Linear Classification

1   Linear Classifiers - Perceptrons

1. You have learned the following perceptron weights for a two attribute $(x_1 \text{and} x_2)$ classification:

$$\boldsymbol{w} = \begin{bmatrix} 0.3 \\ -0.2 \\ 0.8 \end{bmatrix}$$

Using the sign activation function, find the class (+1 or -1) for the following data:

x_1	x_2	Class
+1	+1	
+1	-1	
-1	-1	
-1	+1	

2. Find out appropriate values of the weight vector to represent the following boolean functions.
   a. AND function i.e. a function that will separate following data
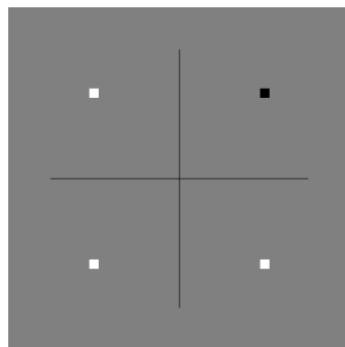
-1 AND -1 = false

-1 AND +1 = false

+1 AND -1 = false

+1 AND +1 = true

   b. OR function

   c. NOT i.e. represent NOT (A and B) function

   d. NOT of a single variable i.e. NOT (A)

e. Can you represent the XOR function. Show graphically why it's not possible

3. You are given the following training data:

x_1	x_2	x_3	Class
-1	1	-1	1
1	1	-1	0

Assume initial weights as $\boldsymbol{w} = \begin{bmatrix} 0.5 \\ -1.0 \\ -0.5 \end{bmatrix}$. The bias (b) or w_0 is given to be: 0.5.

Note that the decision rule in this case would be (hardlimit transfer function)

$$\text{if } w^T x > 0 \text{ then class} = 1 \text{ else class} = 0$$

Apply the perceptron learning algorithm on this dataset and find the final values of the weights vector. Assume a learning rate of 1.

4. You are given the following training data and parameters as shown in figure below:
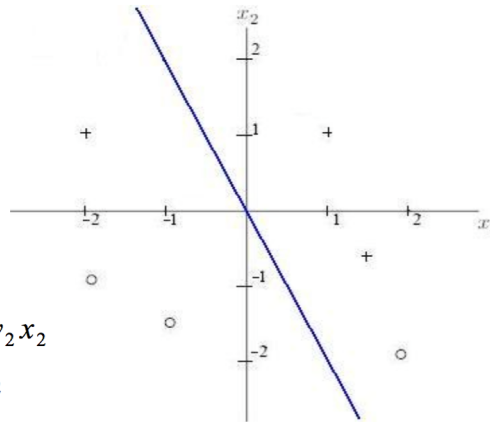
Learning Example

Initial Values:

$\eta = 0.2$

$$w = \begin{pmatrix} 0 \\ 1 \\ 0.5 \end{pmatrix}$$

$$0 = w_0 + w_1 x_1 + w_2 x_2$$
$$= 0 + x_1 + 0.5x_2$$
$$\Rightarrow x_2 = -2x_1$$

Note that the class labels are represented by + or dot symbols. The values of the attributes can be read from the figure.

Apply the perceptron learning algorithm on this dataset and find the final values of the weights vector. Assume a learning rate of 0.2.