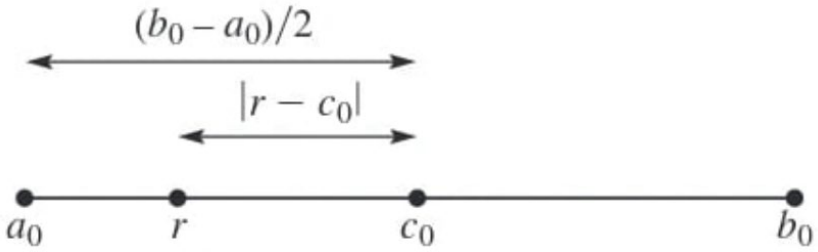
Recap on Bisection

- Bisection method for root finding
- "Almost" linear convergence: $|c_n - r| \leq \frac{b-a}{2^{n+1}}$

$$|x_{n+1} - x^*| \leq C|x_n - x^*|$$
$$\Rightarrow |x_{n+1} - x^*| \leq AC^n$$

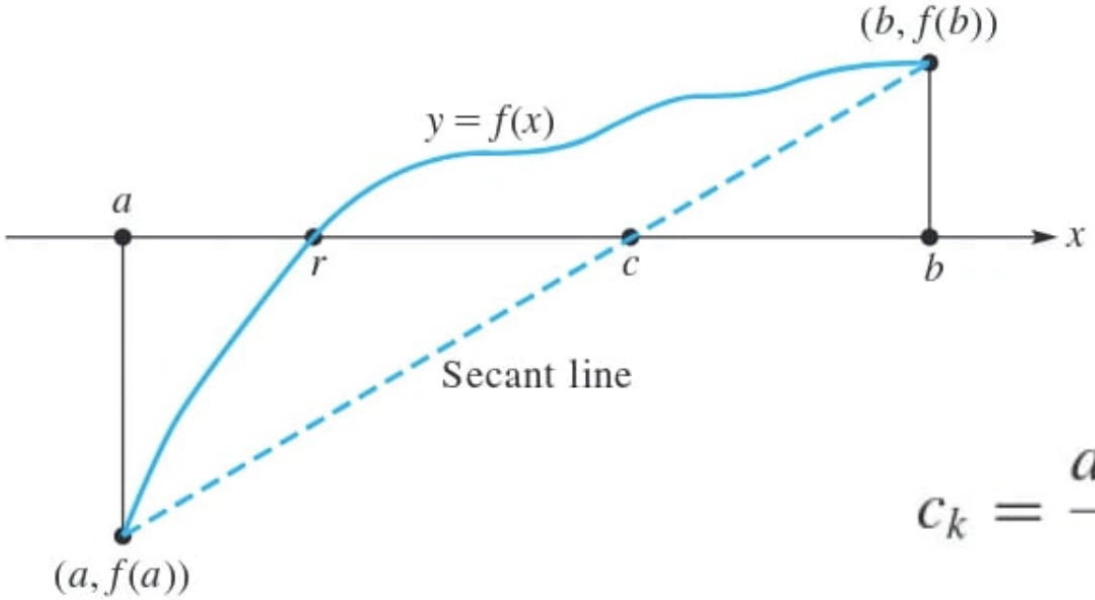
- Today's agenda
  - Bisection variants
  - Matlab tutorial
  - Newton's method



Bisection variants

False position method

Rather than selecting the midpoint, this method uses the point where the secant lines intersect the x-axis.

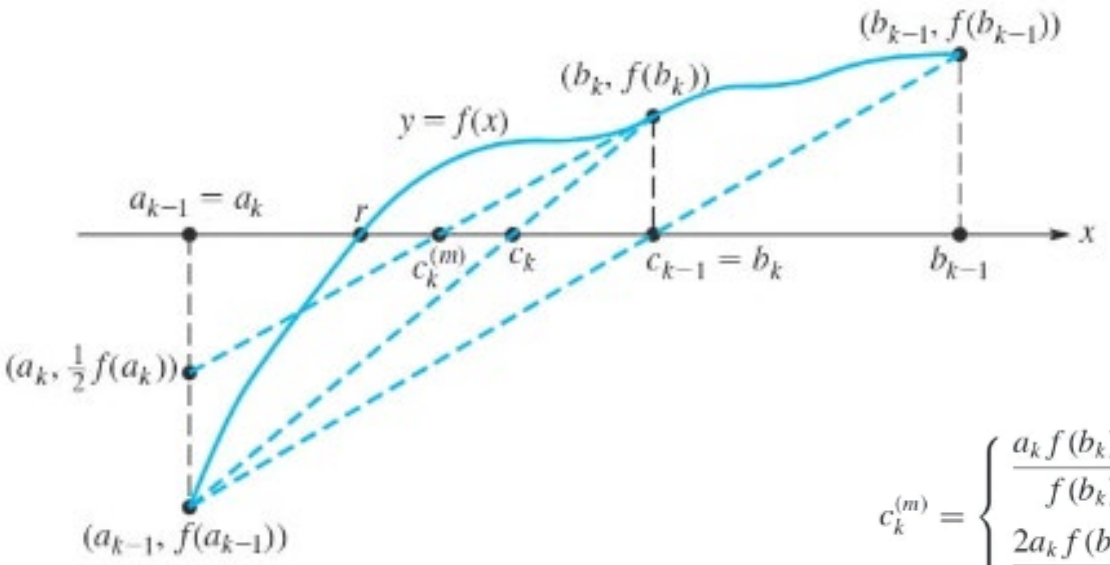


$$c_k = \frac{a_k f(b_k) - b_k f(a_k)}{f(b_k) - f(a_k)}$$

Remarks

- False position method uses the values of $f(a), f(b)$, which is more adaptive to a particular function.
- It may repeatedly select the same endpoint.
- Modified false position method changes the slope of the straight line to get closer to the root.

MFP method



$$c_k^{(m)} = \begin{cases} \dfrac{a_k f(b_k) - 2b_k f(a_k)}{f(b_k) - 2f(a_k)}, & \text{if } f(a_k) f(b_k) < 0 \\ \dfrac{2a_k f(b_k) - b_k f(a_k)}{2f(b_k) - f(a_k)}, & \text{if } f(a_k) f(b_k) > 0 \end{cases}$$

In some cases, superlinear convergence rate can be obtained.

Convergence rate

- Linear convergence: $C \in [0,1)$

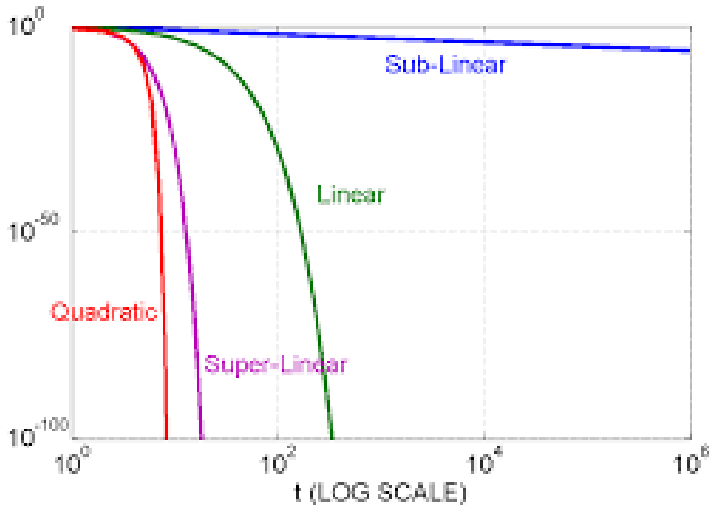
$$|x_{n+1} - x^*| \leq C|x_n - x^*|$$

- Superlinear convergence: $\alpha \in (1,2)$

$$|x_{n+1} - x^*| \leq C|x_n - x^*|^{\alpha}$$

- Quadratic convergence

$$|x_{n+1} - x^*| \leq C|x_n - x^*|^2$$



Matlab tutorial

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Some facts

- Everything in Matlab is a matrix or tensor!
- Matlab does not need any variable declarations, no dimension statements, no storage allocation, no pointers; but better to aware of these for efficiency.
- Programs can be run step by step, with full access to all variables, functions, etc.

Getting help

- To get help on a function type “help function_name”, e.g., “help plot”.
- To find a topic, type “lookfor topic”, e.g., “lookfor matrix”

```
>> lookfor matrix
array2table                - Convert tall matrix to table
array2timetable            - Convert tall matrix to timetable
cell2mat                   - Convert the contents of a cell array into a sin
cov                        - Covariance matrix.
flipdim                    - Flip tall matrix along specified dimension.
ismatrix                   - True if input is a matrix.
mldivide                   - \   Backslash or left matrix divide.
mrdivide                   - /   Slash or right matrix divide.
mtimes                     - *  Matrix multiply.
nnz                        - Number of nonzero matrix elements.
norm                       - tall vector and matrix norms.
pTimesTranspose            - matrix multiply with transposed inputs
str2num                    - Convert string matrix to numeric array.
topkrows                   - Top k sorted rows of a matrix, table, or timeta
validateMatrix             - validateType Possibly deferred check for matrix
extractDataVarsTall        - Extracts data variables for table and matrix in
parseGroupVarsTall         - Parses grouping variables for table and matrix
cov                        - Covariance matrix.
issortedrows               - Check if matrix rows are sorted
sortrows                   - Sort rows of a matrix.
topkrows                   - Top K sorted rows of matrix.
mtimes                     - Matrix multiplication for calendar durations.
ctranspose                 - Transpose a categorical matrix.
ismatrix                   - True if categorical array is a matrix.
```

Matlab's Workspace

- **who**, **whos** – current workspace vars.
- **save** – save workspace vars to *.mat file.
- **load** – load variables from *.mat file.
- **clear all** – clear workspace vars.
- **close all** – close all figures
- **clc** – clear screen
- **clf** – clear figure

Variables

- Variable names:
  - Must start with a letter
  - May contain only letters, digits, and '_'
  - Case sensitive, one & ONE are different
  - Built-in variables/functions are all lower-case.
- Assignment:
  - Variable = number
  - Variable = expression

Basic Commands

- **%** used to denote a comment
- **;** suppresses display of value (when placed at end of a statement)
- **...** continues the statement on next line
- **eps** machine epsilon
- **inf** infinity
- **NaN** not-a number, e.g., 0/0.

Numbers

- To change format of numbers:

  format long, format short, etc.

  See "**help format**".
- Mathematical functions: **sqrt(x)**, **exp(x)**, **cos(x)**, **sin(x)**, **sum(x)**, etc.
- Operations: **+**, **-**, *, **/**,**^**
- Constants: **pi**, **exp(1)**, etc.
- Elementwise operator for vectors, **.**

Arrays and Matrices

- **v = [-2 3 0 4.5 -1.5];** % length 5 row vector.
- **v = v';** % transposes v.
- **v(1);** % first element of v.
- **v(2:4);** % entries 2-4 of v.
- **v([3,5]);** % returns entries 3 & 5.
- **v=[4:-1:2];** % same as v=[4 3 2];
- **a=1:3; b=2:3; c=[a b];** → c = [1 2 3 2 3];

Arrays and Matrices (2)

- **x = linspace(-pi,pi,10);** % creates 10 linearly-spaced elements from –pi to pi.
- **logspace** is similar.
- **A = [1 2 3; 4 5 6];** % creates 2x3 matrix
- **A(1,2)** % the element in row 1, column 2.
- **A(:,2)** % the second column.
- **A(2,:)** % the second row.

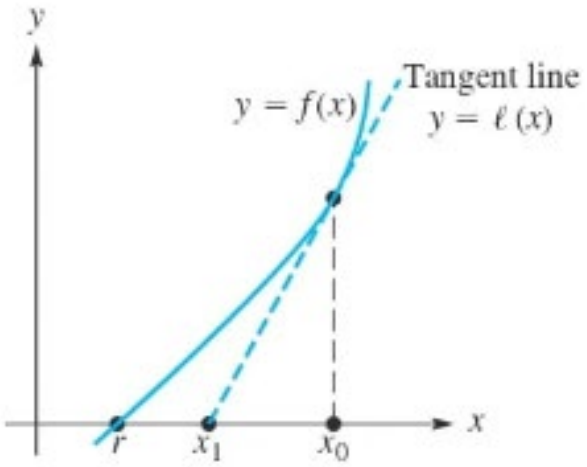
Newton's method

Overview

- Newton's method is also called Newton-Raphson iteration.
- It has a wider spectrum of applications.
- It requires the function to be differentiable.
- Its basic idea is that the graph at a certain point can be well approximated by its tangent.

Interpretation

- Starting from a point $(x_0, f(x_0))$
- Compute the tangent line
- Advance to the next point



Another interpretation

- What correction h should be added to x_0 to obtain the root precisely?
- Taylor series
- Find an approximated value for h

Summary of Newton

- Newton's method returns a sequence of points: $x_0, x_1, \cdots$
- Recursive or inductive definition

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Convergence: $\lim_{n \to \infty} x_n = r$