# Z Specification of Fragmenta

Nuno Amálio

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## Contents

# 1 Generics

**section** *Fragmenta_Generics* **parents** *standard_toolkit*

$\mathrm{acyclic}[X] == \{r : X \leftrightarrow X \mid r^{+} \cap \mathrm{id}\, X = \varnothing\}$

$\mathit{connected}[X] == \{r : X \leftrightarrow X \mid \forall x : \mathrm{dom}\, r;\ y : \mathrm{ran}\, r \bullet x \mapsto y \in r^{+}\}$

$\mathrm{tree}[X] == \{r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \land r \in X \rightarrow\!\!\!\!\!\!+\; X\}$

$\mathit{forest}[X] == \{r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \land (\forall s : X \leftrightarrow X \mid s \subseteq r \land s \in \mathit{connected} \bullet s \in \mathit{tree})\}$

$\mathrm{injrel}[X, Y] == \{r : X \leftrightarrow Y \mid r^{\sim} \in Y \rightarrow\!\!\!\!\!\!+\; X\}$

$\mathrm{antireflexive}[X] == \{r : X \leftrightarrow X \mid r \cap \mathit{id}(\mathrm{dom}\ r) = \varnothing\}$

$[X, Y, Z]$

$\mathrm{flip} : (X \rightarrow Y \rightarrow\!\!\!\!\!\!+\; Z) \rightarrow (Y \rightarrow X \rightarrow\!\!\!\!\!\!+\; Z)$

$\forall f : X \rightarrow Y \rightarrow\!\!\!\!\!\!+\; Z \bullet \mathrm{flip}\, f = (\lambda\, y : Y \bullet \lambda\, x : X \bullet f\ x\ y)$

$[X, Y, Z, W]$

$\mathrm{apply} : (X \rightarrow Z) \rightarrow (Y \rightarrow W) \rightarrow (X \times Y) \rightarrow (Z \times W)$

$\forall f : X \rightarrow Z;\ g : Y \rightarrow W;\ x : X;\ y : Y \bullet \mathrm{apply}\, f\ g\ (x, y) = (f\ x, g\ y)$

$[X, Y]$

$\mathrm{map} : (X \rightarrow Y) \rightarrow \mathbb{P}\, X \rightarrow \mathbb{P}\, Y$
$\mathrm{mapS} : (X \rightarrow Y) \rightarrow \mathrm{seq}\, X \rightarrow \mathrm{seq}\, Y$

$\forall f : X \rightarrow Y \bullet \mathrm{map}\, f \{\} = \{\}$

$\forall f : X \rightarrow Y;\ x : X;\ xs : \mathbb{P}\, X \bullet \mathrm{map}\, f(\{x\} \cup xs) = \{f\ x\} \cup (\mathrm{map}\, f\ xs)$

$\forall f : X \rightarrow Y \bullet \mathrm{mapS}\, f \langle\rangle = \langle\rangle$

$\forall f : X \rightarrow Y;\ x : X;\ xs : \mathrm{seq}\, X \bullet \mathrm{mapS}\, f(\langle x\rangle \frown xs) = \langle f\ x\rangle \frown (\mathrm{mapS}\, f\ xs)$

**function** 10 **leftassoc** $(\_ \boxdot \_)$

$[X]$

$\_ \boxdot \_ : ((X \rightarrow X) \times \mathbb{P}\, X) \rightarrow (X \rightarrow\!\!\!\!\!\!+\; X)$

$\forall f : X \rightarrow X;\ s : \mathbb{P}\, X \bullet f \boxdot s = (\mathrm{id}\ s) \oplus f$

**function**$(\_^{\oplus})$

$$[X]$$
$$\_^{\oplus} : (X \leftrightarrow X) \rightarrow (X \leftrightarrow X)$$
$$\forall r : X \leftrightarrow X \bullet r^{\oplus} = \textbf{if } r \oplus r \comp r = r \textbf{ then } r \textbf{ else } (r \oplus r \comp r)^{\oplus}$$

$$\mathrm{opt}[X] == \{s : \mathbb{P}\, X \mid \# s \leq 1\}$$

$$[X]$$
$$\mathrm{the} : \mathrm{opt}[X] \pfun X$$
$$\forall x : X \bullet \mathrm{the}\,\{x\} = x$$

$$[X, Y]$$
$$\mathit{flatten} : (X \pfun \mathbb{P}\, Y) \rightarrow (X \leftrightarrow Y)$$
$$\forall f : X \pfun \mathbb{P}\, Y \bullet \mathit{flatten}\, f = \{x : \mathrm{dom}\, f;\ y : Y \mid y \in f\, x\}$$

## 2 Graphs

**section** *Fragmenta_Graphs* **parents** *standard_toolkit*, *Fragmenta_Generics*

$$[V, E]$$
$$Gr == \{vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s, t : E \pfun V \mid s \in es \rightarrow vs \land t \in es \rightarrow vs\}$$

$$Ns : Gr \rightarrow \mathbb{P}\, V$$
$$Es : Gr \rightarrow \mathbb{P}\, E$$
$$src, tgt : Gr \rightarrow E \pfun V$$
$$\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \pfun V;\ t : E \pfun V \bullet Ns(vs, es, s, t) = vs$$
$$\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \pfun V;\ t : E \pfun V \bullet Es(vs, es, s, t) = es$$
$$\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \pfun V;\ t : E \pfun V \bullet src(vs, es, s, t) = s$$
$$\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \pfun V;\ t : E \pfun V \bullet tgt(vs, es, s, t) = t$$

$$
\begin{array}{|l}
\varnothing_G : Gr \\
\hline
\varnothing_G = (\varnothing, \varnothing, \varnothing, \varnothing)
\end{array}
$$

$$
\begin{array}{|l}
EsId : Gr \to \mathbb{P}\, E \\
\hline
\forall\, G : Gr \bullet EsId\ G = \{e : Es\ G \mid src\ G\ e = tgt\ G\ e\}
\end{array}
$$

**relation**(adjacent _ )

$$
\begin{array}{|l}
\mathrm{adjacent}\_ : \mathbb{P}(Gr \times V \times V) \\
\hline
\forall\, G : Gr;\ v_1, v_2 : V \bullet \mathrm{adjacent}(G, v_1, v_2) \Leftrightarrow \exists\, e : Es\ G \bullet src\ G\ e = v_1 \wedge tgt\ G\ e = v_2
\end{array}
$$

**function** 10 **leftassoc** $(\_ \circ\!\!\rightarrow\!\!\circ \_)$

$$
\begin{array}{|l}
\_ \circ\!\!\rightarrow\!\!\circ \_ : Gr \times \mathbb{P}\, V \to \mathbb{P}\, E \\
\hline
\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet G \circ\!\!\rightarrow\!\!\circ\ vs = (src\ G)^{\sim} (\!| vs |\!) \cup (tgt\ G)^{\sim} (\!| vs |\!)
\end{array}
$$

**function** 10 **leftassoc** $(\_ \bullet\!\!\leftrightarrow\!\!\bullet \_)$

$$
\begin{array}{|l}
\_ \bullet\!\!\leftrightarrow\!\!\bullet \_ : Gr \times \mathbb{P}\, V \to \mathbb{P}\, E \\
\hline
\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet G \bullet\!\!\leftrightarrow\!\!\bullet\ vs = (src\ G)^{\sim} (\!| vs |\!) \cap (tgt\ G)^{\sim} (\!| vs |\!)
\end{array}
$$

$$
\begin{array}{|l}
rNs : Gr \to \mathbb{P}\, E \to \mathbb{P}\, V \\
\hline
\forall\, G : Gr;\ es : \mathbb{P}\, E \bullet rNs\ G\ es = \mathrm{ran}(es \lhd src\ G) \cup \mathrm{ran}(es \lhd tgt\ G)
\end{array}
$$

**function** 10 **leftassoc** $(\_ \bowtie_{Es} \_)$

$$
\begin{array}{|l}
\_ \bowtie_{Es} \_ : Gr \times \mathbb{P}\, E \to Gr \\
\hline
\forall\, G : Gr;\ es : \mathbb{P}\, E \bullet G \bowtie_{Es}\ es = (rNs\ G\ es, Es\ G \cap es, es \lhd src\ G, es \lhd tgt\ G)
\end{array}
$$

**function** 10 **leftassoc** $(\_ \bowtie_{Ns} \_)$

$$\begin{array}{|l}
_ \bowtie_{Ns} _ : Gr \times \mathbb{P}\, V \to Gr \\
\hline
\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet \\
\quad G \bowtie_{Ns} vs = (Ns\ G \cap vs, G \bullet\!\!\leftrightarrow\!\!\bullet\ vs, (G \bullet\!\!\leftrightarrow\!\!\bullet\ vs) \lhd src\ G, (G \bullet\!\!\leftrightarrow\!\!\bullet\ vs) \lhd tgt\ G)
\end{array}$$

**function** 10 **leftassoc** $(\_ \ominus_{Ns} \_)$

$$\begin{array}{|l}
_ \ominus_{Ns} _ : Gr \times \mathbb{P}\, V \to Gr \\
\hline
\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet \\
\quad G \ominus_{Ns} vs = (Ns\ G \setminus vs, Es\ G \setminus (G \circ\!\!\to\!\!\circ\ vs), (G \circ\!\!\to\!\!\circ\ vs) \ndres src\ G, (G \circ\!\!\to\!\!\circ\ vs) \ndres tgt\ G)
\end{array}$$

$$\begin{array}{|l}
\text{successors} : V \times Gr \to \mathbb{P}\, V \\
\hline
\forall\, v : V;\ G : Gr \bullet \text{successors}(v, G) = \{v_1 : Ns\ G \mid \text{adjacent}(G, v, v_1)\}
\end{array}$$

**function** $(\_^{\rightleftarrows})$

$$\begin{array}{|l}
_^{\rightleftarrows} : Gr \to Gr \\
\hline
\forall\, G : Gr \bullet G^{\rightleftarrows} = (Ns\ G, Es\ G, tgt\ G, src\ G)
\end{array}$$

**function** $(\_^{\leftrightarrow})$

$$\begin{array}{|l}
_^{\leftrightarrow} : Gr \to V \leftrightarrow V \\
\hline
\forall\, G : Gr \bullet G^{\leftrightarrow} = \{v_1, v_2 : Ns\ G \mid \text{adjacent}(G, v_1, v_2)\}
\end{array}$$

**relation** $(\otimes \_)$

$$\begin{array}{|l}
\otimes _ : \mathbb{P}\, Gr \\
\hline
\forall\, G : Gr \bullet \otimes\ G \Leftrightarrow G^{\leftrightarrow} \in \text{acyclic}
\end{array}$$

**relation**$(\boxminus_{Es}\ \_)$
**relation**$(\boxminus\ \_)$

$$\boxminus_{Es}\ \_, \boxminus\ \_ : \mathbb{P}(Gr \times Gr)$$

$$\forall\, G_1, G_2 : Gr \bullet \boxminus_{Es}(G_1, G_2) \Leftrightarrow Es\ G_1 \cap Es\ G_2 = \varnothing$$
$$\forall\, G_1, G_2 : Gr \bullet \boxminus(G_1, G_2) \Leftrightarrow Ns\ G_1 \cap Ns\ G_2 = \varnothing \wedge \boxminus_{Es}(G_1, G_2)$$

**relation**$(\boxplus\ \_)$

$$[I]$$
$$\boxplus\ \_ : \mathbb{P}(I \rightarrowtail\!\!\!\!\!\!+\ Gr)$$

$$\forall\, Gs : I \rightarrowtail\!\!\!\!\!\!+\ Gr \bullet \boxplus\ Gs \Leftrightarrow \forall\, i, j : \mathrm{dom}\ Gs \mid i \neq j \bullet \boxminus(Gs\, i, Gs\, j)$$

**function** 10 **leftassoc** $(\_ \cup_G \_)$

$$\_ \cup_G \_ : Gr \times Gr \rightarrow Gr$$

$$\forall\, G_1, G_2 : Gr \bullet G_1 \cup_G G_2 = (Ns\ G_1 \cup Ns\ G_2, Es\ G_1 \cup Es\ G_2, src\ G_1 \cup src\ G_2, tgt\ G_1 \cup tgt\ G_2)$$

**function** 10 **leftassoc** $(\_ \odot \_)$

$$\_ \odot \_ : Gr \times (V \leftrightarrow V) \rightarrowtail\!\!\!\!\!\!+\ Gr$$

$$\forall\, G : Gr;\ s : V \leftrightarrow V \mid s \in Ns\ G \rightarrowtail\!\!\!\!\!\!+\ Ns\ G \wedge s \in \text{antireflexive} \bullet$$
$$G \odot s = (Ns\ G \setminus \mathrm{dom}\, s, Es\ G, (s \boxdot Ns\ G) \circ (src\ G), (s \boxdot Ns\ G) \circ (tgt\ G))$$

$$GrM == (V \rightarrowtail\!\!\!\!\!\!+\ V) \times (E \rightarrowtail\!\!\!\!\!\!+\ E)$$

$$fV : GrM \rightarrow V \rightarrowtail\!\!\!\!\!\!+\ V$$
$$fE : GrM \rightarrow E \rightarrowtail\!\!\!\!\!\!+\ E$$

$$\forall\, fv : V \rightarrowtail\!\!\!\!\!\!+\ V;\ fe : E \rightarrowtail\!\!\!\!\!\!+\ E \bullet fV(fv, fe) = fv$$
$$\forall\, fv : V \rightarrowtail\!\!\!\!\!\!+\ V;\ fe : E \rightarrowtail\!\!\!\!\!\!+\ E \bullet fE(fv, fe) = fe$$

$$\begin{array}{|l}
\mathrm{gid} : Gr \rightarrow GrM \\
\hline
\forall\, G : Gr \bullet \mathrm{gid}\; G = (id\,(Ns\; G), id\,(Es\; G))
\end{array}$$

$$\begin{array}{|l}
\varnothing_{GM} : GrM \\
\hline
\varnothing_{GM} = (\{\}, \{\})
\end{array}$$

$$\begin{array}{|l}
\mathrm{domg}, \mathrm{codg} : GrM \rightarrow Gr \\
\hline
\forall\, m : GrM;\; G : Gr \bullet \mathrm{domg}\; m = G \Leftrightarrow \mathrm{dom}(fV\; m) = Ns\; G \wedge \mathrm{dom}(fE\; m) = Es\; G \\
\forall\, m : GrM;\; G : Gr \bullet \mathrm{codg}\; m = G \Leftrightarrow \mathrm{ran}(fV\; m) \subseteq Ns\; G \wedge \mathrm{ran}(fE\; m) \subseteq Es\; G
\end{array}$$

**function** 10 **leftassoc** $(\_ \cup_{GM} \_)$

$$\begin{array}{|l}
_ \cup_{GM} _ : GrM \times GrM \rightarrow\!\!\!\!\!+ GrM \\
\bigcup_{GM} : \mathbb{P}\, GrM \rightarrow GrM \\
\hline
\forall\, f, g : GrM \bullet f \cup_{GM} g = (fV\, f \cup fV\; g, fE\, f \cup fE\; g) \\
\bigcup_{GM} \varnothing = \varnothing_{GM} \\
\forall\, f : GrM;\; fs : \mathbb{P}\, GrM \bullet \bigcup_{GM} (\{f\} \cup fs) = f \cup_{GM} (\bigcup_{GM} fs)
\end{array}$$

**function** 10 **leftassoc** $(\_ \rightarrow_G \_)$

$$\begin{array}{|l}
_ \rightarrow_G _ : Gr \times Gr \rightarrow \mathbb{P}\, GrM \\
\hline
\forall\, G_1, G_2 : Gr \bullet G_1 \rightarrow_G G_2 = \{fv : Ns\; G_1 \rightarrow Ns\; G_2;\; fe : Es\; G_1 \rightarrow Es\; G_2 \mid \\
\quad src\; G_2 \circ fe = fv \circ src\; G_1 \wedge tgt\; G_2 \circ fe = fv \circ tgt\; G_1\}
\end{array}$$

**function** 10 **leftassoc** $(\_ \circ_G \_)$

$$\begin{array}{|l}
_ \circ_G _ : GrM \times GrM \rightarrow GrM \\
\hline
\forall\, g, f : GrM \bullet g \circ_G f = (fV\; g \circ fV\, f, fE\; g \circ fE\, f)
\end{array}$$

# 3 Graphs with typing

**section** *Fragmenta_GrswT* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$$GrwT == \{G : Gr;\ t : GrM \mid \mathrm{domg}\, t = G\}$$

$$\begin{array}{|l}
gOf : GrwT \to Gr \\
ty : GrwT \to GrM \\
\hline
\forall G : Gr;\ sm : V \pfun \mathrm{seq}\, V;\ t : GrM \bullet gOf(G, t) = G \\
\forall G : Gr;\ sm : V \pfun \mathrm{seq}\, V;\ t : GrM \bullet ty(G, t) = t
\end{array}$$

$$\begin{array}{|l}
\varnothing_{GwT} : GrwT \\
\hline
\varnothing_{GwT} = (\varnothing_G, \varnothing_{GM})
\end{array}$$

**function** 10 **leftassoc** $(\_ \cup_{GwT} \_)$

$$\begin{array}{|l}
_ \cup_{GwT} _ : GrwT \times GrwT \to GrwT \\
\hline
\forall G_1, G_2 : GrwT \bullet G_1 \cup_{GwT} G_2 = ((gOf\ G_1) \cup_G (gOf\ G_2), (ty\ G_1) \cup_{GM} (ty\ G_2))
\end{array}$$

# 4 SG Element Types

**section** *Fragmenta_SGElemTys* **parents** *standard_toolkit*, *Fragmenta_Generics*

$$SGNT ::= nnrml \mid nabst \mid nprxy \mid nenum \mid nval \mid nvirt \mid nopt$$

$$SGED ::= dbi \mid duni$$

$$SGET ::= einh \mid ecomp\langle\langle SGED\rangle\rangle \mid erel\langle\langle SGED\rangle\rangle \mid ewander \mid eder$$

**relation**$(\_ \prec_{NT} \_)$

$$
\begin{array}{|l}
\_ \prec_{NT} \_ : SGNT \leftrightarrow SGNT \\
\hline
\forall nt_1, nt_2 : SGNT \bullet nt_1 \prec_{NT} nt_2 \Leftrightarrow ((nt_2 = nenum \land nt_1 = nval) \\
\quad \lor (nt_1 = nvirt \land nt_2 = nvirt) \lor (nt_1 = nabst \land nt_2 \in \{nabst, nvirt, nprxy\}) \lor (nt_1 = nnrml \land nt_2 \notin \{nenu \\
\quad \land nt_1 \notin \{nprxy, nenum, nopt\} \land nt_2 \notin \{nopt\}
\end{array}
$$

**relation**$(\_ \leq_{rNT} \_)$

$$
\begin{array}{|l}
\_ \leq_{rNT} \_ : SGNT \leftrightarrow SGNT \\
\hline
\forall nt_1, nt_2 : SGNT \bullet nt_1 \leq_{rNT} nt_2 \Leftrightarrow nt_1 = nt_2 \lor nt_1 = nprxy \\
\quad \lor nt_2 = nabst \land nt_1 \in \{nnrml, nvirt\} \lor nt_2 \in \{nnrml, nopt\}
\end{array}
$$

**relation**$(\_ =_{ET} \_)$

$$
\begin{array}{|l}
\_ =_{ET} \_ : SGET \leftrightarrow SGET \\
\hline
\forall et_1, et_2 : SGET \bullet et_1 =_{ET} et_2 \Leftrightarrow et_1 = et_2 \\
\quad \lor (\forall d_1, d_2 : SGED \bullet et_1 = erel\ d_1 \land et_2 = erel\ d_2 \lor et_1 = ecomp\ d_1 \land et_2 = ecomp\ d_2)
\end{array}
$$

**relation**$(\_ \leq_{ET} \_)$

$$
\begin{array}{|l}
\_ \leq_{ET} \_ : SGET \leftrightarrow SGET \\
\hline
\forall et_1, et_2 : SGET \bullet et_1 \leq_{ET} et_2 \Leftrightarrow einh \notin \{et_1, et_2\} \\
\quad \land (et_1 =_{ET} et_2 \lor et_2 = ewander \\
\quad \lor et_1 = eder \land et_2 \in \mathrm{dom}(erel^{\sim}) \cup \mathrm{dom}(ecomp^{\sim}))
\end{array}
$$

# 5 Multiplicities

**section** $Fragmenta\_Mult$ **parents** $standard\_toolkit, Fragmenta\_Generics, Fragmenta\_SGElemTys$

$MultVal ::= \mathbf{v}\langle\langle \mathbb{N} \rangle\rangle \mid *$

$MultC ::= mr\langle\langle \mathbb{N} \times MultVal \rangle\rangle \mid ms\langle\langle MultVal \rangle\rangle$

**relation**$(\_ \leq_{mv} \_)$

$$\_ \leq_{mv} \_ : MultVal \leftrightarrow MultVal$$

$$\forall m_1, m_2 : MultVal \bullet m_1 \leq_{mv} m_2 \Leftrightarrow m_2 = * \lor \exists j, k : \mathbb{N} \mid m_1 = \mathbf{v}\ j \land m_2 = \mathbf{v}\ k \bullet j \leq k$$

$$Mult == \{mc : MultC \mid \exists lb : \mathbb{N};\ ub : MultVal \bullet mc = mr(lb, ub) \land \mathbf{v}\ lb \leq_{mv} ub \\ \lor \exists mv : MultVal \bullet mc = ms\ mv\}$$

$$MultMany == \{ms\ *, mr(0, *)\}$$

$$MultRange == \{m : MultC \mid \exists k : \mathbb{N} \mid k > 1 \bullet m = ms\ (\mathbf{v}\ k) \\ \lor \exists lb : \mathbb{N};\ umv : MultVal \mid \mathbf{v}\ 2 \leq_{mv} umv \bullet m = mr(lb, umv)\}$$

**relation**$(\_ \between \_)$

$$\_ \between \_ : \mathbb{P}(\mathbb{N} \times (MultVal \times MultVal))$$

$$\forall k : \mathbb{N};\ lb, ub : MultVal \bullet k \between (lb, ub) \Leftrightarrow lb \leq_{mv} \mathbf{v}\ k \land \mathbf{v}\ k \leq_{mv} ub$$

$$mlb, mub : MultC \nrightarrow MultVal$$

$$mlb(ms\ *) = \mathbf{v}\ 0$$

$$\forall k : \mathbb{N} \bullet mlb(ms(\mathbf{v}\ k)) = \mathbf{v}\ k$$

$$\forall k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ k$$

$$\forall mv : MultVal \bullet mub(ms\ mv) = mv$$

$$\forall k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ m$$

**relation**$(\_ \leq_{\mathcal{M}} \_)$

$$\_ \leq_{\mathcal{M}} \_ : MultC \leftrightarrow MultC$$

$$\forall m_1, m_2 : MultC \bullet m_1 \leq_{\mathcal{M}} m_2 \Leftrightarrow mlb\ m_2 \leq_{mv} mlb\ m_1 \land mub\ m_1 \leq_{mv} mub\ m_2$$

**relation**$(\_ \propto \_)$

$$\_ \propto \_ : \mathbb{P}(SGET \times (MultC \times MultC))$$

$$\forall et : SGET;\ m_1, m_2 : MultC \bullet et \propto (m_1, m_2) \Leftrightarrow et = erel\ dbi \lor et = eder \\ \lor et = ecomp\ duni \land m_1 = ms(\mathbf{v}\ 1) \lor et = erel\ duni \land m_1 \in MultMany \\ \lor et = ecomp\ dbi \land m_1 \in \{ms(\mathbf{v}\ 1), mr(0, \mathbf{v}\ 1)\} \\ \lor et = ewander \land (m_1, m_2) \in MultMany \times MultMany$$

**relation**(rbounded_)

$$
\begin{array}{|l}
[X, Y] \\
\hline
\mathrm{rbounded\_} : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\, X \times \mathit{MultC}) \\
\hline
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ m : \mathit{MultC} \bullet \\
\quad \mathrm{rbounded}(r, s, m) \Leftrightarrow \forall\, x : s \bullet \#(r \limg \{x\} \rimg) \between (\mathit{mlb}\ m, \mathit{mub}\ m) \\
\hline
\end{array}
$$

**relation**($r\mathcal{M}Ok\_$)

$$
\begin{array}{|l}
[X, Y] \\
\hline
r\mathcal{M}Ok\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\, X \times \mathbb{P}\, Y \times \mathit{MultC} \times \mathit{MultC}) \\
\hline
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{ms}(\mathbf{v}\ 1), \mathit{ms}(\mathbf{v}\ 1)) \Leftrightarrow r \in s \bij t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mr}(0, \mathbf{v}\ 1), \mathit{mr}(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \pinj t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mr}(0, \mathbf{v}\ 1), \mathit{ms}(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{ms}(\mathbf{v}\ 1), \mathit{mr}(0, \mathbf{v}\ 1)) \Leftrightarrow r^{\sim} \in t \rightarrowtail s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mm}, \mathit{ms}(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrow t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{ms}(\mathbf{v}\ 1), \mathit{mm}) \Leftrightarrow r^{\sim} \in t \rightarrow s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm}_1, \mathit{mm}_2 : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mm}_1, \mathit{mm}_2) \Leftrightarrow r \in s \leftrightarrow t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany};\ \mathit{mr} : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mm}, \mathit{mr}) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r, s, \mathit{mr}) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany};\ \mathit{mr} : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mr}, \mathit{mm}) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r^{\sim}, t, \mathit{mr}) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mm}, \mathit{mr}(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \pfun t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mr}(0, \mathbf{v}\ 1), \mathit{mm}) \Leftrightarrow r^{\sim} \in t \pfun s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mr}_1, \mathit{mr}_2 : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mr}_1, \mathit{mr}_2) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r, s, \mathit{mr}_2) \land \mathrm{rbounded}(r^{\sim}, t, \mathit{mr}_1) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mr} : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mr}, \mathit{ms}(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrow t \land \mathrm{rbounded}(r^{\sim}, t, \mathit{mr}) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mr} : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{ms}(\mathbf{v}\ 1), \mathit{mr}) \Leftrightarrow r^{\sim} \in t \rightarrow s \land \mathrm{rbounded}(r, s, \mathit{mr}) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ m : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, m, \mathit{mr}(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \pfun t \land \mathrm{rbounded}(r^{\sim}, t, m) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ m : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mr}(0, \mathbf{v}\ 1), m) \Leftrightarrow r^{\sim} \in t \pfun s \land \mathrm{rbounded}(r, s, m) \\
\hline
\end{array}
$$

## 6 Structural Graphs

**section** *Fragmenta_SGs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*, *Fragmenta_SGElemTys*, *Fragmenta_Mult*, *Fragmenta_GrswT*

$$SGr_0 == \{G : Gr;\ nt : V \rightarrow\!\!\!\!\!+ SGNT;\ et : E \rightarrow\!\!\!\!\!+ SGET;\ sm, tm : E \rightarrow\!\!\!\!\!+ Mult;\ db : E \rightarrow\!\!\!\!\!+ E \mid nt \in Ns\ G \rightarrow SGNT \land et \in Es\ G \rightarrow SGET\}$$

$$\begin{array}{|l}
gr : SGr_0 \rightarrow Gr \\
sg_Ns : SGr_0 \rightarrow \mathbb{P}\, V \\
sg_Es : SGr_0 \rightarrow \mathbb{P}\, E \\
sg_src, sg_tgt : SGr_0 \rightarrow E \rightarrow\!\!\!\!\!+ V \\
nty : SGr_0 \rightarrow V \rightarrow\!\!\!\!\!+ SGNT \\
ety : SGr_0 \rightarrow E \rightarrow\!\!\!\!\!+ SGET \\
srcm, tgtm : SGr_0 \rightarrow E \rightarrow\!\!\!\!\!+ Mult \\
derb : SGr_0 \rightarrow E \rightarrow\!\!\!\!\!+ E \\
\hline
\forall G : Gr;\ nt : V \rightarrow\!\!\!\!\!+ SGNT;\ et : E \rightarrow\!\!\!\!\!+ SGET;\ sm, tm : E \rightarrow\!\!\!\!\!+ Mult;\ db : E \rightarrow\!\!\!\!\!+ E \bullet \\
\quad gr\,(G, nt, et, sm, tm, db) = G \\
sg_Ns = Ns \circ gr \\
sg_Es = Es \circ gr \\
sg_src = src \circ gr \\
sg_tgt = tgt \circ gr \\
\forall G : Gr;\ nt : V \rightarrow\!\!\!\!\!+ SGNT;\ et : E \rightarrow\!\!\!\!\!+ SGET;\ sm, tm : E \rightarrow\!\!\!\!\!+ Mult;\ db : E \rightarrow\!\!\!\!\!+ E \bullet \\
\quad nty(G, nt, et, sm, tm, db) = nt \\
\forall G : Gr;\ nt : V \rightarrow\!\!\!\!\!+ SGNT;\ et : E \rightarrow\!\!\!\!\!+ SGET;\ sm, tm : E \rightarrow\!\!\!\!\!+ Mult;\ db : E \rightarrow\!\!\!\!\!+ E \bullet \\
\quad ety(G, nt, et, sm, tm, db) = et \\
\forall G : Gr;\ nt : V \rightarrow\!\!\!\!\!+ SGNT;\ et : E \rightarrow\!\!\!\!\!+ SGET;\ sm, tm : E \rightarrow\!\!\!\!\!+ Mult;\ db : E \rightarrow\!\!\!\!\!+ E \bullet \\
\quad srcm(G, nt, et, sm, tm, db) = sm \\
\forall G : Gr;\ nt : V \rightarrow\!\!\!\!\!+ SGNT;\ et : E \rightarrow\!\!\!\!\!+ SGET;\ sm, tm : E \rightarrow\!\!\!\!\!+ Mult;\ db : E \rightarrow\!\!\!\!\!+ E \bullet \\
\quad tgtm(G, nt, et, sm, tm, db) = tm \\
\forall G : Gr;\ nt : V \rightarrow\!\!\!\!\!+ SGNT;\ et : E \rightarrow\!\!\!\!\!+ SGET;\ sm, tm : E \rightarrow\!\!\!\!\!+ Mult;\ db : E \rightarrow\!\!\!\!\!+ E \bullet \\
\quad derb(G, nt, et, sm, tm, db) = db
\end{array}$$

$$\begin{array}{|l}
\varnothing_{SG} : SGr_0 \\
\hline
\varnothing_{SG} = (\varnothing_G, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing)
\end{array}$$

$$\begin{array}{|l}
NsTy : SGr_0 \rightarrow \mathbb{P}\, SGNT \rightarrow \mathbb{P}\, V \\
EsTy : SGr_0 \rightarrow \mathbb{P}\, SGET \rightarrow \mathbb{P}\, E \\
\hline
\forall SG : SGr_0;\ nts : \mathbb{P}\, SGNT \bullet NsTy\ SG\ nts = (nty\ SG)^{\sim} (\!| nts |\!) \\
\forall SG : SGr_0;\ ets : \mathbb{P}\, SGET \bullet EsTy\ SG\ ets = (ety\ SG)^{\sim} (\!| ets |\!)
\end{array}$$

$$EsA, EsW, EsI, EsC, EsD : SGr_0 \to \mathbb{P}\, E$$

$$\forall\, SG : SGr_0 \bullet EsA\ SG = EsTy\ SG\ (erel\ (\!|\ SGED\ |\!) \cup ecomp\ (\!|\ SGED\ |\!))$$
$$EsW = (\text{flip}\ EsTy)\ \{ewander\}$$
$$EsI = (\text{flip}\ EsTy)\ \{einh\}$$
$$EsD = (\text{flip}\ EsTy)\ \{eder\}$$
$$\forall\, SG : SGr_0 \bullet EsC\ SG = EsA\ SG \cup EsW\ SG \cup EsD\ SG$$

$$NsP, NsEther, NsO, NsV : SGr_0 \to \mathbb{P}\, V$$

$$NsP = (\text{flip}\ NsTy)\ \{nprxy\}$$
$$NsEther = (\text{flip}\ NsTy)\ \{nabst, nvirt, nenum\}$$
$$NsO = (\text{flip}\ NsTy)\ \{nopt\}$$
$$NsV = (\text{flip}\ NsTy)\ \{nvirt\}$$

$$\pitchfork : SGr_0 \to Gr$$
$$\prec : SGr_0 \to V \leftrightarrow V$$

$$\forall\, SG : SGr_0 \bullet \pitchfork\ SG = gr\ SG \rhd\!\!\lhd_{Es}\ EsI\ SG$$
$$\prec\ = (\_^{\leftrightarrow}) \circ \pitchfork$$

$$srcma : SGr_0 \to (E \nrightarrow Mult)$$

$$\forall\, SG : SGr_0 \bullet srcma\ SG =$$
$$(srcm\ SG) \oplus (EsTy\ SG\ \{ecomp\ duni\} \times \{ms(\mathbf{v}\ 1)\}) \oplus (EsTy\ SG\ \{erel\ duni\} \times \{ms\ *\})$$

$\mathbf{relation}(\mathcal{M}etysOk\ \_)$

$$\mathcal{M}etysOk\ \_ : \mathbb{P}\, SGr_0$$

$$\forall\, SG : SGr_0 \bullet \mathcal{M}etysOk\ \ SG \Leftrightarrow \forall\, e : EsC\ SG \bullet (ety\ SG\ e) \propto (srcma\ SG\ e, tgtm\ SG\ e)$$

$$\preceq\ : SGr_0 \to V \leftrightarrow V$$

$$\forall\, SG : SGr_0 \bullet \preceq\ SG = (\prec\ SG)^*$$

$$\begin{array}{|l}
srcr, tgtr : SGr_0 \to E \leftrightarrow V \\
src_0^*, src^*, tgt_0^*, tgt^* : SGr_0 \to E \leftrightarrow V \\
\hline
\forall SG : SGr_0 \bullet srcr\, SG = sg_src\, SG \cup (EsW\ SG \lhd sg_tgt\, SG) \\
\forall SG : SGr_0 \bullet tgtr\, SG = sg_tgt\, SG \cup (EsW\ SG \lhd sg_src\, SG) \\
\forall SG : SGr_0 \bullet src_0^*\ SG = EsC\, SG \lhd (srcr\, SG) \\
\forall SG : SGr_0 \bullet src^*\ SG = (src_0^*\ SG) \fatsemi (\preceq SG)^\sim \\
\forall SG : SGr_0 \bullet tgt_0^*\ SG = EsC\, SG \lhd (tgtr\, SG) \\
\forall SG : SGr_0 \bullet tgt^*\ SG = (tgt_0^*\ SG) \fatsemi (\preceq SG)^\sim
\end{array}$$

**function** 10 **leftassoc** $(\_ \circ\!\!\rightarrow\!\!\circ^* \_)$

$$\begin{array}{|l}
_ \circ\!\!\rightarrow\!\!\circ^* _ : SGr_0 \times \mathbb{P}\, V \to \mathbb{P}\, E \\
\hline
\forall SG : SGr_0;\ vs : \mathbb{P}\, V \bullet SG \circ\!\!\rightarrow\!\!\circ^*\ vs = (src^*\ SG)^\sim (\!| vs |\!) \cup (tgt^*\ SG)^\sim (\!| vs |\!)
\end{array}$$

**relation**(optsVoluntary_)

$$\begin{array}{|l}
\text{optsVoluntary_} : \mathbb{P}\, SGr_0 \\
\hline
\forall SG : SGr_0 \bullet \\
\quad \text{optsVoluntary}\ SG \Leftrightarrow (ety\, SG)\, (\!| (SG \circ\!\!\rightarrow\!\!\circ^*\ (NsO\ SG)) \setminus (EsI\ SG) |\!) \subseteq \{ewander\}
\end{array}$$

**relation**(inhOk_)

$$\begin{array}{|l}
\text{inhOk_} : \mathbb{P}\, SGr_0 \\
\hline
\forall SG : SGr_0 \bullet \text{inhOk}\ SG \\
\quad \Leftrightarrow (\forall v, v' : sg_Ns\, SG \mid (v, v') \in (\prec SG) \bullet nty\, SG\, v \prec_{NT} nty\, SG\, v') \\
\quad \land \oslash(\pitchfork SG) \land ((\pitchfork SG) \ominus_{Ns} (NsV\ SG))^{\leftrightarrow} \in sg_Ns\, SG \nrightarrow sg_Ns\, SG
\end{array}$$

$$SGr == \{SG : SGr_0 \mid \{srcma\, SG, tgtm\, SG\} \subseteq EsC\, SG \to Mult \land \mathrm{dom}(derb\, SG) = EsD\, SG$$
$$\land\ \mathcal{M}etysOk\ SG \land \text{optsVoluntary}\ SG \land \text{inhOk}\ SG\}$$

**relation**(etherealAreInherited_)

$$
\begin{array}{|l}
\text{etherealAreInherited\_} : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \text{etherealAreInherited}\, SG \Leftrightarrow NsEther\, SG \subseteq \operatorname{ran}(\prec SG)
\end{array}
$$

**relation**(derivedOk _ )

$$
\begin{array}{|l}
\text{derivedOk\_} : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \text{derivedOk}\, SG \Leftrightarrow \operatorname{ran}(derb\, SG) \subseteq EsA\, SG \\
\quad \land (\forall\, e : EsD\, SG \bullet (sg\_src\, SG\, e, ((sg\_src\, SG) \circ (derb\, SG))\, e) \in (\preceq SG) \\
\qquad \land (sg\_tgt\, SG\, e, ((sg\_tgt\, SG) \circ (derb\, SG))\, e) \in (\preceq SG))
\end{array}
$$

$TSGr == \{\, SG : SGr \mid \text{etherealAreInherited}\, SG \land \text{derivedOk}\, SG \,\}$

**relation**($\boxminus_{SGs}$ _ )

$$
\begin{array}{|l}
\boxminus_{SGs}\_ : \mathbb{P}(SGr \times SGr) \\
\hline
\forall\, SG_1, SG_2 : SGr \bullet \boxminus_{SGs}(SG_1, SG_2) \Leftrightarrow \boxminus(gr\, SG_1, gr\, SG_2)
\end{array}
$$

**function** 10 **leftassoc** (_ $\cup_{SG}$ _)

$$
\begin{array}{|l}
\_ \cup_{SG} \_ : SGr \times SGr \nrightarrow SGr \\
\hline
\forall\, SG_1, SG_2 : SGr \bullet SG_1 \cup_{SG} SG_2 = (gr\, SG_1 \cup_G gr\, SG_2, nty\, SG_1 \cup nty\, SG_2, \\
\quad ety\, SG_1 \cup ety\, SG_2, srcm\, SG_1 \cup srcm\, SG_2, tgtm\, SG_1 \cup tgtm\, SG_2, derb\, SG_1 \cup derb\, SG_2)
\end{array}
$$

**function** 10 **leftassoc** (_ $\odot^{SG}$ _)

$$
\begin{array}{|l}
\_ \odot^{SG} \_ : SGr \times (V \nrightarrow V) \nrightarrow SGr \\
\hline
\forall\, SG : SGr;\ s : V \nrightarrow V \mid s \in NsP\, SG \nrightarrow sg\_Ns\, SG \bullet \\
\quad SG \odot^{SG} s = (gr\, SG \odot s, (\operatorname{dom} s \setminus \operatorname{ran} s) \lhd nty\, SG, ety\, SG, srcm\, SG, tgtm\, SG, derb\, SG)
\end{array}
$$

**function** 10 **leftassoc** (_ $\rightarrow_{SG}$ _)

$$
\begin{array}{|l}
\_ \rightarrow_{SG} \_ : SGr \times SGr \rightarrow \mathbb{P}\ GrM \\
\hline
\forall\ SG_s, SG_t : SGr \bullet \\
\quad SG_s \rightarrow_{SG} SG_t = \{fv : sg\_Ns\ SG_s \rightarrow sg\_Ns\ SG_t;\ fe : EsC\ SG_s \rightarrow EsC\ SG_t \mid \\
\qquad fv \circ src^*\ SG_s \subseteq src^*\ SG_t \circ fe \wedge fv \circ tgt^*\ SG_s \subseteq tgt^*\ SG_t \circ fe \\
\qquad \wedge\ fv \circ \preceq SG_s \subseteq \preceq SG_t \circ fv\}
\end{array}
$$

**relation**$(\_ \Rrightarrow^{SG} \_)$

$$
\begin{array}{|l}
\_ \Rrightarrow^{SG} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\ SG_s, SG_t : SGr;\ m : GrM \bullet (SG_s, m) \Rrightarrow^{SG} SG_t \Leftrightarrow m \in SG_s \rightarrow_{SG} SG_t
\end{array}
$$

**function** 10 **leftassoc** $(\_ \rightarrow_{G2SG} \_)$

$$
\begin{array}{|l}
\_ \rightarrow_{G2SG} \_ : Gr \times SGr \rightarrow \mathbb{P}\ GrM \\
\hline
\forall\ G : Gr;\ SG : SGr \bullet G \rightarrow_{G2SG} SG = \{fv : Ns\ G \rightarrow sg\_Ns\ SG;\ fe : Es\ G \rightarrow EsC\ SG \mid \\
\quad fv \circ src\ G \subseteq src^*\ SG \circ fe \wedge fv \circ tgt\ G \subseteq tgt^*\ SG \circ fe\}
\end{array}
$$

**relation**$(\_ \Rrightarrow^{GwT} \_)$

$$
\begin{array}{|l}
\_ \Rrightarrow^{GwT} \_ : (GrwT \leftrightarrow SGr) \\
\hline
\forall\ GwT : GrwT;\ SG : SGr \bullet GwT \Rrightarrow^{GwT} SG \Leftrightarrow (ty\ GwT) \in (gOf\ GwT) \rightarrow_{G2SG} SG
\end{array}
$$

$$
\begin{array}{|l}
totaliseForDer : GrM \times SGr \rightarrow GrM \\
\hline
\forall\ m : GrM;\ SG : SGr \bullet totaliseForDer(m, SG) = (fV\ m, ((derb\ SG) \boxdot (EsC\ SG))\ ⨾\ fE\ m)
\end{array}
$$

$$
\begin{array}{|l}
insOf : GrM \times SGr \times \mathbb{P}\ V \rightarrow \mathbb{P}\ V \\
iesOf : GrM \times \mathbb{P}\ E \rightarrow \mathbb{P}\ E \\
igRMEs : GrwT \times \mathbb{P}\ E \rightarrow Gr \\
igRMNsEs : GrwT \times SGr \times \mathbb{P}\ V \times \mathbb{P}\ E \rightarrow Gr \\
\hline
\forall\ m : GrM;\ SG : SGr;\ mns : \mathbb{P}\ V \bullet insOf(m, SG, mns) = (fV\ m)^{\sim} (\!| (\prec SG)^{\sim} (\!| mns |\!) |\!) \\
\forall\ m : GrM;\ mes : \mathbb{P}\ E \bullet iesOf(m, mes) = (fE\ m)^{\sim} (\!| mes |\!) \\
\forall\ GwT : GrwT;\ mes : \mathbb{P}\ E \bullet igRMEs(GwT, mes) = (gOf\ GwT) \bowtie_{Es} iesOf((ty\ GwT), mes) \\
\forall\ GwT : GrwT;\ SG : SGr;\ mns : \mathbb{P}\ V;\ mes : \mathbb{P}\ E \bullet \\
\quad igRMNsEs(GwT, SG, mns, mes) = igRMEs(GwT, mes) \bowtie_{Ns} insOf(ty\ GwT, SG, mns)
\end{array}
$$

**relation**$(\mathrm{inverted_E}\_)$

$$
\begin{array}{|l}
\mathrm{inverted_E}\_ : \mathbb{P}(GrwT \times SGr \times E) \\
gOfwei, igRMEsW : GrwT \times SGr \times E \to Gr \\
gOfweis : GrwT \times SGr \times \mathbb{P}\, E \to Gr \\
\hline
\forall\, G : Gr;\ m : GrM;\ SG : SGr;\ e : E \bullet \\
\quad \mathrm{inverted_E}((G, m), SG, e) \Leftrightarrow ((sg\_tgt\ SG) \circ (fE\ m))e = ((fV\ m) \circ (src\ G))e \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E \bullet \\
\quad gOfwei(GwT, SG, e) = \mathbf{if}\ \mathrm{inverted_E}(GwT, SG, e)\ \mathbf{then}\ ((gOf\ GwT) \rhd\!\!\lhd_{Es} \{e\})^{\rightleftarrows}\ \mathbf{else}(gOf\ GwT) \rhd\!\!\lhd_{Es} \{e\} \\
\forall\, GwT : GrwT;\ SG : SGr \bullet gOfweis(GwT, SG, \{\}) = \varnothing_G \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E;\ es : \mathbb{P}\, E \bullet \\
\quad gOfweis(GwT, SG, \{e\} \cup es) = gOfwei(GwT, SG, e) \cup_G gOfweis(GwT, SG, es) \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E \mid e \in EsD\ SG \bullet igRMEsW(GwT, SG, e) = \\
\quad igRMNsEs(GwT, SG, \{(sg\_src\ SG)(derb\ SG\ e), (sg\_tgt\ SG)(derb\ SG\ e)\}, \{derb\ SG\ e\}) \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E \mid e \notin EsW\ SG \bullet igRMEsW(GwT, SG, e) = igRMEs(GwT, \{e\}) \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E \mid e \in EsW\ SG \bullet igRMEsW(GwT, SG, e) = \\
\quad gOfweis(GwT, SG, ((fE \circ ty)\ GwT)^{\sim} (\!|\{e\}|\!))
\end{array}
$$

**relation**$(\_ \sqsupseteq^{SG} \_)$
**relation**$(\_ \sqsupseteq^{SG_0} \_)$
**relation**$(\_ \sqsupseteq_{NT} \_)$
**relation**$(\_ \sqsupseteq_{ET} \_)$
**relation**$(\_ \sqsupseteq_{\mathcal{M}} \_)$

$$
\begin{array}{|l}
\_ \sqsupseteq_{NT} \_, \_ \sqsupseteq_{ET} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq_{NT} SG_a \Leftrightarrow \forall\, n : sg\_Ns\ SG_c \bullet (nty\ SG_c)\ n \leq_{rNT} ((nty\ SG_a) \circ (fV\ m))\ n \\
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq_{ET} SG_a \Leftrightarrow \forall\, e : EsC\ SG_c \bullet (ety\ SG_c)\ e \leq_{ET} ((ety\ SG_a) \circ (fE\ m))\ e
\end{array}
$$

$$
\begin{array}{|l}
\_ \sqsupseteq_{\mathcal{M}} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq_{\mathcal{M}} SG_a \Leftrightarrow \forall\, e : EsC\ SG_c \setminus EsD\ SG_c \bullet (srcma\ SG_c)\ e \leq_{\mathcal{M}} ((srcma\ SG_a) \circ (fE\ m))\ e \\
\qquad\qquad \wedge\ (tgtm\ SG_c)\ e \leq_{\mathcal{M}} ((tgtm\ SG_a) \circ (fE\ m))\ e
\end{array}
$$

$$\_\sqsupseteq^{SG}\_, \_\sqsupseteq^{SG_0}\_ : \mathbb{P}((SGr \times GrM) \times SGr)$$

$$\forall SG_c, SG_a : SGr;\ m : GrM \bullet$$
$$(SG_c, m) \sqsupseteq^{SG_0} SG_a \Leftrightarrow (SG_c, m) \sqsupseteq_{NT} SG_a \land (SG_c, m) \sqsupseteq_{ET} SG_a \land (SG_c, m) \sqsupseteq_{\mathcal{M}} SG_a$$

$$\forall SG_c, SG_a : SGr;\ m : GrM \bullet$$
$$(SG_c, m) \sqsupseteq^{SG} SG_a \Leftrightarrow \exists m' : GrM \mid m' = totaliseForDer(m, SG_c) \bullet$$
$$m' \in SG_c \to_{SG} SG_a \land (SG_c, m') \sqsupseteq^{SG_0} SG_a$$

**relation**$(\_ \sqsupset^{SG} \_)$
**relation**$(\_ \sqsupset^{SG_0} \_)$
**relation**$(\_ \sqsupset_{AEs} \_)$
**relation**(_OkRefinedIn_)
**relation**$(\_ \sqsupset_{ANNs} \_)$

$$\_\sqsupset_{ANNs}\_ : \mathbb{P}(GrM \times SGr)$$

$$\forall SG_a : SGr;\ m : GrM \bullet$$
$$m \sqsupset_{ANNs} SG_a \Leftrightarrow \forall nn : NsTy\, SG_a\{nnrml\} \bullet (\preceq SG_a) (\!| \{nn\} |\!) \cap \mathrm{ran}(fV\ m) = \varnothing$$

$$\_\mathrm{OkRefinedIn}\_ : \mathbb{P}((SGr \times E) \times (SGr \times GrM))$$
$$\_\sqsupset_{AEs}\_ : \mathbb{P}((SGr \times GrM) \times SGr)$$

$$\forall SG_c, SG_a : SGr;\ m : GrM;\ ae : E \bullet$$
$$(SG_a, ae)\mathrm{OkRefinedIn}(SG_c, m) \Leftrightarrow$$
$$\exists r : V \leftrightarrow V;\ s, t : \mathbb{P}\ V \mid r = (\preceq SG_c) \mathbin{⨾} igRMEsW((gr\ SG_c, m), SG_a, ae)^{\leftrightarrow} \mathbin{⨾} (\preceq SG_c)^{\sim}$$
$$\land\ s = insOf(m, SG_a, (sg\_src\ SG_a (\!| \{ae\} |\!))) \setminus ((NsEther\ SG_c) \setminus \mathrm{dom}\ r)$$
$$\land\ t = insOf(m, SG_a, (sg\_tgt\ SG_a (\!| \{ae\} |\!))) \setminus ((NsEther\ SG_c) \setminus \mathrm{ran}\ r)$$
$$\bullet\ r \in s \leftrightarrow t \land r \neq \varnothing$$

$$\forall SG_c, SG_a : SGr;\ m : GrM \bullet$$
$$(SG_c, m) \sqsupset_{AEs} SG_a \Leftrightarrow \forall e : (EsA\ SG_a) \bullet (SG_a, e)\mathrm{OkRefinedIn}(SG_c, m)$$

$$\_\sqsupset^{SG}\_, \_\sqsupset^{SG_0}\_ : \mathbb{P}((SGr \times GrM) \times SGr)$$

$$\forall SG_c, SG_a : SGr;\ m : GrM \bullet$$
$$(SG_c, m) \sqsupset^{SG_0} SG_a \Leftrightarrow (SG_c, m) \sqsupseteq^{SG_0} SG_a \land m \sqsupset_{ANNs} SG_a \land (SG_c, m) \sqsupset_{AEs} SG_a$$

$$\forall SG_c, SG_a : SGr;\ m : GrM \bullet$$
$$(SG_c, m) \sqsupset^{SG} SG_a \Leftrightarrow \exists m' : GrM \mid m' = totaliseForDer(m, SG_c) \bullet$$
$$m' \in SG_c \to_{SG} SG_a \land (SG_c, m') \sqsupset^{SG_0} SG_a$$

**relation**$(\_ \Supset^{SG} \_)$
**relation**$(\_ \Supset_{\mathcal{M}} \_)$
**relation**$(\_ \Supset_{FI} \_)$
**relation**$(\_ \Supset_{PNS} \_)$
**relation**$(\_\ ME\mathcal{M}Ok\ \_)$

$$
\begin{array}{l}
\_\,\mathit{ME}\mathcal{M}\mathit{Ok}\,\_ : \mathbb{P}((\mathit{SGr} \times E) \times \mathit{GrwT}) \\
\hline
\forall \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr};\ \mathit{me} : E \bullet (\mathit{SG}, \mathit{me})\ \mathit{ME}\mathcal{M}\mathit{Ok}\ \mathit{GwT} \Leftrightarrow \\
\quad \exists\, r : V \leftrightarrow V;\ s, t : \mathbb{P}\ V \mid r = \mathit{igRMEsW}(\mathit{GwT}, \mathit{SG}, \mathit{me})^{\leftrightarrow} \\
\qquad \land\ s = \mathit{insOf}(\mathit{ty}\ \mathit{GwT}, \mathit{SG}, (\mathit{src}^*\ \mathit{SG})\,(\!|\,\{\mathit{me}\}\,|\!)) \\
\qquad \land\ t = \mathit{insOf}(\mathit{ty}\ \mathit{GwT}, \mathit{SG}, (\mathit{tgt}^*\ \mathit{SG})\,(\!|\,\{\mathit{me}\}\,|\!)) \\
\qquad\quad \bullet\ r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{srcma}\ \mathit{SG}\ \mathit{me}, \mathit{tgtm}\ \mathit{SG}\ \mathit{me})
\end{array}
$$

$$
\begin{array}{l}
\_ \ni_{\mathcal{M}} \_ : \mathit{GrwT} \leftrightarrow \mathit{SGr} \\
\_ \ni_{FI} \_ : \mathit{GrwT} \leftrightarrow \mathit{SGr} \\
\_ \ni_{PNS} \_ : \mathit{GrwT} \leftrightarrow \mathit{SGr} \\
\hline
\forall \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr} \bullet \mathit{GwT} \ni_{\mathcal{M}} \mathit{SG} \Leftrightarrow \forall\, \mathit{me} : \mathit{EsC}\ \mathit{SG} \bullet (\mathit{SG}, \mathit{me})\ \mathit{ME}\mathcal{M}\mathit{Ok}\ \mathit{GwT} \\
\forall \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr} \bullet \mathit{GwT} \ni_{FI} \mathit{SG} \Leftrightarrow ((\mathit{fV} \circ \mathit{ty})\,\mathit{GwT})^{\sim}\,(\!|\mathit{NsEther}\ \mathit{SG}\,|\!) = \varnothing \\
\forall \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr} \bullet \\
\quad \mathit{GwT} \ni_{PNS} \mathit{SG} \Leftrightarrow \mathit{igRMEs}(\mathit{GwT}, \mathit{EsTy}\ \mathit{SG}\ \{\mathit{ecomp}\ \mathit{dbi}, \mathit{ecomp}\ \mathit{duni}\})^{\leftrightarrow} \in \mathrm{injrel}
\end{array}
$$

$$
\begin{array}{l}
\_ \ni^{SG} \_ : \mathit{GrwT} \leftrightarrow \mathit{SGr} \\
\hline
\forall \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr} \bullet \\
\quad \mathit{GwT} \ni^{SG} \mathit{SG} \Leftrightarrow \mathit{GwT} \Rrightarrow^{\mathit{GwT}} \mathit{SG} \land \mathit{GwT} \ni_{\mathcal{M}} \mathit{SG} \land \mathit{GwT} \ni_{FI} \mathit{SG} \land \mathit{GwT} \ni_{PNS} \mathit{SG}
\end{array}
$$

# 7 Fragments

**section** *Fragmenta_Frs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_SGs*, *Fragmenta_GrswT*

$$
\begin{array}{l}
\mathit{Fr}_0 == \{\mathit{SG} : \mathit{SGr}_0;\ \mathit{esr} : \mathbb{P}\,E;\ \mathit{sr}, \mathit{tr} : E \nrightarrow V \mid \mathit{esr} \cap (\mathit{sg\_Es}\ \mathit{SG}) = \varnothing \\
\quad \land\ \mathit{sr} \in \mathit{esr} \rightarrowtail\!\!\!\rightarrow (\mathit{NsP}\ \mathit{SG}) \land \mathit{tr} \in \mathit{esr} \rightarrow (V \setminus (\mathit{NsO}\ \mathit{SG}))\}
\end{array}
$$

$$
\begin{array}{l}
\mathit{fSG} : \mathit{Fr}_0 \rightarrow \mathit{SGr} \\
\mathit{EsR} : \mathit{Fr}_0 \rightarrow \mathbb{P}\,E \\
\mathit{srcR}, \mathit{tgtR} : \mathit{Fr}_0 \rightarrow E \nrightarrow V \\
\hline
\forall \mathit{SG} : \mathit{SGr};\ \mathit{esr} : \mathbb{P}\,E;\ \mathit{sr}, \mathit{tr} : E \nrightarrow V \bullet \mathit{fSG}(\mathit{SG}, \mathit{esr}, \mathit{sr}, \mathit{tr}) = \mathit{SG} \\
\forall \mathit{SG} : \mathit{SGr};\ \mathit{esr} : \mathbb{P}\,E;\ \mathit{sr}, \mathit{tr} : E \nrightarrow V \bullet \mathit{EsR}(\mathit{SG}, \mathit{esr}, \mathit{sr}, \mathit{tr}) = \mathit{esr} \\
\forall \mathit{SG} : \mathit{SGr};\ \mathit{esr} : \mathbb{P}\,E;\ \mathit{sr}, \mathit{tr} : E \nrightarrow V \bullet \mathit{srcR}(\mathit{SG}, \mathit{esr}, \mathit{sr}, \mathit{tr}) = \mathit{sr} \\
\forall \mathit{SG} : \mathit{SGr};\ \mathit{esr} : \mathbb{P}\,E;\ \mathit{sr}, \mathit{tr} : E \nrightarrow V \bullet \mathit{tgtR}(\mathit{SG}, \mathit{esr}, \mathit{sr}, \mathit{tr}) = \mathit{tr}
\end{array}
$$

$$
\begin{array}{|l}
\mathit{fLEs}, \mathit{fEs}, \mathit{fEsC} : \mathit{Fr}_0 \rightarrow \mathbb{P}\, E \\
\mathit{fLNs}, \mathit{fRNs}, \mathit{fNs} : \mathit{Fr}_0 \rightarrow \mathbb{P}\, V \\
\mathit{srcF}, \mathit{tgtF} : \mathit{Fr}_0 \rightarrow E \nrightarrow V \\
\hline
\mathit{fLEs} = (\mathit{sg\_Es} \circ \mathit{fSG}) \\
\forall F : \mathit{Fr}_0 \bullet \mathit{fEs}\, F = \mathit{fLEs}\, F \cup \mathit{EsR}\, F \\
\mathit{fEsC} = \mathit{EsC} \circ \mathit{fSG} \\
\mathit{fLNs} = \mathit{sg\_Ns} \circ \mathit{fSG} \\
\mathit{fRNs} = \operatorname{ran} \circ \mathit{tgtR} \\
\forall F : \mathit{Fr}_0 \bullet \mathit{fNs}\, F = \mathit{fLNs}\, F \cup \mathit{fRNs}\, F \\
\forall F : \mathit{Fr}_0 \bullet \mathit{srcF}\, F = (\mathit{sg\_src} \circ \mathit{fSG})\, F \cup \mathit{srcR}\, F \\
\forall F : \mathit{Fr}_0 \bullet \mathit{tgtF}\, F = (\mathit{sg\_tgt} \circ \mathit{fSG})\, F \cup \mathit{tgtR}\, F
\end{array}
$$

$$
\begin{array}{|l}
\overset{G}{\leftrightsquigarrow} : \mathit{Fr}_0 \rightarrow \mathit{Gr} \\
\leftrightsquigarrow : \mathit{Fr}_0 \rightarrow V \leftrightarrow V \\
\hline
\forall F : \mathit{Fr}_0 \bullet \overset{G}{\leftrightsquigarrow}\, F = ((\mathit{NsP} \circ \mathit{fSG}) F \cup \mathit{fRNs}\, F, \mathit{EsR}\, F, \mathit{srcR}\, F, \mathit{tgtR}\, F) \\
\forall F : \mathit{Fr}_0 \bullet \leftrightsquigarrow\, F = (\overset{G}{\leftrightsquigarrow}\, F)^{\leftrightarrow}
\end{array}
$$

**function** 10 **leftassoc** $(\_ \cup_F \_)$

$$
\begin{array}{|l}
\varnothing_F : \mathit{Fr}_0 \\
\_ \cup_F \_ : \mathit{Fr}_0 \times \mathit{Fr}_0 \rightarrow \mathit{Fr}_0 \\
\bigcup_F : \mathbb{P}\, \mathit{Fr}_0 \rightarrow \mathit{Fr}_0 \\
\hline
\varnothing_F = (\varnothing_{SG}, \varnothing, \varnothing, \varnothing) \\
\forall F_1, F_2 : \mathit{Fr}_0 \bullet F_1 \cup_F F_2 = \\
\quad (\mathit{fSG}\, F_1 \cup_{SG} \mathit{fSG}\, F_2, \mathit{EsR}\, F_1 \cup \mathit{EsR}\, F_2, \mathit{srcR}\, F_1 \cup \mathit{srcR}\, F_2, \mathit{tgtR}\, F_1 \cup \mathit{tgtR}\, F_2) \\
\bigcup_F \{\} = \varnothing_F \\
\forall F : \mathit{Fr}_0;\ \mathit{Fs} : \mathbb{P}\, \mathit{Fr}_0 \bullet \bigcup_F (\{F\} \cup \mathit{Fs}) = F \cup_F (\bigcup_F \mathit{Fs})
\end{array}
$$

$$
\begin{array}{|l}
\rightsquigarrow\ : Fr_0 \twoheadrightarrow V \twoheadrightarrow V \\
\circledbullet^{SG} : Fr_0 \twoheadrightarrow SGr \\
rEsR : Fr_0 \twoheadrightarrow \mathbb{P}\, E \\
\circledbullet : Fr_0 \twoheadrightarrow Fr_0 \\
\hline
\forall\, F : Fr_0 \bullet \rightsquigarrow F = (\leftrightsquigarrow F) \rhd (fLNs\ F) \\
\forall\, F : Fr_0 \bullet \circledbullet^{SG} F = (fSG\ F) \odot^{SG} (\rightsquigarrow F) \\
\forall\, F : Fr_0 \bullet rEsR\ F = \mathrm{dom}((srcR\ F) \unrhd \mathrm{dom}(\rightsquigarrow F)) \\
\forall\, F : Fr_0 \bullet \circledbullet F = (\circledbullet^{SG} F, rEsR\ F, (rEsR\ F) \lhd (srcR\ F), (rEsR\ F) \lhd (tgtR\ F))
\end{array}
$$

$Fr_a == \{F : Fr_0 \mid \oslash(\overset{G}{\leftrightsquigarrow} F)\}$

$Fr == \{F : Fr_a \mid \circledbullet^{SG} F \in SGr\}$

**relation**(refsLocal_)

$$
\begin{array}{|l}
\mathrm{refsLocal\_} : \mathbb{P}\, Fr_0 \\
\hline
\forall\, F : Fr_0 \bullet \mathrm{refsLocal}\ F \Leftrightarrow fRNs\ F \subseteq fLNs\ F
\end{array}
$$

$TFr == \{F : Fr_a \mid \mathrm{refsLocal}\ F \wedge \circledbullet^{SG} F \in TSGr\}$

**relation**($\boxminus$ _)
**relation**($\boxplus$ _)

$$
\begin{array}{|l}
\boxminus\_ : Fr \leftrightarrow Fr \\
\hline
\forall\, F_1, F_2 : Fr \bullet \boxminus(F_1, F_2) \Leftrightarrow fLNs\ F_1 \cap fLNs\ F_2 = \varnothing \wedge fEs\ F_1 \cap fEs\ F_2 = \varnothing
\end{array}
$$

$$
\begin{array}{|l|}
\hline
[I] \\
\hline
\boxplus\_ : \mathbb{P}(I \twoheadrightarrow Fr) \\
\hline
\forall\, Fs : I \twoheadrightarrow Fr \bullet \boxplus\ Fs \Leftrightarrow \forall\, i, j : \mathrm{dom}\, Fs \mid i \neq j \bullet \boxminus(Fs\ i, Fs\ j) \\
\hline
\end{array}
$$

**relation**(_ $\subseteq^{rs}$ _)
**relation**(_ $\Mapsto$ _)

$$\_ \subseteq^{rs} \_ : Fr \leftrightarrow Fr$$
$$\_ \mapsto \_ : Fr \leftrightarrow Fr$$

$$\forall F_1, F_2 : Fr \bullet F_1 \subseteq^{rs} F_2 \Leftrightarrow \operatorname{ran}(tgtR\ F_1) \cap fLNs\ F_2 \neq \varnothing$$
$$\forall F_1, F_2 : Fr \bullet F_1 \mapsto F_2 \Leftrightarrow F_1 \subseteq^{rs} F_2 \land \neg\, (F_2 \subseteq^{rs} F_1)$$

**function** 10 **leftassoc** $(\_ \rightrightarrows_\bullet \_)$

$$\_ \rightrightarrows_\bullet \_ : GrM \times (Fr \times Fr) \to GrM$$

$$\forall m : GrM;\ F_s, F_t : Fr_0 \bullet$$
$$m \rightrightarrows_\bullet (F_s, F_t) = (((\rightsquigarrow F_s)^{\oplus} \boxdot (fLNs\ F_s))^{\sim} \mathbin{;} (fV\ m) \mathbin{;} ((\rightsquigarrow F_t)^{\oplus} \boxdot (fLNs\ F_t)), fE\ m)$$

**function** 1 **leftassoc** $(\_ \to_F \_)$

$$\_ \to_F \_ : Fr \times Fr \to \mathbb{P}\ GrM$$

$$\forall F_s, F_t : Fr \bullet F_s \to_F F_t = \{fv : fLNs\ F_s \to fLNs\ F_t;\ fe : fEsC\ F_s \to fEsC\ F_t \mid$$
$$(\circledbullet^{SG} F_s, (fv, fe) \rightrightarrows_\bullet (F_s, F_t)) \Rrightarrow^{SG} \circledbullet^{SG} F_t\}$$

**relation** $(\_ \Rrightarrow^F \_)$

$$\_ \Rrightarrow^F \_ : (Fr \times GrM) \leftrightarrow Fr$$

$$\forall m : GrM;\ F_s, F_t : Fr_0 \bullet (F_s, m) \Rrightarrow^F F_t \Leftrightarrow m \in F_s \to_F F_t$$

**relation** $(\_ \sqsupseteq^F \_)$

$$\_ \sqsupseteq^F \_ : (Fr \times GrM) \leftrightarrow Fr$$

$$\forall F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupseteq^F F_a \Leftrightarrow (F_c, m) \Rrightarrow^F F_a$$
$$\land\ (\circledbullet^{SG} F_c, m \rightrightarrows_\bullet (F_c, F_a)) \sqsupseteq^{SG_0} (\circledbullet^{SG} F_a)$$

**relation** $(\_ \sqsupset^F \_)$

$$
\begin{array}{|l}
\_ \sqsupset^{F} \_ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupset^{F} F_a \Leftrightarrow (F_c, m) \Rrightarrow^{F} F_a \\
\quad \land\ (\circledbullet^{SG} F_c, m \rightrightarrows_{\bullet} (F_c, F_a)) \sqsupset^{SG_0} (\circledbullet^{SG} F_a)
\end{array}
$$

**relation**$(\_ \Supset^{F} \_)$

$$
\begin{array}{|l}
\_ \Supset^{F} \_ : GrwT \leftrightarrow Fr \\
\hline
\forall GwT : GrwT;\ F : Fr \bullet GwT \Supset^{F} F \Leftrightarrow GwT \Supset^{SG} \circledbullet^{SG} F
\end{array}
$$

## 8 Global Fragment Graphs

**section** *Fragmenta_GFGs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$GFGr == \{G : Gr \mid \oslash(G \bowtie_{Es} (Es\ G \setminus EsId\ G))\}$

**function**$(\_^{\dashrightarrow})$

$$
\begin{array}{|l}
\_^{\dashrightarrow} : GFGr \to V \leftrightarrow V \\
\hline
\forall GFG : GFGr \bullet GFG^{\dashrightarrow} = (GFG^{\leftrightarrow})^{+}
\end{array}
$$

## 9 Models

**section** *Fragmenta_Mdls* **parents** *standard_toolkit*, *Fragmenta_Frs*, *Fragmenta_GFGs*

$Mdl_0 == \{GFG : GFGr;\ fd : V \nrightarrow Fr \mid fd \in Ns\ GFG \to Fr \land \boxplus\ fd\}$

$$
\begin{array}{|l}
mGFG : Mdl_0 \to GFGr \\
mFD : Mdl_0 \to V \nrightarrow Fr \\
\hline
\forall GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mGFG(GFG, fd) = GFG \\
\forall GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mFD(GFG, fd) = fd
\end{array}
$$

$$mUFs : Mdl_0 \rightarrow Fr$$
$$mUFs = \bigcup_F \circ \operatorname{ran} \circ mFD$$

$$\text{from} : Mdl_0 \rightarrow V \nrightarrow V$$
$$\forall M : Mdl_0;\ v : V \bullet \text{from}\ M\ v = (\mu\, vf : (Ns \circ mGFG)M \mid v \in fLNs(mFD\ M\ vf))$$

**relation**$(\Uparrow \_)$

$$\Uparrow \_ : \mathbb{P}\, Mdl_0$$
$$\forall M : Mdl_0 \bullet \Uparrow M \Leftrightarrow \exists\, UF : Fr_0 \mid UF = mUFs\ M \bullet$$
$$\quad \forall p : (NsP \circ fSG)\, UF \bullet (from\ M\ p, from\ M\ (\leftrightsquigarrow\ UF\ p)) \in ((\_^{\dashrightarrow}) \circ mGFG)M$$

$$Mdl == \{M : Mdl_0 \mid (mUFs\ M) \in TFr \wedge \Uparrow M\}$$

$$\odot^M : Mdl \rightarrow Fr$$
$$\forall M : Mdl_0 \bullet \odot^M = \odot \circ mUFs$$

**function** 1 **leftassoc** $(\_ \rightarrow_M \_)$
**relation**$(\_ \Rrightarrow^M \_)$

$$\_ \rightarrow_M \_ : Mdl \times Mdl \rightarrow \mathbb{P}\, GrM$$
$$\_ \Rrightarrow^M \_ : \mathbb{P}((Mdl \times \mathbb{P}\, GrM) \times Mdl)$$
$$\forall M_s, M_t : Mdl \bullet M_s \rightarrow_M M_t = \{m : GrM \mid$$
$$\quad \exists\, UF_s, UF_t : Fr_0 \mid UF_s = mUFs\ M_s \wedge UF_t = mUFs\ M_t \bullet m \in UF_s \rightarrow_F UF_t\}$$
$$\forall M_s, M_t : Mdl;\ ms : \mathbb{P}\, GrM \bullet (M_s, ms) \Rrightarrow^M M_t \Leftrightarrow \bigcup_{GM} ms \in M_s \rightarrow_M M_t$$

**relation**$(\_ \sqsupset^M \_)$

$$\_ \sqsupset^M \_ : (Mdl \times \mathbb{P}\, GrM) \leftrightarrow Mdl$$
$$\forall M_c, M_a : Mdl_0;\ ms : \mathbb{P}\, GrM \bullet (M_c, ms) \sqsupset^M M_a$$
$$\quad \Leftrightarrow \exists\, UF_c, UF_a : Fr_0 \mid UF_c = mUFs\ M_c \wedge UF_a = mUFs\ M_a \bullet (UF_c, \bigcup_{GM} ms) \sqsupset^F UF_a$$

**relation**$( \_ \Supset^M \_ )$

$$\_ \Supset^M \_ : GrwT \leftrightarrow Mdl$$

$$\forall\, GwT : GrwT;\ M : Mdl \bullet GwT \Supset^M M \Leftrightarrow GwT \Supset^F mUFs\, M$$