Part 1:

0.1 Representing Sentences in First - Order Logic

•) $I(x)$: x has an Internet Connection | x, y: student

•) $C(x,y)$: x and y have chatted over Internet.

1. Exactly one Student in your class has a Internet connection.

•) $\exists x\,(I(x) \land \forall y\,(I(y) \to y = x))$. Using Uniqueness quantifier : $\exists!\, x\, I(x)$

2. Everyone except one student in your class has an Internet connection.

•) $\exists x\,(\neg I(x) \land \forall y\,(I(y) \to y = x))$. Using Uniqueness quatifier : $\exists! x\, \neg I(x)$

3. Everyone in your class with an internet Connection has chatted over the Internet with at least one other student in your class.

•) $\forall x\,(I(x) \to \exists y\,(C(x,y) \land x \neq y))$.

4. Some one in your class has an Internet Connection but has not chatted with anyone else in your class.

•) $\exists x\,(I(x) \land \forall y\, \neg C(x,y))$.

5. There are two Student in your class who have not chatted with each other over the Internet.

•) $\exists x\, \exists y\,(x \neq y \land \neg C(x,y))$

6. There is a Student in your class who has chatted with everyone in your class over the Internet.

•) $\exists x\, \forall y\,(x \neq y \,;\, C(x,y))$

7. There are at least two Students in your class who have not chatted with the Same person in your class.

•) $\exists x\, \exists y\,(x \neq y \land \exists z\,(x \neq y \land y \neq z \land \neg C(x,z) \land \neg C(y,z)))$

8. There are two Students in the class who have chatted with everyone else in your class

•) $\exists x\, \exists y\,(x \neq y \land \forall z\,(C(x,z) \land C(y,z)))$

Q.2 Validity and Satisfiability.

a) Big ∨ Dump ∨ (Big ⇒ Dump) : valid.

then: (Big ∨ Dump ∨ ¬Big ∨ Dump)

then (Big ∨ ¬ Big) ∨ Dump

$\underbrace{\text{Big} \vee \neg \text{Big}}_{\text{True}}$ ∨ Dump : Satisfiability + Valid.

b) (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire) : (valid)

then ¬ (Smoke ⇒ Fire) ∨ ((Smoke ∧ Heat) ⇒ Fire)

then: ¬ (Smoke ⇒ Fire) ∨ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

then :(¬ (Smoke ⇒ Fire) ∨ (Smoke ⇒ Fire)) ∨ (Heat ⇒ Fire)

Then: $\underbrace{\neg(\text{Smoke} \Rightarrow \text{Fire}) \vee (\text{Smoke} \Rightarrow \text{Fire})}_{\text{True}}$ ∨ (Heat ⇒ Fire) Satisfiability + Valid.

0.3 Models : Solved by finding a truth assignment to the propositional variable (A, B, C, D..) that make it true is a model.

1) $(A \wedge B) \vee (B \wedge C)$

2) $A \vee B$

3) $A \Leftrightarrow B \Leftrightarrow C$

A	B	C	D	$A \wedge B$	$B \vee C$	$(A \wedge B) \vee (B \vee C)$	$A \vee B$	$A \Leftrightarrow B$	$A \Leftrightarrow B \Leftrightarrow C$
0	0	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	1	0
0	0	1	0	0	0	0	0	1	1
0	0	1	1	0	0	0	0	1	1
0	1	0	0	0	0	0	1	0	1
0	1	0	1	0	0	0	1	0	1
0	1	1	0	0	1	1	1	0	0
0	1	1	1	0	1	1	1	0	0
1	0	0	0	0	0	0	1	0	1
1	0	0	1	0	0	0	1	0	1
1	0	1	0	0	0	0	1	0	0
1	0	1	1	0	0	0	1	0	0
1	1	0	0	1	0	1	1	1	0
1	1	0	1	1	0	1	1	1	0
1	1	1	0	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1

1) $(A \wedge B) \vee (B \vee C)$ has 6 models.

2) $A \vee B$ has 12 model.

3) $A \Leftrightarrow B \Leftrightarrow C$ has 8 model

Q. 4. Unification

1) $Q(y, Gee(A,B)), Q(Gee(x,x), y)$.

2) $Older(Father(y), y), Older(Father(x), John)$.

3) $Knows(Father(y), y), Knows(x,x)$.

Answer:

1) $Q(y, Gee(A,B)), Q(Gee(x,x), y)$.

Progressive unification.

∘) $Q(y, Gee(A,B)), Q(Gee(x,\cancel{x})\ y) : \{y / Gee(x,x)\}$,

∘) $Q(Gee(x,x), Gee(A,B)), Q(Gee(x,x), Gee(x,x)) : \{y / Gee(x,x)\}$.

∘) $Q(Gee(x,x), Gee(A,B)), Q(Gee(x,x), Gee(x,x)) : \{y / Gee(x,x); x/A\}$,

∘) $Q(Gee(A,A), Gee(A,B)), Q(Gee(A,A), Gee(A,A) : \{y / Gee(x,x), x/A\}$

∴ Cannot unify constant A with constan B.

2) $Older(Father(y), y), Older(Father(x), John)$

Progressive unification

∘) $Older(Father(y), y), Older(Father(x), John) : \{y / John\}$

∘) $Older(Father(John), John), Older(Father(x), John) : \{y / John\}$

∘) $Older(Father(John), John), Older(Father(x), John) : \{y / John; x / John\}$

∘) $Older(Father(John), John), Older(Father(John), John) : \{y / John; x / John\}$

∴ unify when $\{x / John; y / John\}$

3) Knows (Father (y), y) , Knows (x, x)

Progressive Unification:

Knows (father (y), y), Know (x, x) : {x / Father (y)}

Knows (Father (y), y) , Know (Father (y), Father (y)) : {x / Father (y)}

∴ Cannot unify variable y with Father (y), which is a term referring to variable y

Q.5. Inference in First Order Logic

a) Translate the six statement above into first order logic using these predicates

1) $\forall x \forall y \, (Child(x) \wedge Candy(y) \rightarrow Loves(x,y))$

2) $\forall x \; \exists y \, (Candy(y) \wedge Loves(x,y) \rightarrow \neg Fanatic(x))$

3) $\forall x \, \exists y \, (Eats(x,y) \wedge Pumkin(y) \rightarrow Fanatic(x))$

4) $\forall x \, \forall y \, (Pumkin(y) \wedge Buys(x,y) \rightarrow Carves(x,y) \vee Eats(x,y))$

5) $\exists y \, (Pumkin(y) \wedge Buys(John, y))$

6) $Candy(Life\,savers)$

b) FOL → CNF

1) $\neg Child \vee \neg Candy(y) \vee Loves(x,y)$

2) $\neg Candy \vee \neg Loves(x,y) \vee \neg Fanatic(x)$

3) $\neg Eats(x,y) \vee \neg Pumkin(y) \vee Fanatic(x)$

4) $\neg Pumkin(y) \vee \neg Buys(x,y) \vee Carves(x,y) \vee Eats(x,y)$

5) a) $Pumkin(y)$ b) $Buys(John, y)$.

6) $Candy(Life\ Savers.)$.

c) Prove using resolution by refutation that

If John is a child, then John carves some pumkin

•) $\exists y (Child(John) \wedge Pumkin(y) \rightarrow Carves(John, y))$

FOL → CNF: $\neg Child(John) \vee \neg Pumkin(y) \vee Carves(John, y)$

(1) → (6) From Q.5 b.

(7) $Child(John)$ $\{x / John\}$

(8) $Pumkin(y)$ $\{x / John\}$

(9) $\neg Carves(John, y)$ $\{x / John\}$

(10) $\neg Candy(y) \vee Loves(John, y)$ $\{(7), (1), x / John\}$

(11) $\neg Fanatic(John)$ $\{(10); (2); x / John\}$

(12) $\neg Eats(John, y) \vee \neg Pumkin(y)$ $\{(11), 13); x / John\}$

(13) $\neg Buys(John, y) \vee \neg Pumkin(y) \vee Carves(John, y)$ $\{(12), (4); x / John\}$

(14) $Carves(John, y)$ $\{(5), (13); x / John\}$

(15) Contration $\{(14); (9); x / John\}$

Valid.