MATH 4334 Section 001
Fall 2022
Final
12/12/2022
Time Limit: 120 Minutes

Name (Print): ____________________

Student ID: ____________________

Major: ____________________

This exam contains 10 pages (including this cover page) and 9 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

This is a closed-book, closed-note exam. A scientific calculator and a SINGLE A4 page are allowed.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit. If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Points	Score
1	10	
2	12	
3	18	
4	6	
5	12	
6	10	
7	10	
8	12	
9	10	
Total:	100	

1. (10 points) Circle ONE correct answer of the multiple-choice questions (no need to show steps)

(a) Suppose that a hypothetical binary computer stores floating point numbers in 16-bit words. Bit 1 is used for the sign of the number, bit 2 for the sign of the exponent, bits 3-4 for

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
s	exp			mantissa											

the magnitude of the exponent, and the remaining twelve bits for the magnitude of the mantissa. What is machine epsilon for this computer?

i. 2^{-4}
ii. 2^{-8}
iii. 2^{-12}
iv. 2^{-16}

(b) If the bisection method is applied to the function $f(x) = (x+1)(x-1)(x-3)$ with initial interval $[-4, 4]$, how many roots of $f(x)$ will it find?

i. 3
ii. 2
iii. 1
iv. 0

(c) A sufficient condition for the Gauss-Jacobi method to converge for the linear system $Ax = b$ is

i. A is non-singular.
ii. A is strictly diagonally dominant.
iii. A is symmetric positive definite.
iv. The spectral radius of A is strictly less than 1.

(d) Which assertion is true?

i. Every real symmetric matrix is similar to a diagonal matrix.
ii. Every $n \times n$ matrix has n distinct eigenvalues.
iii. The eigenvectors associated with different eigenvalues are orthogonal to each other.
iv. The spectral radius of a matrix is a norm.

(e) For an $m \times n$ matrix A whose element is a_{ij}, its infinity norm is defined as

i. $\max_{1\le j\le n} \sum_{i=1}^{m} |a_{ij}|$.
ii. $\max_{1\le i\le m} \sum_{j=1}^{n} |a_{ij}|$.
iii. $\sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}^2}$.
iv. the largest singular value of A in magnitude.

2. (a) (3 points) Define the Newton's method for finding a root. When does it fail to find a root of a function.

(b) (3 points) Given a set of n data points (x_i, y_i) for $i = 1, 2, \cdots, n$, determine the order of interpolating polynomials. Define the Vandermonde matrix for polynomial interpolation. State ONE of its drawbacks.

(c) (3 points) What is condition number? How is it computed? Why is it useful?

(d) (3 points) Describe ONE advantage and ONE disadvantage of using iterative methods (e.g., Gauss-Jacobi and Gauss-Seidel) to solve a linear system as opposed to direct methods (e.g., Gauss elimination and LU decomposition).

3. (a) (4 points) What is the Taylor series for $\ln(\frac{1+x}{1-x})$?

   (b) (2 points) Determine what value of x to use in part (a) if we wish to compute $\ln 2$.

   (c) (4 points) Approximate $\ln 2$ by four terms of the Taylor series in (a), round your answer to 4 decimal places.

   (d) (8 points) Apply Newton's method to the equation $e^x = 2$ to compute $\ln 2$. Starting from $x_0 = 1$, list all the intermediate steps until the absolute error of the current solution and the answer in part (c) is less than 0.0001.

4. (6 points) Develop the divided-difference table from the given data.

x	0	1	3	2	5
$f(x)$	2	1	5	6	-183

Write down the interpolating polynomial and rearrange it for fast computation without simplifying.

5. (a) (4 points) Given the data find an approximate value of $\sin 0.705$ by linear interpolation.

x	$\sin x$
0.70	0.64422
0.71	0.65183

(b) (4 points) Use a calculator to find the exact value of $\sin 0.705$ rounding to 5 decimal places and compute the absolute error of part (a).

(c) (4 points) How accurately can we determine $\sin(x)$ by linear interpolation for x in $[0, 2]$ with $h = 0.01$? Hint: give an upper bound of the interpolation error?

6. (10 points) Consider the integral $\int_0^1 \sin(\pi x^2/2)dx$. Suppose that we wish to integrate numerically with an error of magnitude less than 10^{-3}. How many subintervals are needed if we wish to use the composite trapezoid rule?

7. (10 points) Determine A, B, C, D for a formula of the form

$$Af(-h) + Bf(0) + Cf(h) = hDf'(h) + \int_{-h}^{h} f(t)dt$$

that is accurate for polynomials of as high degree as possible.

8. (12 points) Solve the following linear system using Gaussian elimination with scaled partial pivoting.

$$\begin{pmatrix} 2 & -1 & 3 & 7 \\ 4 & 4 & 0 & 7 \\ 2 & 1 & 1 & 3 \\ 6 & 5 & 4 & 17 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 15 \\ 11 \\ 7 \\ 31 \end{pmatrix}$$

Show the scale array, tell how the pivot rows are selected, and carry out the computation. Include the index array for each step. There are no fractions in the correct solution, except for certain ratios that must be looked at to select pivots.

9. (10 points) Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & -1 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$$

Determine a unit lower triangular matrix L and an upper triangular matrix U such that $A = LU$.