Z Specification of Fragmenta

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Contents

1 Generics

section *Fragmenta_Generics* **parents** *standard_toolkit*

$\mathrm{acyclic}[X] == \{r : X \leftrightarrow X \mid r^{+} \cap \mathrm{id}\, X = \varnothing\}$

$connected[X] == \{r : X \leftrightarrow X \mid \forall x : \mathrm{dom}\, r;\ y : \mathrm{ran}\, r \bullet x \mapsto y \in r^{+}\}$

$\mathrm{tree}[X] == \{r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \land r \in X \pfun X\}$

$forest[X] == \{r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \land (\forall s : X \leftrightarrow X \mid s \subseteq r \land s \in connected \bullet s \in tree)\}$

$\mathrm{injrel}[X, Y] == \{r : X \leftrightarrow Y \mid r^{\sim} \in Y \pfun X\}$

$\mathrm{antireflexive}[X] == \{r : X \leftrightarrow X \mid r \cap id(\mathrm{dom}\ r) = \varnothing\}$

$[X, Y, Z]$

$\mathrm{flip} : (X \to Y \pfun Z) \to (Y \to X \pfun Z)$

$\forall f : X \to Y \pfun Z \bullet \mathrm{flip}\, f = (\lambda\, y : Y \bullet \lambda\, x : X \bullet f\ x\ y)$

$[X, Y, Z, W]$

$\mathrm{apply} : (X \to Z) \to (Y \to W) \to (X \times Y) \to (Z \times W)$

$\forall f : X \to Z;\ g : Y \to W;\ x : X;\ y : Y \bullet \mathrm{apply}\, f\ g\ (x, y) = (f\ x, g\ y)$

$[X, Y]$

$\mathrm{map} : (X \to Y) \to \mathbb{P}\, X \to \mathbb{P}\, Y$
$\mathrm{mapS} : (X \to Y) \to \mathrm{seq}\, X \to \mathrm{seq}\ Y$

$\forall f : X \to Y \bullet \mathrm{map}\, f \{\} = \{\}$

$\forall f : X \to Y;\ x : X;\ xs : \mathbb{P}\, X \bullet \mathrm{map}\, f (\{x\} \cup xs) = \{f\ x\} \cup (\mathrm{map}\, f\ xs)$

$\forall f : X \to Y \bullet \mathrm{mapS}\, f \langle\rangle = \langle\rangle$

$\forall f : X \to Y;\ x : X;\ xs : \mathrm{seq}\, X \bullet \mathrm{mapS}\, f (\langle x \rangle \frown xs) = \langle f\ x \rangle \frown (\mathrm{mapS}\, f\ xs)$

function 10 **leftassoc** $(\_ \boxdot \_)$

$[X]$

$\_ \boxdot \_ : ((X \to X) \times \mathbb{P}\, X) \to (X \pfun X)$

$\forall f : X \to X;\ s : \mathbb{P}\, X \bullet f \boxdot s = (\mathrm{id}\ s) \oplus f$

function$(\_^{\oplus})$

$$
\begin{array}{l}
[X] \\
\hline
\_^{\oplus} : (X \leftrightarrow X) \rightarrow (X \leftrightarrow X) \\
\hline
\forall r : X \leftrightarrow X \bullet r^{\oplus} = \mathbf{if}\ r \oplus r \mathbin{;} r = r\ \mathbf{then}\ r\ \mathbf{else}\ (r \oplus r \mathbin{;} r)^{\oplus}
\end{array}
$$

$$\mathrm{opt}[X] == \{s : \mathbb{P}\, X \mid \# s \leq 1\}$$

$$
\begin{array}{l}
[X] \\
\hline
\mathrm{the} : \mathrm{opt}[X] \rightarrow\!\!\!\!\!\!+\ X \\
\hline
\forall x : X \bullet \mathrm{the}\,\{x\} = x
\end{array}
$$

$$
\begin{array}{l}
[X, Y] \\
\hline
\mathit{flatten} : (X \rightarrow\!\!\!\!\!\!+\ \mathbb{P}\, Y) \rightarrow (X \leftrightarrow Y) \\
\hline
\forall f : X \rightarrow\!\!\!\!\!\!+\ \mathbb{P}\, Y \bullet \mathit{flatten}\, f = \{x : \mathrm{dom}\, f;\ y : Y \mid y \in f\, x\}
\end{array}
$$

2 Graphs

section *Fragmenta_Graphs* **parents** *standard_toolkit*, *Fragmenta_Generics*

$[V, E]$

$$Gr == \{vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s, t : E \rightarrow\!\!\!\!\!\!+\ V \mid s \in es \rightarrow vs \land t \in es \rightarrow vs\}$$

$$
\begin{array}{l}
Ns : Gr \rightarrow \mathbb{P}\, V \\
Es : Gr \rightarrow \mathbb{P}\, E \\
src, tgt : Gr \rightarrow E \rightarrow\!\!\!\!\!\!+\ V \\
\hline
\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!\!+\ V;\ t : E \rightarrow\!\!\!\!\!\!+\ V \bullet Ns(vs, es, s, t) = vs \\
\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!\!+\ V;\ t : E \rightarrow\!\!\!\!\!\!+\ V \bullet Es(vs, es, s, t) = es \\
\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!\!+\ V;\ t : E \rightarrow\!\!\!\!\!\!+\ V \bullet src(vs, es, s, t) = s \\
\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!\!+\ V;\ t : E \rightarrow\!\!\!\!\!\!+\ V \bullet tgt(vs, es, s, t) = t
\end{array}
$$

$$\emptyset_G : Gr$$
$$\emptyset_G = (\emptyset, \emptyset, \emptyset, \emptyset)$$

$$EsId : Gr \rightarrow \mathbb{P}\, E$$
$$\forall\, G : Gr \bullet EsId\ G = \{ e : Es\ G \mid src\ G\ e = tgt\ G\ e \}$$

relation(adjacent _)

$$\mathrm{adjacent}\_ : \mathbb{P}(Gr \times V \times V)$$
$$\forall\, G : Gr;\ v_1, v_2 : V \bullet \mathrm{adjacent}(G, v_1, v_2) \Leftrightarrow \exists\, e : Es\ G \bullet src\ G\ e = v_1 \land tgt\ G\ e = v_2$$

function 10 **leftassoc** $(\_ \circ\!\!\rightarrow\!\!\circ \_)$

$$\_ \circ\!\!\rightarrow\!\!\circ \_ : Gr \times \mathbb{P}\, V \rightarrow \mathbb{P}\, E$$
$$\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet G \circ\!\!\rightarrow\!\!\circ\ vs = (src\ G)^{\sim} (\!| vs |\!) \cup (tgt\ G)^{\sim} (\!| vs |\!)$$

function 10 **leftassoc** $(\_ \bullet\!\!\leftrightarrow\!\!\bullet \_)$

$$\_ \bullet\!\!\leftrightarrow\!\!\bullet \_ : Gr \times \mathbb{P}\, V \rightarrow \mathbb{P}\, E$$
$$\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet G \bullet\!\!\leftrightarrow\!\!\bullet\ vs = (src\ G)^{\sim} (\!| vs |\!) \cap (tgt\ G)^{\sim} (\!| vs |\!)$$

$$rNs : Gr \rightarrow \mathbb{P}\, E \rightarrow \mathbb{P}\, V$$
$$\forall\, G : Gr;\ es : \mathbb{P}\, E \bullet rNs\ G\ es = \mathrm{ran}(es \lhd src\ G) \cup \mathrm{ran}(es \lhd tgt\ G)$$

function 10 **leftassoc** $(\_ \bowtie_{Es} \_)$

$$\_ \bowtie_{Es} \_ : Gr \times \mathbb{P}\, E \rightarrow Gr$$
$$\forall\, G : Gr;\ es : \mathbb{P}\, E \bullet G \bowtie_{Es} es = (rNs\ G\ es, Es\ G \cap es, es \lhd src\ G, es \lhd tgt\ G)$$

function 10 **leftassoc** $(\_ \bowtie_{Ns} \_)$

$$\_ \bowtie_{Ns} \_ : Gr \times \mathbb{P}\ V \rightarrow Gr$$

$$\begin{array}{l} \forall\, G : Gr;\ vs : \mathbb{P}\ V \bullet \\ \quad G \bowtie_{Ns} vs = (Ns\ G \cap vs, G \bullet\!\!\leftrightarrow\!\!\bullet\ vs, (G \bullet\!\!\leftrightarrow\!\!\bullet\ vs) \lhd src\ G, (G \bullet\!\!\leftrightarrow\!\!\bullet\ vs) \lhd tgt\ G) \end{array}$$

function 10 **leftassoc** $(\_ \ominus_{Ns} \_)$

$$\_ \ominus_{Ns} \_ : Gr \times \mathbb{P}\ V \rightarrow Gr$$

$$\begin{array}{l} \forall\, G : Gr;\ vs : \mathbb{P}\ V \bullet \\ \quad G \ominus_{Ns} vs = (Ns\ G \setminus vs, Es\ G \setminus (G \circ\!\!\rightarrow\!\!\circ\ vs), (G \circ\!\!\rightarrow\!\!\circ\ vs) \mathbin{\lhd\mkern-14mu-} src\ G, (G \circ\!\!\rightarrow\!\!\circ\ vs) \mathbin{\lhd\mkern-14mu-} tgt\ G) \end{array}$$

$$\mathrm{successors} : V \times Gr \rightarrow \mathbb{P}\ V$$

$$\forall\, v : V;\ G : Gr \bullet \mathrm{successors}(v, G) = \{v_1 : Ns\ G \mid \mathrm{adjacent}(G, v, v_1)\}$$

function $(\_^{\rightleftarrows})$

$$\_^{\rightleftarrows} : Gr \rightarrow Gr$$

$$\forall\, G : Gr \bullet G^{\rightleftarrows} = (Ns\ G, Es\ G, tgt\ G, src\ G)$$

function $(\_^{\leftrightarrow})$

$$\_^{\leftrightarrow} : Gr \rightarrow V \leftrightarrow V$$

$$\forall\, G : Gr \bullet G^{\leftrightarrow} = \{v_1, v_2 : Ns\ G \mid \mathrm{adjacent}(G, v_1, v_2)\}$$

relation $(\otimes\ \_)$

$$\otimes\ \_ : \mathbb{P}\ Gr$$

$$\forall\, G : Gr \bullet \otimes\ G \Leftrightarrow G^{\leftrightarrow} \in \mathrm{acyclic}$$

relation$(\boxminus_{Es}\ \_\,)$
relation$(\boxminus\ \_\,)$

$$\begin{array}{|l}
\boxminus_{Es}\ _, \boxminus\ _ : \mathbb{P}(Gr \times Gr) \\
\hline
\forall\, G_1, G_2 : Gr \bullet \boxminus_{Es}(G_1, G_2) \Leftrightarrow Es\ G_1 \cap Es\ G_2 = \varnothing \\
\forall\, G_1, G_2 : Gr \bullet \boxminus(G_1, G_2) \Leftrightarrow Ns\ G_1 \cap Ns\ G_2 = \varnothing \wedge \boxminus_{Es}(G_1, G_2)
\end{array}$$

relation$(\boxplus\ \_\,)$

$$\begin{array}{|l|}
\hline
[I] \\
\boxplus\ _ : \mathbb{P}(I \nrightarrow Gr) \\
\hline
\forall\, Gs : I \nrightarrow Gr \bullet \boxplus\ Gs \Leftrightarrow \forall\, i, j : \mathrm{dom}\ Gs \mid i \neq j \bullet \boxminus(Gs\ i, Gs\ j) \\
\hline
\end{array}$$

function 10 **leftassoc** $(\_ \cup_G \_)$

$$\begin{array}{|l}
_ \cup_G _ : Gr \times Gr \rightarrow Gr \\
\hline
\forall\, G_1, G_2 : Gr \bullet G_1 \cup_G G_2 = (Ns\ G_1 \cup Ns\ G_2, Es\ G_1 \cup Es\ G_2, src\ G_1 \cup src\ G_2, tgt\ G_1 \cup tgt\ G_2)
\end{array}$$

function 10 **leftassoc** $(\_ \odot \_)$

$$\begin{array}{|l}
_ \odot _ : Gr \times (V \leftrightarrow V) \nrightarrow Gr \\
\hline
\forall\, G : Gr;\ s : V \leftrightarrow V \mid s \in Ns\ G \nrightarrow Ns\ G \wedge s \in \mathrm{antireflexive} \bullet \\
\quad G \odot s = (Ns\ G \setminus \mathrm{dom}\, s, Es\ G, (s \boxdot Ns\ G) \circ (src\ G), (s \boxdot Ns\ G) \circ (tgt\ G))
\end{array}$$

$$GrM == (V \nrightarrow V) \times (E \nrightarrow E)$$

$$\begin{array}{|l}
fV : GrM \rightarrow V \nrightarrow V \\
fE : GrM \rightarrow E \nrightarrow E \\
\hline
\forall\, fv : V \nrightarrow V;\ fe : E \nrightarrow E \bullet fV(fv, fe) = fv \\
\forall\, fv : V \nrightarrow V;\ fe : E \nrightarrow E \bullet fE(fv, fe) = fe
\end{array}$$

$$\begin{array}{|l}
\mathrm{gid} : Gr \to GrM \\
\hline
\forall G : Gr \bullet \mathrm{gid}\ G = (id\ (Ns\ G), id\ (Es\ G))
\end{array}$$

$$\begin{array}{|l}
\varnothing_{GM} : GrM \\
\hline
\varnothing_{GM} = (\{\}, \{\})
\end{array}$$

$$\begin{array}{|l}
\mathrm{domg}, \mathrm{codg} : GrM \to Gr \\
\hline
\forall m : GrM;\ G : Gr \bullet \mathrm{domg}\ m = G \Leftrightarrow \mathrm{dom}(fV\ m) = Ns\ G \land \mathrm{dom}(fE\ m) = Es\ G \\
\forall m : GrM;\ G : Gr \bullet \mathrm{codg}\ m = G \Leftrightarrow \mathrm{ran}(fV\ m) \subseteq Ns\ G \land \mathrm{ran}(fE\ m) \subseteq Es\ G
\end{array}$$

function 10 **leftassoc** $(\_ \cup_{GM} \_)$

$$\begin{array}{|l}
_ \cup_{GM} _ : GrM \times GrM \nrightarrow GrM \\
\bigcup_{GM} : \mathbb{P}\ GrM \to GrM \\
\hline
\forall f, g : GrM \bullet f \cup_{GM} g = (fV\ f \cup fV\ g, fE\ f \cup fE\ g) \\
\bigcup_{GM} \varnothing = \varnothing_{GM} \\
\forall f : GrM;\ fs : \mathbb{P}\ GrM \bullet \bigcup_{GM} (\{f\} \cup fs) = f \cup_{GM} (\bigcup_{GM} fs)
\end{array}$$

function 10 **leftassoc** $(\_ \to_G \_)$

$$\begin{array}{|l}
_ \to_G _ : Gr \times Gr \to \mathbb{P}\ GrM \\
\hline
\forall G_1, G_2 : Gr \bullet G_1 \to_G G_2 = \{fv : Ns\ G_1 \to Ns\ G_2;\ fe : Es\ G_1 \to Es\ G_2\ | \\
\quad src\ G_2 \circ fe = fv \circ src\ G_1 \land tgt\ G_2 \circ fe = fv \circ tgt\ G_1\}
\end{array}$$

function 10 **leftassoc** $(\_ \circ_G \_)$

$$\begin{array}{|l}
_ \circ_G _ : GrM \times GrM \to GrM \\
\hline
\forall g, f : GrM \bullet g \circ_G f = (fV\ g \circ fV\ f, fE\ g \circ fE\ f)
\end{array}$$

3 Graphs with typing

section *Fragmenta_GrswT* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$$GrwT == \{G : Gr;\ t : GrM \mid \operatorname{domg} t = G\}$$

$$\begin{array}{|l}
gOf : GrwT \rightarrow Gr \\
ty : GrwT \rightarrow GrM \\
\hline
\forall G : Gr;\ sm : V \leftrightarrow \operatorname{seq} V;\ t : GrM \bullet gOf(G, t) = G \\
\forall G : Gr;\ sm : V \leftrightarrow \operatorname{seq} V;\ t : GrM \bullet ty(G, t) = t
\end{array}$$

$$\begin{array}{|l}
\varnothing_{GwT} : GrwT \\
\hline
\varnothing_{GwT} = (\varnothing_G, \varnothing_{GM})
\end{array}$$

function 10 **leftassoc** $(\_ \cup_{GwT} \_)$

$$\begin{array}{|l}
_ \cup_{GwT} _ : GrwT \times GrwT \rightarrow GrwT \\
\hline
\forall G_1, G_2 : GrwT \bullet G_1 \cup_{GwT} G_2 = ((gOf\ G_1) \cup_G (gOf\ G_2), (ty\ G_1) \cup_{GM} (ty\ G_2))
\end{array}$$

4 SG Element Types

section *Fragmenta_SGElemTys* **parents** *standard_toolkit*, *Fragmenta_Generics*

$$SGNT ::= nnrml \mid nabst \mid nprxy \mid nenum \mid nval \mid nvirt \mid nopt$$

$$SGED ::= dbi \mid duni$$

$$SGET ::= einh \mid ecomp\langle\langle SGED\rangle\rangle \mid erel\langle\langle SGED\rangle\rangle \mid ewander \mid eder$$

relation $(\_ \prec_{NT} \_)$

$$
\begin{array}{|l}
\_ \prec_{NT} \_ : SGNT \leftrightarrow SGNT \\
\hline
\forall nt_1, nt_2 : SGNT \bullet nt_1 \prec_{NT} nt_2 \Leftrightarrow ((nt_2 = nenum \land nt_1 = nval) \\
\quad \lor (nt_1 = nvirt \land nt_2 = nvirt) \lor (nt_1 = nabst \land nt_2 \in \{nabst, nvirt, nprxy\})) \\
\quad \land nt_1 \notin \{nprxy, nenum, nopt\} \land nt_2 \notin \{nopt\}
\end{array}
$$

relation$(\_ \leq_{rNT} \_)$

$$
\begin{array}{|l}
\_ \leq_{rNT} \_ : SGNT \leftrightarrow SGNT \\
\hline
\forall nt_1, nt_2 : SGNT \bullet nt_1 \leq_{rNT} nt_2 \Leftrightarrow nt_1 = nt_2 \lor nt_1 = nprxy \\
\quad \lor nt_2 = nabst \land nt_1 \in \{nnrml, nvirt\} \lor nt_2 \in \{nnrml, nopt\}
\end{array}
$$

relation$(\_ =_{ET} \_)$

$$
\begin{array}{|l}
\_ =_{ET} \_ : SGET \leftrightarrow SGET \\
\hline
\forall et_1, et_2 : SGET \bullet et_1 =_{ET} et_2 \Leftrightarrow et_1 = et_2 \\
\quad \lor (\forall d_1, d_2 : SGED \bullet et_1 = erel\ d_1 \land et_2 = erel\ d_2 \lor et_1 = ecomp\ d_1 \land et_2 = ecomp\ d_2)
\end{array}
$$

relation$(\_ \leq_{ET} \_)$

$$
\begin{array}{|l}
\_ \leq_{ET} \_ : SGET \leftrightarrow SGET \\
\hline
\forall et_1, et_2 : SGET \bullet et_1 \leq_{ET} et_2 \Leftrightarrow einh \notin \{et_1, et_2\} \\
\quad \land (et_1 =_{ET} et_2 \lor et_2 = ewander \\
\quad \lor et_1 = eder \land et_2 \in \mathrm{dom}(erel^{\sim}) \cup \mathrm{dom}(ecomp^{\sim}))
\end{array}
$$

5 Multiplicities

section *Fragmenta_Mult* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_SGElemTys*

$MultVal ::= \mathbf{v}\langle\langle \mathbb{N} \rangle\rangle \mid *$

$MultC ::= mr\langle\langle \mathbb{N} \times MultVal \rangle\rangle \mid ms\langle\langle MultVal \rangle\rangle$

relation$(\_ \leq_{mv} \_)$

$$
\begin{array}{|l}
\_ \leq_{mv} \_ : MultVal \leftrightarrow MultVal \\
\hline
\forall m_1, m_2 : MultVal \bullet m_1 \leq_{mv} m_2 \Leftrightarrow m_2 = * \lor \exists j, k : \mathbb{N} \mid m_1 = \mathbf{v}\ j \land m_2 = \mathbf{v}\ k \bullet j \leq k
\end{array}
$$

$Mult == \{mc : MultC \mid \exists lb : \mathbb{N};\ ub : MultVal \bullet mc = mr(lb, ub) \land \mathbf{v}\ lb \leq_{mv} ub$
$\quad \lor \exists mv : MultVal \bullet mc = ms\ mv\}$

$MultMany == \{ms\ *, mr(0, *)\}$

$MultRange == \{m : MultC \mid \exists k : \mathbb{N} \mid k > 1 \bullet m = ms\ (\mathbf{v}\ k)$
$\quad \lor \exists lb : \mathbb{N};\ umv : MultVal \mid \mathbf{v}\ 2 \leq_{mv} umv \bullet m = mr(lb, umv)\}$

relation$(\_ \between \_)$

$$
\begin{array}{|l}
\_ \between \_ : \mathbb{P}(\mathbb{N} \times (MultVal \times MultVal)) \\
\hline
\forall k : \mathbb{N};\ lb, ub : MultVal \bullet k \between (lb, ub) \Leftrightarrow lb \leq_{mv} \mathbf{v}\ k \land \mathbf{v}\ k \leq_{mv} ub
\end{array}
$$

$$
\begin{array}{|l}
mlb, mub : MultC \nrightarrow MultVal \\
\hline
mlb(ms\ *) = \mathbf{v}\ 0 \\
\forall k : \mathbb{N} \bullet mlb(ms(\mathbf{v}\ k)) = \mathbf{v}\ k \\
\forall k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ k \\
\forall mv : MultVal \bullet mub(ms\ mv) = mv \\
\forall k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ m
\end{array}
$$

relation$(\_ \leq_{\mathcal{M}} \_)$

$$
\begin{array}{|l}
\_ \leq_{\mathcal{M}} \_ : MultC \leftrightarrow MultC \\
\hline
\forall m_1, m_2 : MultC \bullet m_1 \leq_{\mathcal{M}} m_2 \Leftrightarrow mlb\ m_2 \leq_{mv} mlb\ m_1 \land mub\ m_1 \leq_{mv} mub\ m_2
\end{array}
$$

relation$(\_ \propto \_)$

$$
\begin{array}{|l}
\_ \propto \_ : \mathbb{P}(SGET \times (MultC \times MultC)) \\
\hline
\forall et : SGET;\ m_1, m_2 : MultC \bullet et \propto (m_1, m_2) \Leftrightarrow et = erel\ dbi \lor et = eder \\
\quad \lor et = ecomp\ duni \land m_1 = ms(\mathbf{v}\ 1) \lor et = erel\ duni \land m_1 \in MultMany \\
\quad \lor et = ecomp\ dbi \land m_1 \in \{ms(\mathbf{v}\ 1), mr(0, \mathbf{v}\ 1)\} \\
\quad \lor et = ewander \land (m_1, m_2) \in MultMany \times MultMany
\end{array}
$$

relation(rbounded_)

$$
\begin{array}{l}
[X, Y] \\
\hline
\mathrm{rbounded}\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\, X \times \mathit{MultC}) \\
\hline
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ m : \mathit{MultC} \bullet \\
\quad \mathrm{rbounded}(r, s, m) \Leftrightarrow \forall\, x : s \bullet \#(r \mathrel{(\!|} \{x\} \mathrel{|\!)}) \between (\mathit{mlb}\ m, \mathit{mub}\ m)
\end{array}
$$

relation($r\mathcal{M}\mathit{Ok}\,\_$)

$$
\begin{array}{l}
[X, Y] \\
\hline
r\mathcal{M}\mathit{Ok}\,\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\, X \times \mathbb{P}\, Y \times \mathit{MultC} \times \mathit{MultC}) \\
\hline
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{ms}(\mathbf{v}\ 1), \mathit{ms}(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail\!\!\!\!\twoheadrightarrow t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{mr}(0, \mathbf{v}\ 1), \mathit{mr}(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail\!\!\!\!\!\!\mapsto t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{mr}(0, \mathbf{v}\ 1), \mathit{ms}(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{ms}(\mathbf{v}\ 1), \mathit{mr}(0, \mathbf{v}\ 1)) \Leftrightarrow r^{\sim} \in t \rightarrowtail s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{mm}, \mathit{ms}(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrow t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{ms}(\mathbf{v}\ 1), \mathit{mm}) \Leftrightarrow r^{\sim} \in t \rightarrow s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm}_1, \mathit{mm}_2 : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{mm}_1, \mathit{mm}_2) \Leftrightarrow r \in s \leftrightarrow t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany};\ \mathit{mr} : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{mm}, \mathit{mr}) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r, s, \mathit{mr}) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany};\ \mathit{mr} : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{mr}, \mathit{mm}) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r^{\sim}, t, \mathit{mr}) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{mm}, \mathit{mr}(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \nrightarrow t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{mr}(0, \mathbf{v}\ 1), \mathit{mm}) \Leftrightarrow r^{\sim} \in t \nrightarrow s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mr}_1, \mathit{mr}_2 : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{mr}_1, \mathit{mr}_2) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r, s, \mathit{mr}_2) \land \mathrm{rbounded}(r^{\sim}, t, \mathit{mr}_1) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mr} : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{mr}, \mathit{ms}(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrow t \land \mathrm{rbounded}(r^{\sim}, t, \mathit{mr}) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mr} : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{ms}(\mathbf{v}\ 1), \mathit{mr}) \Leftrightarrow r^{\sim} \in t \rightarrow s \land \mathrm{rbounded}(r, s, \mathit{mr}) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ m : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, m, \mathit{mr}(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \nrightarrow t \land \mathrm{rbounded}(r^{\sim}, t, m) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ m : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{mr}(0, \mathbf{v}\ 1), m) \Leftrightarrow r^{\sim} \in t \nrightarrow s \land \mathrm{rbounded}(r, s, m)
\end{array}
$$

6 Structural Graphs

section *Fragmenta_SGs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*, *Fragmenta_SGElemTys*, *Fragmenta_Mult*, *Fragmenta_GrswT*

$$SGr_0 == \{G : Gr;\ nt : V \leftrightarrow SGNT;\ et : E \leftrightarrow SGET;\ sm, tm : E \leftrightarrow Mult;\ db : E \leftrightarrow E \mid nt \in Ns\ G \rightarrow SGNT \wedge et \in Es\ G \rightarrow SGET\}$$

$$
\begin{array}{|l}
gr : SGr_0 \rightarrow Gr \\
sg\_Ns : SGr_0 \rightarrow \mathbb{P}\ V \\
sg\_Es : SGr_0 \rightarrow \mathbb{P}\ E \\
sg\_src, sg\_tgt : SGr_0 \rightarrow E \leftrightarrow V \\
nty : SGr_0 \rightarrow V \leftrightarrow SGNT \\
ety : SGr_0 \rightarrow E \leftrightarrow SGET \\
srcm, tgtm : SGr_0 \rightarrow E \leftrightarrow Mult \\
derb : SGr_0 \rightarrow E \leftrightarrow E \\
\hline
\forall G : Gr;\ nt : V \leftrightarrow SGNT;\ et : E \leftrightarrow SGET;\ sm, tm : E \leftrightarrow Mult;\ db : E \leftrightarrow E \bullet \\
\quad gr\ (G, nt, et, sm, tm, db) = G \\
sg\_Ns = Ns \circ gr \\
sg\_Es = Es \circ gr \\
sg\_src = src \circ gr \\
sg\_tgt = tgt \circ gr \\
\forall G : Gr;\ nt : V \leftrightarrow SGNT;\ et : E \leftrightarrow SGET;\ sm, tm : E \leftrightarrow Mult;\ db : E \leftrightarrow E \bullet \\
\quad nty(G, nt, et, sm, tm, db) = nt \\
\forall G : Gr;\ nt : V \leftrightarrow SGNT;\ et : E \leftrightarrow SGET;\ sm, tm : E \leftrightarrow Mult;\ db : E \leftrightarrow E \bullet \\
\quad ety(G, nt, et, sm, tm, db) = et \\
\forall G : Gr;\ nt : V \leftrightarrow SGNT;\ et : E \leftrightarrow SGET;\ sm, tm : E \leftrightarrow Mult;\ db : E \leftrightarrow E \bullet \\
\quad srcm(G, nt, et, sm, tm, db) = sm \\
\forall G : Gr;\ nt : V \leftrightarrow SGNT;\ et : E \leftrightarrow SGET;\ sm, tm : E \leftrightarrow Mult;\ db : E \leftrightarrow E \bullet \\
\quad tgtm(G, nt, et, sm, tm, db) = tm \\
\forall G : Gr;\ nt : V \leftrightarrow SGNT;\ et : E \leftrightarrow SGET;\ sm, tm : E \leftrightarrow Mult;\ db : E \leftrightarrow E \bullet \\
\quad derb(G, nt, et, sm, tm, db) = db
\end{array}
$$

$$
\begin{array}{|l}
\emptyset_{SG} : SGr_0 \\
\hline
\emptyset_{SG} = (\emptyset_G, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset)
\end{array}
$$

$$
\begin{array}{|l}
NsTy : SGr_0 \rightarrow \mathbb{P}\ SGNT \rightarrow \mathbb{P}\ V \\
EsTy : SGr_0 \rightarrow \mathbb{P}\ SGET \rightarrow \mathbb{P}\ E \\
\hline
\forall SG : SGr_0;\ nts : \mathbb{P}\ SGNT \bullet NsTy\ SG\ nts = (nty\ SG)^{\sim} (\!| nts |\!) \\
\forall SG : SGr_0;\ ets : \mathbb{P}\ SGET \bullet EsTy\ SG\ ets = (ety\ SG)^{\sim} (\!| ets |\!)
\end{array}
$$

$$
\begin{array}{l}
EsA, EsW, EsI, EsC, EsD : SGr_0 \to \mathbb{P}\, E \\
\hline
\forall\, SG : SGr_0 \bullet EsA\; SG = EsTy\; SG\; (erel \llparenthesis SGED \rrparenthesis \cup ecomp \llparenthesis SGED \rrparenthesis) \\
EsW = (\mathrm{flip}\; EsTy)\; \{ewander\} \\
EsI = (\mathrm{flip}\; EsTy)\; \{einh\} \\
EsD = (\mathrm{flip}\; EsTy)\; \{eder\} \\
\forall\, SG : SGr_0 \bullet EsC\; SG = EsA\; SG \cup EsW\; SG \cup EsD\; SG
\end{array}
$$

$$
\begin{array}{l}
NsP, NsEther, NsO, NsV : SGr_0 \to \mathbb{P}\, V \\
\hline
NsP = (\mathrm{flip}\; NsTy)\; \{nprxy\} \\
NsEther = (\mathrm{flip}\; NsTy)\; \{nabst, nvirt, nenum\} \\
NsO = (\mathrm{flip}\; NsTy)\; \{nopt\} \\
NsV = (\mathrm{flip}\; NsTy)\; \{nvirt\}
\end{array}
$$

$$
\begin{array}{l}
\pitchfork : SGr_0 \to Gr \\
\prec : SGr_0 \to V \leftrightarrow V \\
\hline
\forall\, SG : SGr_0 \bullet \pitchfork\, SG = gr\; SG \rhd\!\!\lhd_{Es}\; EsI\; SG \\
\prec\; = (\_^{\leftrightarrow}) \circ \pitchfork
\end{array}
$$

$$
\begin{array}{l}
srcma : SGr_0 \to (E \pfun Mult) \\
\hline
\forall\, SG : SGr_0 \bullet srcma\; SG = \\
\quad (srcm\; SG) \oplus (EsTy\; SG\; \{ecomp\; duni\} \times \{ms(\mathbf{v}\; 1)\}) \oplus (EsTy\; SG\; \{erel\; duni\} \times \{ms\; *\})
\end{array}
$$

relation($\mathcal{M}etysOk\, \_$)

$$
\begin{array}{l}
\mathcal{M}etysOk\, \_ : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \mathcal{M}etysOk\;\; SG \Leftrightarrow \forall\, e : EsC\; SG \bullet (ety\; SG\; e) \propto (srcma\; SG\; e, tgtm\; SG\; e)
\end{array}
$$

$$
\begin{array}{l}
\preceq : SGr_0 \to V \leftrightarrow V \\
\hline
\forall\, SG : SGr_0 \bullet \preceq\; SG = (\prec\; SG)^*
\end{array}
$$

$$
\begin{array}{|l}
srcr, tgtr : SGr_0 \to E \leftrightarrow V \\
src_0^*, src^*, tgt_0^*, tgt^* : SGr_0 \to E \leftrightarrow V \\
\hline
\forall SG : SGr_0 \bullet srcr\, SG = sg\_src\, SG \cup (EsW\ SG \lhd sg\_tgt\, SG) \\
\forall SG : SGr_0 \bullet tgtr\, SG = sg\_tgt\, SG \cup (EsW\ SG \lhd sg\_src\, SG) \\
\forall SG : SGr_0 \bullet src_0^*\ SG = EsC\, SG \lhd (srcr\, SG) \\
\forall SG : SGr_0 \bullet src^*\ SG = (src_0^*\ SG) \fatsemi (\preceq SG)^\sim \\
\forall SG : SGr_0 \bullet tgt_0^*\ SG = EsC\, SG \lhd (tgtr\, SG) \\
\forall SG : SGr_0 \bullet tgt^*\ SG = (tgt_0^*\ SG) \fatsemi (\preceq SG)^\sim
\end{array}
$$

function 10 **leftassoc** $(\_ \circ\!\!\rightarrow\!\!\circ^* \_)$

$$
\begin{array}{|l}
\_ \circ\!\!\rightarrow\!\!\circ^* \_ : SGr_0 \times \mathbb{P}\, V \to \mathbb{P}\, E \\
\hline
\forall SG : SGr_0;\ vs : \mathbb{P}\, V \bullet SG \circ\!\!\rightarrow\!\!\circ^*\ vs = (src^*\ SG)^\sim (\!|\, vs \,|\!) \cup (tgt^*\ SG)^\sim (\!|\, vs \,|\!)
\end{array}
$$

relation(optsVoluntary_)

$$
\begin{array}{|l}
\mathrm{optsVoluntary}\_ : \mathbb{P}\, SGr_0 \\
\hline
\forall SG : SGr_0 \bullet \\
\quad \mathrm{optsVoluntary}\, SG \Leftrightarrow (ety\, SG)\, (\!|\, (SG \circ\!\!\rightarrow\!\!\circ^* (NsO\, SG)) \setminus (EsI\ SG) \,|\!) \subseteq \{ewander\}
\end{array}
$$

relation(inhOk_)

$$
\begin{array}{|l}
\mathrm{inhOk}\_ : \mathbb{P}\, SGr_0 \\
\hline
\forall SG : SGr_0 \bullet \mathrm{inhOk}\, SG \\
\quad \Leftrightarrow (\forall v, v' : sg\_Ns\, SG \mid (v, v') \in (\prec SG) \bullet nty\, SG\, v \prec_{NT} nty\, SG\, v') \\
\quad \wedge \oslash(\pitchfork SG) \wedge ((\pitchfork SG) \ominus_{Ns} (NsV\ SG))^{\leftrightarrow} \in sg\_Ns\, SG \nrightarrow sg\_Ns\, SG
\end{array}
$$

$$
\begin{array}{l}
SGr == \{SG : SGr_0 \mid \{srcma\, SG, tgtm\, SG\} \subseteq EsC\, SG \to Mult \wedge \mathrm{dom}(derb\, SG) = EsD\, SG \\
\quad \wedge \mathcal{M}etysOk\ SG \wedge \mathrm{optsVoluntary}\, SG \wedge \mathrm{inhOk}\, SG\}
\end{array}
$$

relation(etherealAreInherited_)

$$\begin{array}{|l}
\mathrm{etherealAreInherited}_ : \mathbb{P}\, SGr_0 \\
\hline
\forall SG : SGr_0 \bullet \mathrm{etherealAreInherited}\, SG \Leftrightarrow NsEther\, SG \subseteq \mathrm{ran}(\prec SG)
\end{array}$$

relation(derivedOk _)

$$\begin{array}{|l}
\mathrm{derivedOk}_ : \mathbb{P}\, SGr_0 \\
\hline
\forall SG : SGr_0 \bullet \mathrm{derivedOk}\, SG \Leftrightarrow \mathrm{ran}(derb\, SG) \subseteq EsA\, SG \\
\quad \wedge (\forall e : EsD\, SG \bullet (sg_src\, SG\, e, ((sg_src\, SG) \circ (derb\, SG))\, e) \in (\preceq SG) \\
\qquad \wedge (sg_tgt\, SG\, e, ((sg_tgt\, SG) \circ (derb\, SG))\, e) \in (\preceq SG))
\end{array}$$

$$TSGr == \{ SG : SGr \mid \mathrm{etherealAreInherited}\, SG \wedge \mathrm{derivedOk}\, SG \}$$

relation($\boxminus_{SGs}$ _)

$$\begin{array}{|l}
\boxminus_{SGs_} : \mathbb{P}(SGr \times SGr) \\
\hline
\forall SG_1, SG_2 : SGr \bullet \boxminus_{SGs}(SG_1, SG_2) \Leftrightarrow \boxminus(gr\, SG_1, gr\, SG_2)
\end{array}$$

function 10 **leftassoc** ($\_ \cup_{SG} \_$)

$$\begin{array}{|l}
_ \cup_{SG} _ : SGr \times SGr \nrightarrow SGr \\
\hline
\forall SG_1, SG_2 : SGr \bullet SG_1 \cup_{SG} SG_2 = (gr\, SG_1 \cup_G gr\, SG_2, nty\, SG_1 \cup nty\, SG_2, \\
\quad ety\, SG_1 \cup ety\, SG_2, srcm\, SG_1 \cup srcm\, SG_2, tgtm\, SG_1 \cup tgtm\, SG_2, derb\, SG_1 \cup derb\, SG_2)
\end{array}$$

function 10 **leftassoc** ($\_ \odot^{SG} \_$)

$$\begin{array}{|l}
_ \odot^{SG} _ : SGr \times (V \nrightarrow V) \nrightarrow SGr \\
\hline
\forall SG : SGr;\ s : V \nrightarrow V \mid s \in NsP\, SG \nrightarrow sg_Ns\, SG \bullet \\
\quad SG \odot^{SG} s = (gr\, SG \odot s, (\mathrm{dom}\ s \setminus \mathrm{ran}\ s) \lhd nty\, SG, ety\, SG, srcm\, SG, tgtm\, SG, derb\, SG)
\end{array}$$

function 10 **leftassoc** ($\_ \rightarrow_{SG} \_$)

$$\_ \rightarrow_{SG} \_ : SGr \times SGr \rightarrow \mathbb{P}\, GrM$$

$$\forall\, SG_s, SG_t : SGr \bullet$$
$$SG_s \rightarrow_{SG} SG_t = \{fv : sg\_Ns\, SG_s \rightarrow sg\_Ns\, SG_t;\ fe : EsC\, SG_s \rightarrow EsC\, SG_t \mid$$
$$fv \circ src^*\ SG_s \subseteq src^*\ SG_t \circ fe \land fv \circ tgt^*\ SG_s \subseteq tgt^*\ SG_t \circ fe$$
$$\land\, fv \circ \preceq SG_s \subseteq \preceq SG_t \circ fv\}$$

relation$(\_ \Rrightarrow^{SG} \_)$

$$\_ \Rrightarrow^{SG} \_ : \mathbb{P}((SGr \times GrM) \times SGr)$$

$$\forall\, SG_s, SG_t : SGr;\ m : GrM \bullet (SG_s, m) \Rrightarrow^{SG} SG_t \Leftrightarrow m \in SG_s \rightarrow_{SG} SG_t$$

function 10 **leftassoc** $(\_ \rightarrow_{G2SG} \_)$

$$\_ \rightarrow_{G2SG} \_ : Gr \times SGr \rightarrow \mathbb{P}\, GrM$$

$$\forall\, G : Gr;\ SG : SGr \bullet G \rightarrow_{G2SG} SG = \{fv : Ns\, G \rightarrow sg\_Ns\, SG;\ fe : Es\, G \rightarrow EsC\, SG \mid$$
$$fv \circ src\, G \subseteq src^*\ SG \circ fe \land fv \circ tgt\, G \subseteq tgt^*\ SG \circ fe\}$$

relation$(\_ \Rrightarrow^{GwT} \_)$

$$\_ \Rrightarrow^{GwT} \_ : (GrwT \leftrightarrow SGr)$$

$$\forall\, GwT : GrwT;\ SG : SGr \bullet GwT \Rrightarrow^{GwT} SG \Leftrightarrow (ty\ GwT) \in (gOf\ GwT) \rightarrow_{G2SG} SG$$

$$totaliseForDer : GrM \times SGr \rightarrow GrM$$

$$\forall\, m : GrM;\ SG : SGr \bullet totaliseForDer(m, SG) = (fV\ m, ((derb\, SG) \boxdot (EsC\, SG)) \mathbin{⨾} fE\ m)$$

$$insOf : GrM \times SGr \times \mathbb{P}\, V \rightarrow \mathbb{P}\, V$$
$$iesOf : GrM \times \mathbb{P}\, E \rightarrow \mathbb{P}\, E$$
$$igRMEs : GrwT \times \mathbb{P}\, E \rightarrow Gr$$
$$igRMNsEs : GrwT \times SGr \times \mathbb{P}\, V \times \mathbb{P}\, E \rightarrow Gr$$

$$\forall\, m : GrM;\ SG : SGr;\ mns : \mathbb{P}\, V \bullet insOf(m, SG, mns) = (fV\ m)^{\sim} (\!|(\prec SG)^{\sim} (\!| mns |\!) |\!)$$

$$\forall\, m : GrM;\ mes : \mathbb{P}\, E \bullet iesOf(m, mes) = (fE\ m)^{\sim} (\!| mes |\!)$$

$$\forall\, GwT : GrwT;\ mes : \mathbb{P}\, E \bullet igRMEs(GwT, mes) = (gOf\ GwT) \Join_{Es} iesOf((ty\ GwT), mes)$$

$$\forall\, GwT : GrwT;\ SG : SGr;\ mns : \mathbb{P}\, V;\ mes : \mathbb{P}\, E \bullet$$
$$igRMNsEs(GwT, SG, mns, mes) = igRMEs(GwT, mes) \Join_{Ns} insOf(ty\ GwT, SG, mns)$$

relation$(\mathrm{inverted}_{\mathrm{E}} \_)$

$$
\begin{array}{|l}
\mathrm{inverted}_{\mathrm{E}} \_ : \mathbb{P}(\mathit{GrwT} \times \mathit{SGr} \times E) \\
\mathit{gOfwei}, \mathit{igRMEsW} : \mathit{GrwT} \times \mathit{SGr} \times E \rightarrow \mathit{Gr} \\
\mathit{gOfweis} : \mathit{GrwT} \times \mathit{SGr} \times \mathbb{P}\, E \rightarrow \mathit{Gr} \\
\hline
\forall\, G : \mathit{Gr};\ m : \mathit{GrM};\ \mathit{SG} : \mathit{SGr};\ e : E \bullet \\
\quad \mathrm{inverted}_{\mathrm{E}}((G, m), \mathit{SG}, e) \Leftrightarrow ((\mathit{sg\_tgt}\ \mathit{SG}) \circ (\mathit{fE}\ m))e = ((\mathit{fV}\ m) \circ (\mathit{src}\ G))e \\
\forall\, \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr};\ e : E \bullet \\
\quad \mathit{gOfwei}(\mathit{GwT}, \mathit{SG}, e) = \mathbf{if}\ \mathrm{inverted}_{\mathrm{E}}(\mathit{GwT}, \mathit{SG}, e)\ \mathbf{then}\ ((\mathit{gOf}\ \mathit{GwT}) \rhd\!\lhd_{\mathit{Es}} \{e\})^{\rightleftarrows}\ \mathbf{else} (\mathit{gOf}\ \mathit{GwT}) \rhd\!\lhd_{\mathit{Es}} \{e\} \\
\forall\, \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr} \bullet \mathit{gOfweis}(\mathit{GwT}, \mathit{SG}, \{\}) = \varnothing_G \\
\forall\, \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr};\ e : E;\ \mathit{es} : \mathbb{P}\, E \bullet \\
\quad \mathit{gOfweis}(\mathit{GwT}, \mathit{SG}, \{e\} \cup \mathit{es}) = \mathit{gOfwei}(\mathit{GwT}, \mathit{SG}, e) \cup_G \mathit{gOfweis}(\mathit{GwT}, \mathit{SG}, \mathit{es}) \\
\forall\, \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr};\ e : E \mid e \in \mathit{EsD}\ \mathit{SG} \bullet \mathit{igRMEsW}(\mathit{GwT}, \mathit{SG}, e) = \\
\quad \mathit{igRMNsEs}(\mathit{GwT}, \mathit{SG}, \{(\mathit{sg\_src}\ \mathit{SG})(\mathit{derb}\ \mathit{SG}\ e), (\mathit{sg\_tgt}\ \mathit{SG})(\mathit{derb}\ \mathit{SG}\ e)\}, \{\mathit{derb}\ \mathit{SG}\ e\}) \\
\forall\, \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr};\ e : E \mid e \notin \mathit{EsW}\ \mathit{SG} \bullet \mathit{igRMEsW}(\mathit{GwT}, \mathit{SG}, e) = \mathit{igRMEs}(\mathit{GwT}, \{e\}) \\
\forall\, \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr};\ e : E \mid e \in \mathit{EsW}\ \mathit{SG} \bullet \mathit{igRMEsW}(\mathit{GwT}, \mathit{SG}, e) = \\
\quad \mathit{gOfweis}(\mathit{GwT}, \mathit{SG}, ((\mathit{fE} \circ \mathit{ty})\ \mathit{GwT})^{\sim} (\!| \{e\} |\!))
\end{array}
$$

relation$(\_ \sqsupseteq^{SG} \_)$
relation$(\_ \sqsupseteq^{SG_0} \_)$
relation$(\_ \sqsupseteq_{NT} \_)$
relation$(\_ \sqsupseteq_{ET} \_)$
relation$(\_ \sqsupseteq_{\mathcal{M}} \_)$

$$
\begin{array}{|l}
\_ \sqsupseteq_{NT} \_, \_ \sqsupseteq_{ET} \_ : \mathbb{P}((\mathit{SGr} \times \mathit{GrM}) \times \mathit{SGr}) \\
\hline
\forall\, \mathit{SG}_c, \mathit{SG}_a : \mathit{SGr};\ m : \mathit{GrM} \bullet \\
\quad (\mathit{SG}_c, m) \sqsupseteq_{NT}\ \mathit{SG}_a \Leftrightarrow \forall\, n : \mathit{sg\_Ns}\ \mathit{SG}_c \bullet (\mathit{nty}\ \mathit{SG}_c)\ n \leq_{rNT} ((\mathit{nty}\ \mathit{SG}_a) \circ (\mathit{fV}\ m))\ n \\
\forall\, \mathit{SG}_c, \mathit{SG}_a : \mathit{SGr};\ m : \mathit{GrM} \bullet \\
\quad (\mathit{SG}_c, m) \sqsupseteq_{ET}\ \mathit{SG}_a \Leftrightarrow \forall\, e : \mathit{EsC}\ \mathit{SG}_c \bullet (\mathit{ety}\ \mathit{SG}_c)\ e \leq_{ET} ((\mathit{ety}\ \mathit{SG}_a) \circ (\mathit{fE}\ m))\ e
\end{array}
$$

$$
\begin{array}{|l}
\_ \sqsupseteq_{\mathcal{M}} \_ : \mathbb{P}((\mathit{SGr} \times \mathit{GrM}) \times \mathit{SGr}) \\
\hline
\forall\, \mathit{SG}_c, \mathit{SG}_a : \mathit{SGr};\ m : \mathit{GrM} \bullet \\
\quad (\mathit{SG}_c, m) \sqsupseteq_{\mathcal{M}}\ \mathit{SG}_a \Leftrightarrow \forall\, e : \mathit{EsC}\ \mathit{SG}_c \setminus \mathit{EsD}\ \mathit{SG}_c \bullet (\mathit{srcma}\ \mathit{SG}_c)\ e \leq_{\mathcal{M}} ((\mathit{srcma}\ \mathit{SG}_a) \circ (\mathit{fE}\ m))\ e \\
\qquad\qquad \wedge (\mathit{tgtm}\ \mathit{SG}_c)\ e \leq_{\mathcal{M}} ((\mathit{tgtm}\ \mathit{SG}_a) \circ (\mathit{fE}\ m))\ e
\end{array}
$$

$$
\begin{array}{|l}
\_ \sqsupseteq^{SG} \_, \_ \sqsupseteq^{SG_0} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq^{SG_0}\ SG_a \Leftrightarrow (SG_c, m) \sqsupseteq_{NT}\ SG_a \land (SG_c, m) \sqsupseteq_{ET}\ SG_a \land (SG_c, m) \sqsupseteq_{\mathcal{M}}\ SG_a \\
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq^{SG}\ SG_a \Leftrightarrow \exists m' : GrM \mid m' = totaliseForDer(m, SG_c) \bullet \\
\qquad m' \in SG_c \to_{SG} SG_a \land (SG_c, m') \sqsupseteq^{SG_0}\ SG_a
\end{array}
$$

relation$(\_ \sqsupset^{SG} \_)$
relation$(\_ \sqsupset^{SG_0} \_)$
relation$(\_ \sqsupset_{AEs} \_)$
relation(_OkRefinedIn_)
relation$(\_ \sqsupset_{ANNs} \_)$

$$
\begin{array}{|l}
\_ \sqsupset_{ANNs} \_ : \mathbb{P}(GrM \times SGr) \\
\hline
\forall SG_a : SGr;\ m : GrM \bullet \\
\quad m \sqsupset_{ANNs} SG_a \Leftrightarrow \forall nn : NsTy\ SG_a\{nnrml\} \bullet (\preceq\ SG_a) \limg \{nn\} \rimg \cap \operatorname{ran}(fV\ m) = \varnothing
\end{array}
$$

$$
\begin{array}{|l}
\_\mathrm{OkRefinedIn}\_ : \mathbb{P}((SGr \times E) \times (SGr \times GrM)) \\
\_ \sqsupset_{AEs} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM;\ ae : E \bullet \\
\quad (SG_a, ae)\mathrm{OkRefinedIn}(SG_c, m) \Leftrightarrow \\
\qquad \exists r : V \leftrightarrow V;\ s, t : \mathbb{P}\ V \mid r = (\preceq\ SG_c) \comp igRMEsW((gr\ SG_c, m), SG_a, ae)^{\leftrightarrow} \comp (\preceq\ SG_c)^{\sim} \\
\qquad\quad \land\ s = insOf(m, SG_a, (sg\_src\ SG_a \limg \{ae\} \rimg)) \setminus ((NsEther\ SG_c) \setminus \operatorname{dom} r) \\
\qquad\quad \land\ t = insOf(m, SG_a, (sg\_tgt\ SG_a \limg \{ae\} \rimg)) \setminus ((NsEther\ SG_c) \setminus \operatorname{ran} r) \\
\qquad\qquad \bullet\ r \in s \leftrightarrow t \land r \neq \varnothing \\
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset_{AEs}\ SG_a \Leftrightarrow \forall e : (EsA\ SG_a) \bullet (SG_a, e)\mathrm{OkRefinedIn}(SG_c, m)
\end{array}
$$

$$
\begin{array}{|l}
\_ \sqsupset^{SG} \_, \_ \sqsupset^{SG_0} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset^{SG_0}\ SG_a \Leftrightarrow (SG_c, m) \sqsupseteq^{SG_0}\ SG_a \land m \sqsupset_{ANNs} SG_a \land (SG_c, m) \sqsupset_{AEs}\ SG_a \\
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset^{SG}\ SG_a \Leftrightarrow \exists m' : GrM \mid m' = totaliseForDer(m, SG_c) \bullet \\
\qquad m' \in SG_c \to_{SG} SG_a \land (SG_c, m') \sqsupset^{SG_0}\ SG_a
\end{array}
$$

relation$(\_ \Supset^{SG} \_)$
relation$(\_ \Supset_{\mathcal{M}} \_)$
relation$(\_ \Supset_{FI} \_)$
relation$(\_ \Supset_{PNS} \_)$
relation$(\_\ ME\mathcal{M}Ok\ \_)$

$$
\begin{array}{l}
\_ \, ME\mathcal{M}Ok \, \_ : \mathbb{P}((SGr \times E) \times GrwT) \\
\hline
\forall GwT : GrwT; \; SG : SGr; \; me : E \bullet (SG, me) \; ME\mathcal{M}Ok \; GwT \Leftrightarrow \\
\quad \exists r : V \leftrightarrow V; \; s, t : \mathbb{P} \, V \mid r = igRMEsW(GwT, SG, me)^{\leftrightarrow} \\
\qquad \land s = insOf(ty \; GwT, SG, (src^{*} \; SG) \, (\!| \{me\} |\!)) \\
\qquad \land t = insOf(ty \; GwT, SG, (tgt^{*} \; SG) \, (\!| \{me\} |\!)) \\
\qquad \quad \bullet r\mathcal{M}Ok(r, s, t, srcma \; SG \; me, tgtm \; SG \; me)
\end{array}
$$

$$
\begin{array}{l}
\_ \ni_{\mathcal{M}} \_ : GrwT \leftrightarrow SGr \\
\_ \ni_{FI} \_ : GrwT \leftrightarrow SGr \\
\_ \ni_{PNS} \_ : GrwT \leftrightarrow SGr \\
\hline
\forall GwT : GrwT; \; SG : SGr \bullet GwT \ni_{\mathcal{M}} SG \Leftrightarrow \forall me : EsC \; SG \bullet (SG, me) \; ME\mathcal{M}Ok \; GwT \\
\forall GwT : GrwT; \; SG : SGr \bullet GwT \ni_{FI} SG \Leftrightarrow ((fV \circ ty) GwT)^{\sim} \, (\!| NsEther \; SG |\!) = \varnothing \\
\forall GwT : GrwT; \; SG : SGr \bullet \\
\quad GwT \ni_{PNS} SG \Leftrightarrow igRMEs(GwT, EsTy \; SG \, \{ecomp \; dbi, ecomp \; duni\})^{\leftrightarrow} \in \mathrm{injrel}
\end{array}
$$

$$
\begin{array}{l}
\_ \ni^{SG} \_ : GrwT \leftrightarrow SGr \\
\hline
\forall GwT : GrwT; \; SG : SGr \bullet \\
\quad GwT \ni^{SG} SG \Leftrightarrow GwT \Rrightarrow^{GwT} SG \land GwT \ni_{\mathcal{M}} SG \land GwT \ni_{FI} SG \land GwT \ni_{PNS} SG
\end{array}
$$

7 Fragments

section *Fragmenta_Frs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_SGs*, *Fragmenta_GrswT*

$$
\begin{array}{l}
Fr_0 == \{SG : SGr; \; esr : \mathbb{P} \, E; \; sr, tr : E \rightarrow\!\!\!\!\!\!+ \; V \mid esr \cap (sg\_Es \; SG) = \varnothing \\
\quad \land sr \in esr \rightarrowtail\!\!\!\!\rightarrow (NsP \; SG) \land tr \in esr \rightarrow (V \setminus (NsO \; SG))\}
\end{array}
$$

$$
\begin{array}{l}
fSG : Fr_0 \rightarrow SGr \\
EsR : Fr_0 \rightarrow \mathbb{P} \, E \\
srcR, tgtR : Fr_0 \rightarrow E \rightarrow\!\!\!\!\!\!+ \; V \\
\hline
\forall SG : SGr; \; esr : \mathbb{P} \, E; \; sr, tr : E \rightarrow\!\!\!\!\!\!+ \; V \bullet fSG(SG, esr, sr, tr) = SG \\
\forall SG : SGr; \; esr : \mathbb{P} \, E; \; sr, tr : E \rightarrow\!\!\!\!\!\!+ \; V \bullet EsR(SG, esr, sr, tr) = esr \\
\forall SG : SGr; \; esr : \mathbb{P} \, E; \; sr, tr : E \rightarrow\!\!\!\!\!\!+ \; V \bullet srcR(SG, esr, sr, tr) = sr \\
\forall SG : SGr; \; esr : \mathbb{P} \, E; \; sr, tr : E \rightarrow\!\!\!\!\!\!+ \; V \bullet tgtR(SG, esr, sr, tr) = tr
\end{array}
$$

$$
\begin{array}{|l}
fLEs, fEs, fEsC : Fr_0 \rightarrow \mathbb{P}\, E \\
fLNs, fRNs, fNs : Fr_0 \rightarrow \mathbb{P}\, V \\
srcF, tgtF : Fr_0 \rightarrow E \nrightarrow V \\
\hline
fLEs = (sg\_Es \circ fSG) \\
\forall F : Fr_0 \bullet fEs\ F = fLEs\ F \cup EsR\ F \\
fEsC = EsC \circ fSG \\
fLNs = sg\_Ns \circ fSG \\
fRNs = \mathrm{ran} \circ tgtR \\
\forall F : Fr_0 \bullet fNs\ F = fLNs\ F \cup fRNs\ F \\
\forall F : Fr_0 \bullet srcF\ F = (sg\_src \circ fSG)\ F \cup srcR\ F \\
\forall F : Fr_0 \bullet tgtF\ F = (sg\_tgt \circ fSG)\ F \cup tgtR\ F
\end{array}
$$

$$
\begin{array}{|l}
\overset{G}{\leftrightsquigarrow} : Fr_0 \rightarrow Gr \\
\leftrightsquigarrow : Fr_0 \rightarrow V \leftrightarrow V \\
\hline
\forall F : Fr_0 \bullet \overset{G}{\leftrightsquigarrow}\ F = ((NsP \circ fSG)F \cup fRNs\ F, EsR\ F, srcR\ F, tgtR\ F) \\
\forall F : Fr_0 \bullet \leftrightsquigarrow\ F = (\overset{G}{\leftrightsquigarrow}\ F)^{\leftrightarrow}
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_F \_)$

$$
\begin{array}{|l}
\varnothing_F : Fr_0 \\
\_ \cup_F \_ : Fr_0 \times Fr_0 \rightarrow Fr_0 \\
\bigcup_F : \mathbb{P}\, Fr_0 \rightarrow Fr_0 \\
\hline
\varnothing_F = (\varnothing_{SG}, \varnothing, \varnothing, \varnothing) \\
\forall F_1, F_2 : Fr_0 \bullet F_1 \cup_F\ F_2 = \\
\quad (fSG\ F_1 \cup_{SG}\ fSG\ F_2, EsR\ F_1 \cup\ EsR\ F_2, srcR\ F_1 \cup\ srcR\ F_2, tgtR\ F_1 \cup\ tgtR\ F_2) \\
\bigcup_F \{\} = \varnothing_F \\
\forall F : Fr_0;\ Fs : \mathbb{P}\, Fr_0 \bullet \bigcup_F (\{F\} \cup Fs) = F \cup_F (\bigcup_F Fs)
\end{array}
$$

$$\rightsquigarrow \,:\, Fr_0 \pfun V \pfun V$$
$$\circledbullet^{SG} : Fr_0 \pfun SGr$$
$$rEsR : Fr_0 \pfun \mathbb{P}\, E$$
$$\circledbullet : Fr_0 \pfun Fr_0$$

$$\forall F : Fr_0 \bullet \rightsquigarrow F = (\leftrightsquigarrow F) \rhd (fLNs\, F)$$
$$\forall F : Fr_0 \bullet \circledbullet^{SG} F = (fSG\, F) \odot^{SG} (\rightsquigarrow F)$$
$$\forall F : Fr_0 \bullet rEsR\, F = \mathrm{dom}((srcR\, F) \unrhd \mathrm{dom}(\rightsquigarrow F))$$
$$\forall F : Fr_0 \bullet \circledbullet F = (\circledbullet^{SG} F, rEsR\, F, (rEsR\, F) \lhd (srcR\, F), (rEsR\, F) \lhd (tgtR\, F))$$

$$Fr_a == \{F : Fr_0 \mid \otimes(\overset{G}{\leftrightsquigarrow} F)\}$$
$$Fr == \{F : Fr_a \mid \circledbullet^{SG} F \in SGr\}$$

relation(refsLocal_)

$$\text{refsLocal\_} : \mathbb{P}\, Fr_0$$

$$\forall F : Fr_0 \bullet \text{refsLocal}\, F \Leftrightarrow fRNs\, F \subseteq fLNs\, F$$

$$TFr == \{F : Fr_a \mid \text{refsLocal}\, F \wedge \circledbullet^{SG} F \in TSGr\}$$

relation($\boxminus$ _)
relation($\boxplus$ _)

$$\boxminus \_ : Fr \leftrightarrow Fr$$

$$\forall F_1, F_2 : Fr \bullet \boxminus(F_1, F_2) \Leftrightarrow fLNs\, F_1 \cap fLNs\, F_2 = \varnothing \wedge fEs\, F_1 \cap fEs\, F_2 = \varnothing$$

$[I]$

$$\boxplus \_ : \mathbb{P}(I \pfun Fr)$$

$$\forall Fs : I \pfun Fr \bullet \boxplus\, Fs \Leftrightarrow \forall i, j : \mathrm{dom}\, Fs \mid i \neq j \bullet \boxminus(Fs\, i, Fs\, j)$$

relation(_ $\subseteq^{rs}$ _)
relation(_ $\mapsto$ _)

$$
\begin{array}{|l}
\_ \subseteq^{rs} \_ : Fr \leftrightarrow Fr \\
\_ \mapsto \_ : Fr \leftrightarrow Fr \\
\hline
\forall F_1, F_2 : Fr \bullet F_1 \subseteq^{rs} F_2 \Leftrightarrow \mathrm{ran}(tgtR\, F_1) \cap fLNs\, F_2 \neq \varnothing \\
\forall F_1, F_2 : Fr \bullet F_1 \mapsto F_2 \Leftrightarrow F_1 \subseteq^{rs} F_2 \land \neg\, (F_2 \subseteq^{rs} F_1)
\end{array}
$$

function 10 **leftassoc** $(\_ \rightrightarrows_{\bullet} \_)$

$$
\begin{array}{|l}
\_ \rightrightarrows_{\bullet} \_ : GrM \times (Fr \times Fr) \to GrM \\
\hline
\forall m : GrM;\ F_s, F_t : Fr_0 \bullet \\
\quad m \rightrightarrows_{\bullet} (F_s, F_t) = (((\rightsquigarrow F_s)^{\oplus} \boxdot (fLNs\, F_s))^{\sim} \mathbin{⨾} (fV\ m) \mathbin{⨾} ((\rightsquigarrow F_t)^{\oplus} \boxdot (fLNs\, F_t)), fE\ m)
\end{array}
$$

function 1 **leftassoc** $(\_ \to_F \_)$

$$
\begin{array}{|l}
\_ \to_F \_ : Fr \times Fr \to \mathbb{P}\, GrM \\
\hline
\forall F_s, F_t : Fr \bullet F_s \to_F F_t = \{ fv : fLNs\, F_s \to fLNs\, F_t;\ fe : fEsC\, F_s \to fEsC\, F_t\ | \\
\quad (\circledbullet^{SG} F_s, (fv, fe) \rightrightarrows_{\bullet} (F_s, F_t)) \Rrightarrow^{SG} \circledbullet^{SG} F_t \}
\end{array}
$$

relation $(\_ \Rrightarrow^F \_)$

$$
\begin{array}{|l}
\_ \Rrightarrow^F \_ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall m : GrM;\ F_s, F_t : Fr_0 \bullet (F_s, m) \Rrightarrow^F F_t \Leftrightarrow m \in F_s \to_F F_t
\end{array}
$$

relation $(\_ \sqsupseteq^F \_)$

$$
\begin{array}{|l}
\_ \sqsupseteq^F \_ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupseteq^F F_a \Leftrightarrow (F_c, m) \Rrightarrow^F F_a \\
\quad \land (\circledbullet^{SG} F_c, m \rightrightarrows_{\bullet} (F_c, F_a)) \sqsupseteq^{SG_0} (\circledbullet^{SG} F_a)
\end{array}
$$

relation $(\_ \sqsupset^F \_)$

$$
\begin{array}{|l}
\_ \sqsupset^{F} \_ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupset^{F} F_a \Leftrightarrow (F_c, m) \Rrightarrow^{F} F_a \\
\quad \land (\circledbullet^{SG} F_c, m \rightrightarrows_{\bullet} (F_c, F_a)) \sqsupset^{SG_0} (\circledbullet^{SG} F_a)
\end{array}
$$

relation$(\_ \Supset^{F} \_)$

$$
\begin{array}{|l}
\_ \Supset^{F} \_ : GrwT \leftrightarrow Fr \\
\hline
\forall GwT : GrwT;\ F : Fr \bullet GwT \Supset^{F} F \Leftrightarrow GwT \Supset^{SG} \circledbullet^{SG} F
\end{array}
$$

8 Global Fragment Graphs

section *Fragmenta_GFGs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$$GFGr == \{G : Gr \mid \otimes(G \bowtie_{Es} (Es\ G \setminus EsId\ G))\}$$

function$(\_ ^{\dashrightarrow})$

$$
\begin{array}{|l}
\_ ^{\dashrightarrow} : GFGr \rightarrow V \leftrightarrow V \\
\hline
\forall GFG : GFGr \bullet GFG ^{\dashrightarrow} = (GFG ^{\leftrightarrow})^{+}
\end{array}
$$

9 Models

section *Fragmenta_Mdls* **parents** *standard_toolkit*, *Fragmenta_Frs*, *Fragmenta_GFGs*

$$Mdl_0 == \{GFG : GFGr;\ fd : V \nrightarrow Fr \mid fd \in Ns\ GFG \rightarrow Fr \land \boxplus\ fd\}$$

$$
\begin{array}{|l}
mGFG : Mdl_0 \rightarrow GFGr \\
mFD : Mdl_0 \rightarrow V \nrightarrow Fr \\
\hline
\forall GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mGFG(GFG, fd) = GFG \\
\forall GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mFD(GFG, fd) = fd
\end{array}
$$

$$
\begin{array}{|l}
mUFs : Mdl_0 \rightarrow Fr \\
\hline
mUFs = \bigcup_F \circ \operatorname{ran} \circ mFD
\end{array}
$$

$$
\begin{array}{|l}
\mathrm{from} : Mdl_0 \rightarrow V \twoheadrightarrow V \\
\hline
\forall\, M : Mdl_0;\ v : V \bullet \mathrm{from}\, M\, v = (\mu\, vf : (Ns \circ mGFG)M \mid v \in fLNs(mFD\, M\, vf))
\end{array}
$$

relation$(\Uparrow \_)$

$$
\begin{array}{|l}
\Uparrow \_ : \mathbb{P}\, Mdl_0 \\
\hline
\forall\, M : Mdl_0 \bullet \Uparrow M \Leftrightarrow \exists\, UF : Fr_0 \mid UF = mUFs\, M \bullet \\
\quad \forall\, p : (NsP \circ fSG)\, UF \bullet (from\ M\ p, from\ M\ (\leftrightsquigarrow\ UF\ p)) \in ((\_ ^{\dashrightarrow}) \circ mGFG)M
\end{array}
$$

$$Mdl == \{M : Mdl_0 \mid (mUFs\, M) \in TFr \land \Uparrow M\}$$

$$
\begin{array}{|l}
\circledbullet^{M} : Mdl \rightarrow Fr \\
\hline
\forall\, M : Mdl_0 \bullet \circledbullet^{M} = \circledbullet \circ mUFs
\end{array}
$$

function 1 **leftassoc** $(\_ \rightarrow_M \_)$
relation$(\_ \Rrightarrow^M \_)$

$$
\begin{array}{|l}
\_ \rightarrow_M \_ : Mdl \times Mdl \rightarrow \mathbb{P}\, GrM \\
\_ \Rrightarrow^M \_ : \mathbb{P}((Mdl \times \mathbb{P}\, GrM) \times Mdl) \\
\hline
\forall\, M_s, M_t : Mdl \bullet M_s \rightarrow_M M_t = \{m : GrM \mid \\
\quad \exists\, UF_s, UF_t : Fr_0 \mid UF_s = mUFs\, M_s \land UF_t = mUFs\, M_t \bullet m \in UF_s \rightarrow_F UF_t\} \\
\forall\, M_s, M_t : Mdl;\ ms : \mathbb{P}\, GrM \bullet (M_s, ms) \Rrightarrow^M M_t \Leftrightarrow \bigcup_{GM} ms \in M_s \rightarrow_M M_t
\end{array}
$$

relation$(\_ \sqsupset^M \_)$

$$
\begin{array}{|l}
\_ \sqsupset^M \_ : (Mdl \times \mathbb{P}\, GrM) \leftrightarrow Mdl \\
\hline
\forall\, M_c, M_a : Mdl_0;\ ms : \mathbb{P}\, GrM \bullet (M_c, ms) \sqsupset^M M_a \\
\quad \Leftrightarrow \exists\, UF_c, UF_a : Fr_0 \mid UF_c = mUFs\, M_c \land UF_a = mUFs\, M_a \bullet (UF_c, \bigcup_{GM} ms) \sqsupset^F UF_a
\end{array}
$$

relation$(\_ \Supset^M \_)$

$$\begin{array}{|l}
_ \Supset^M _ : GrwT \leftrightarrow Mdl \\
\hline
\forall\, GwT : GrwT;\ M : Mdl \bullet GwT \Supset^M M \Leftrightarrow GwT \Supset^F mUFs\, M
\end{array}$$