Z Specification of Fragmenta

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Contents

1 Generics

section *Fragmenta_Generics* **parents** *standard_toolkit*

$\mathrm{acyclic}[X] == \{r : X \leftrightarrow X \mid r^{+} \cap \mathrm{id}\, X = \varnothing\}$

$connected[X] == \{r : X \leftrightarrow X \mid \forall x : \mathrm{dom}\, r;\ y : \mathrm{ran}\, r \bullet x \mapsto y \in r^{+}\}$

$tree[X] == \{r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \land r \in X \rightarrowtail X\}$

$forest[X] == \{r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \land (\forall s : X \leftrightarrow X \mid s \subseteq r \land s \in connected \bullet s \in tree)\}$

$\mathrm{injrel}[X, Y] == \{r : X \leftrightarrow Y \mid r^{\sim} \in Y \rightarrowtail X\}$

$\mathrm{antireflexive}[X] == \{r : X \leftrightarrow X \mid r \cap id(\mathrm{dom}\ r) = \varnothing\}$

$[X, Y, Z]$

$\mathrm{flip} : (X \rightarrow Y \rightarrowtail Z) \rightarrow (Y \rightarrow X \rightarrowtail Z)$

$\forall f : X \rightarrow Y \rightarrowtail Z \bullet \mathrm{flip}\, f = (\lambda\, y : Y \bullet \lambda\, x : X \bullet f\ x\ y)$

$[X, Y, Z, W]$

$\mathrm{apply} : (X \rightarrow Z) \rightarrow (Y \rightarrow W) \rightarrow (X \times Y) \rightarrow (Z \times W)$

$\forall f : X \rightarrow Z;\ g : Y \rightarrow W;\ x : X;\ y : Y \bullet \mathrm{apply}\, f\ g\ (x, y) = (f\ x, g\ y)$

$[X, Y]$

$\mathrm{map} : (X \rightarrow Y) \rightarrow \mathbb{P}\, X \rightarrow \mathbb{P}\, Y$
$\mathrm{mapS} : (X \rightarrow Y) \rightarrow \mathrm{seq}\, X \rightarrow \mathrm{seq}\, Y$

$\forall f : X \rightarrow Y \bullet \mathrm{map}\, f \{\} = \{\}$

$\forall f : X \rightarrow Y;\ x : X;\ xs : \mathbb{P}\, X \bullet \mathrm{map}\, f (\{x\} \cup xs) = \{f\ x\} \cup (\mathrm{map}\, f\ xs)$

$\forall f : X \rightarrow Y \bullet \mathrm{mapS}\, f \langle\rangle = \langle\rangle$

$\forall f : X \rightarrow Y;\ x : X;\ xs : \mathrm{seq}\, X \bullet \mathrm{mapS}\, f (\langle x \rangle \frown xs) = \langle f\ x \rangle \frown (\mathrm{mapS}\, f\ xs)$

function 10 **leftassoc** $(\_ \boxdot \_)$

$[X]$

$\_ \boxdot \_ : ((X \rightarrow X) \times \mathbb{P}\, X) \rightarrow (X \rightarrowtail X)$

$\forall f : X \rightarrow X;\ s : \mathbb{P}\, X \bullet f \boxdot s = (\mathrm{id}\ s) \oplus f$

function$(\_^{\oplus})$

$$[X]$$
$$\_^{\oplus} : (X \leftrightarrow X) \rightarrow (X \leftrightarrow X)$$
$$\forall r : X \leftrightarrow X \bullet r^{\oplus} = \textbf{if } r \oplus r \mathbin{;} r = r \textbf{ then } r \textbf{ else } (r \oplus r \mathbin{;} r)^{\oplus}$$

$\mathrm{opt}[X] == \{s : \mathbb{P}\, X \mid \# s \leq 1\}$

$$[X]$$
$$\mathrm{the} : \mathrm{opt}[X] \rightarrow\!\!\!\!\!\!\!+\; X$$
$$\forall x : X \bullet \mathrm{the}\,\{x\} = x$$

$$[X, Y]$$
$$\mathit{flatten} : (X \rightarrow\!\!\!\!\!\!\!+\; \mathbb{P}\, Y) \rightarrow (X \leftrightarrow Y)$$
$$\forall f : X \rightarrow\!\!\!\!\!\!\!+\; \mathbb{P}\, Y \bullet \mathit{flatten}\, f = \{x : \mathrm{dom}\, f;\ y : Y \mid y \in f\, x\}$$

2 Graphs

section *Fragmenta_Graphs* **parents** *standard_toolkit*, *Fragmenta_Generics*

$[V, E]$

$\mathit{Gr} == \{vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s, t : E \rightarrow\!\!\!\!\!\!\!+\; V \mid s \in es \rightarrow vs \land t \in es \rightarrow vs\}$

$$\mathit{Ns} : \mathit{Gr} \rightarrow \mathbb{P}\, V$$
$$\mathit{Es} : \mathit{Gr} \rightarrow \mathbb{P}\, E$$
$$\mathit{src}, \mathit{tgt} : \mathit{Gr} \rightarrow E \rightarrow\!\!\!\!\!\!\!+\; V$$
$$\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!\!\!+\; V;\ t : E \rightarrow\!\!\!\!\!\!\!+\; V \bullet \mathit{Ns}(vs, es, s, t) = vs$$
$$\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!\!\!+\; V;\ t : E \rightarrow\!\!\!\!\!\!\!+\; V \bullet \mathit{Es}(vs, es, s, t) = es$$
$$\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!\!\!+\; V;\ t : E \rightarrow\!\!\!\!\!\!\!+\; V \bullet \mathit{src}(vs, es, s, t) = s$$
$$\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!\!\!+\; V;\ t : E \rightarrow\!\!\!\!\!\!\!+\; V \bullet \mathit{tgt}(vs, es, s, t) = t$$

$$\varnothing_G : Gr$$
$$\varnothing_G = (\varnothing, \varnothing, \varnothing, \varnothing)$$

$$EsId : Gr \rightarrow \mathbb{P}\, E$$
$$\forall\, G : Gr \bullet EsId\ G = \{e : Es\ G \mid src\ G\ e = tgt\ G\ e\}$$

relation(adjacent _)

$$\mathrm{adjacent}\_ : \mathbb{P}(Gr \times V \times V)$$
$$\forall\, G : Gr;\ v_1, v_2 : V \bullet \mathrm{adjacent}(G, v_1, v_2) \Leftrightarrow \exists\, e : Es\ G \bullet src\ G\ e = v_1 \wedge tgt\ G\ e = v_2$$

$$EsIncident : Gr \rightarrow \mathbb{P}\, V \rightarrow \mathbb{P}\, E$$
$$\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet EsIncident\ G\ vs = (src\ G)^{\sim} (\!| vs |\!) \cup (tgt\ G)^{\sim} (\!| vs |\!)$$

function 10 **leftassoc** (_ ⋈ _)

$$\_ \bowtie \_ : Gr \times \mathbb{P}\, E \rightarrow Gr$$
$$\forall\, G : Gr;\ es : \mathbb{P}\, E \bullet G \bowtie es = (Ns\ G, Es\ G \cap es, es \lhd src\ G, es \lhd tgt\ G)$$

function 10 **leftassoc** (_ ∇ _)

$$\_ \nabla \_ : Gr \times \mathbb{P}\, V \rightarrow Gr$$
$$\forall\, G, G' : Gr;\ vs : \mathbb{P}\, V \bullet G \nabla vs = G' \Leftrightarrow$$
$$\exists\, es : \mathbb{P}\, E \bullet es = \{e : Es\ G \mid \{src\ G\ e, tgt\ G\ e\} \subseteq Ns\ G \cap vs\} \wedge G' = (Ns\ G \cap vs, es, es \lhd src\ G, es \lhd tgt\ G)$$

$$\mathrm{successors} : V \times Gr \rightarrow \mathbb{P}\, V$$
$$\forall\, v : V;\ G : Gr \bullet \mathrm{successors}(v, G) = \{v_1 : Ns\ G \mid \mathrm{adjacent}(G, v, v_1)\}$$

function($\_^{\rightleftarrows}$)

$$
\begin{array}{|l}
\_^{\rightleftarrows} : Gr \rightarrow Gr \\
\hline
\forall\, G : Gr \bullet G^{\rightleftarrows} = (Ns\ G, Es\ G, tgt\ G, src\ G)
\end{array}
$$

function$(\_^{\leftrightarrow})$

$$
\begin{array}{|l}
\_^{\leftrightarrow} : Gr \rightarrow V \leftrightarrow V \\
\hline
\forall\, G : Gr \bullet G^{\leftrightarrow} = \{v_1, v_2 : Ns\ G \mid \mathrm{adjacent}(G, v_1, v_2)\}
\end{array}
$$

relation$(\otimes\ \_)$

$$
\begin{array}{|l}
\otimes\_ : \mathbb{P}\ Gr \\
\hline
\forall\, G : Gr \bullet \otimes\ G \Leftrightarrow G^{\leftrightarrow} \in \mathrm{acyclic}
\end{array}
$$

relation$(\boxminus_{Es}\ \_)$
relation$(\boxminus\ \_)$

$$
\begin{array}{|l}
\boxminus_{Es}\ \_, \boxminus\ \_ : \mathbb{P}(Gr \times Gr) \\
\hline
\forall\, G_1, G_2 : Gr \bullet \boxminus_{Es}(G_1, G_2) \Leftrightarrow Es\ G_1 \cap Es\ G_2 = \varnothing \\
\forall\, G_1, G_2 : Gr \bullet \boxminus(G_1, G_2) \Leftrightarrow Ns\ G_1 \cap Ns\ G_2 = \varnothing \wedge \boxminus_{Es}(G_1, G_2)
\end{array}
$$

relation$(\boxplus\ \_)$

$$
\begin{array}{|l}
[I] \\
\hline
\boxplus\_ : \mathbb{P}(I \rightarrow\!\!\!\!\!+ Gr) \\
\hline
\forall\, Gs : I \rightarrow\!\!\!\!\!+ Gr \bullet \boxplus\ Gs \Leftrightarrow \forall\, i, j : \mathrm{dom}\ Gs \mid i \neq j \bullet \boxminus(Gs\ i, Gs\ j) \\
\hline
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_G \_)$

$$
\begin{array}{|l}
\_ \cup_G \_ : Gr \times Gr \rightarrow Gr \\
\hline
\forall G_1, G_2 : Gr \bullet G_1 \cup_G G_2 = (Ns\ G_1 \cup Ns\ G_2, Es\ G_1 \cup Es\ G_2, src\ G_1 \cup src\ G_2, tgt\ G_1 \cup tgt\ G_2)
\end{array}
$$

function 10 **leftassoc** $(\_ \odot \_)$

$$
\begin{array}{|l}
\_ \odot \_ : Gr \times (V \leftrightarrow V) \nrightarrow Gr \\
\hline
\forall G : Gr;\ s : V \leftrightarrow V \mid s \in Ns\ G \nrightarrow Ns\ G \wedge s \in \text{antireflexive} \bullet \\
\quad G \odot s = (Ns\ G \setminus \operatorname{dom} s, Es\ G, (s \boxdot Ns\ G) \circ (src\ G), (s \boxdot Ns\ G) \circ (tgt\ G))
\end{array}
$$

$$
\begin{array}{|l}
replaceGfun : (E \nrightarrow V) \rightarrow (V \nrightarrow V) \rightarrow (E \nrightarrow V) \\
\hline
\forall f : E \nrightarrow V;\ sub : V \nrightarrow V \bullet \\
\quad replaceGfun\, f\ sub = f \oplus \{e : \operatorname{dom} f;\ v : V \mid (f\ e) \in \operatorname{dom} sub \wedge sub\ (f\ e) = v\}
\end{array}
$$

$$
\begin{array}{|l}
replaceG : Gr \rightarrow (V \nrightarrow V) \rightarrow Gr \\
\hline
\forall G : Gr;\ sub : V \nrightarrow V \bullet replaceG\ G\ sub = (Ns\ G \setminus \operatorname{dom} sub \cup \operatorname{ran}(Ns\ G \lhd sub), Es\ G, \\
\quad replaceGfun\ (src\ G)\ sub, replaceGfun\ (tgt\ G)\ sub)
\end{array}
$$

$$GrM == (V \nrightarrow V) \times (E \nrightarrow E)$$

$$
\begin{array}{|l}
fV : GrM \rightarrow V \nrightarrow V \\
fE : GrM \rightarrow E \nrightarrow E \\
\hline
\forall fv : V \nrightarrow V;\ fe : E \nrightarrow E \bullet fV(fv, fe) = fv \\
\forall fv : V \nrightarrow V;\ fe : E \nrightarrow E \bullet fE(fv, fe) = fe
\end{array}
$$

$$
\begin{array}{|l}
\text{gid} : Gr \rightarrow GrM \\
\hline
\forall G : Gr \bullet \text{gid}\ G = (id\ (Ns\ G), id\ (Es\ G))
\end{array}
$$

$$
\begin{array}{|l}
\varnothing_{GM} : GrM \\
\hline
\varnothing_{GM} = (\{\}, \{\})
\end{array}
$$

$$
\begin{array}{|l}
\text{domg}, \text{codg} : GrM \rightarrow Gr \\
\hline
\forall m : GrM;\ G : Gr \bullet \text{domg}\ m = G \Leftrightarrow \operatorname{dom}(fV\ m) = Ns\ G \wedge \operatorname{dom}(fE\ m) = Es\ G \\
\forall m : GrM;\ G : Gr \bullet \text{codg}\ m = G \Leftrightarrow \operatorname{ran}(fV\ m) \subseteq Ns\ G \wedge \operatorname{ran}(fE\ m) \subseteq Es\ G
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_{GM} \_)$

$$
\begin{array}{l}
\_ \cup_{GM} \_ : GrM \times GrM \nrightarrow GrM \\
\bigcup_{GM} : \mathbb{P}\, GrM \rightarrow GrM \\
\hline
\forall f, g : GrM \bullet f \cup_{GM} g = (fV\, f \cup fV\, g, fE\, f \cup fE\, g) \\
\bigcup_{GM} \varnothing = \varnothing_{GM} \\
\forall f : GrM;\ fs : \mathbb{P}\, GrM \bullet \bigcup_{GM} (\{f\} \cup fs) = f \cup_{GM} (\bigcup_{GM} fs)
\end{array}
$$

function 10 **leftassoc** $(\_ \rightarrow_G \_)$

$$
\begin{array}{l}
\_ \rightarrow_G \_ : Gr \times Gr \rightarrow \mathbb{P}\, GrM \\
\hline
\forall G_1, G_2 : Gr \bullet G_1 \rightarrow_G G_2 = \{fv : Ns\, G_1 \rightarrow Ns\, G_2;\ fe : Es\, G_1 \rightarrow Es\, G_2 \mid \\
\quad src\, G_2 \circ fe = fv \circ src\, G_1 \wedge tgt\, G_2 \circ fe = fv \circ tgt\, G_1\}
\end{array}
$$

function 10 **leftassoc** $(\_ \circ_G \_)$

$$
\begin{array}{l}
\_ \circ_G \_ : GrM \times GrM \rightarrow GrM \\
\hline
\forall g, f : GrM \bullet g \circ_G f = (fV\, g \circ fV\, f, fE\, g \circ fE\, f)
\end{array}
$$

$$
\begin{array}{l}
restrictGToTyEdges : GrM \times Gr \times \mathbb{P}\, E \rightarrow Gr \\
\hline
\forall m : GrM;\ G : Gr;\ tes : \mathbb{P}\, E \bullet restrictGToTyEdges(m, G, tes) = G \bowtie (fE\, m)^{\sim} (\!| tes |\!)
\end{array}
$$

3 Graphs with typing

section *Fragmenta_GrswT* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$GrwT == \{G : Gr;\ t : GrM \mid \mathrm{domg}\, t = G\}$

$$
\begin{array}{|l}
gOf : GrwT \to Gr \\
ty : GrwT \to GrM \\
\hline
\forall\, G : Gr;\ sm : V \rightarrowtail\!\!\!\!\!\!+ \ \operatorname{seq} V;\ t : GrM \bullet gOf(G, t) = G \\
\forall\, G : Gr;\ sm : V \rightarrowtail\!\!\!\!\!\!+ \ \operatorname{seq} V;\ t : GrM \bullet ty(G, t) = t
\end{array}
$$

$$
\begin{array}{|l}
\varnothing_{GwT} : GrwT \\
\hline
\varnothing_{GwT} = (\varnothing_G, \varnothing_{GM})
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_{GwT} \_)$

$$
\begin{array}{|l}
\_ \cup_{GwT} \_ : GrwT \times GrwT \to GrwT \\
\hline
\forall\, G_1, G_2 : GrwT \bullet G_1 \cup_{GwT} G_2 = ((gOf\ G_1) \cup_G (gOf\ G_2), (ty\ G_1) \cup_{GM} (ty\ G_2))
\end{array}
$$

4 SG Element Types

section *Fragmenta_SGElemTys* **parents** *standard_toolkit*, *Fragmenta_Generics*

$SGNT ::= nnrml \mid nabst \mid nprxy \mid nenum \mid nval \mid nvirt \mid nopt$

$SGED ::= dbi \mid duni$

$SGET ::= einh \mid ecomp\langle\!\langle SGED \rangle\!\rangle \mid erel\langle\!\langle SGED \rangle\!\rangle \mid ewander \mid eder$

relation$(\_ \prec_{NT} \_)$

$$
\begin{array}{|l}
\_ \prec_{NT} \_ : SGNT \leftrightarrow SGNT \\
\hline
\forall\, nt_1, nt_2 : SGNT \bullet nt_1 \prec_{NT} nt_2 \Leftrightarrow (nt_2 = nenum \Leftrightarrow nt_1 = nval) \\
\quad \land (nt_1 = nvirt \Rightarrow nt_2 = nvirt) \land (nt_1 = nabst \Rightarrow nt_2 \in \{nabst, nvirt, nprxy\}) \\
\quad \land nt_1 \notin \{nprxy, nenum\} \land nt_2 \notin \{nopt\}
\end{array}
$$

relation$(\_ \leq_{rNT} \_)$

$$\begin{array}{|l} \_ \leq_{rNT} \_ : SGNT \leftrightarrow SGNT \\ \hline \forall nt_1, nt_2 : SGNT \bullet nt_1 \leq_{rNT} nt_2 \Leftrightarrow nt_1 = nt_2 \lor nt_1 = nprxy \\ \quad \lor nt_2 = nabst \land nt_1 \in \{nnrml, nvirt\} \lor nt_2 \in \{nnrml, nopt\} \end{array}$$

relation$(\_ =_{ET} \_)$

$$\begin{array}{|l} \_ =_{ET} \_ : SGET \leftrightarrow SGET \\ \hline \forall et_1, et_2 : SGET \bullet et_1 =_{ET} et_2 \Leftrightarrow et_1 = et_2 \\ \quad \lor (\forall d_1, d_2 : SGED \bullet et_1 = erel\ d_1 \land et_2 = erel\ d_2 \lor et_1 = ecomp\ d_1 \land et_2 = ecomp\ d_2) \end{array}$$

relation$(\_ \leq_{ET} \_)$

$$\begin{array}{|l} \_ \leq_{ET} \_ : SGET \leftrightarrow SGET \\ \hline \forall et_1, et_2 : SGET \bullet et_1 \leq_{ET} et_2 \Leftrightarrow et_2 = ewander \land et_1 \neq einh \\ \quad \lor et_1 = eder \land et_2 \in \mathrm{dom}(erel^{\sim}) \cup \mathrm{dom}(ecomp^{\sim}) \lor et_1 =_{ET} et_2 \end{array}$$

5 Multiplicities

section *Fragmenta_Mult* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_SGElemTys*

$MultVal ::= \mathbf{v}\langle\langle \mathbb{N} \rangle\rangle \mid *$

$MultC ::= mr\langle\langle \mathbb{N} \times MultVal \rangle\rangle \mid ms\langle\langle MultVal \rangle\rangle$

relation$(\_ \leq_{mv} \_)$

$$\begin{array}{|l} \_ \leq_{mv} \_ : MultVal \leftrightarrow MultVal \\ \hline \forall m_1, m_2 : MultVal \bullet m_1 \leq_{mv} m_2 \Leftrightarrow m_2 = * \lor \exists j, k : \mathbb{N} \mid m_1 = \mathbf{v}\ j \land m_2 = \mathbf{v}\ k \bullet j \leq k \end{array}$$

$Mult == \{mc : MultC \mid \exists lb : \mathbb{N};\ ub : MultVal \bullet mc = mr(lb, ub) \land \mathbf{v}\ lb \leq_{mv} ub$
$\quad \lor \exists mv : MultVal \bullet mc = ms\ mv\}$

$MultMany == \{ms\ *, mr(0, *)\}$

$MultRange == \{m : MultC \mid \exists k : \mathbb{N} \mid k \geq 0 \bullet m = ms\ (\mathbf{v}\ k)$
$\quad \lor \exists lb : \mathbb{N};\ umv : MultVal \mid \mathbf{v}\ 2 \leq_{mv} umv \bullet m = mr(lb, umv)\}$

relation($\_ \between \_$)

$$\begin{array}{|l}
_ \between _ : \mathbb{P}(\mathbb{N} \times (\mathit{MultVal} \times \mathit{MultVal})) \\
\hline
\forall k : \mathbb{N};\ lb, ub : \mathit{MultVal} \bullet k \between (lb, ub) \Leftrightarrow lb \leq_{mv} \mathbf{v}\ k \land \mathbf{v}\ k \leq_{mv} ub
\end{array}$$

$$\begin{array}{|l}
mlb, mub : \mathit{MultC} \nrightarrow \mathit{MultVal} \\
\hline
mlb(ms\, *) = \mathbf{v}\ 0 \\
\forall k : \mathbb{N} \bullet mlb(ms(\mathbf{v}\ k)) = \mathbf{v}\ k \\
\forall k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ k \\
\forall mv : \mathit{MultVal} \bullet mub(ms\ mv) = mv \\
\forall k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ m
\end{array}$$

relation($\_ \leq_{\mathcal{M}} \_$)

$$\begin{array}{|l}
_ \leq_{\mathcal{M}} _ : \mathit{MultC} \leftrightarrow \mathit{MultC} \\
\hline
\forall m_1, m_2 : \mathit{MultC} \bullet m_1 \leq_{\mathcal{M}} m_2 \Leftrightarrow mlb\ m_2 \leq_{mv} mlb\ m_1 \land mub\ m_1 \leq_{mv} mub\ m_2
\end{array}$$

relation($\_ \propto \_$)

$$\begin{array}{|l}
_ \propto _ : \mathbb{P}(\mathit{SGET} \times (\mathit{MultC} \times \mathit{MultC})) \\
\hline
\forall et : \mathit{SGET};\ m_1, m_2 : \mathit{MultC} \bullet et \propto (m_1, m_2) \Leftrightarrow et = erel\ dbi \lor et = eder \\
\quad \lor et = ecomp\ duni \land m_1 = ms(\mathbf{v}\ 1) \lor et = erel\ duni \land m_1 \in \mathit{MultMany} \\
\quad \lor et = ecomp\ dbi \land m_1 \in \{ms(\mathbf{v}\ 1), mr(0, \mathbf{v}\ 1)\} \\
\quad \lor et = ewander \land (m_1, m_2) \in \mathit{MultMany} \times \mathit{MultMany}
\end{array}$$

relation(rbounded_)

$$\begin{array}{|l}
[X, Y] \\
\hline
\mathrm{rbounded}_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\, X \times \mathit{MultC}) \\
\hline
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ m : \mathit{MultC} \bullet \\
\quad \mathrm{rbounded}(r, s, m) \Leftrightarrow \forall x : s \bullet \#(r \limg \{x\} \rimg) \between (mlb\ m, mub\ m) \\
\hline
\end{array}$$

relation$(r\mathcal{M}Ok\_)$

$$
\begin{array}{l}
[X, Y] \\
\hline
r\mathcal{M}Ok\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\, X \times \mathbb{P}\, Y \times \mathit{MultC} \times \mathit{MultC}) \\
\hline
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, ms(\mathbf{v}\ 1), ms(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail\!\!\!\!\rightarrow t \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr(0, \mathbf{v}\ 1), mr(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail\!\!\!\!\!\!\mapsto t \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr(0, \mathbf{v}\ 1), ms(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail t \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, ms(\mathbf{v}\ 1), mr(0, \mathbf{v}\ 1)) \Leftrightarrow r^{\sim} \in t \rightarrowtail s \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mm : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mm, ms(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrow t \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mm : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, ms(\mathbf{v}\ 1), mm) \Leftrightarrow r^{\sim} \in t \rightarrow s \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mm_1, mm_2 : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mm_1, mm_2) \Leftrightarrow r \in s \leftrightarrow t \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mm : \mathit{MultMany};\ mr : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mm, mr) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r, s, mr) \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mm : \mathit{MultMany};\ mr : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr, mm) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r^{\sim}, t, mr) \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mm : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mm, mr(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \nrightarrow t \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mm : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr(0, \mathbf{v}\ 1), mm) \Leftrightarrow r^{\sim} \in t \nrightarrow s \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mr_1, mr_2 : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr_1, mr_2) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r, s, mr_2) \land \mathrm{rbounded}(r^{\sim}, t, mr_1) \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mr : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr, ms(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrow t \land \mathrm{rbounded}(r^{\sim}, t, mr) \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mr : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, ms(\mathbf{v}\ 1), mr) \Leftrightarrow r^{\sim} \in t \rightarrow s \land \mathrm{rbounded}(r, s, mr) \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ m : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, m, mr(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \nrightarrow t \land \mathrm{rbounded}(r^{\sim}, t, m) \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ m : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr(0, \mathbf{v}\ 1), m) \Leftrightarrow r^{\sim} \in t \nrightarrow s \land \mathrm{rbounded}(r, s, m)
\end{array}
$$

6 Structural Graphs

section *Fragmenta_SGs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*, *Fragmenta_SGElemTys*, *Fragmenta_Mult*, *Fragmenta_GrswT*

$$SGr_0 == \{G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ db : E \pfun E \mid nt \in Ns\, G \fun SGNT \land et \in Es\, G \fun SGET\}$$

$$\begin{array}{l}
gr : SGr_0 \fun Gr \\
sg_Ns : SGr_0 \fun \mathbb{P}\, V \\
sg_Es : SGr_0 \fun \mathbb{P}\, E \\
sg_src, sg_tgt : SGr_0 \fun E \pfun V \\
nty : SGr_0 \fun V \pfun SGNT \\
ety : SGr_0 \fun E \pfun SGET \\
srcm, tgtm : SGr_0 \fun E \pfun Mult \\
derb : SGr_0 \fun E \pfun E \\
\hline
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ db : E \pfun E \bullet \\
\quad gr\,(G, nt, et, sm, tm, db) = G \\
sg_Ns = Ns \circ gr \\
sg_Es = Es \circ gr \\
sg_src = src \circ gr \\
sg_tgt = tgt \circ gr \\
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ db : E \pfun E \bullet \\
\quad nty(G, nt, et, sm, tm, db) = nt \\
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ db : E \pfun E \bullet \\
\quad ety(G, nt, et, sm, tm, db) = et \\
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ db : E \pfun E \bullet \\
\quad srcm(G, nt, et, sm, tm, db) = sm \\
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ db : E \pfun E \bullet \\
\quad tgtm(G, nt, et, sm, tm, db) = tm \\
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ db : E \pfun E \bullet \\
\quad derb(G, nt, et, sm, tm, db) = db
\end{array}$$

$$\begin{array}{l}
\varnothing_{SG} : SGr_0 \\
\hline
\varnothing_{SG} = (\varnothing_G, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing)
\end{array}$$

$$\begin{array}{l}
NsTy : SGr_0 \fun \mathbb{P}\, SGNT \fun \mathbb{P}\, V \\
EsTy : SGr_0 \fun \mathbb{P}\, SGET \fun \mathbb{P}\, E \\
\hline
\forall SG : SGr_0;\ nts : \mathbb{P}\, SGNT \bullet NsTy\, SG\, nts = (nty\, SG)^\sim \limg nts \rimg \\
\forall SG : SGr_0;\ ets : \mathbb{P}\, SGET \bullet EsTy\, SG\, ets = (ety\, SG)^\sim \limg ets \rimg
\end{array}$$

$$EsA, EsW, EsI, EsC, EsD : SGr_0 \to \mathbb{P}\, E$$

$$\forall\, SG : SGr_0 \bullet EsA\ SG = EsTy\ SG\, (erel \,(\!|\, SGED \,|\!)\, \cup ecomp \,(\!|\, SGED \,|\!))$$

$$EsW = (\text{flip}\ EsTy)\, \{ewander\}$$

$$EsI = (\text{flip}\ EsTy)\, \{einh\}$$

$$EsD = (\text{flip}\ EsTy)\, \{eder\}$$

$$\forall\, SG : SGr_0 \bullet EsC\ SG = EsA\ SG \cup EsW\ SG \cup EsD\ SG$$

$$NsP, NsEther, NsO, NsSeq : SGr_0 \to \mathbb{P}\, V$$

$$NsP = (\text{flip}\ NsTy)\, \{nprxy\}$$

$$NsEther = (\text{flip}\ NsTy)\, \{nabst, nvirt, nenum\}$$

$$NsO = (\text{flip}\ NsTy)\, \{nopt\}$$

$$\pitchfork : SGr_0 \to Gr$$
$$\prec : SGr_0 \to V \leftrightarrow V$$

$$\forall\, SG : SGr_0 \bullet \pitchfork\ SG = gr\ SG \bowtie EsI\ SG$$

$$\prec = (\_^{\leftrightarrow}) \circ \pitchfork$$

$$srcma : SGr_0 \to (E \nrightarrow Mult)$$

$$\forall\, SG : SGr_0 \bullet srcma\ SG =$$
$$(srcm\ SG) \oplus (EsTy\ SG\, \{ecomp\ duni\} \times \{ms(\mathbf{v}\ 1)\}) \oplus (EsTy\ SG\, \{erel\ duni\} \times \{ms\, *\})$$

relation($\mathcal{M}etysOk\, \_$)

$$\mathcal{M}etysOk\, \_ : \mathbb{P}\, SGr_0$$

$$\forall\, SG : SGr_0 \bullet \mathcal{M}etysOk\ \ SG \Leftrightarrow \forall\, e : EsC\ SG \bullet (ety\ SG\ e) \propto (srcma\ SG\ e, tgtm\ SG\ e)$$

relation(optsVoluntary_)

$$\text{optsVoluntary}\_ : \mathbb{P}\, SGr_0$$

$$\forall\, SG : SGr_0 \bullet$$
$$\text{optsVoluntary}\ SG \Leftrightarrow (ety\ SG)\,(\!|\, EsIncident(gr\ SG)(NsO\ SG) \,|\!) \subseteq \{ewander\}$$

relation(inhNtysOk_)

$$
\begin{array}{|l}
\mathrm{inhNtysOk\_} : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \\
\quad \mathrm{inhNtysOk}\, SG \Leftrightarrow \forall\, v, v' : sg\_Ns\, SG \mid (v, v') \in (\prec SG) \bullet nty\, SG\, v \prec_{NT} nty\, SG\, v'
\end{array}
$$

relation(seqsOk_)

$$
\begin{aligned}
SGr == \{SG : SGr_0 \mid &\{srcma\, SG, tgtm\, SG\} \subseteq EsC\, SG \to Mult \wedge \mathrm{dom}(derb\, SG) = EsD\, SG \\
&\wedge \mathcal{M}etysOk\ SG \wedge \mathrm{optsVoluntary}\, SG \wedge \mathrm{inhNtysOk}\, SG \wedge \circleddash(\pitchfork SG)\}
\end{aligned}
$$

$$
\begin{array}{|l}
\preceq : SGr \to V \leftrightarrow V \\
\hline
\forall\, SG : SGr \bullet \preceq SG = (\prec SG)^{\,*}
\end{array}
$$

$$
\begin{array}{|l}
srcr, tgtr : SGr \to E \leftrightarrow V \\
src_0^*, src^*, tgt_0^*, tgt^* : SGr \to E \leftrightarrow V \\
\hline
\forall\, SG : SGr \bullet srcr\, SG = sg\_src\, SG \cup (EsW\ SG \lhd sg\_tgt\, SG) \\
\forall\, SG : SGr \bullet tgtr\, SG = sg\_tgt\, SG \cup (EsW\ SG \lhd sg\_src\, SG) \\
\forall\, SG : SGr \bullet src_0^*\ SG = EsC\, SG \lhd (srcr\, SG) \\
\forall\, SG : SGr \bullet src^*\ SG = (src_0^*\ SG) \mathbin{⨾} (\preceq SG)^{\sim} \\
\forall\, SG : SGr \bullet tgt_0^*\ SG = EsC\, SG \lhd (tgtr\, SG) \\
\forall\, SG : SGr \bullet tgt^*\ SG = (tgt_0^*\ SG) \mathbin{⨾} (\preceq SG)^{\sim}
\end{array}
$$

relation(etherealAreInherited _)

$$
\begin{array}{|l}
\mathrm{etherealAreInherited\_} : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \mathrm{etherealAreInherited}\, SG \Leftrightarrow NsEther\, SG \subseteq \mathrm{ran}(\prec SG)
\end{array}
$$

relation(derivedOk _)

$$
\begin{array}{|l}
\mathrm{derivedOk}\_ : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \mathrm{derivedOk}\, SG \Leftrightarrow \mathrm{ran}(derb\; SG) \subseteq EsA\; SG \\
\quad \land (\forall\, e : EsD\; SG \bullet (sg\_src\; SG\; e, ((sg\_src\; SG) \circ (derb\; SG))\; e) \in (\preceq SG) \\
\qquad \land (sg\_tgt\; SG\; e, ((sg\_tgt\; SG) \circ (derb\; SG))\; e) \in (\preceq SG))
\end{array}
$$

$$TSGr == \{\, SG : SGr \mid \mathrm{etherealAreInherited}\, SG \land \mathrm{derivedOk}\, SG \,\}$$

relation$(\boxminus_{SGs}\,\_)$

$$
\begin{array}{|l}
\boxminus_{SGs}\_ : \mathbb{P}(SGr \times SGr) \\
\hline
\forall\, SG_1, SG_2 : SGr \bullet \boxminus_{SGs}(SG_1, SG_2) \Leftrightarrow \boxminus(gr\; SG_1, gr\; SG_2)
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_{SG} \_)$

$$
\begin{array}{|l}
\_ \cup_{SG} \_ : SGr \times SGr \pfun SGr \\
\hline
\forall\, SG_1, SG_2 : SGr \bullet SG_1 \cup_{SG} SG_2 = (gr\; SG_1 \cup_G gr\; SG_2, nty\; SG_1 \cup nty\; SG_2, \\
\quad ety\; SG_1 \cup ety\; SG_2, srcm\; SG_1 \cup srcm\; SG_2, tgtm\; SG_1 \cup tgtm\; SG_2, derb\; SG_1 \cup derb\; SG_2)
\end{array}
$$

function 10 **leftassoc** $(\_ \odot^{SG} \_)$

$$
\begin{array}{|l}
\_ \odot^{SG} \_ : SGr \times (V \pfun V) \pfun SGr \\
\hline
\forall\, SG : SGr;\; s : V \pfun V \mid s \in NsP\; SG \pfun sg\_Ns\; SG \land s \in antireflexive \bullet \\
\quad SG \odot^{SG} s = (gr\; SG \odot s, (dom\; s) \ndres nty\; SG, ety\; SG, srcm\; SG, tgtm\; SG, derb\; SG)
\end{array}
$$

function 10 **leftassoc** $(\_ \rightarrow_{SG} \_)$

$$
\begin{array}{|l}
\_ \rightarrow_{SG} \_ : SGr \times SGr \rightarrow \mathbb{P}\; GrM \\
\hline
\forall\, SG_s, SG_t : SGr \bullet \\
\quad SG_s \rightarrow_{SG} SG_t = \{fv : sg\_Ns\; SG_s \rightarrow sg\_Ns\; SG_t;\; fe : EsC\; SG_s \rightarrow EsC\; SG_t \mid \\
\qquad fv \circ src^*\; SG_s \subseteq src^*\; SG_t \circ fe \land fv \circ tgt^*\; SG_s \subseteq tgt^*\; SG_t \circ fe \\
\qquad \land fv \circ \preceq SG_s \subseteq \preceq SG_t \circ fv\}
\end{array}
$$

relation$(\_ \Rrightarrow^{SG} \_)$

$$\begin{array}{|l}
_ \Rrightarrow^{SG} _ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_s, SG_t : SGr;\ m : GrM \bullet (SG_s, m) \Rrightarrow^{SG} SG_t \Leftrightarrow m \in SG_s \rightarrow_{SG} SG_t
\end{array}$$

function 10 **leftassoc** $(\_ \rightarrow_{G2SG} \_)$

$$\begin{array}{|l}
_ \rightarrow_{G2SG} _ : Gr \times SGr \rightarrow \mathbb{P}\ GrM \\
\hline
\forall G : Gr;\ SG : SGr \bullet G \rightarrow_{G2SG} SG = \{fv : Ns\ G \rightarrow sg_Ns\ SG;\ fe : Es\ G \rightarrow EsC\ SG \mid \\
\quad fv \circ src\ G \subseteq src^*\ SG \circ fe \wedge fv \circ tgt\ G \subseteq tgt^*\ SG \circ fe\}
\end{array}$$

relation$(\_ \Rrightarrow^{GwT} \_)$

$$\begin{array}{|l}
_ \Rrightarrow^{GwT} _ : (GrwT \leftrightarrow SGr) \\
\hline
\forall GwT : GrwT;\ SG : SGr \bullet GwT \Rrightarrow^{GwT} SG \Leftrightarrow (ty\ GwT) \in (gOf\ GwT) \rightarrow_{G2SG} SG
\end{array}$$

$$\begin{array}{|l}
totaliseForDer : GrM \times SGr \rightarrow GrM \\
\hline
\forall m : GrM;\ SG : SGr \bullet totaliseForDer(m, SG) = (fV\ m, ((derb\ SG) \boxdot (EsC\ SG))\ ;\ fE\ m)
\end{array}$$

$$\begin{array}{|l}
insOf : GrM \times SGr \times \mathbb{P}\ V \rightarrow \mathbb{P}\ V \\
iesOf : GrM \times \mathbb{P}\ E \rightarrow \mathbb{P}\ E \\
igRMEs : GrwT \times \mathbb{P}\ E \rightarrow Gr \\
igRMNsEs : GrwT \times SGr \times \mathbb{P}\ V \times \mathbb{P}\ E \rightarrow Gr \\
\hline
\forall m : GrM;\ SG : SGr;\ mns : \mathbb{P}\ V \bullet insOf(m, SG, mns) = (fV\ m)^{\sim} (\!| (\prec SG)^{\sim} (\!| mns |\!) |\!) \\
\forall m : GrM;\ mes : \mathbb{P}\ E \bullet iesOf(m, mes) = (fE\ m)^{\sim} (\!| mes |\!) \\
\forall GwT : GrwT;\ mes : \mathbb{P}\ E \bullet igRMEs(GwT, mes) = (gOf\ GwT) \bowtie iesOf((ty\ GwT), mes) \\
\forall GwT : GrwT;\ SG : SGr;\ mns : \mathbb{P}\ V;\ mes : \mathbb{P}\ E \bullet \\
\quad igRMNsEs(GwT, SG, mns, mes) = igRMEs(GwT, mes) \nabla insOf(ty\ GwT, SG, mns)
\end{array}$$

relation$(\mathrm{inverted_E}\_)$

$$
\begin{array}{|l}
\mathrm{inverted}_{\mathrm{E}\_} : \mathbb{P}(GrwT \times SGr \times E) \\
gOfwei, igRMEsW : GrwT \times SGr \times E \to Gr \\
gOfweis : GrwT \times SGr \times \mathbb{P}\, E \to Gr \\
\hline
\forall\, G : Gr;\ m : GrM;\ SG : SGr;\ e : E \bullet \\
\quad \mathrm{inverted}_{\mathrm{E}}((G, m), SG, e) \Leftrightarrow ((sg\_tgt\ SG) \circ (fE\ m))e = ((fV\ m) \circ (src\ G))e \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E \bullet \\
\quad gOfwei(GwT, SG, e) = \mathbf{if}\ \mathrm{inverted}_{\mathrm{E}}(GwT, SG, e)\ \mathbf{then}\ ((gOf\ GwT) \bowtie \{e\})^{\rightleftarrows}\ \mathbf{else}(gOf\ GwT) \bowtie \{e\} \\
\forall\, GwT : GrwT;\ SG : SGr \bullet gOfweis(GwT, SG, \{\}) = \varnothing_G \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E;\ es : \mathbb{P}\, E \bullet \\
\quad gOfweis(GwT, SG, \{e\} \cup es) = gOfwei(GwT, SG, e) \cup_G gOfweis(GwT, SG, es) \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E \mid e \in EsD\ SG \bullet igRMEsW(GwT, SG, e) = \\
\quad igRMNsEs(GwT, SG, \{(sg\_src\ SG)(derb\ SG\ e), (sg\_tgt\ SG)(derb\ SG\ e)\}, \{derb\ SG\ e\}) \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E \mid e \notin EsW\ SG \bullet igRMEsW(GwT, SG, e) = igRMEs(GwT, \{e\}) \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E \mid e \in EsW\ SG \bullet igRMEsW(GwT, SG, e) = \\
\quad gOfweis(GwT, SG, ((fE \circ ty)\ GwT)^{\sim} (\!| \{e\} |\!))
\end{array}
$$

relation$(\_ \sqsupseteq^{SG} \_)$
relation$(\_ \sqsupseteq^{SG_0} \_)$
relation$(\_ \sqsupseteq_{NT} \_)$
relation$(\_ \sqsupseteq_{ET} \_)$
relation$(\_ \sqsupseteq_{\mathcal{M}} \_)$

$$
\begin{array}{|l}
\_ \sqsupseteq_{NT} \_, \_ \sqsupseteq_{ET} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq_{NT}\ SG_a \Leftrightarrow \forall\, n : sg\_Ns\ SG_c \bullet (nty\ SG_c)\ n \leq_{rNT} ((nty\ SG_a) \circ (fV\ m))\ n \\
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq_{ET}\ SG_a \Leftrightarrow \forall\, e : EsC\ SG_c \bullet (ety\ SG_c)\ e \leq_{ET} ((ety\ SG_a) \circ (fE\ m))\ e
\end{array}
$$

$$
\begin{array}{|l}
\_ \sqsupseteq_{\mathcal{M}} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq_{\mathcal{M}}\ SG_a \Leftrightarrow \forall\, e : EsC\ SG_c \setminus EsD\ SG_c \bullet (srcma\ SG_c)\ e \leq_{\mathcal{M}} ((srcma\ SG_a) \circ (fE\ m))\ e \\
\qquad\qquad \wedge\ (tgtm\ SG_c)\ e \leq_{\mathcal{M}} ((tgtm\ SG_a) \circ (fE\ m))\ e
\end{array}
$$

$$
\begin{array}{l}
\_ \sqsupseteq^{SG} \_, \_ \sqsupseteq^{SG_0} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq^{SG_0}\ SG_a \Leftrightarrow (SG_c, m) \sqsupseteq_{NT}\ SG_a \wedge (SG_c, m) \sqsupseteq_{ET}\ SG_a \wedge (SG_c, m) \sqsupseteq_{\mathcal{M}}\ SG_a \\
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq^{SG}\ SG_a \Leftrightarrow \exists\, m' : GrM \mid m' = \mathit{totaliseForDer}(m, SG_c) \bullet \\
\qquad m' \in SG_c \to_{SG} SG_a \wedge (SG_c, m') \sqsupseteq^{SG_0}\ SG_a
\end{array}
$$

relation$(\_ \sqsupset^{SG} \_)$
relation$(\_ \sqsupset^{SG_0} \_)$
relation$(\_ \sqsupset_{AEs} \_)$
relation(_OkRefinedIn_)
relation$(\_ \sqsupset_{ANNs} \_)$

$$
\begin{array}{l}
\_ \sqsupset_{ANNs} \_ : \mathbb{P}(GrM \times SGr) \\
\hline
\forall\, SG_a : SGr;\ m : GrM \bullet \\
\quad m \sqsupset_{ANNs} SG_a \Leftrightarrow \forall\, nn : \mathit{NsTy}\, SG_a\{\mathit{nnrml}\} \bullet (\preceq\ SG_a) (\!|\, \{nn\} \,|\!) \cap \mathrm{ran}(\mathit{fV}\ m) = \varnothing
\end{array}
$$

$$
\begin{array}{l}
\_\mathrm{OkRefinedIn}\_ : \mathbb{P}((SGr \times E) \times (SGr \times GrM)) \\
\_ \sqsupset_{AEs} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM;\ ae : E \bullet \\
\quad (SG_a, ae)\mathrm{OkRefinedIn}(SG_c, m) \Leftrightarrow \\
\qquad \exists\, r : V \leftrightarrow V;\ s, t : \mathbb{P}\ V \mid r = (\preceq\ SG_c) \mathbin{⨾} \mathit{igRMEsW}((\mathit{gr}\ SG_c, m), SG_a, ae)^{\leftrightarrow} \mathbin{⨾} (\preceq\ SG_c)^{\sim} \\
\qquad\quad \wedge\ s = \mathit{insOf}(m, SG_a, (\mathit{sg\_src}\ SG_a (\!|\, \{ae\} \,|\!))) \setminus ((\mathit{NsEther}\ SG_c) \setminus \mathrm{dom}\ r) \\
\qquad\quad \wedge\ t = \mathit{insOf}(m, SG_a, (\mathit{sg\_tgt}\ SG_a (\!|\, \{ae\} \,|\!))) \setminus ((\mathit{NsEther}\ SG_c) \setminus \mathrm{ran}\ r) \\
\qquad\qquad \bullet\ r \in s \leftrightarrow t \wedge r \neq \varnothing \\
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset_{AEs}\ SG_a \Leftrightarrow \forall\, e : (\mathit{EsA}\ SG_a) \bullet (SG_a, e)\mathrm{OkRefinedIn}(SG_c, m)
\end{array}
$$

$$
\begin{array}{l}
\_ \sqsupset^{SG} \_, \_ \sqsupset^{SG_0} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset^{SG_0}\ SG_a \Leftrightarrow (SG_c, m) \sqsupseteq^{SG_0}\ SG_a \wedge m \sqsupset_{ANNs} SG_a \wedge (SG_c, m) \sqsupset_{AEs}\ SG_a \\
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset^{SG}\ SG_a \Leftrightarrow \exists\, m' : GrM \mid m' = \mathit{totaliseForDer}(m, SG_c) \bullet \\
\qquad m' \in SG_c \to_{SG} SG_a \wedge (SG_c, m') \sqsupset^{SG_0}\ SG_a
\end{array}
$$

relation$(\_ \Supset^{SG} \_)$
relation$(\_ \Supset_{\mathcal{M}} \_)$
relation$(\_ \Supset_{FI} \_)$
relation$(\_ \Supset_{PNS} \_)$
relation$(\_\ \mathit{ME\mathcal{M}Ok}\ \_)$

$$
\begin{array}{l}
\_\ ME\mathcal{M}Ok\ \_ : \mathbb{P}((SGr \times E) \times GrwT) \\
\hline
\forall GwT : GrwT;\ SG : SGr;\ me : E \bullet (SG, me)\ ME\mathcal{M}Ok\ GwT \Leftrightarrow \\
\quad \exists r : V \leftrightarrow V;\ s, t : \mathbb{P}\ V \mid r = igRMEsW(GwT, SG, me)^{\leftrightarrow} \\
\qquad \land s = insOf(ty\ GwT, SG, (src^*\ SG) (\!|\ \{me\}\ |\!)) \\
\qquad \land t = insOf(ty\ GwT, SG, (tgt^*\ SG) (\!|\ \{me\}\ |\!)) \\
\qquad\quad \bullet r\mathcal{M}Ok(r, s, t, srcma\ SG\ me, tgtm\ SG\ me)
\end{array}
$$

$$
\begin{array}{l}
\_ \ni_{\mathcal{M}} \_ : GrwT \leftrightarrow SGr \\
\_ \ni_{FI} \_ : GrwT \leftrightarrow SGr \\
\_ \ni_{PNS} \_ : GrwT \leftrightarrow SGr \\
\hline
\forall GwT : GrwT;\ SG : SGr \bullet GwT \ni_{\mathcal{M}} SG \Leftrightarrow \forall me : EsC\ SG \bullet (SG, me)\ ME\mathcal{M}Ok\ GwT \\
\forall GwT : GrwT;\ SG : SGr \bullet GwT \ni_{FI} SG \Leftrightarrow ((fV \circ ty) GwT)^{\sim} (\!| NsEther\ SG |\!) = \varnothing \\
\forall GwT : GrwT;\ SG : SGr \bullet \\
\quad GwT \ni_{PNS} SG \Leftrightarrow igRMEs(GwT, EsTy\ SG\ \{ecomp\ dbi, ecomp\ duni\})^{\leftrightarrow} \in \mathrm{injrel}
\end{array}
$$

$$
\begin{array}{l}
\_ \ni^{SG} \_ : GrwT \leftrightarrow SGr \\
\hline
\forall GwT : GrwT;\ SG : SGr \bullet \\
\quad GwT \ni^{SG} SG \Leftrightarrow GwT \Rrightarrow^{GwT} SG \land GwT \ni_{\mathcal{M}} SG \land GwT \ni_{FI} SG \land GwT \ni_{PNS} SG
\end{array}
$$

7 Fragments

section *Fragmenta_Frs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_SGs*, *Fragmenta_GrswT*

$$
\begin{array}{l}
Fr_0 == \{SG : SGr;\ esr : \mathbb{P}\ E;\ sr, tr : E \rightarrowtail\!\!\!\!\!+ V \mid esr \cap (sg\_Es\ SG) = \varnothing \\
\quad \land sr \in esr \rightarrowtail\!\!\!\!\rightarrow (NsP\ SG) \land tr \in esr \rightarrow V\}
\end{array}
$$

$$
\begin{array}{l}
fSG : Fr_0 \rightarrow SGr \\
EsR : Fr_0 \rightarrow \mathbb{P}\ E \\
srcR, tgtR : Fr_0 \rightarrow E \nrightarrow V \\
\hline
\forall SG : SGr;\ esr : \mathbb{P}\ E;\ sr, tr : E \nrightarrow V \bullet fSG(SG, esr, sr, tr) = SG \\
\forall SG : SGr;\ esr : \mathbb{P}\ E;\ sr, tr : E \nrightarrow V \bullet EsR(SG, esr, sr, tr) = esr \\
\forall SG : SGr;\ esr : \mathbb{P}\ E;\ sr, tr : E \nrightarrow V \bullet srcR(SG, esr, sr, tr) = sr \\
\forall SG : SGr;\ esr : \mathbb{P}\ E;\ sr, tr : E \nrightarrow V \bullet tgtR(SG, esr, sr, tr) = tr
\end{array}
$$

$$
\begin{array}{l}
\mathit{fLEs}, \mathit{fEs}, \mathit{fEsC} : \mathit{Fr}_0 \rightarrow \mathbb{P}\, E \\
\mathit{fLNs}, \mathit{fRNs}, \mathit{fNs} : \mathit{Fr}_0 \rightarrow \mathbb{P}\, V \\
\mathit{srcF}, \mathit{tgtF} : \mathit{Fr}_0 \rightarrow E \nrightarrow V \\
\hline
\mathit{fLEs} = (\mathit{sg\_Es} \circ \mathit{fSG}) \\
\forall\, F : \mathit{Fr}_0 \bullet \mathit{fEs}\, F = \mathit{fLEs}\, F \cup \mathit{EsR}\, F \\
\mathit{fEsC} = \mathit{EsC} \circ \mathit{fSG} \\
\mathit{fLNs} = \mathit{sg\_Ns} \circ \mathit{fSG} \\
\mathit{fRNs} = \mathrm{ran} \circ \mathit{tgtR} \\
\forall\, F : \mathit{Fr}_0 \bullet \mathit{fNs}\, F = \mathit{fLNs}\, F \cup \mathit{fRNs}\, F \\
\forall\, F : \mathit{Fr}_0 \bullet \mathit{srcF}\, F = (\mathit{sg\_src} \circ \mathit{fSG})\, F \cup \mathit{srcR}\, F \\
\forall\, F : \mathit{Fr}_0 \bullet \mathit{tgtF}\, F = (\mathit{sg\_tgt} \circ \mathit{fSG})\, F \cup \mathit{tgtR}\, F
\end{array}
$$

$$
\begin{array}{l}
\overset{G}{\leftrightsquigarrow} : \mathit{Fr}_0 \rightarrow \mathit{Gr} \\
\leftrightsquigarrow : \mathit{Fr}_0 \rightarrow V \leftrightarrow V \\
\hline
\forall\, F : \mathit{Fr}_0 \bullet \overset{G}{\leftrightsquigarrow}\, F = ((\mathit{NsP} \circ \mathit{fSG}) F \cup \mathit{fRNs}\, F, \mathit{EsR}\, F, \mathit{srcR}\, F, \mathit{tgtR}\, F) \\
\forall\, F : \mathit{Fr}_0 \bullet \leftrightsquigarrow\, F = (\overset{G}{\leftrightsquigarrow}\, F)^{\Leftrightarrow}
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_F \_)$

$$
\begin{array}{l}
\varnothing_F : \mathit{Fr}_0 \\
\_ \cup_F \_ : \mathit{Fr}_0 \times \mathit{Fr}_0 \rightarrow \mathit{Fr}_0 \\
\bigcup_F : \mathbb{P}\, \mathit{Fr}_0 \rightarrow \mathit{Fr}_0 \\
\hline
\varnothing_F = (\varnothing_{SG}, \varnothing, \varnothing, \varnothing) \\
\forall\, F_1, F_2 : \mathit{Fr}_0 \bullet F_1 \cup_F F_2 = \\
\quad (\mathit{fSG}\, F_1 \cup_{SG} \mathit{fSG}\, F_2, \mathit{EsR}\, F_1 \cup \mathit{EsR}\, F_2, \mathit{srcR}\, F_1 \cup \mathit{srcR}\, F_2, \mathit{tgtR}\, F_1 \cup \mathit{tgtR}\, F_2) \\
\bigcup_F \{\} = \varnothing_F \\
\forall\, F : \mathit{Fr}_0;\ \mathit{Fs} : \mathbb{P}\, \mathit{Fr}_0 \bullet \bigcup_F (\{F\} \cup \mathit{Fs}) = F \cup_F (\bigcup_F \mathit{Fs})
\end{array}
$$

$$
\begin{array}{l}
\rightsquigarrow : Fr_0 \rightarrowtail V \rightarrowtail V \\
\circledbullet^{SG} : Fr_0 \rightarrowtail SGr \\
rEsR : Fr_0 \rightarrowtail \mathbb{P}\, E \\
\circledbullet : Fr_0 \rightarrowtail Fr_0 \\
\hline
\forall F : Fr_0 \bullet \rightsquigarrow F = (\leftrightsquigarrow F) \rhd (fLNs\, F) \\
\forall F : Fr_0 \bullet \circledbullet^{SG} F = (fSG\, F) \odot^{SG} (\rightsquigarrow F) \\
\forall F : Fr_0 \bullet rEsR\, F = \mathrm{dom}((srcR\, F) \rhd \mathrm{dom}(\rightsquigarrow F)) \\
\forall F : Fr_0 \bullet \circledbullet F = (\circledbullet^{SG} F, rEsR\, F, (rEsR\, F) \lhd (srcR\, F), (rEsR\, F) \lhd (tgtR\, F))
\end{array}
$$

$Fr_a == \{F : Fr_0 \mid \otimes(\overset{G}{\leftrightsquigarrow} F)\}$

$Fr == \{F : Fr_a \mid \circledbullet^{SG} F \in SGr\}$

relation(refsLocal_)

$$
\begin{array}{l}
\mathrm{refsLocal\_} : \mathbb{P}\, Fr_0 \\
\hline
\forall F : Fr_0 \bullet \mathrm{refsLocal}\, F \Leftrightarrow fRNs\, F \subseteq fLNs\, F
\end{array}
$$

$TFr == \{F : Fr_a \mid \mathrm{refsLocal}\, F \land \circledbullet^{SG} F \in TSGr\}$

relation($\boxminus$ _)
relation($\boxplus$ _)

$$
\begin{array}{l}
\boxminus\_ : Fr \leftrightarrow Fr \\
\hline
\forall F_1, F_2 : Fr \bullet \boxminus(F_1, F_2) \Leftrightarrow \boxminus_{SGs}(fSG\, F_1, fSG\, F_2) \land EsR\, F_1 \cap EsR\, F_2 = \varnothing
\end{array}
$$

$$
\begin{array}{l}
[I] \\
\hline
\boxplus\_ : \mathbb{P}(I \rightarrowtail Fr) \\
\hline
\forall Fs : I \rightarrowtail Fr \bullet \boxplus\, Fs \Leftrightarrow \forall i, j : \mathrm{dom}\, Fs \mid i \neq j \bullet \boxminus(Fs\, i, Fs\, j)
\end{array}
$$

relation(_ $\subseteq^{rs}$ _)
relation(_ $\Mapsto$ _)

$$
\begin{array}{|l}
\_ \subseteq^{rs} \_ : Fr \leftrightarrow Fr \\
\_ \mapsto \_ : Fr \leftrightarrow Fr \\
\hline
\forall F_1, F_2 : Fr \bullet F_1 \subseteq^{rs} F_2 \Leftrightarrow \mathrm{ran}(tgtR\ F_1) \subseteq fLNs\ F_2 \\
\forall F_1, F_2 : Fr \bullet F_1 \mapsto F_2 \Leftrightarrow F_1 \subseteq^{rs} F_2 \wedge \neg\,(F_2 \subseteq^{rs} F_1)
\end{array}
$$

function 10 **leftassoc** $(\_ \rightrightarrows_\bullet \_)$

$$
\begin{array}{|l}
\_ \rightrightarrows_\bullet \_ : GrM \times (Fr \times Fr) \rightarrow GrM \\
\hline
\forall m : GrM;\ F_s, F_t : Fr_0 \bullet \\
\quad m \rightrightarrows_\bullet (F_s, F_t) = (((\rightsquigarrow F_s)^{\oplus} \boxdot (fLNs\ F_s))^{\sim} \fatsemi (fV\ m) \fatsemi ((\rightsquigarrow F_t)^{\oplus} \boxdot (fLNs\ F_t)), fE\ m)
\end{array}
$$

function 1 **leftassoc** $(\_ \rightarrow_F \_)$

$$
\begin{array}{|l}
\_ \rightarrow_F \_ : Fr \times Fr \rightarrow \mathbb{P}\ GrM \\
\hline
\forall F_s, F_t : Fr \bullet F_s \rightarrow_F F_t = \{fv : fLNs\ F_s \rightarrow fLNs\ F_t;\ fe : fEsC\ F_s \rightarrow fEsC\ F_t \mid \\
\quad (\circledbullet^{SG} F_s, (fv, fe) \rightrightarrows_\bullet (F_s, F_t)) \Rrightarrow^{SG} \circledbullet^{SG} F_t\}
\end{array}
$$

relation $(\_ \Rrightarrow^F \_)$

$$
\begin{array}{|l}
\_ \Rrightarrow^F \_ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall m : GrM;\ F_s, F_t : Fr_0 \bullet (F_s, m) \Rrightarrow^F F_t \Leftrightarrow m \in F_s \rightarrow_F F_t
\end{array}
$$

relation $(\_ \sqsupseteq^F \_)$

$$
\begin{array}{|l}
\_ \sqsupseteq^F \_ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupseteq^F F_a \Leftrightarrow (F_c, m) \Rrightarrow^F F_a \\
\quad \wedge (\circledbullet^{SG} F_c, m \rightrightarrows_\bullet (F_c, F_a)) \sqsupseteq^{SG_0} (\circledbullet^{SG} F_a)
\end{array}
$$

relation $(\_ \sqsupset^F \_)$

$$
\begin{array}{|l}
\_ \sqsupset^F \_ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupset^F F_a \Leftrightarrow (F_c, m) \Rrightarrow^F F_a \\
\quad \land (\circledcirc^{SG} F_c, m \rightrightarrows_\bullet (F_c, F_a)) \sqsupset^{SG_0} (\circledcirc^{SG} F_a)
\end{array}
$$

relation$(\_ \Supset^F \_)$

$$
\begin{array}{|l}
\_ \Supset^F \_ : GrwT \leftrightarrow Fr \\
\hline
\forall GwT : GrwT;\ F : Fr \bullet GwT \Supset^F F \Leftrightarrow GwT \Supset^{SG} \circledcirc^{SG} F
\end{array}
$$

8 Global Fragment Graphs

section *Fragmenta_GFGs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$GFGr == \{G : Gr \mid \circleddash(G \bowtie (Es\ G \setminus EsId\ G))\}$

function$(\_^{\dashrightarrow})$

$$
\begin{array}{|l}
\_^{\dashrightarrow} : GFGr \rightarrow V \leftrightarrow V \\
\hline
\forall GFG : GFGr \bullet GFG^{\dashrightarrow} = (GFG^{\leftrightarrow})^+
\end{array}
$$

9 Models

section *Fragmenta_Mdls* **parents** *standard_toolkit*, *Fragmenta_Frs*, *Fragmenta_GFGs*

$Mdl_0 == \{GFG : GFGr;\ fd : V \nrightarrow Fr \mid fd \in Ns\ GFG \rightarrow Fr \land \boxplus\ fd\}$

$$
\begin{array}{|l}
mGFG : Mdl_0 \rightarrow GFGr \\
mFD : Mdl_0 \rightarrow V \nrightarrow Fr \\
\hline
\forall GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mGFG(GFG, fd) = GFG \\
\forall GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mFD(GFG, fd) = fd
\end{array}
$$

$$\begin{array}{|l} mUFs : Mdl_0 \rightarrow Fr \\ \hline mUFs = \bigcup_F \circ \operatorname{ran} \circ mFD \end{array}$$

$$\begin{array}{|l} \text{from} : Mdl_0 \rightarrow V \nrightarrow V \\ \hline \forall\, M : Mdl_0;\ v : V \bullet \text{from}\, M\, v = (\mu\, vf : (Ns \circ mGFG) M \mid v \in fLNs(mFD\, M\, vf)) \end{array}$$

relation$(\Uparrow \_)$

$$\begin{array}{|l} \Uparrow \_ : \mathbb{P}\, Mdl_0 \\ \hline \forall\, M : Mdl_0 \bullet \Uparrow M \Leftrightarrow \exists\, UF : Fr_0 \mid UF = mUFs\, M \bullet \\ \quad \forall\, p : (NsP \circ fSG)\, UF \bullet (from\, M\, p, from\, M\, (\leftrightsquigarrow\ UF\, p)) \in ((\_^{\dashrightarrow}) \circ mGFG) M \end{array}$$

$$Mdl == \{M : Mdl_0 \mid (mUFs\, M) \in TFr \wedge \Uparrow M\}$$

$$\begin{array}{|l} \odot^M : Mdl \rightarrow Fr \\ \hline \forall\, M : Mdl_0 \bullet \odot^M = \odot \circ mUFs \end{array}$$

function 1 **leftassoc** $(\_ \rightarrow_M \_)$
relation$(\_ \Rrightarrow^M \_)$

$$\begin{array}{|l} \_ \rightarrow_M \_ : Mdl \times Mdl \rightarrow \mathbb{P}\, GrM \\ \_ \Rrightarrow^M \_ : \mathbb{P}((Mdl \times \mathbb{P}\, GrM) \times Mdl) \\ \hline \forall\, M_s, M_t : Mdl \bullet M_s \rightarrow_M M_t = \{m : GrM \mid \\ \quad \exists\, UF_s, UF_t : Fr_0 \mid UF_s = mUFs\, M_s \wedge UF_t = mUFs\, M_t \bullet m \in UF_s \rightarrow_F UF_t\} \\ \forall\, M_s, M_t : Mdl;\ ms : \mathbb{P}\, GrM \bullet (M_s, ms) \Rrightarrow^M M_t \Leftrightarrow \bigcup_{GM} ms \in M_s \rightarrow_M M_t \end{array}$$

relation$(\_ \sqsupset^M \_)$

$$\begin{array}{|l} \_ \sqsupset^M \_ : (Mdl \times \mathbb{P}\, GrM) \leftrightarrow Mdl \\ \hline \forall\, M_c, M_a : Mdl_0;\ ms : \mathbb{P}\, GrM \bullet (M_c, ms) \sqsupset^M M_a \\ \quad \Leftrightarrow \exists\, UF_c, UF_a : Fr_0 \mid UF_c = mUFs\, M_c \wedge UF_a = mUFs\, M_a \bullet (UF_c, \bigcup_{GM} ms) \sqsupset^F UF_a \end{array}$$

relation$(\_ \Supset^M \_)$

$$\begin{array}{|l}
_ \Supset^M _ : GrwT \leftrightarrow Mdl \\
\hline
\forall\, GwT : GrwT;\ M : Mdl \bullet GwT \Supset^M M \Leftrightarrow GwT \Supset^F mUFs\, M
\end{array}$$