MidTerm Review

Anurag Nagar

Machine Learning Class

Outline

Topics Covered

List of topics covered so far:

- Inductive Learning - Linear Regression
- Information Based Learning - Entropy and Information Gain, Decision Trees
- Probability and naive Bayes Classifier
- Perceptron
- Artificial Neural Network

Deep Learning will **not** be included

Outline

Inductive Learning

What is Statistical Learning?

- A dataset is available with following data for each instance:
  target variable Y

  feature vector $X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$

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$$Y = f(X) + \epsilon$$

where ϵ is the error.

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  where ϵ is the error.

- For example, Y could be the GPA, and X_1 could be the SAT score, X_2 could be percent attendance, and X_3 could be the hours studied every week.

Inductive Learning

Function Estimation

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Function Estimation

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- You would like $\hat{f}$ to be such that it minimizes the mean squared error over all examples i

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}(Y_i - \hat{f}(x_i))^2$$

Inductive Learning

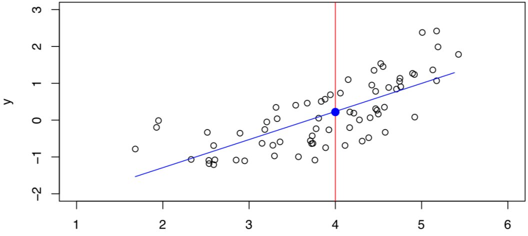


Figure: Linear Model

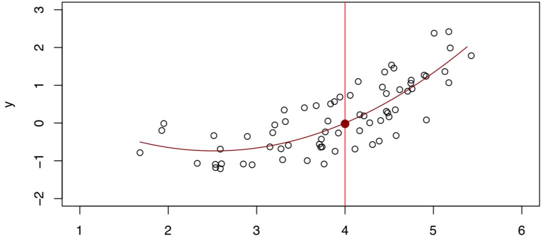
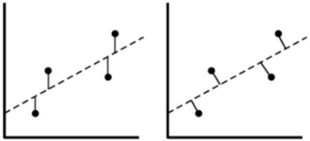


Figure: Quadratic Model

Example 1

In the figures below, the circles represent the data points, and the dashed line represents the function fitted to the line. Which of the following shows the correct value of error for each point?



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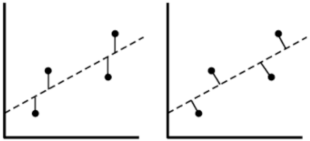

The one on the left as it shows the distance between prediction and actual value.

Example 2

For predicting the output variable Y as a function of input variable x, you design a linear model of the following form:

$$Y = a + bx$$

What do a and b represent?

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a represents the Y-intercept and b represents the slope.

Example 3

For the following dataset, x represents the independent variable, and y is the output variable.

x	1	10	20
y	1	100	400

You propose a linear regression equation as below:

$$Y = -140 + 30x$$

What will be the value of Mean Squared Error (MSE)?

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What will be the value of Mean Squared Error (MSE)?

The predicted values for each of the points would be: -110, 160, and 460. So the MSE would be:

$$MSE = \frac{1}{3}[(1+110)^2 + (100-160)^2 + (400-460)^2] = 6507$$

Outline

Decision Trees

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1. Root Node
2. Leaf Node
3. Internal Node
4. Jump Node

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Decision Trees

In a decision tree, which node contains all of the input data?

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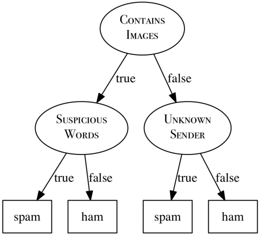
Decision Trees

Suppose you have an email dataset with three Boolean attributes - Suspicious Words, Uknown Sender, and Contains Images, and an Boolean output which could be spam or ham You create the decision tree below.



What would be the predicted classification for Suspicious Words = true, Unknown Sender = True, and Contains Images = False

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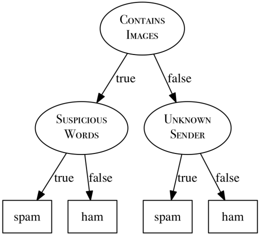

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Spam

Entropy

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$$H(\text{dataset}) = -\sum_{i=1}^{k} p_i log_2(p_i)$$

Entropy

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$$\begin{aligned}
H(suit) &= -\sum_{l \in \{\heartsuit,\clubsuit,\diamondsuit,\spadesuit\}} P(suit = l) \times log_2(P(suit = l)) \\
&= -\big((P(\heartsuit) \times log_2(P(\heartsuit))) + (P(\clubsuit) \times log_2(P(\clubsuit))) \\
&\quad + (P(\diamondsuit) \times log_2(P(\diamondsuit))) + (P(\spadesuit) \times log_2(P(\spadesuit)))\big) \\
&= -\Big(\left({}^{13}/_{52} \times log_2({}^{13}/_{52})\right) + \left({}^{13}/_{52} \times log_2({}^{13}/_{52})\right) \\
&\quad + \left({}^{13}/_{52} \times log_2({}^{13}/_{52})\right) + \left({}^{13}/_{52} \times log_2({}^{13}/_{52})\right)\Big) \\
&= -\big((0.25 \times -2) + (0.25 \times -2) \\
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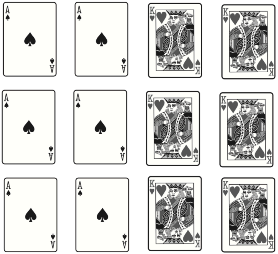
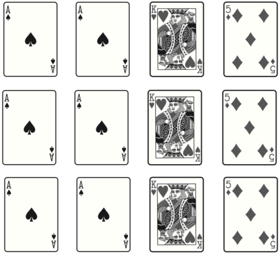
&= 2\ bits
\end{aligned}$$

Entropy

Which of the following has lower entropy?





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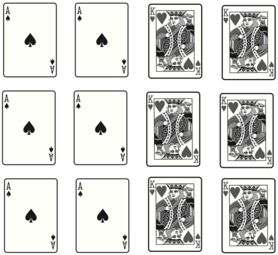

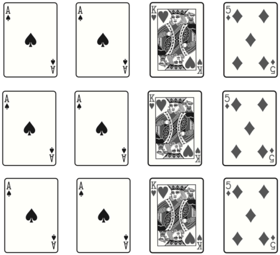

Left one has entropy of 1.00 and right has entropy of 1.50

Information Gain

In the following dataset, which attribute out of X, Y, and Z would give the best information gain

X	Y	Z	C
1	1	1	I
1	1	0	I
0	0	1	II
1	0	0	II

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Best Attribute: Y

Information Gain

- Go through some examples of creating decision trees using ID3

Information Gain

- Go through some examples of creating decision trees using ID3
- No need to study alternate information measures, noise, overfitting, etc

Outline

Probability

Conditional Probability:

$$P(X = a|Y = b) = \frac{P(X = a, Y = b)}{P(Y = b)}$$
$$= \frac{P(X = a)P(Y = b|X = a)}{P(Y = b)}$$

Probability

10% of patients entering a doctor's office have liver disease, and 5% of patients walking in are alcoholics. Of those that are diagnosed with liver disease, 7% are alcoholics. Given a patient walking into the office is alcoholic, what is the probability that he will be diagnosed with liver disease.

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A = having liver disease
B = being alcoholic

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{0.07 \times 0.10}{0.05} = 0.14$$

14%

Probability

From a standard deck of playing cards, someone draws a random card and tells you it's a face card. What is the probability that it is a king of any suit?

Hint: J, Q, K are called face cards.

Probability

From a standard deck of playing cards, someone draws a random card and tells you it's a face card. What is the probability that it is a king of any suit?
Hint: J, Q, K are called face cards.

A = card being a king
B = drawing a face card

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{1.00 \times (4/52)}{(12/52)} = 1/3$$

33.3%

Probability

In an experiment if there are n possible outcomes which are denoted by random variable X taking values $X_1, X_2, \ldots, X_n$ with respected probability values $p_1, p_2, \ldots, p_n$, the expected value of X is :

$$E(X) = \mu(X) = \sum_{i=1}^{n} p_i X_i$$

and variance of X is:

$$\begin{aligned} var(X) &= E[(X-\mu)^2] \\ &= E[X^2] - E[X]^2 \end{aligned}$$

Expected Value

An insurance company sells a policy that costs \$150, and pays \$5000 for major accident, and \$1000 for a minor accident only once. The probability of major accident is 0.005 and that of minor incident is 0.08. What is the expected value of the profit/loss for each policy sold by the company.

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There could be three cases for the company

1. No accident, profit = \$150, probability $= 1 - 0.005 - 0.08 = 0.915$
2. Major accident, profit = \$ (150 - 5000) = -\$4850, probability = 0.005
3. Minor accident, profit = \$ (150 - 1000) = -\$850, probability = 0.08

$$
\begin{aligned}
E(X) &= 150 * 0.915 - 4850 * 0.005 - 850 * 0.08 \\
&= 45
\end{aligned}
$$

Variance

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$E(X) = p = 0.2$

X	+1	-1
p(X)	0.6	0.4
$(X - E(X))^2)$	$(1 - 0.2)^2$	$(-1 - 0.2)^2$

$$\begin{aligned} \text{var}(X) &= 0.6 * (0.8)^2 + 0.4 * (1.2)^2 \\ &= 0.96 \end{aligned}$$

Outline

Naive Bayes Classifier

Suppose there are two classed C_1 and C_2, and we are given data $X = (X_1, X_2, \ldots, X_n)$, the probability of it belonging to C_1 and C_2 is:

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$$P(C_1|X) = \frac{P(X|C_1)P(C_1)}{P(X)} \propto P(X|C_1)P(C_1)$$

$$P(C_2|X) = \frac{P(X|C_2)P(C_2)}{P(X)} \propto P(X|C_2)P(C_2)$$

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$$P(C_2|X) = \frac{P(X|C_2)P(C_2)}{P(X)} \propto P(X|C_2)P(C_2)$$

The Maximum a Posteriori (MAP) hypothesis compares $P(X|C_i)P(C_i)$ for each class C_i and assigns X to the class that has the maximum value.

Naive Assumption

In many cases, it's not possible to evaluate $P(X|C_1) = P((X_1, X_2, \ldots, X_n)|C_i)$ because of lack of training data.
In such cases, we make the assumption that each of the features $X_1, X_2, \ldots, X_n$ is independent of each other for every class C_i (called the naive Bayes assumption)

$$\begin{aligned} P(X|C_1) &= P((X_1, X_2, \ldots, X_n)|C_i) \\ &= P(X_1|C_i)P(X_2|C_i)\ldots P(X_n|C_i) \end{aligned}$$

Naive Bayes Classifier

We have a small dataset consisting of three classes *well*, *cold*, *allergy* and three words *sneeze*, *cough*, *fever*. The distribution of the words for each class is shown below:

Prob	**Well**	**Cold**	**Allergy**
$P(c_i)$	0.9	0.05	0.05
$P(\text{sneeze}\mid c_i)$	0.1	0.9	0.9
$P(\text{cough}\mid c_i)$	0.1	0.8	0.7
$P(\text{fever}\mid c_i)$	0.01	0.7	0.4

What would be the predicted tag for the a text containing words W $=$ *sneeze*, *cough*, $\neg$ *fever*

Naive Bayes Classifier

$$\begin{aligned} P(well|W) &= P(sneeze|well)P(cough|well)P(\neg fever|well)P(well) \\ &= 0.1 \times 0.1 \times 0.99 \times 0.9 \\ &= 0.0089 \end{aligned}$$

$$\begin{aligned} P(cold|W) &= P(sneeze|cold)P(cough|cold)P(\neg fever|cold)P(cold) \\ &= 0.9 \times 0.8 \times 0.3 \times 0.05 \\ &= 0.01 \end{aligned}$$

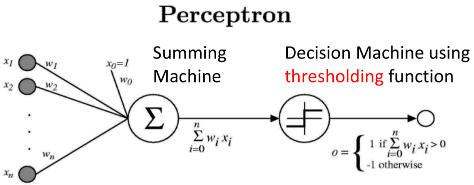
$$\begin{aligned} P(allergy|W) &= P(sneeze|allergy)P(cough|allergy)P(\neg fever|allergy)P(allergy) \\ &= 0.9 \times 0.7 \times 0.6 \times 0.05 \\ &= 0.019 \end{aligned}$$

The predicted class is **allergy**

Outline

Perceptron

Perceptron is a single unit of computation, that consists of an addition operation followed by an activation operation.



$$o(x_1, \ldots, x_n) = \begin{cases} 1 & \text{if } w_0 + w_1 x_1 + \cdots + w_n x_n > 0 \\ -1 & \text{otherwise.} \end{cases}$$

Sometimes we'll use simpler vector notation:

$$o(\vec{x}) = \begin{cases} 1 & \text{if } \vec{w} \cdot \vec{x} > 0 \\ -1 & \text{otherwise.} \end{cases}$$

Note the classification rule above.

Perceptron

Another way of binary classification

Linear Classification

- Two equivalent ways:

$$w^T x > \theta \text{ where } w = \begin{pmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{pmatrix} x = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix} \text{ for class } +1$$

$$w^T x > 0 \text{ where } w = \begin{pmatrix} w_0 \\ w_1 \\ w_2 \\ \dots \\ w_n \end{pmatrix} x = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix} \text{ for class } +1$$

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Go through **AND, OR** classification using Perceptron from class slides.
You could be asked similar questions for AND NOT, NOT OR, etc

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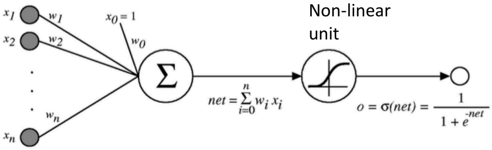
Neural Networks

In neural nets, each neuron uses a non-linear activation function, such as sigmoid.



$\sigma(x)$ is the sigmoid function

$$\frac{1}{1 + e^{-x}}$$

Nice property: $\frac{d\sigma(x)}{dx} = \sigma(x)(1 - \sigma(x))$ Really useful result

Note the properties of sigmoid and its derivative.

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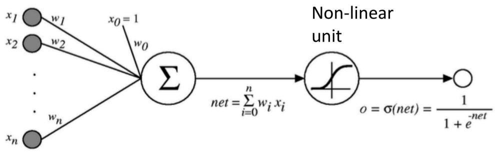

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Backpropagation Algorithm for Sigmoid Activation

Backpropagation Algorithm

Initialize all weights to small random numbers

Until convergence, Do

For each training example, Do

1. Input it to network and compute network outputs — Forward pass
2. For each output unit k

$$\delta_k \leftarrow o_k(1 - o_k)(t_k - o_k)$$

3. For each hidden unit h

$$\delta_h \leftarrow o_h(1 - o_h) \sum_{k \in outputs} w_{h,k}\delta_k$$

4. Update each network weight $w_{i,j}$ — Backward pass

$$w_{i,j} \leftarrow w_{i,j} + \Delta w_{i,j}$$

where $\Delta w_{i,j} = \eta \delta_j x_{i,j}$

Some possible questions

Go through a simple example of backpropagation from class slides and questions.

Calculate square error at the output layer

Compute weight updates for backward pass

Given number of neurons in each layer, find number of connections.