Assignment 3 – OpenMP-Style Parallel Loops with Numba

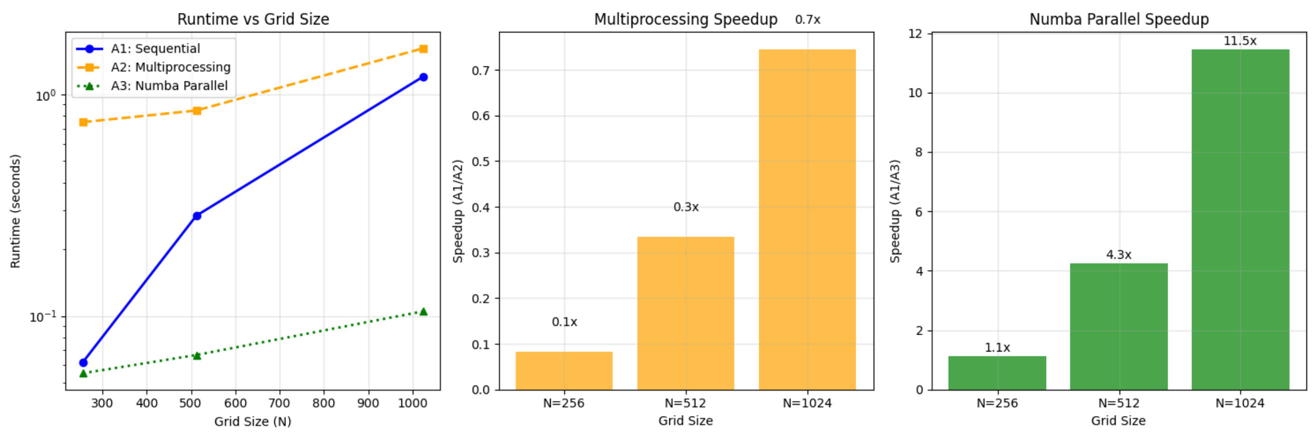
2D Heat Diffusion Solver: A1 vs A2 vs A3 Comparison (T = 500)

Assignment Comparison

- **A1 (Sequential)**: Pure Python/NumPy implementation
- **A2 (Multiprocessing)**: Process-based parallelism with shared memory
- **A3 (Numba Parallel)**: JIT compilation with OpenMP-style `prange` loops

Performance Results

Runtime Comparison



Results Table

N	Type	Runtime (sec)	Speedup vs A1	JIT Warmup (sec)
256	Numba Parallel	0.750	0.08×	0.000
256	Sequential	0.062	1.00×	0.000
256	Numba Parallel	0.055	1.12×	0.426
512	Numba Parallel	0.846	0.34×	0.000
512	Sequential	0.284	1.00×	0.000
512	Numba Parallel	0.067	4.25×	0.001
1024	Numba Parallel	1.615	0.75×	0.000
1024	Sequential	1.204	1.00×	0.000
1024	Numba Parallel	0.105	11.46×	0.001

Performance Analysis

Best speedup achieved: **11.5×** at **N=1024**

- **N=256**: Sequential = 0.062s, Numba = 0.055s → **1.1× speedup**
- **N=512**: Sequential = 0.284s, Numba = 0.067s → **4.3× speedup**
- **N=1024**: Sequential = 1.204s, Numba = 0.105s → **11.5× speedup**

Analysis:

- **JIT Compilation**: Numba compiles Python to optimized machine code with LLVM backend.
- **Parallel Execution**: `prange` enables OpenMP-like parallelization across CPU cores.
- **Memory Layout**: C-contiguous arrays optimize cache performance and SIMD vectorization.
- **Fastmath**: Aggressive floating-point optimizations improve computational throughput.

Scaling Characteristics:

- **Compute-bound**: Larger grids show better speedup as parallelism overhead is amortized.
- **Memory bandwidth**: Performance ultimately limited by memory access patterns.
- **Thread overhead**: Smaller grids may show diminishing returns due to thread management costs.

JIT Compilation Cost

- **Warm-up Phase**: 5 iterations to trigger JIT compilation and optimization
- **One-time Cost**: JIT compilation happens once per function signature
- **Measurement**: Warm-up time excluded from performance benchmarks
- **Production**: In real applications, JIT cost is amortized over many calls

Technical Implementation

Numba Kernel

```
@njit(parallel=True, fastmath=True, cache=True)
def jacobi_kernel(u, u_new):
    N = u.shape[0]
    for i in prange(1, N-1):  # Parallel loop
        for j in range(1, N-1):
            u_new[i,j] = 0.25 * (u[i-1,j] + u[i+1,j] +
                                 u[i,j-1] + u[i,j+1])
```

Key Optimizations

- **C-contiguous arrays**: `order='C'` for optimal memory layout
- **Pre-allocation**: Arrays reused across iterations
- **Fast math**: Aggressive floating-point optimizations
- **Caching**: Compiled functions cached for subsequent runs

Comparison with Previous Assignments

- **A1 (Sequential)**: Pure Python numpy operations
- **A2 (Multiprocessing)**: Process-based parallelism with shared memory
- **A3 (Numba)**: Thread-based parallelism with JIT compilation

Trade-offs

- **Compilation overhead**: JIT warm-up vs immediate execution
- **Memory sharing**: Threads vs processes
- **Scaling**: Thread synchronization vs process communication

Conclusion

Numba provides an excellent balance of performance and simplicity for computational kernels. The `@njit(parallel=True)` decorator enables OpenMP-like parallelization with minimal code changes, while LLVM compilation delivers near-C performance from Python. For iterative algorithms like Jacobi, the JIT compilation cost is easily amortized, making Numba an attractive option for scientific computing.