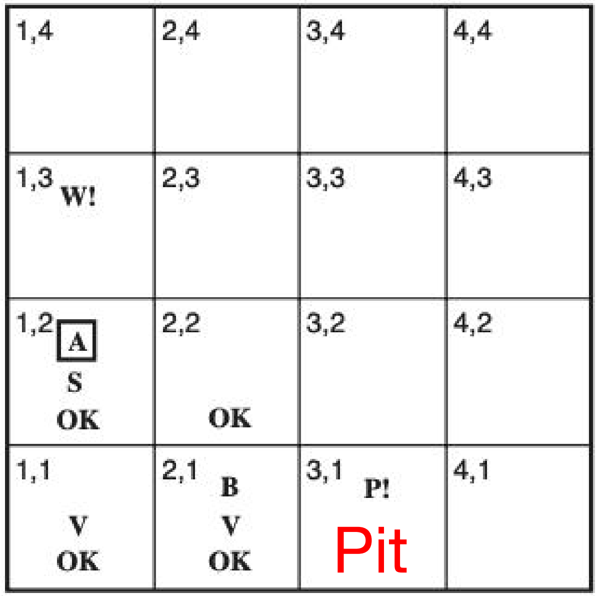
- The agent return from [2,1] to [1,1] and then moves to [1, 2]

  - R11: ¬B1,2
  - R12: B1,2 ⇔ (P1,1 ∨ P2,2 ∨ P1,3)
  - R13: ¬P2,2
  - R14: ¬P1,3

  - R3: B2,1 ⇔ (P1,1 ∨ P2,2 ∨ P3,1)
  - R5: B2,1 → R15: P1,1 ∨ P2,2 ∨ P3,1

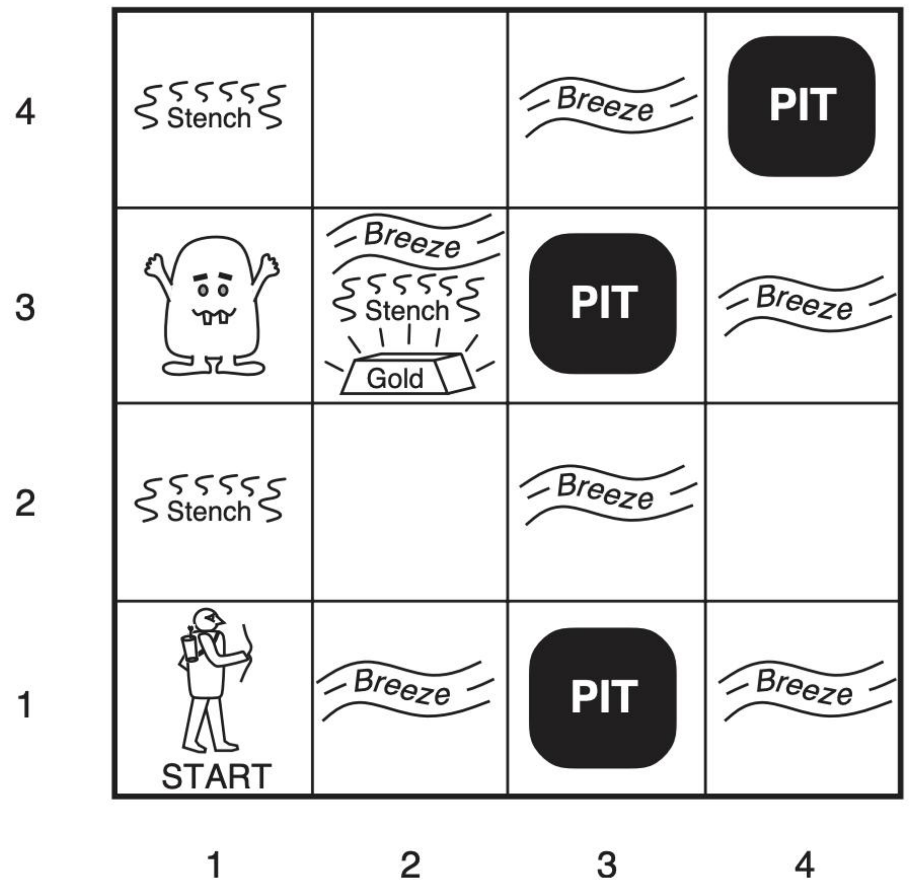
  - R16: P1,1 ∨ P3,1 (resolve R13 with R15)

  - R17: P3,1 （resolve R1 with R16)



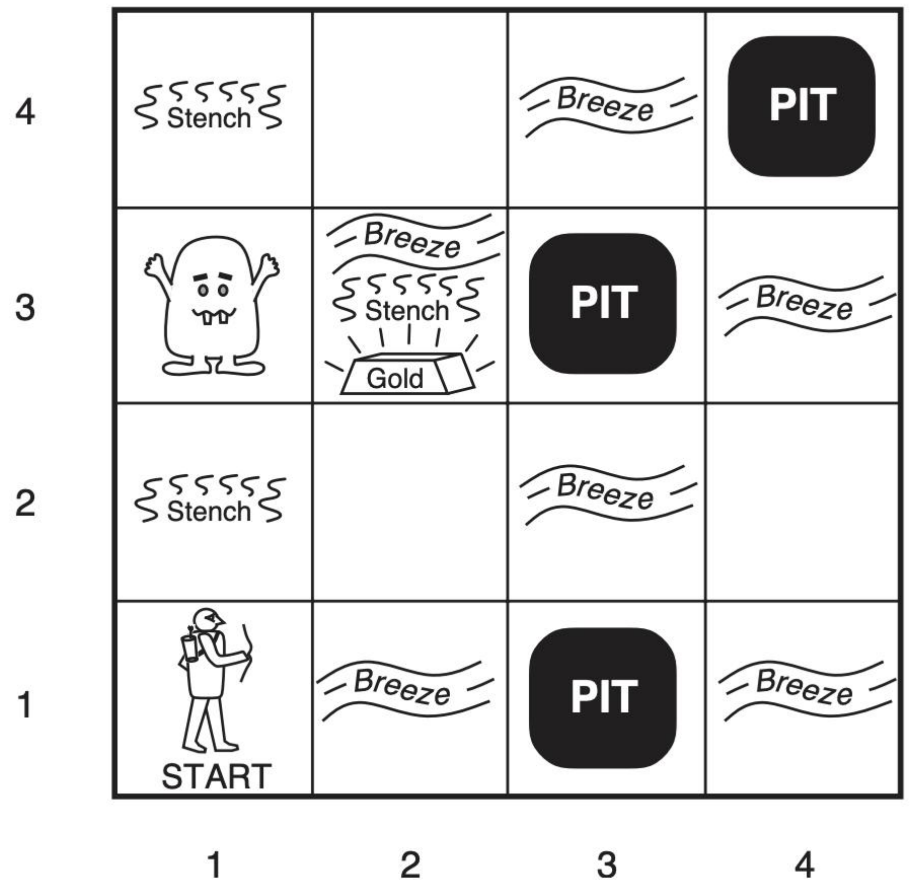
The current state of the world

- Collecting axioms:
  - ¬P1,1
  - ¬W1,1
  - For each square, it knows that the square is breezy if and only if a neighboring square has a pit:
    - B1,1 ⇔ (P1,2 ∨ P2,1)
    - ...
  - Square is smelly if and only if a neighboring square has a wumpus.
    - S1,1 ⇔ (W1,2 ∨ W2,1)
    - ...



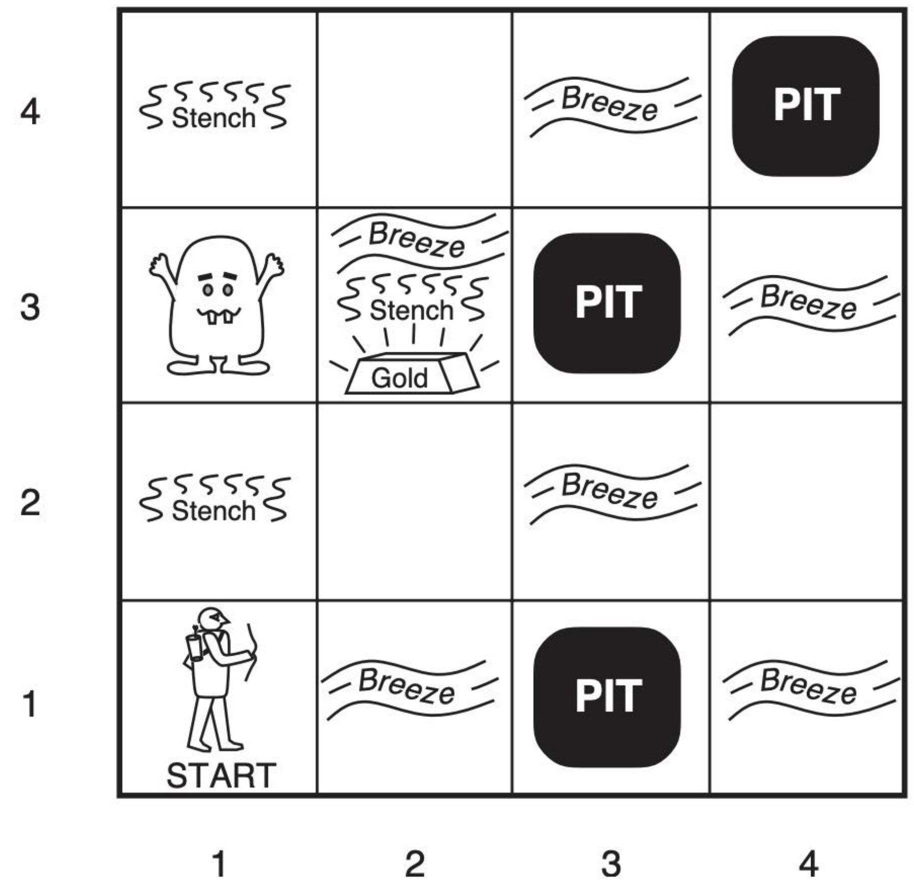
The current state of the world

- There is exactly one wumpus
  - $W1,1 \lor W1,2 \lor ..., W4,3 \lor W4,4$
- There is at most one wumpus
  - $\neg W1,1 \lor \neg W1,2$
  - $\neg W1,1 \lor \neg W1,3$
  - ...
  - $\neg W4,3 \lor \neg W4,4$



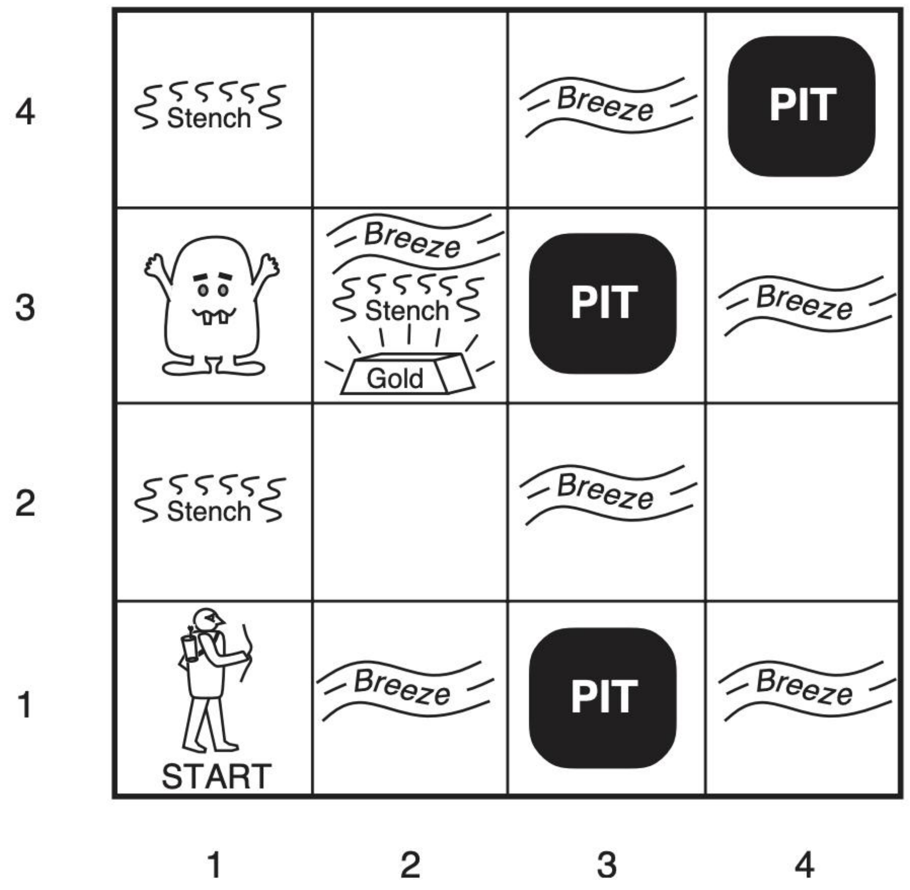
The current state of the world

- Fluent: associating propositions with time steps extends to any aspect of the world that changes over time
  - $L^0_{1,1}$: the agent is at square [1,1] at time 0
- Consider time stamp for percepts
  - $\neg Stench^3$
  - $Stench^4$



The current state of the world

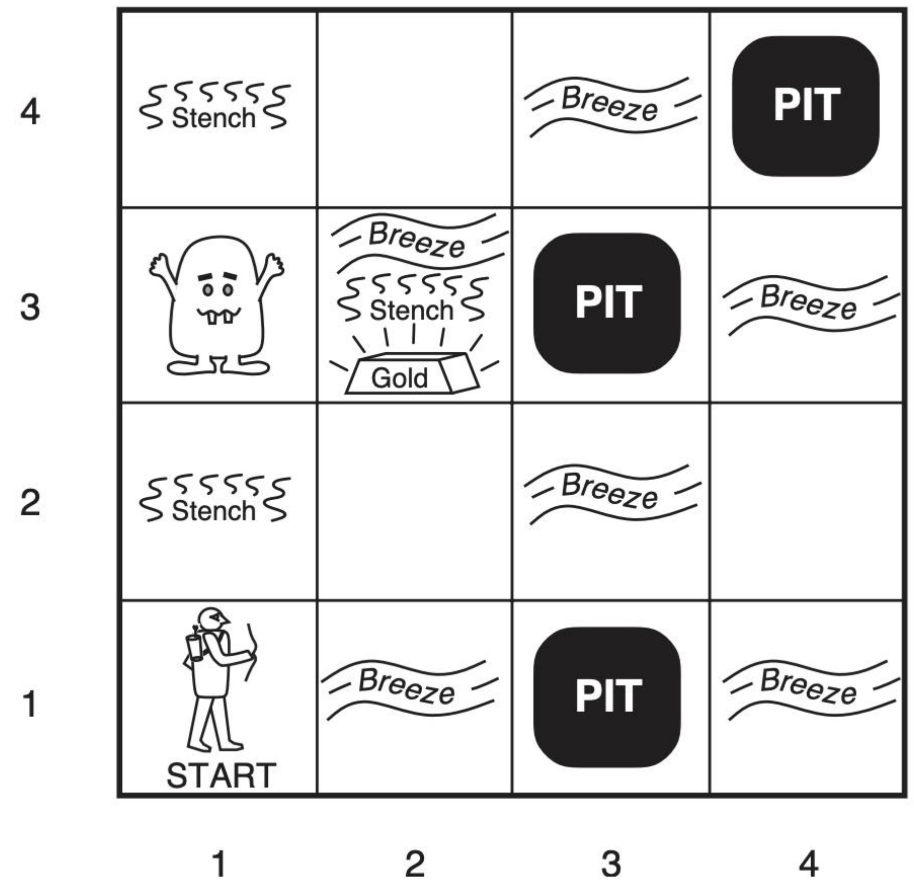
- For any time step t and any square [x, y], we have
  - $L^t_{x,y} \Rightarrow (Breeze^t \Leftrightarrow B_{x,y})$
  - $L^t_{x,y} \Rightarrow (Stench^t \Leftrightarrow S_{x,y})$



The current state of the world

- Proposition symbols for the occurrences of actions
  - $Forward_0$

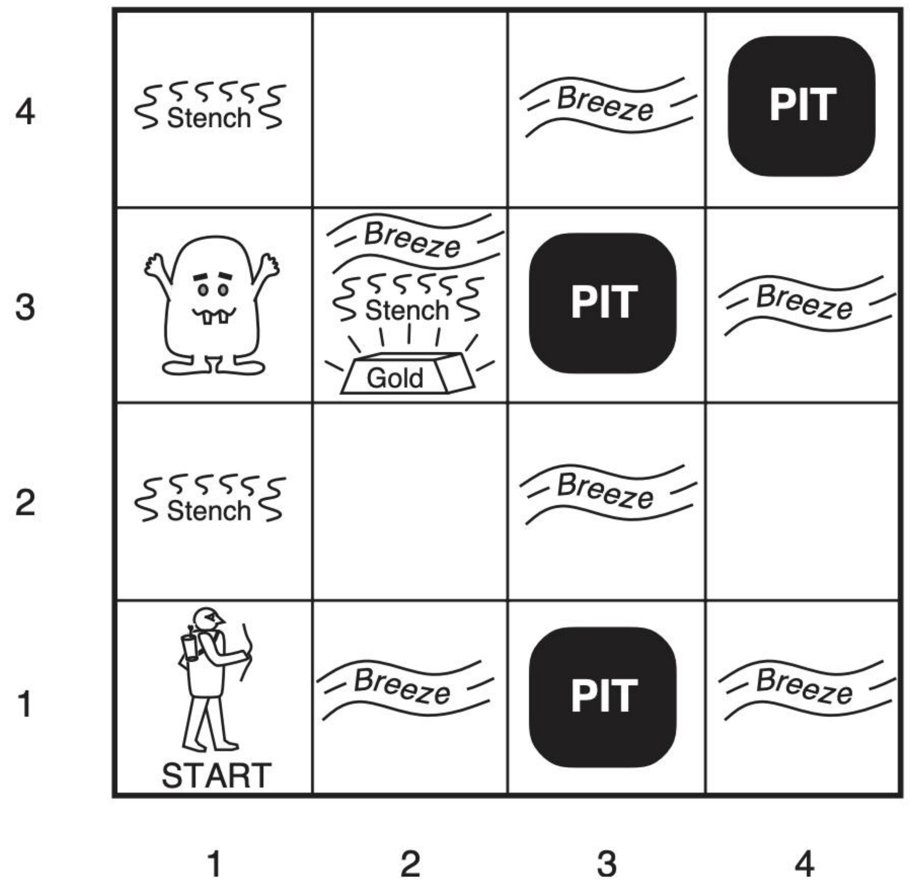
- The percept for a given time step happens first, followed by the action for that time step, followed by a transition to the next time step.



The current state of the world

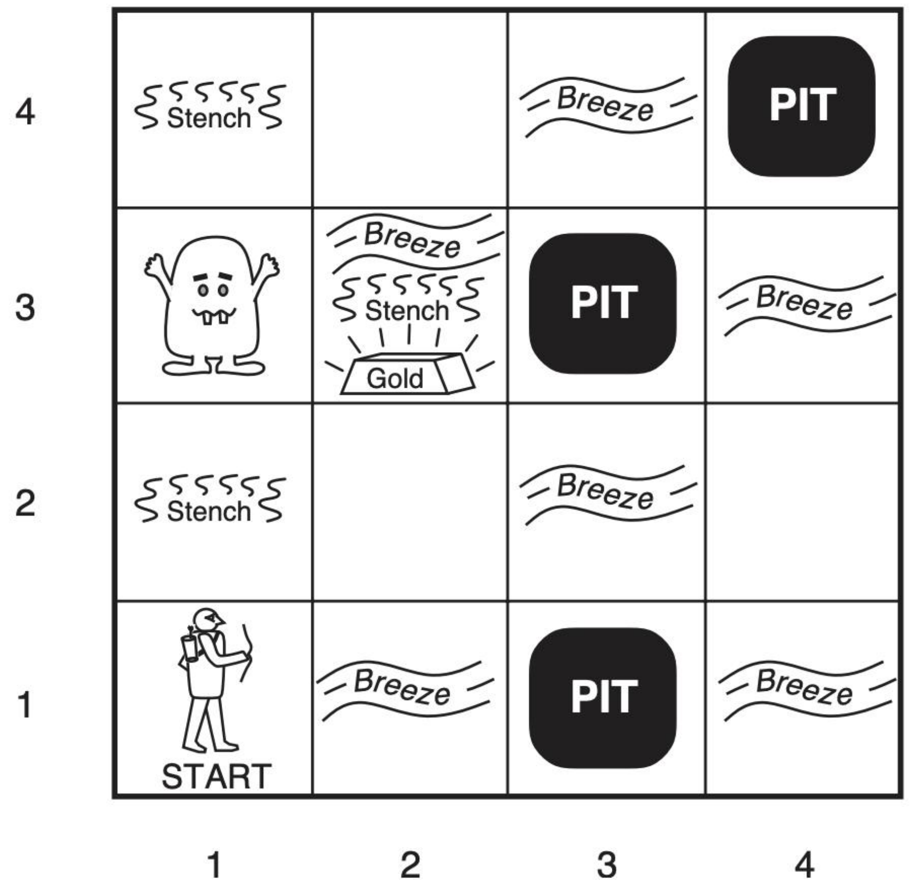
- Effect axioms: describe how the world changes,
  - $L^0_{1,1} \land FaceEast_0 \land Forward_0 \Rightarrow (L^1_{2,1} \land \neg L^1_{1,1})$

- Need to specify for each action, time step, location and orientation



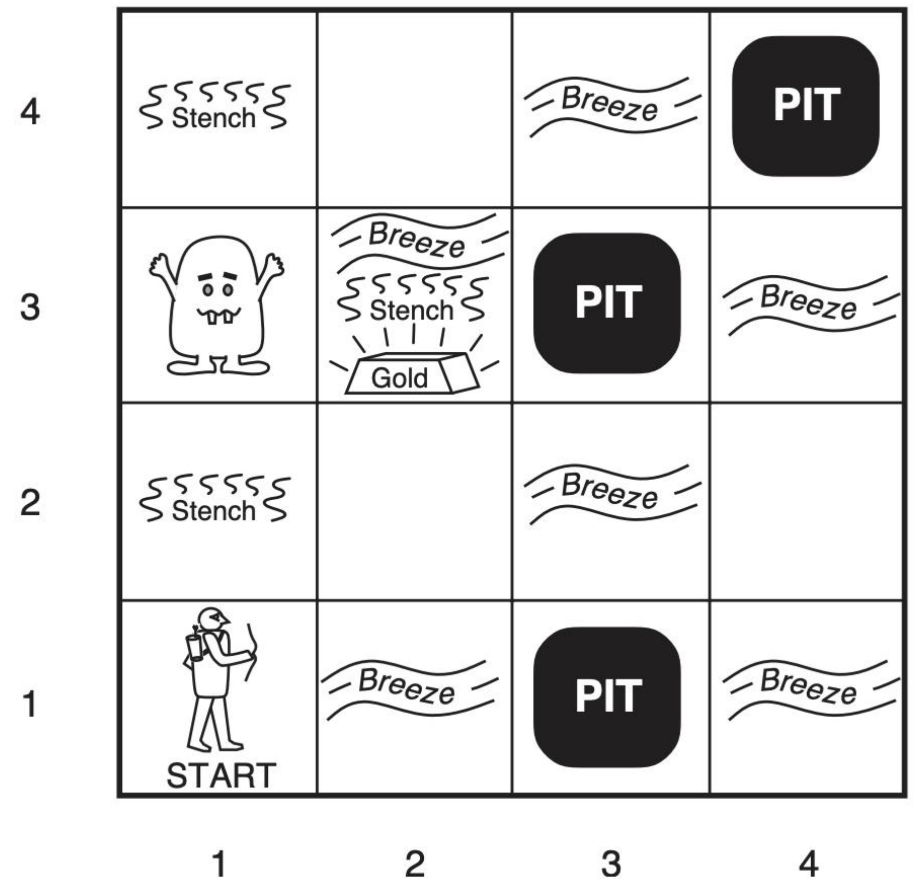
The current state of the world

- Frame problem:
  - ASK(KB, $HaveArrow^1$)
  - The information has been lost because the effect axiom fails to state what remains unchanged as the result of an action
- Add frame axioms:
  - $Forward^t \Rightarrow (HaveArrow^t \Leftrightarrow HaveArrow^{t+1})$
  - $Froward^t \Rightarrow (WumpusAlive^t \Leftrightarrow WumpusAlive^{t+1})$
- For m actions and n fluents, needs O(mn)



The current state of the world

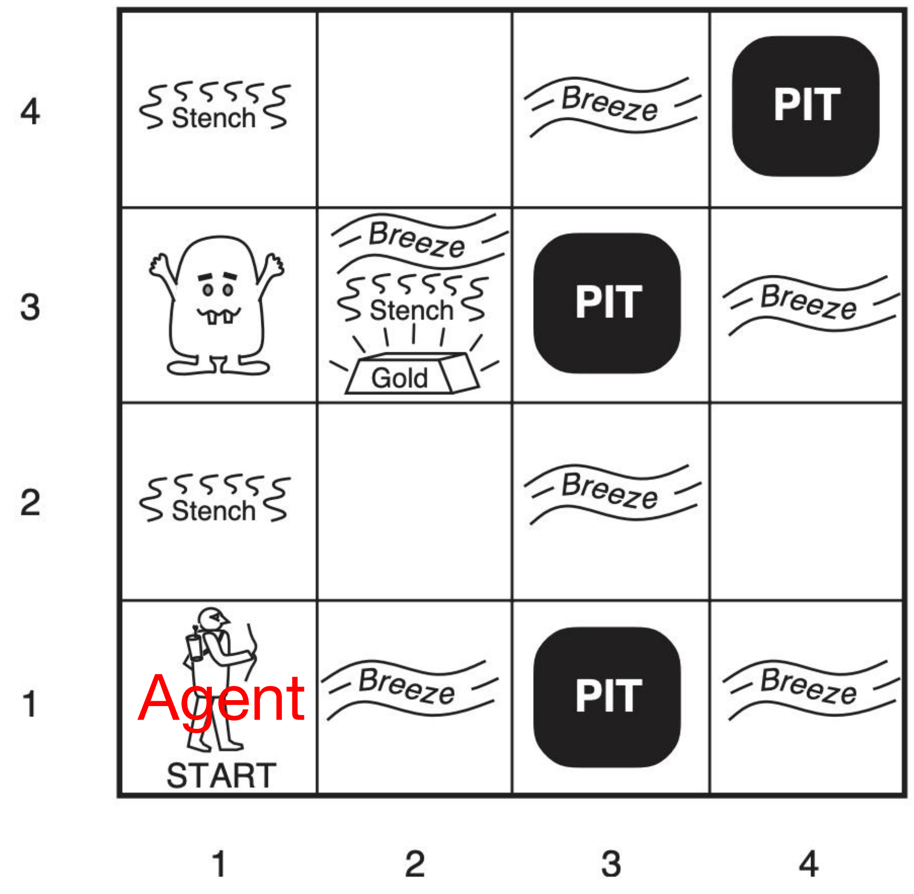
- Writing axioms about fluents
- Successor-state axiom:
  - $F^{t+1} \Leftrightarrow \text{ActionCausesF}^{t} \lor (F^{t} \land \text{ActionCausesNotF}^{t})$
  - ...
  - $\text{HaveArrow}^{t+1} \Leftrightarrow (\text{HaveArrow}^{t} \Leftrightarrow \neg\text{Shoot}^{t})$
  - $L^{t+1}_{1,1} \Leftrightarrow (L^{t}_{1,1} \land (\neg\text{Forward}^{t} \lor \text{Bump}^{t+1})) \lor (L^{t}_{1,2} \land (\text{South}^{t} \land \text{Forward}^{t})) \lor (L^{t}_{2,1} \land (\text{West}^{t} \land \text{Forward}^{t}))$



The current state of the world

- $\neg Stench^0 \wedge \neg Breeze^0 \wedge \neg Glitter^0 \wedge \neg Bump^0 \wedge \neg Screem^0$

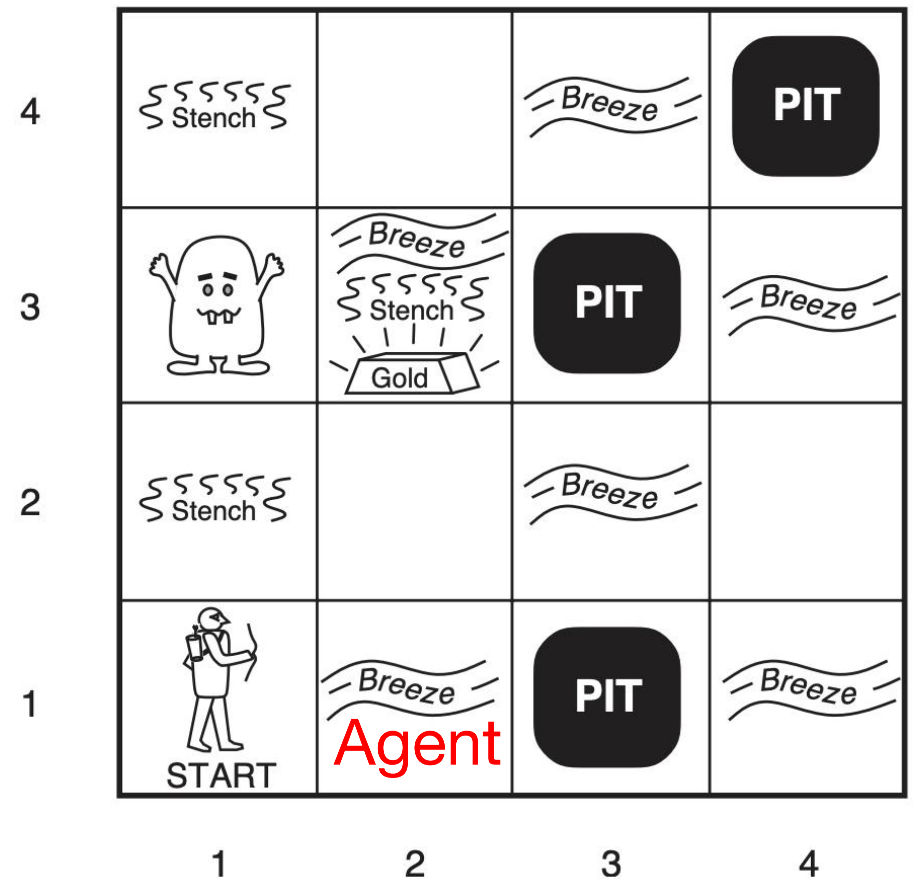
- $Forward^0$



The current state of the world

- $\neg Stench^1 \wedge Breeze^1 \wedge \neg Glitter^1 \wedge \neg Bump^1 \wedge \neg Screem^1$

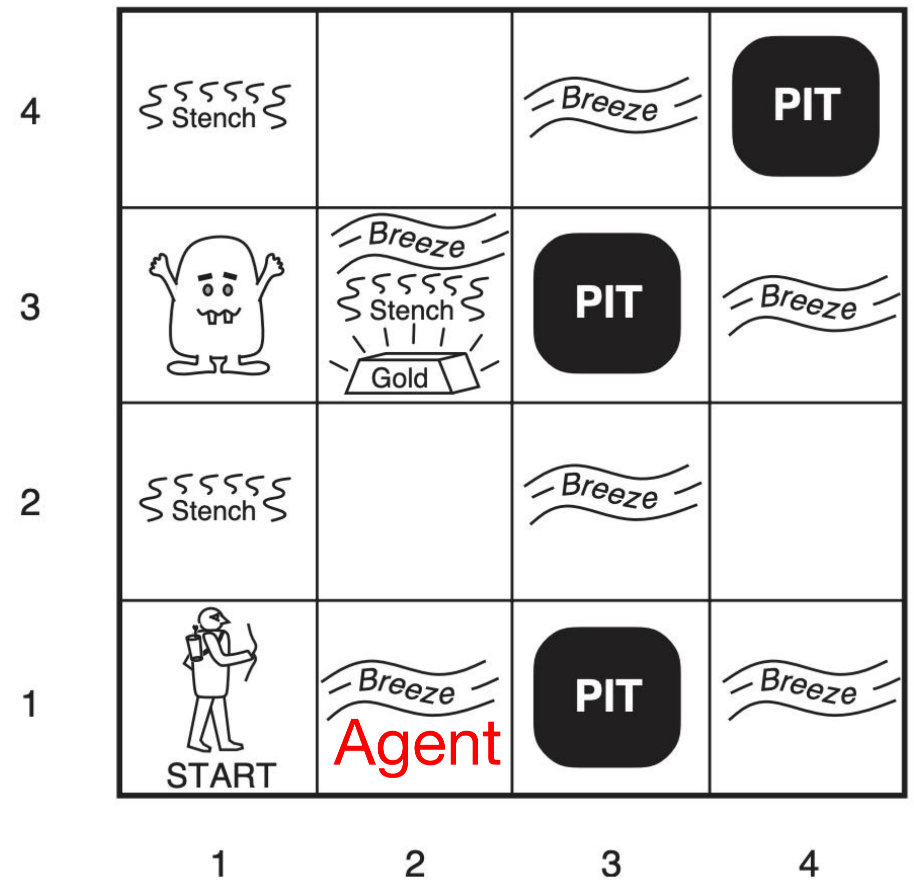
- $TurnRight^1$



The current state of the world

- ¬Stench^2 ∧ Breeze^2 ∧ ¬Glitter^2 ∧ ¬Bump^2 ∧ ¬Screem^2

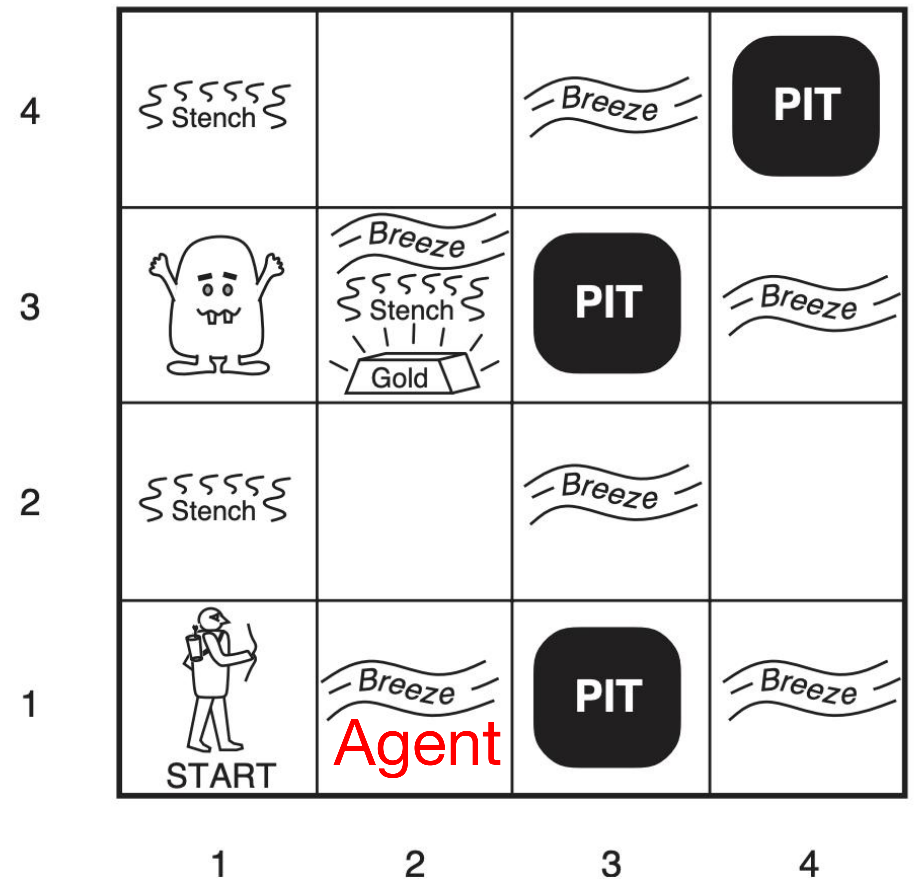
- TurnRight^2



The current state of the world

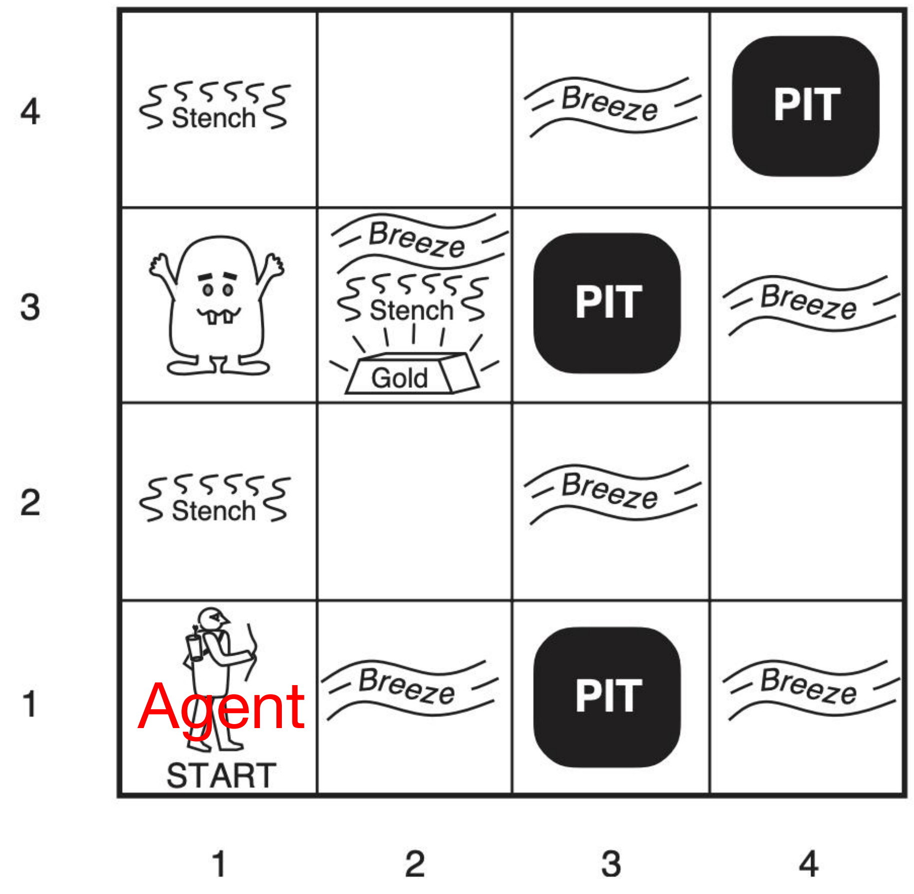
- $\neg Stench^3 \wedge Breeze^3 \wedge \neg Glitter^3 \wedge \neg Bump^3 \wedge \neg Screem^3$

- $Forward^3$



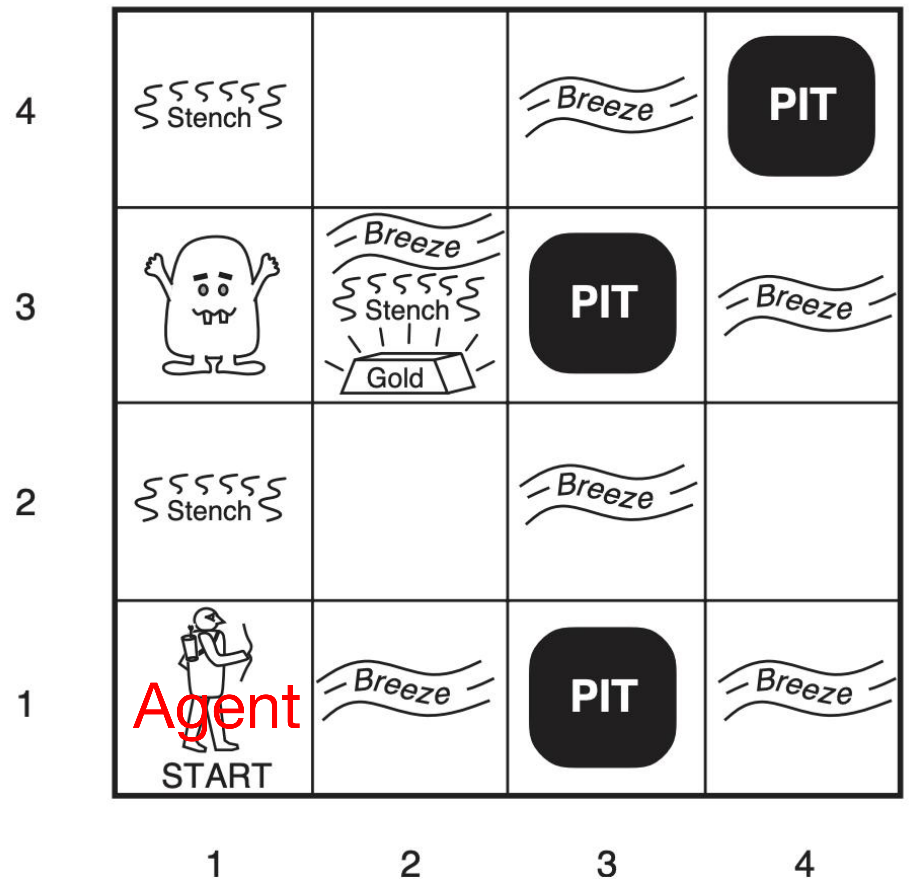
The current state of the world

- $\neg Stench^4 \land \neg Breeze^4 \land \neg Glitter^4 \land \neg Bump^4 \land \neg Screem^4$
- $TurnRight^4$



The current state of the world

- $\neg Stench^5 \wedge \neg Breeze^5 \wedge \neg Glitter^5 \wedge \neg Bump^5 \wedge \neg Screem^5$
- $Forward^5$



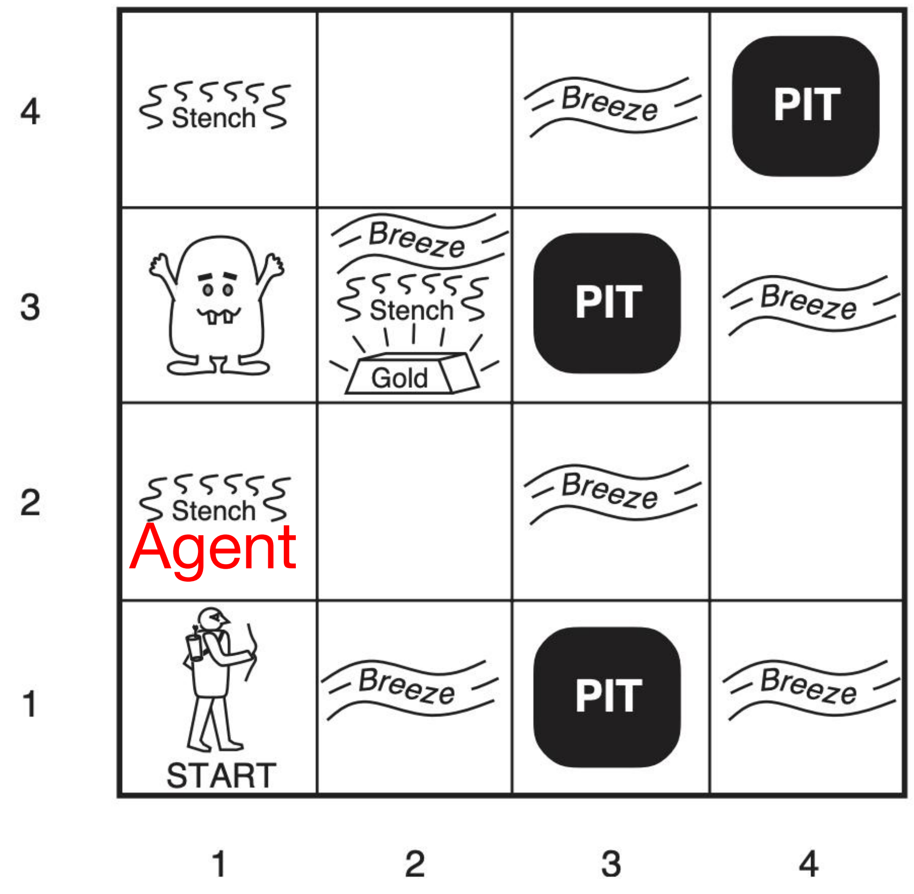
The current state of the world

- $Stench^6 \land \neg Breeze^6 \land \neg Glitter^6 \land \neg Bump^6 \land \neg Screem^6$

- $Ask(KB, L^6_{1,2}) = True$

- $Ask(KB, W_{1,3}) = True$

- $Ask(KB, P_{3,1}) = True$



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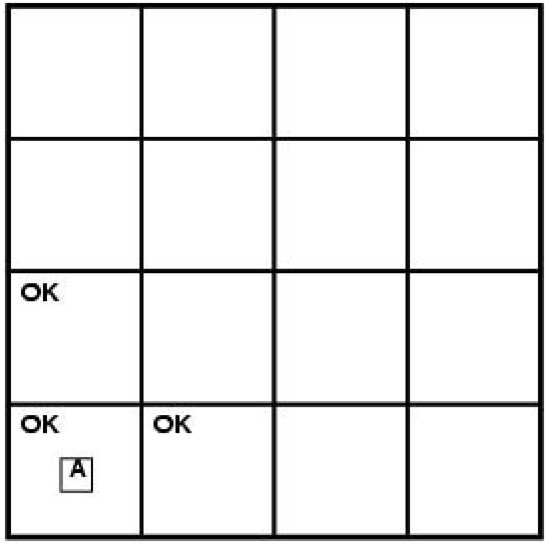

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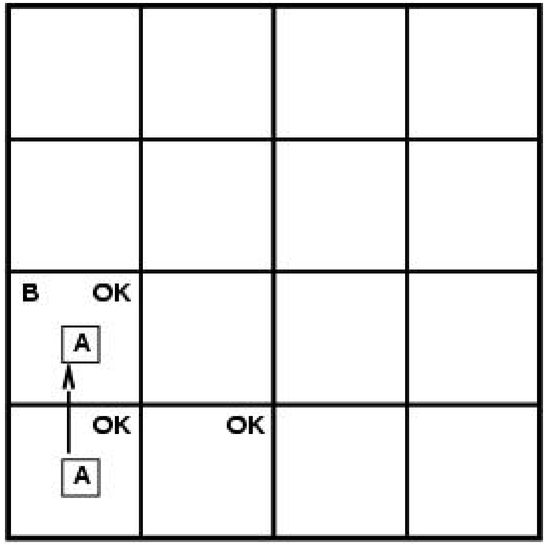

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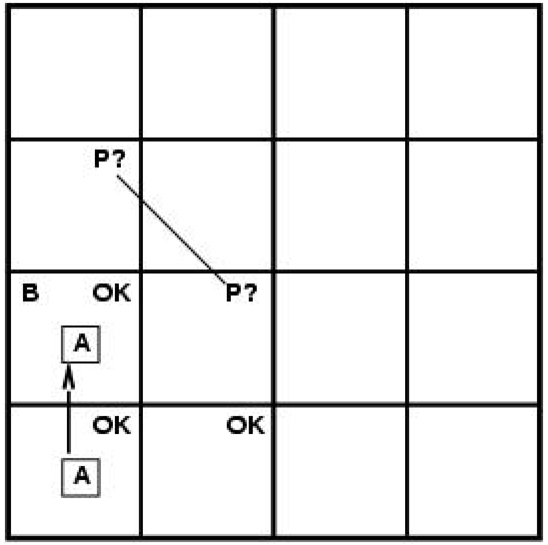

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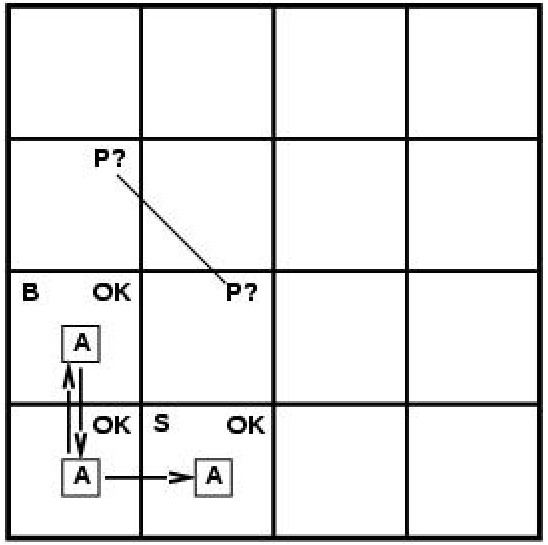

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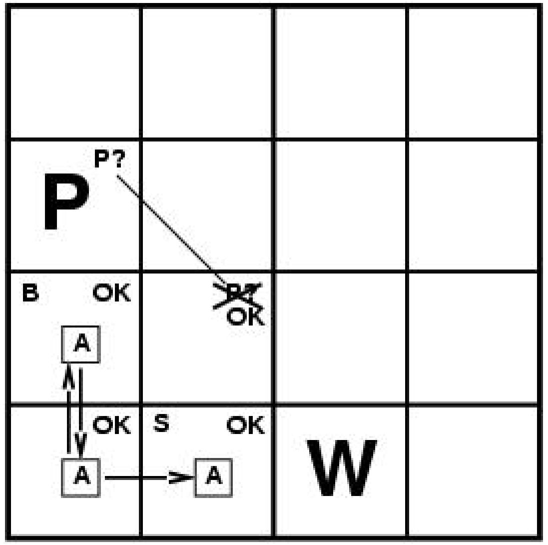

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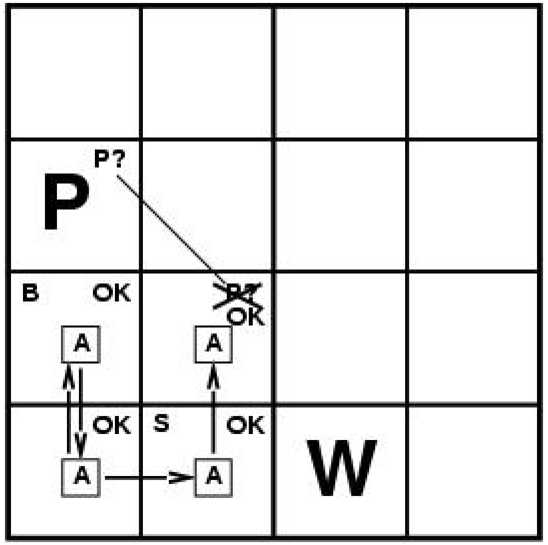

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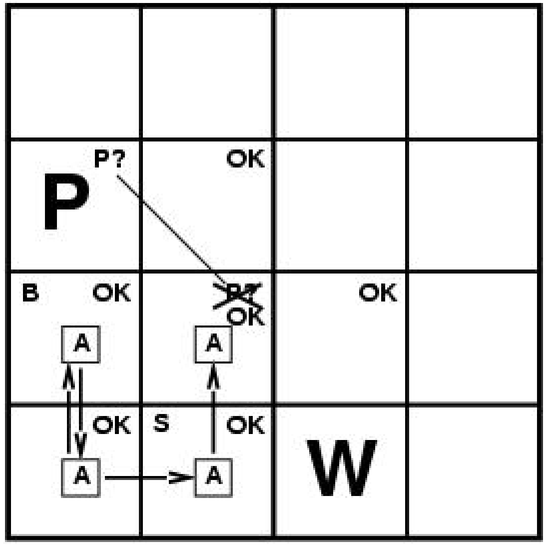

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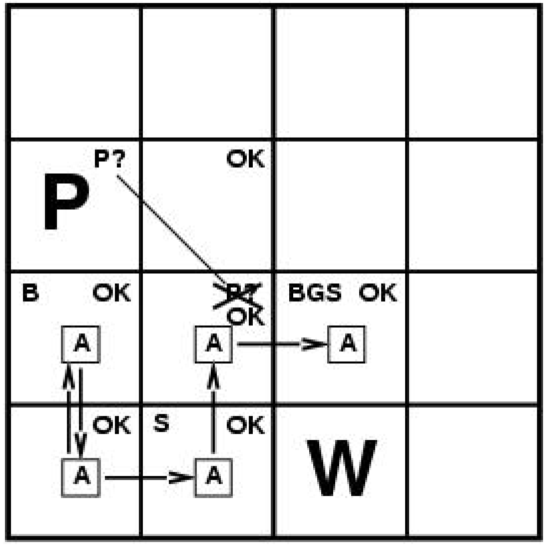
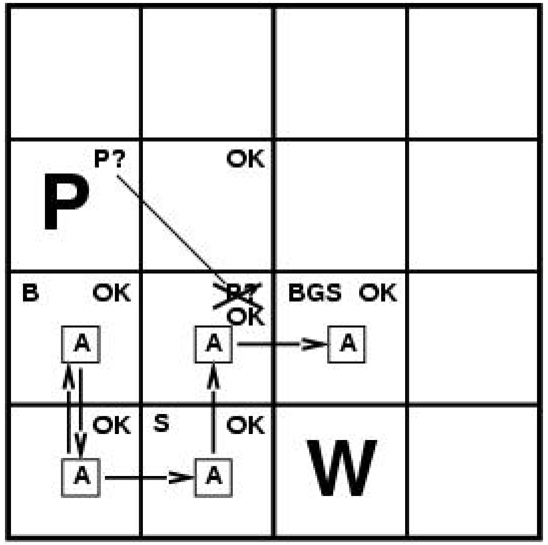


The current state of the world

- In each case where the agent draws a conclusion from the available Information, that conclusion is guaranteed to be correct if the available Information is correct.
- This is a fundamental property of logical reasoning



A Hybrid Agent

- The agent program maintains and updates a knowledge base as well as a current plan.

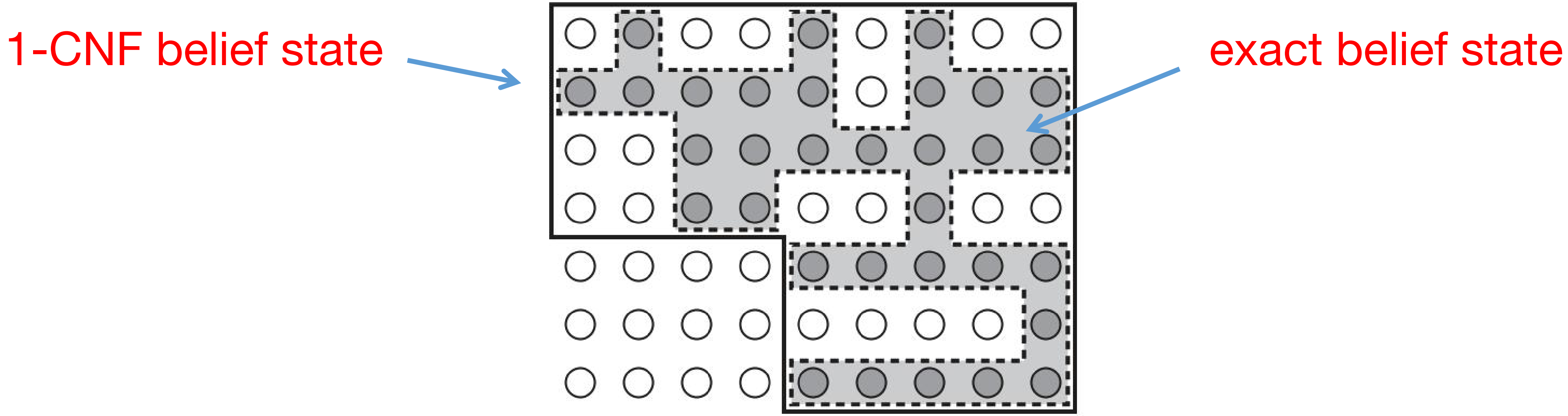
- A* search
  - First, if there is a glitter, the program constructs a plan to grab the gold, follow a route back to the initial location.
  - Otherwise, if there is no current plan, the program plans a route to the closest safe square that it has not visited yet, making sure the route goes through only safe squares.

Logical State Estimation

- Constant update time of the knowledge base
  - Using a cache to store the results of inference.
- Belief state:
  - the short-term memory of the agent.
  - all of the information the agent has remembered from the previous time at time t
  - every possible set of physical states.
- The set of states of at time 1:
  - $WumpusAlive \wedge L^1_{2,1} \wedge B_{2,1} \wedge (P_{2,1} \vee P_{2,2})$
- State estimation
  - The process of updating the belief state as new percepts arrive

Logical State Estimation

- Approximate state estimation
    - represent belief states as conjunctions of literals, that is, 1-CNF formulas
- Prove X_t and $\neg X_t$ for each symbol X_t given the belief state at t - 1



Example

- Which of the following is correct?
  - False $\models$ True True
  - True $\models$ False False
  - $(A \wedge B) \models (A \Leftrightarrow B)$ True
  - $A \Leftrightarrow B \models A \vee B$ False
  - $A \Leftrightarrow B \models \neg A \vee B$ True
  - $(A \vee B) \wedge \neg(A \Rightarrow B)$ is satisfiable True
  - $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable True

Example

- Consider a vocabulary with only four propositions, A, B, C, D. How many models are there for the following sentences?
  - $B \lor C$ 12
  - $\neg A \lor \neg B \lor \neg C \lor \neg D$ 15

Example

- Prove, or find a counterexample to, each of the following assertions

1. If $\alpha \vDash \gamma$ or $\beta \vDash \gamma$ (or both) then $(\alpha \land \beta) \vDash \gamma$ True

2. If $\alpha \vDash (\beta \land \gamma)$ then $\alpha \vDash \gamma$ and $\alpha \vDash \beta$ True

3. If $\alpha \vDash (\beta \lor \gamma)$ then $\alpha \vDash \gamma$ or $\alpha \vDash \beta$ (or both) False

Example

- A propositional 2-CNF expression is a conjunction of clauses, each containing exactly 2 literals, e.g.,

$$(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G)$$

1. Prove using resolution that the above sentence entails G

- Add $\neg G$. Resolve with the last two clauses to produce $\neg C$ and $\neg D$.
- Resolve with the second and third produce $\neg A$ and $\neg B$
- Resolve with the first clause to produce the empty clause

Example

- A propositional 2-CNF expression is a conjunction of clauses, each containing exactly 2 literals, e.g.,

$$(A \lor B) \land (\neg A \lor C) \land (\neg B \lor D) \land (\neg C \lor G) \land (\neg D \lor G)$$

2. Two clauses are semantically distinct if they are not logically equivalent. How many semantically distinct 2-CNF clauses can be constructed from n proposition symbols?

- There are $C(2n, 2) = (2n)(2n-1)/2 = 2n^2-n$ clauses with two different literals
- $\neg A \lor A$ ... are equivalent. So we have $2n^2-2n+1$
- Add $A \lor A$ ..., so we have $2n^2+1$