Z Specification of Fragmenta

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Contents

1 Generics

section *Fragmenta_Generics* **parents** *standard_toolkit*

$\mid\quad \mathbb{R} : \mathbb{P}\,\mathbb{A}$

$acyclic[X] == \{r : X \leftrightarrow X \mid r^{+} \cap \operatorname{id} X = \varnothing\}$

$connected[X] == \{r : X \leftrightarrow X \mid \forall\, x : \operatorname{dom} r;\ y : \operatorname{ran} r \bullet x \mapsto y \in r^{+}\}$

$tree[X] == \{r : X \leftrightarrow X \mid r \in \text{acyclic} \wedge r \in X \nrightarrow X\}$

$forest[X] == \{r : X \leftrightarrow X \mid r \in \text{acyclic} \wedge (\forall\, s : X \leftrightarrow X \mid s \subseteq r \wedge s \in connected \bullet s \in tree)\}$

$injrel[X, Y] == \{r : X \leftrightarrow Y \mid r^{\sim} \in Y \nrightarrow X\}$

$antireflexive[X] == \{r : X \leftrightarrow X \mid r \cap (id\, X) = \varnothing\}$

$$
\begin{array}{l}
\underline{\quad[X, Y, Z]} \\
\quad \operatorname{flip} : (X \to Y \nrightarrow Z) \to (Y \to X \nrightarrow Z) \\
\overline{} \\
\quad \forall\, f : X \to Y \nrightarrow Z \bullet \operatorname{flip} f = (\lambda\, y : Y \bullet \lambda\, x : X \bullet f\, x\, y) \\
\end{array}
$$

$$
\begin{array}{l}
\underline{\quad[X, Y, Z, W]} \\
\quad \operatorname{apply} : (X \to Z) \to (Y \to W) \to (X \times Y) \to (Z \times W) \\
\overline{} \\
\quad \forall\, f : X \to Z;\ g : Y \to W;\ x : X;\ y : Y \bullet \operatorname{apply} f\, g\, (x, y) = (f\, x, g\, y) \\
\end{array}
$$

relation$(\_ =_{p} \_)$
relation$(\_ \subseteq_{p} \_)$

$$
\begin{array}{l}
\underline{\quad[X, Y]} \\
\quad \_ =_{p} \_ : \mathbb{P}((\mathbb{P}\, X \times \mathbb{P}\, Y) \times (\mathbb{P}\, X \times \mathbb{P}\, Y)) \\
\quad \_ \subseteq_{p} \_ : \mathbb{P}((\mathbb{P}\, X \times \mathbb{P}\, Y) \times (\mathbb{P}\, X \times \mathbb{P}\, Y)) \\
\overline{} \\
\quad \forall\, xs, zs : \mathbb{P}\, X;\ ys, ws : \mathbb{P}\, Y \bullet (xs, ys) =_{p} (zs, ws) \Leftrightarrow xs = zs \wedge ys = ws \\
\\
\quad \forall\, xs, zs : \mathbb{P}\, X;\ ys, ws : \mathbb{P}\, Y \bullet (xs, ys) \subseteq_{p} (zs, ws) \Leftrightarrow xs \subseteq zs \wedge ys \subseteq ws \\
\end{array}
$$

$$
\begin{array}{|l}
\hline [X, Y] \\
\hline
\quad \mathrm{map} : (X \to Y) \to \mathbb{P}\, X \to \mathbb{P}\, Y \\
\quad \mathrm{mapS} : (X \to Y) \to \mathrm{seq}\, X \to \mathrm{seq}\, Y \\
\hline
\quad \forall f : X \to Y \bullet \mathrm{map}\, f\{\} = \{\} \\[4pt]
\quad \forall f : X \to Y;\; x : X;\; xs : \mathbb{P}\, X \bullet \mathrm{map}\, f(\{x\} \cup xs) = \{f\, x\} \cup (\mathrm{map}\, f\, xs) \\[4pt]
\quad \forall f : X \to Y \bullet \mathrm{mapS}\, f\langle\rangle = \langle\rangle \\[4pt]
\quad \forall f : X \to Y;\; x : X;\; xs : \mathrm{seq}\, X \bullet \mathrm{mapS}\, f(\langle x\rangle \frown xs) = \langle f\, x\rangle \frown (\mathrm{mapS}\, f\, xs) \\
\hline
\end{array}
$$

function 10 **leftassoc** $(\_ \, \boxdot \, \_)$

$$
\begin{array}{|l}
\hline [X] \\
\hline
\quad \_ \, \boxdot \, \_ : ((X \to X) \times \mathbb{P}\, X) \to (X \nrightarrow X) \\
\hline
\quad \forall f : X \to X;\; s : \mathbb{P}\, X \bullet f \boxdot s = (\mathrm{id}\; s) \oplus f \\
\hline
\end{array}
$$

function$(\_^{\oplus})$

$$
\begin{array}{|l}
\hline [X] \\
\hline
\quad \_^{\oplus} : (X \leftrightarrow X) \to (X \leftrightarrow X) \\
\hline
\quad \forall r : X \leftrightarrow X \bullet r^{\oplus} = \textbf{if }\, r \oplus r \,\mathbin{\fatsemi}\, r = r \,\textbf{ then }\, r \,\textbf{ else }\, (r \oplus r \,\mathbin{\fatsemi}\, r)^{\oplus} \\
\hline
\end{array}
$$

$\mathrm{opt}[X] == \{s : \mathbb{P}\, X \mid \#\, s \le 1\}$

$$
\begin{array}{|l}
\hline [X] \\
\hline
\quad \mathrm{the} : \mathrm{opt}[X] \nrightarrow X \\
\hline
\quad \forall x : X \bullet \mathrm{the}\, \{x\} = x \\
\hline
\end{array}
$$

$$
\begin{array}{|l}
\hline [X, Y] \\
\hline
\quad \mathit{flatten} : (X \nrightarrow \mathbb{P}\, Y) \to (X \leftrightarrow Y) \\
\hline
\quad \forall f : X \nrightarrow \mathbb{P}\, Y \bullet \mathit{flatten}\, f = \{x : \mathrm{dom}\, f;\; y : Y \mid y \in f\, x\} \\
\hline
\end{array}
$$

2 Graphs

section *Fragmenta_Graphs* **parents** *standard_toolkit, Fragmenta_Generics*

$[V, E]$

$Gr == \{vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s, t : E \nrightarrow V \mid s \in es \rightarrow vs \wedge t \in es \rightarrow vs\}$

$Ns : Gr \rightarrow \mathbb{P}\, V$
$Es : Gr \rightarrow \mathbb{P}\, E$
$src, tgt : Gr \rightarrow E \nrightarrow V$

$\forall\, vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \nrightarrow V;\ t : E \nrightarrow V \bullet Ns(vs, es, s, t) = vs$

$\forall\, vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \nrightarrow V;\ t : E \nrightarrow V \bullet Es(vs, es, s, t) = es$

$\forall\, vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \nrightarrow V;\ t : E \nrightarrow V \bullet src(vs, es, s, t) = s$

$\forall\, vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \nrightarrow V;\ t : E \nrightarrow V \bullet tgt(vs, es, s, t) = t$

$nNatS, nIntS, nRealS : V$
$to\mathbb{Z} : V \rightarrow \mathrm{opt}[\mathbb{Z}]$
$to\mathbb{R} : V \rightarrow \mathrm{opt}[\mathbb{R}]$

$\varnothing_G : Gr$

$\varnothing_G = (\varnothing, \varnothing, \varnothing, \varnothing)$

$Els : Gr \rightarrow (\mathbb{P}\, V \times \mathbb{P}\, E)$
$EsId : Gr \rightarrow \mathbb{P}\, E$

$\forall\, G : Gr \bullet Els\ G = (Ns\ G, Es\ G)$

$\forall\, G : Gr \bullet EsId\ G = \{e : Es\ G \mid src\ G\ e = tgt\ G\ e\}$

---

relation$(adjacent\ \_)$

$adjacent\_ : \mathbb{P}(Gr \times V \times V)$

$\forall\, G : Gr;\ v_1, v_2 : V \bullet adjacent(G, v_1, v_2) \Leftrightarrow \exists\, e : Es\ G \bullet src\ G\ e = v_1 \wedge tgt\ G\ e = v_2$

relation($adjacent_E \ \_$)

$$
\begin{array}{|l}
adjacent_E \ \_ : \mathbb{P}(Gr \times E \times E) \\
\hline
\forall\, e_1, e_2 : E;\ G : Gr \bullet adjacent_E(G, e_1, e_2) \Leftrightarrow tgt\ G\ e_1 = src\ G\ e_2
\end{array}
$$

function 10 **leftassoc** ($\_ \circ\!\!\twoheadrightarrow_{Es} \_$)

$$
\begin{array}{|l}
\_ \circ\!\!\twoheadrightarrow_{Es} \_ : Gr \times \mathbb{P}\ V \to \mathbb{P}\ E \\
\hline
\forall\, G : Gr;\ vs : \mathbb{P}\ V \bullet G \circ\!\!\twoheadrightarrow_{Es} vs = (src\ G)^{\sim}(\!|vs|\!) \cup (tgt\ G)^{\sim}(\!|vs|\!)
\end{array}
$$

function 10 **leftassoc** ($\_ \bullet\!\!\leftrightarrow\!\!\bullet \_$)

$$
\begin{array}{|l}
\_ \bullet\!\!\leftrightarrow\!\!\bullet \_ : Gr \times \mathbb{P}\ V \to \mathbb{P}\ E \\
\hline
\forall\, G : Gr;\ vs : \mathbb{P}\ V \bullet G \bullet\!\!\leftrightarrow\!\!\bullet vs = (src\ G)^{\sim}(\!|vs|\!) \cap (tgt\ G)^{\sim}(\!|vs|\!)
\end{array}
$$

function 10 **leftassoc** ($\_ \circ\!\!\twoheadrightarrow_{Ns} \_$)

$$
\begin{array}{|l}
\_ \circ\!\!\twoheadrightarrow_{Ns} \_ : Gr \times \mathbb{P}\ E \to \mathbb{P}\ V \\
\hline
\forall\, G : Gr;\ es : \mathbb{P}\ E \bullet G \circ\!\!\twoheadrightarrow_{Ns} es = \mathrm{ran}(es \lhd src\ G) \cup \mathrm{ran}(es \lhd tgt\ G)
\end{array}
$$

function 10 **leftassoc** ($\_ \bowtie_{Es} \_$)

$$
\begin{array}{|l}
\_ \bowtie_{Es} \_ : Gr \times \mathbb{P}\ E \to Gr \\
\hline
\forall\, G : Gr;\ es : \mathbb{P}\ E \bullet G \bowtie_{Es} es = (G \circ\!\!\twoheadrightarrow_{Ns} es, Es\ G \cap es, es \lhd src\ G, es \lhd tgt\ G)
\end{array}
$$

function 10 **leftassoc** ($\_ \bowtie_{Ns} \_$)

$$\underline{} \bowtie_{Ns} \underline{} : Gr \times \mathbb{P}\ V \to Gr$$

$$\forall\, G : Gr;\ vs : \mathbb{P}\ V \bullet$$
$$\quad G \bowtie_{Ns}\ vs = (Ns\ G \cap vs,\ G \bullet\!\!\leftrightarrow\!\!\bullet\ vs,\ (G \bullet\!\!\leftrightarrow\!\!\bullet\ vs) \lhd src\ G,\ (G \bullet\!\!\leftrightarrow\!\!\bullet\ vs) \lhd tgt\ G)$$

function 10 **leftassoc** $(\underline{} \ominus_{Ns} \underline{})$

$$\underline{} \ominus_{Ns} \underline{} : Gr \times \mathbb{P}\ V \to Gr$$

$$\forall\, G : Gr;\ vs : \mathbb{P}\ V \bullet$$
$$\quad G \ominus_{Ns}\ vs = (Ns\ G \setminus vs,\ Es\ G \setminus (G \circ\!\!\rightarrow\!\!\circ_{Es}\ vs),\ (G \circ\!\!\rightarrow\!\!\circ_{Es}\ vs) \lhd src\ G,\ (G \circ\!\!\rightarrow\!\!\circ_{Es}\ vs) \lhd tgt\ G)$$

$$successors : V \times Gr \to \mathbb{P}\ V$$

$$\forall\, v : V;\ G : Gr \bullet successors(v, G) = \{ v_1 : Ns\ G \mid adjacent(G, v, v_1) \}$$

function$(\underline{}^{\rightleftarrows})$

$$\underline{}^{\rightleftarrows} : Gr \to Gr$$

$$\forall\, G : Gr \bullet G^{\rightleftarrows} = (Ns\ G,\ Es\ G,\ tgt\ G,\ src\ G)$$

function$(\underline{}^{\leftrightarrow})$

$$\underline{}^{\leftrightarrow} : Gr \to V \leftrightarrow V$$

$$\forall\, G : Gr \bullet G^{\leftrightarrow} = \{ v_1, v_2 : Ns\ G \mid adjacent(G, v_1, v_2) \}$$

function$(\underline{}^{\leftrightarrow_E})$

$$\underline{}^{\leftrightarrow_E} : Gr \to E \leftrightarrow E$$

$$\forall\, G : Gr \bullet G^{\leftrightarrow_E} = \{ e_1, e_2 : Es\ G \mid adjacent_E(G, e_1, e_2) \}$$

relation$(\oslash\ \underline{})$

$$\begin{array}{l} \oslash\_ : \mathbb{P}\, Gr \\ \hline \forall\, G : Gr \bullet \oslash\, G \Leftrightarrow G^{\leftrightarrow_E} \in \mathrm{acyclic} \end{array}$$

relation($\boxminus_{Es}\_$)
relation($\boxminus\_$)

$$\begin{array}{l} \boxminus_{Es}\_, \boxminus\_ : \mathbb{P}(Gr \times Gr) \\ \hline \forall\, G_1, G_2 : Gr \bullet \boxminus_{Es}(G_1, G_2) \Leftrightarrow Es\, G_1 \cap Es\, G_2 = \varnothing \\ \forall\, G_1, G_2 : Gr \bullet \boxminus(G_1, G_2) \Leftrightarrow Ns\, G_1 \cap Ns\, G_2 = \varnothing \wedge \boxminus_{Es}(G_1, G_2) \end{array}$$

relation($\boxplus\_$)

$$\begin{array}{l} {}=[I]=\!\!=\!\!=\!\!=\!\!=\!\!=\!\!=\!\!=\!\!=\!\!=\!\!=\!\!=\!\!=\!\!=\!\!=\!\!=\!\!= \\ \boxplus\_ : \mathbb{P}(I \nrightarrow Gr) \\ \hline \forall\, Gs : I \nrightarrow Gr \bullet \boxplus\, Gs \Leftrightarrow \forall\, i, j : \mathrm{dom}\ Gs \mid i \neq j \bullet \boxminus(Gs\, i, Gs\, j) \end{array}$$

function 10 **leftassoc** ($\_ \cup_G \_$)

$$\begin{array}{l} \_ \cup_G \_ : Gr \times Gr \to Gr \\ \hline \forall\, G_1, G_2 : Gr \bullet G_1 \cup_G G_2 = (Ns\, G_1 \cup Ns\, G_2, Es\, G_1 \cup Es\, G_2, src\, G_1 \cup src\, G_2, tgt\, G_1 \cup tgt\, G_2) \end{array}$$

function 10 **leftassoc** ($\_ \odot \_$)

$$\begin{array}{l} \_ \odot \_ : Gr \times (V \leftrightarrow V) \nrightarrow Gr \\ \hline \forall\, G : Gr;\ s : V \leftrightarrow V \mid s \in Ns\, G \nrightarrow Ns\, G \wedge s \in \mathrm{antireflexive} \bullet \\ \quad G \odot s = (Ns\, G \setminus \mathrm{dom}\, s, Es\, G, (s \boxdot Ns\, G) \circ (src\, G), (s \boxdot Ns\, G) \circ (tgt\, G)) \end{array}$$

$$GrM == (V \nrightarrow V) \times (E \nrightarrow E)$$

$$fV : GrM \to V \nrightarrow V$$
$$fE : GrM \to E \nrightarrow E$$

$$\forall\, fv : V \nrightarrow V;\ fe : E \nrightarrow E \bullet fV(fv, fe) = fv$$

$$\forall\, fv : V \nrightarrow V;\ fe : E \nrightarrow E \bullet fE(fv, fe) = fe$$

$$\mathrm{gid} : Gr \to GrM$$

$$\forall\, G : Gr \bullet \mathrm{gid}\ G = (id\,(Ns\ G), id\,(Es\ G))$$

$$\varnothing_{GM} : GrM$$

$$\varnothing_{GM} = (\{\}, \{\})$$

$$\mathrm{domg}, \mathrm{codg} : GrM \to (\mathbb{P}\ V \times \mathbb{P}\ E)$$

$$\forall\, m : GrM \bullet \mathrm{domg}\ m = (\mathrm{dom}(fV\ m), \mathrm{dom}(fE\ m))$$

$$\forall\, m : GrM \bullet \mathrm{codg}\ m = (\mathrm{ran}(fV\ m), \mathrm{ran}(fE\ m))$$

function 10 **leftassoc** $(\_ \cup_{GM} \_)$

$$\_ \cup_{GM} \_ : GrM \times GrM \nrightarrow GrM$$
$$\bigcup\nolimits_{GM} : \mathbb{P}\ GrM \to GrM$$

$$\forall\, f, g : GrM \bullet f \cup_{GM} g = (fV\ f \cup fV\ g, fE\ f \cup fE\ g)$$

$$\bigcup\nolimits_{GM} \varnothing = \varnothing_{GM}$$

$$\forall\, f : GrM;\ fs : \mathbb{P}\ GrM \bullet \bigcup\nolimits_{GM} (\{f\} \cup fs) = f \cup_{GM} (\bigcup\nolimits_{GM} fs)$$

function 10 **leftassoc** $(\_ \to_G \_)$

$$\_ \to_G \_ : Gr \times Gr \to \mathbb{P}\ GrM$$

$$\forall\, G_1, G_2 : Gr \bullet G_1 \to_G G_2 = \{fv : Ns\ G_1 \to Ns\ G_2;\ fe : Es\ G_1 \to Es\ G_2 \mid$$
$$src\ G_2 \circ fe = fv \circ src\ G_1 \wedge tgt\ G_2 \circ fe = fv \circ tgt\ G_1\}$$

function 10 **leftassoc** $(\_ \nrightarrow_G \_)$

$$\_ \twoheadrightarrow_G \_ : Gr \times Gr \to \mathbb{P}\, GrM$$

$$\forall\, G_1, G_2 : Gr \bullet G_1 \to_G G_2 = \{fv : Ns\ G_1 \twoheadrightarrow Ns\ G_2;\ fe : Es\ G_1 \twoheadrightarrow Es\ G_2 \mid$$
$$src\ G_2 \circ fe = fv \circ ((\mathrm{dom}\, fe) \lhd (src\ G_1)) \wedge tgt\ G_2 \circ fe = fv \circ ((\mathrm{dom}\, fe) \lhd (tgt\ G_1))\}$$

function 10 **leftassoc** $(\_ \circ_G \_)$

$$\_ \circ_G \_ : GrM \times GrM \to GrM$$

$$\forall\, g, f : GrM \bullet g \circ_G f = (fV\ g \circ fV\ f, fE\ g \circ fE\ f)$$

3 Graphs with typing

section *Fragmenta_GrswT* **parents** *standard_toolkit, Fragmenta_Generics, Fragmenta_Graphs*

$$GrwT == \{G : Gr;\ t : GrM \mid \mathrm{domg}\ t =_p Els\ G\}$$

function$(\_^G)$
function$(\_^T)$

$$\_^G : GrwT \to Gr$$
$$\_^T : GrwT \to GrM$$

$$\forall\, G : Gr;\ t : GrM \bullet (G, t)^G = G$$

$$\forall\, G : Gr;\ t : GrM \bullet (G, t)^T = t$$

$$\varnothing_{GwT} : GrwT$$

$$\varnothing_{GwT} = (\varnothing_G, \varnothing_{GM})$$

function 10 **leftassoc** $(\_ \cup_{GwT} \_)$

$$\_ \cup_{GwT} \_ : GrwT \times GrwT \to GrwT$$

$$\forall\, G_1, G_2 : GrwT \bullet G_1 \cup_{GwT} G_2 = (G_1{}^G \cup_G G_2{}^G, G_1{}^T \cup_{GM} G_2{}^T)$$

4 SG Element Types

section *Fragmenta_SGElemTys* **parents** *standard_toolkit*, *Fragmenta_Generics*

$SGNT ::= nnrml \mid nabst \mid nprxy \mid nenum \mid nval \mid nvirt$
$SGED ::= dbi \mid duni$
$SGET ::= einh \mid ecomp\langle\!\langle SGED \rangle\!\rangle \mid erel\langle\!\langle SGED \rangle\!\rangle \mid eder \mid epath \mid evcnt$

relation$(\_ \prec_{NT} \_)$

$\prec_{NT}$, an ordering relation on $SGNT$, indicates the node types that may be in an inheritance relation:

$$\_ \prec_{NT} \_ : SGNT \leftrightarrow SGNT$$
$$\forall nt_1, nt_2 : SGNT \bullet nt_1 \prec_{NT} nt_2 \Leftrightarrow nt_1 \neq nprxy \wedge nt_2 \neq nval$$
$$\wedge\ nt_1 = nabst \Rightarrow nt_2 \in \{nabst, nvirt, nprxy\} \wedge (nt_1 = nenum \Rightarrow nt_2 \in \{nvirt, nprxy\})$$

Above, $\prec_{NT}$ stipulates that (i) proxies must not inherit, (ii) values must not be inherited, (iii) abstract nodes may inherit from abstract and virtuals only, (iv) enumerations may only inherit virtuals or proxies

$\leq_{rNT}$ and $\leq_{ET}$ are ordering relations on sets $SGNT$ and $SGET$, respectively, indicating node and edge types that can be refinement-related, respectively:

relation$(\_ \leq_{rNT} \_)$

$$\_ \leq_{rNT} \_ : SGNT \leftrightarrow SGNT$$
$$\forall nt_1, nt_2 : SGNT \bullet nt_1 \leq_{rNT} nt_2 \Leftrightarrow nt_1 = nt_2 \vee nt_1 = nprxy \vee nt_1 = nnrml \wedge nt_2 = nprxy$$
$$\vee\ nt_2 \in \{nabst, nvirt\} \wedge nt_1 \in \{nnrml, nvirt, nabst\} \vee nt_2 = nnrml$$

relation$(\_ =_{ET} \_)$

$$\_ =_{ET} \_ : SGET \leftrightarrow SGET$$
$$\forall et_1, et_2 : SGET \bullet et_1 =_{ET} et_2 \Leftrightarrow et_1 = et_2$$
$$\vee\ (\forall d_1, d_2 : SGED \bullet et_1 = erel\ d_1 \wedge et_2 = erel\ d_2 \vee et_1 = ecomp\ d_1 \wedge et_2 = ecomp\ d_2)$$

relation$(\_ \leq_{ET} \_)$

$$\_ \leq_{ET} \_ : SGET \leftrightarrow SGET$$
$$\forall et_1, et_2 : SGET \bullet et_1 \leq_{ET} et_2 \Leftrightarrow \neg\ \{einh, eder, epath, evcnt\} \subseteq \{et_1, et_2\}$$
$$\wedge\ (et_1 =_{ET} et_2 \vee et_1 \in \mathrm{dom}(ecomp^{\sim}) \wedge et_2 = erel\ dbi$$
$$\vee\ et_1 = ecomp\ duni \wedge et_2 = erel\ duni)$$

5 Multiplicities

section *Fragmenta_Mult* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_SGElemTys*

$MultVal ::= \mathbf{v}\langle\!\langle \mathbb{N} \rangle\!\rangle \mid *$

$MultC ::= mr\langle\!\langle \mathbb{N} \times MultVal \rangle\!\rangle \mid ms\langle\!\langle MultVal \rangle\!\rangle$

relation($\_ =_{mv} \_$)

$\_ =_{mv} \_ : MultVal \leftrightarrow MultVal$

$\forall\, m_1, m_2 : MultVal \bullet m_1 =_{mv} m_2 \Leftrightarrow \{m_1, m_2\} \subseteq \{*\} \vee \exists\, n : \mathbb{N} \bullet m_1 = \mathbf{v}\ n \wedge m_2 = \mathbf{v}\ n$

function 10 **leftassoc** ($\_ *_{mv} \_$)

$\_ *_{mv} \_ : MultVal \times MultVal \rightarrow MultVal$

$\forall\, m : MultVal \bullet * *_{mv} m = *$

$\forall\, m : MultVal \bullet m *_{mv} * = *$

$\forall\, n_1, n_2 : \mathbb{N} \bullet (\mathbf{v}\ n_1) *_{mv} (\mathbf{v}\ n_2) = \mathbf{v}(n_1 * n_2)$

$mlbn : MultC \rightarrow \mathbb{N}$
$mlb, mub : MultC \rightarrow MultVal$

$mlbn(ms\, *) = 0$

$\forall\, k : \mathbb{N} \bullet mlbn(ms(\mathbf{v}\ k)) = k$

$\forall\, k, m : \mathbb{N} \bullet mlbn(mr(k, \mathbf{v}\ m)) = k$

$\forall\, mc : MultC \bullet mlb\ mc = \mathbf{v}(mlbn\ mc)$

$\forall\, mv : MultVal \bullet mub(ms\ mv) = mv$

$\forall\, k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ m$

function 10 **leftassoc** ($\_ *_{mr} \_$)

$$\_ *_{mr} \_ : MultC \times MultC \nrightarrow MultC$$

$$\forall\, n_1, n_2 : \mathbb{N};\ mv_1, mv_2 : MultVal \bullet mr(n_1, mv_1) *_{mr} mr(n_2, mv_2) = mr(n_1 * n_2, mv_1 *_{mv} mv_2)$$

relation$(\_ \leq_{mv} \_)$

$$\_ \leq_{mv} \_ : MultVal \leftrightarrow MultVal$$

$$\forall\, m_1, m_2 : MultVal \bullet m_1 =_{mv} m_2 \Leftrightarrow \{m_1, m_2\} \subseteq \{*\}$$
$$\qquad \lor ((mlb \circ ms)\, m_1 = (mlb \circ ms)\, m_2 \land (mub \circ ms)\, m_1 = (mub \circ ms)\, m_2)$$
$$\forall\, m_1, m_2 : MultVal \bullet m_1 \leq_{mv} m_2 \Leftrightarrow m_2 = * \lor \exists\, j, k : \mathbb{N} \mid m_1 = \mathbf{v}\, j \land m_2 = \mathbf{v}\, k \bullet j \leq k$$

$MultCOk == \{m : MultC \mid \exists\, lb : \mathbb{N};\ ub : MultVal \bullet m = mr(lb, ub) \land \mathbf{v}\, lb \leq_{mv} ub$
$\quad \lor \exists\, mv : MultVal \bullet m = ms\, mv\, \}$

$MultCMany == \{ms *, mr(0, *)\}$

$Mult == \{\, ms : \mathbb{P}_1\, MultCOk \mid \#\, ms > 1 \Rightarrow (\forall\, m : ms \bullet m \notin MultCMany)\,\}$

$$mk : \mathbb{N} \to MultC$$
$$mks : \mathbb{N} \to Mult$$
$$mrs : \mathbb{N} \times MultVal \to Mult$$
$$mopt : MultC$$
$$mopts, mmanys : Mult$$

$$\forall\, k : \mathbb{N} \bullet mk\, k = ms(\mathbf{v}\, k) \land mks\, k = \{mk\, k\}$$

$$mopt = mr(0, \mathbf{v}\, 1) \land mopts = \{mopt\} \land mmanys = \{ms *\}$$

$$\forall\, lb : \mathbb{N};\ ub : MultVal \bullet mrs(lb, ub) = \{mr(lb, ub)\}$$

$MultMany == \{\, s : Mult \mid \exists\, m : MultC \bullet s = \{m\} \land m \in MultCMany\,\}$

$MultRange == \{s : Mult \mid \exists\, m : MultC \bullet s = \{m\} \land (\exists\, k : \mathbb{N} \mid k > 1 \bullet m = ms(\mathbf{v}\, k)$
$\quad \lor \exists\, lb : \mathbb{N};\ umv : MultVal \mid \mathbf{v}\, 2 \leq_{mv} umv \bullet m = mr(lb, umv))\}$

$MultEither == \{s : Mult \mid \#\, s > 1\}$

$MultLBZ == \{ms : Mult \mid \exists\, m : ms \bullet mlbn\, m = 0\}$

relation$(\_ \between \_)$
relation$(\_ \cdots \_)$

$\_ \between \_ : \mathbb{P}(\mathbb{N} \times \mathit{MultC})$

$\forall k : \mathbb{N};\ m : \mathit{MultC} \bullet k \between m \Leftrightarrow \mathit{mlb}\ m \leq_{mv} \mathbf{v}\ k \wedge \mathbf{v}\ k \leq_{mv} \mathit{mub}\ m$

$\_ \cdots \_ : \mathbb{P}(\mathbb{N} \times \mathit{Mult})$

$\forall k : \mathbb{N};\ m : \mathit{MultC} \bullet k \cdots \{\, m \,\} \Leftrightarrow k \between m$
$\forall k : \mathbb{N};\ m : \mathit{MultC};\ sms : \mathit{Mult} \bullet k \cdots (\{\, m \,\} \cup sms) \Leftrightarrow k \between m \vee k \cdots sms$

relation$(\_ \leq_{\mathcal{M}c} \_)$

$\_ \leq_{\mathcal{M}c} \_ : \mathit{MultC} \leftrightarrow \mathit{MultC}$

$\forall m_1, m_2 : \mathit{MultC} \bullet m_1 \leq_{\mathcal{M}c} m_2 \Leftrightarrow \mathit{mlb}\ m_2 \leq_{mv} \mathit{mlb}\ m_1 \wedge \mathit{mub}\ m_1 \leq_{mv} \mathit{mub}\ m_2$

relation$(\_ \leq_{\mathcal{M}} \_)$

$\_ \leq_{\mathcal{M}} \_ : \mathit{Mult} \leftrightarrow \mathit{Mult}$

$\forall m_1, m_2 : \mathit{Mult} \bullet m_1 \leq_{\mathcal{M}} m_2 \Leftrightarrow \forall mc_1 : m_1 \bullet \exists mc_2 : m_2 \bullet mc_1 \leq_{\mathcal{M}c} mc_2$

relation$(\_ \propto \_)$

$\_ \propto \_ : \mathbb{P}(\mathit{SGET} \times (\mathit{Mult} \times \mathit{Mult}))$

$\forall et : \mathit{SGET};\ m_1, m_2 : \mathit{Mult} \bullet et \propto (m_1, m_2) \Leftrightarrow et = \mathit{erel}\ \mathit{dbi} \vee et = \mathit{eder}$
$\quad \vee\ et = \mathit{ecomp}\ \mathit{duni} \wedge m_1 = \mathit{mks}\ 1$
$\quad \vee\ et = \mathit{erel}\ \mathit{duni} \wedge m_1 \in \mathit{MultMany}$
$\quad \vee\ et = \mathit{ecomp}\ \mathit{dbi} \wedge m_1 \in \{\mathit{mks}\ 1, \mathit{mopts}\}$

relation$(\mathrm{rbounded}\_)$
relation$(\mathrm{eitherbounded}\_)$

$[X, Y]$
$\mathrm{rbounded}\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\,X \times \mathit{MultC})$
$\mathrm{eitherbounded}\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\,X \times \mathit{Mult})$

$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ m : \mathit{MultC} \bullet$
$\quad \mathrm{rbounded}(r, s, m) \Leftrightarrow \forall x : s \bullet \#(r \,(\!|\, \{x\} \,|\!)\,) \between m$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ ms : \mathit{Mult} \bullet$
$\quad \mathrm{eitherbounded}(r, s, ms) \Leftrightarrow \forall x : s \bullet \#(r \,(\!|\, \{x\} \,|\!)\,) \cdots ms$

13

relation$(r\mathcal{M}Ok\_)$

$[X, Y]$

$r\mathcal{M}Ok\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\,X \times \mathbb{P}\,Y \times Mult \times Mult)$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : Mult \bullet r\mathcal{M}Ok(r, s, t, mks\,0, m) \Leftrightarrow s \lhd r = \{\}$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : Mult \bullet r\mathcal{M}Ok(r, s, t, m, mks\,0) \Leftrightarrow r \rhd t = \{\}$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet r\mathcal{M}Ok(r, s, t, mks\,1, mks\,1) \Leftrightarrow r \in s \rightarrowtail t$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet r\mathcal{M}Ok(r, s, t, mopts, mks\,1) \Leftrightarrow r \in s \rightarrowtail t$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet r\mathcal{M}Ok(r, s, t, mks\,1, mopts) \Leftrightarrow r^{\sim} \in t \rightarrowtail s$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany \bullet r\mathcal{M}Ok(r, s, t, mm, mks\,1) \Leftrightarrow r \in s \rightarrow t$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany \bullet r\mathcal{M}Ok(r, s, t, mks\,1, mm) \Leftrightarrow r^{\sim} \in t \rightarrow s$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mr : MultRange \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mr, mks\,1) \Leftrightarrow r \in s \rightarrow t \land \text{rbounded}(r^{\sim}, t, \text{the}\, mr)$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mr : MultRange \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mks\,1, mr) \Leftrightarrow r^{\sim} \in t \rightarrow s \land \text{rbounded}(r, s, \text{the}\, mr)$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ me : MultEither \bullet$
$\quad r\mathcal{M}Ok(r, s, t, me, mks\,1) \Leftrightarrow r \in s \rightarrow t \land \text{eitherbounded}(r^{\sim}, t, me)$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ me : MultEither \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mks\,1, me) \Leftrightarrow r^{\sim} \in t \rightarrow s \land \text{eitherbounded}(r, s, me)$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet r\mathcal{M}Ok(r, s, t, mopts, mopts) \Leftrightarrow r \in s \leftrightarrow\!\!\!\rightarrow t$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany \bullet r\mathcal{M}Ok(r, s, t, mm, mopts) \Leftrightarrow r \in s \twoheadrightarrow t$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mopts, mm) \Leftrightarrow r^{\sim} \in t \twoheadrightarrow s$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : MultRange \bullet$
$\quad r\mathcal{M}Ok(r, s, t, m, mopts) \Leftrightarrow r \in s \twoheadrightarrow t \land \text{rbounded}(r^{\sim}, t, \text{the}\, m)$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : MultRange \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mopts, m) \Leftrightarrow r^{\sim} \in t \twoheadrightarrow s \land \text{rbounded}(r, s, \text{the}\, m)$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : MultEither \bullet$
$\quad r\mathcal{M}Ok(r, s, t, m, mopts) \Leftrightarrow r \in s \twoheadrightarrow t \land \text{eitherbounded}(r^{\sim}, t, m)$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : MultEither \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mopts, m) \Leftrightarrow r^{\sim} \in t \twoheadrightarrow s \land \text{eitherbounded}(r, s, m)$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm_1, mm_2 : MultMany \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mm_1, mm_2) \Leftrightarrow r \in s \leftrightarrow t$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany;\ mr : MultRange \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mm, mr) \Leftrightarrow r \in s \leftrightarrow t \land \text{rbounded}(r, s, \text{the}\, mr)$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany;\ mr : MultRange \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mr, mm) \Leftrightarrow r \in s \leftrightarrow t \land \text{rbounded}(r^{\sim}, t, \text{the}\, mr)$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany;\ me : MultEither \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mm, me) \Leftrightarrow r \in s \leftrightarrow t \land \text{eitherbounded}(r, s, me)$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany;\ me : MultEither \bullet$
$\quad r\mathcal{M}Ok(r, s, t, me, mm) \Leftrightarrow r \in s \leftrightarrow t \land \text{eitherbounded}(r^{\sim}, t, me)$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mr_1, mr_2 : MultRange \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mr_1, mr_2) \Leftrightarrow r \in s \leftrightarrow t \land \text{rbounded}(r, s, \text{the}\, mr_2) \land \text{rbounded}(r^{\sim}, t, \text{the}\, mr_1)$

$\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ me_1, me_2 : MultEither \bullet$
$\quad r\mathcal{M}Ok(r, s, t, me_1, me_2) \Leftrightarrow r \in s \leftrightarrow t \land \text{eitherbounded}(r, s, me_2) \land \text{eitherbounded}(r^{\sim}, t, me_1)$

6 Path Expressions

section *Fragmenta_PEs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$PEA ::= edg\langle\!\langle E \rangle\!\rangle \mid einv\langle\!\langle E \rangle\!\rangle$

$PEC ::= eat\langle\!\langle PEA \rangle\!\rangle \mid eresd\langle\!\langle V \times PEA \rangle\!\rangle \mid eresr\langle\!\langle PEA \times V \rangle\!\rangle$

$PE ::= ec\langle\!\langle PEC \rangle\!\rangle \mid scmp\langle\!\langle PEC \times PE \rangle\!\rangle$

$ePEA : PEA \to E$

$\forall\, e : E \bullet ePEA(edg\ e) = e$

$\forall\, e : E \bullet ePEA(einv\ e) = e$

$startEA_C : PEC \to PEA$

$startEA : PE \to PEA$

$\forall\, pea : PEA \bullet startEA_C(eat\ pea) = pea$

$\forall\, v : V;\ pea : PEA \bullet startEA_C(eresd\ (v, pea)) = pea$

$\forall\, v : V;\ pea : PEA \bullet startEA_C(eresr\ (pea, v)) = pea$

$\forall\, pec : PEC \bullet startEA(ec\ pec) = startEA_C\ pec$

$\forall\, pec : PEC;\ pe : PE \bullet startEA(scmp\ (pec, pe)) = startEA_C\ pec$

$endEA_C : PEC \to PEA$

$endEA : PE \to PEA$

$\forall\, pea : PEA \bullet endEA_C(eat\ pea) = pea$

$\forall\, v : V;\ pea : PEA \bullet endEA_C(eresd\ (v, pea)) = pea$

$\forall\, v : V;\ pea : PEA \bullet endEA_C(eresr\ (pea, v)) = pea$

$\forall\, pec : PEC \bullet endEA(ec\ pec) = endEA_C(pec)$

$\forall\, pec : PEC;\ pe : PE \bullet endEA(scmp\ (pec, pe)) = endEA\ pe$

$$srcPEA : Gr \rightarrow PEA \rightarrow V$$
$$srcPEC : Gr \rightarrow PEC \rightarrow V$$
$$srcPE : Gr \rightarrow PE \rightarrow V$$

$\forall\, G : Gr;\ e : E \bullet srcPEA\, G\,(edg\ e) = src\ G\ e$

$\forall\, G : Gr;\ e : E \bullet srcPEA\, G\,(einv\ e) = tgt\ G\ e$

$\forall\, G : Gr;\ v : V;\ pea : PEA \bullet srcPEC\, G\,(eat\ pea) = srcPEA\, G\ pea$

$\forall\, G : Gr;\ v : V;\ pea : PEA \bullet srcPEC\, G\,(eresd\,(v, pea)) = srcPEA\, G\ pea$

$\forall\, G : Gr;\ v : V;\ pea : PEA \bullet srcPEC\, G\,(eresr\,(pea, v)) = srcPEA\, G\ pea$

$\forall\, G : Gr;\ pec : PEC \bullet srcPE\, G\,(ec\ pec) = srcPEC\, G\ pec$

$\forall\, G : Gr;\ pec : PEC;\ pe : PE \bullet srcPE\, G\,(scmp\,(pec, pe)) = srcPEC\, G\ pec$

$$tgtPEA : Gr \rightarrow PEA \rightarrow V$$
$$tgtPEC : Gr \rightarrow PEC \rightarrow V$$
$$tgtPE : Gr \rightarrow PE \rightarrow V$$

$\forall\, G : Gr;\ e : E \bullet tgtPEA\, G\,(edg\ e) = tgt\ G\ e$

$\forall\, G : Gr;\ e : E \bullet tgtPEA\, G\,(einv\ e) = src\ G\ e$

$\forall\, G : Gr;\ v : V;\ pea : PEA \bullet tgtPEC\, G\,(eat\ pea) = tgtPEA\, G\ pea$

$\forall\, G : Gr;\ v : V;\ pea : PEA \bullet tgtPEC\, G\,(eresd\,(v, pea)) = tgtPEA\, G\ pea$

$\forall\, G : Gr;\ v : V;\ pea : PEA \bullet tgtPEC\, G\,(eresr\,(pea, v)) = tgtPEA\, G\ pea$

$\forall\, G : Gr;\ pec : PEC \bullet tgtPE\, G\,(ec\ pec) = tgtPEC\, G\ pec$

$\forall\, G : Gr;\ pec : PEC;\ pe : PE \bullet tgtPE\, G\,(scmp\,(pec, pe)) = tgtPE\, G\ pe$

$$rsrcPE : PE \rightarrow E$$
$$rtgtPE : PE \rightarrow E$$

$\forall\, pe : PE \bullet rsrcPE\ pe = ePEA\,(startEA\ pe)$

$\forall\, pe : PE \bullet rtgtPE\ pe = ePEA\,(endEA\ pe)$

7 Structural Graphs

section *Fragmenta_SGs* **parents** *standard_toolkit, Fragmenta_Generics, Fragmenta_Graphs, Fragmenta_SGElemTys, Fragmenta_Mult, Fragmenta_PEs, Fragmenta_GrswT*

Z Type *VCI* represents the information associated with a value constraint edge: an operator such as equality (*SGVCEOP*) and an optional edge, which is either one of the edges or the node or none if the constraint edge refers to the values of the node itself.

$SGVCEOP ::= eq \mid neq \mid leq \mid geq \mid lt \mid gt$

$VCI == SGVCEOP \times opt[E]$

$SGr_0 == \{ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;$
$\qquad d : E \leftrightarrow E;\ vci : E \nrightarrow VCI \mid nt \in Ns\ G \rightarrow SGNT \wedge et \in Es\ G \rightarrow SGET \wedge d \in Es\ G \leftrightarrow Es\ G \}$

$gr : SGr_0 \rightarrow Gr$
$sg\_Ns : SGr_0 \rightarrow \mathbb{P}\ V$
$sg\_Es : SGr_0 \rightarrow \mathbb{P}\ E$
$sg\_src, sg\_tgt : SGr_0 \rightarrow E \nrightarrow V$
$nty : SGr_0 \rightarrow V \nrightarrow SGNT$
$ety : SGr_0 \rightarrow E \nrightarrow SGET$
$srcm, tgtm : SGr_0 \rightarrow E \nrightarrow Mult$
$pe : SGr_0 \rightarrow E \nrightarrow PE$
$ds : SGr_0 \rightarrow E \leftrightarrow E$
$vcei : SGr_0 \rightarrow E \nrightarrow SGVCEOP \times opt[E]$

$\forall\ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E;$
$\quad vci : E \nrightarrow VCI \bullet gr\,(G, nt, et, sm, tm, p, d, vci) = G$
$sg\_Ns = Ns \circ gr$
$sg\_Es = Es \circ gr$
$sg\_src = src \circ gr$
$sg\_tgt = tgt \circ gr$
$\forall\ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E;$
$\quad vci : E \nrightarrow VCI \bullet nty(G, nt, et, sm, tm, p, d, vci) = nt$
$\forall\ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E;$
$\quad vci : E \nrightarrow VCI \bullet ety(G, nt, et, sm, tm, p, d, vci) = et$
$\forall\ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E;$
$\quad vci : E \nrightarrow VCI \bullet srcm(G, nt, et, sm, tm, p, d, vci) = sm$
$\forall\ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E;$
$\quad vci : E \nrightarrow VCI \bullet tgtm(G, nt, et, sm, tm, p, d, vci) = tm$
$\forall\ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E;$
$\quad vci : E \nrightarrow VCI \bullet pe(G, nt, et, sm, tm, p, d, vci) = p$
$\forall\ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E;$
$\quad vci : E \nrightarrow VCI \bullet ds(G, nt, et, sm, tm, p, d, vci) = d$
$\forall\ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E;$
$\quad vci : E \nrightarrow VCI \bullet vcei(G, nt, et, sm, tm, p, d, vci) = vci$

$\varnothing_{SG} : SGr_0$

$\varnothing_{SG} = (\varnothing_G, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing)$

$NsTy : SGr_0 \rightarrow \mathbb{P}\ SGNT \rightarrow \mathbb{P}\ V$
$EsTy : SGr_0 \rightarrow \mathbb{P}\ SGET \rightarrow \mathbb{P}\ E$

$\forall\ SG : SGr_0;\ nts : \mathbb{P}\ SGNT \bullet NsTy\ SG\ nts = (nty\ SG)^{\sim}\,(\!|nts|\!)$

$\forall\ SG : SGr_0;\ ets : \mathbb{P}\ SGET \bullet EsTy\ SG\ ets = (ety\ SG)^{\sim}\,(\!|ets|\!)$

$EsA, EsI, EsM, EsD, EsVCnt, EsPa, EsPaCnt : SGr_0 \to \mathbb{P}\, E$

$EsA = (\text{flip } EsTy)\,(erel\, (\!|\, SGED\, |\!)\, \cup ecomp\, (\!|\, SGED\, |\!))$

$EsI = (\text{flip } EsTy)\, \{einh\}$

$EsD = (\text{flip } EsTy)\, \{eder\}$

$EsPa = (\text{flip } EsTy)\, \{epath\}$

$EsVCnt = (\text{flip } EsTy)\, \{evcnt\}$

$\forall\, SG : SGr_0 \bullet EsM\ SG = EsA\ SG \cup EsD\ SG$

$\forall\, SG : SGr_0 \bullet EsPaCnt\ SG = EsD\ SG \cup EsPa\ SG$

$NsN, NsP, NsEther, NsVi, NsVa : SGr_0 \to \mathbb{P}\, V$

$NsN = (\text{flip } NsTy)\, \{nnrml\}$

$NsP = (\text{flip } NsTy)\, \{nprxy\}$

$NsEther = (\text{flip } NsTy)\, \{nabst, nvirt, nenum\}$

$NsVi = (\text{flip } NsTy)\, \{nvirt\}$

$NsVa = (\text{flip } NsTy)\, \{nval\}$

$tpe : SGr_0 \to E \nrightarrow PE$

$\forall\, SG : SGr_0 \bullet tpe\ SG = \{e : sg\_Es\ SG \bullet (e, (ec \circ eat \circ edg)\ e)\} \oplus pe\ SG$

$\Vdash : SGr_0 \to Gr$

$\prec\, : SGr_0 \to V \leftrightarrow V$

$\forall\, SG : SGr_0 \bullet \Vdash SG = gr\ SG \bowtie_{Es} EsI\ SG$

$\prec\, = (\_ \overset{\leftrightarrow}{}) \circ \Vdash$

$srcma : SGr_0 \to (E \nrightarrow Mult)$

$\forall\, SG : SGr_0 \bullet srcma\ SG =$
  $(srcm\ SG) \oplus (EsTy\ SG\, \{ecomp\ duni\} \times \{mks\ 1\}) \oplus (EsTy\ SG\, \{erel\ duni\} \times \{\{ms * \}\})$

relation$(\mathcal{M}etysOk\,\_)$

$$\mathcal{M}etysOk\_ : \mathbb{P}\,SGr_0$$

$$\forall\,SG : SGr_0 \bullet \mathcal{M}etysOk\ SG \Leftrightarrow \forall\,e : EsM\ SG \bullet (ety\ SG\ e) \propto (srcma\ SG\ e, tgtm\ SG\ e)$$

$$\preceq : SGr_0 \rightarrow V \leftrightarrow V$$

$$\forall\,SG : SGr_0 \bullet \preceq SG = (\prec SG)^*$$

$$rsrc, rtgt : SGr_0 \rightarrow \mathbb{P}\,E \rightarrow E \leftrightarrow V$$
$$src_M^*, src^*, tgt_M^*, tgt^* : SGr_0 \rightarrow E \leftrightarrow V$$

$$\forall\,SG : SGr_0;\ es : \mathbb{P}\,E \bullet rsrc\ SG\ es = es \lhd (sg\_src\ SG)$$

$$\forall\,SG : SGr_0;\ es : \mathbb{P}\,E \bullet rtgt\ SG\ es = es \lhd (sg\_tgt\ SG)$$

$$\forall\,SG : SGr_0 \bullet src_M^*\ SG = (rsrc\ SG\ (EsA\ SG)) \,\fatsemi\, (\preceq SG)^{\sim}$$

$$\forall\,SG : SGr_0 \bullet src^*\ SG = (rsrc\ SG\ (EsA\ SG \cup EsPaCnt\ SG)) \,\fatsemi\, (\preceq SG)^{\sim}$$

$$\forall\,SG : SGr_0 \bullet tgt_M^*\ SG = (rtgt\ SG\ (EsA\ SG)) \,\fatsemi\, (\preceq SG)^{\sim}$$

$$\forall\,SG : SGr_0 \bullet tgt^*\ SG = (rtgt\ SG\ (EsA\ SG \cup EsPaCnt\ SG)) \,\fatsemi\, (\preceq SG)^{\sim}$$

Predicate VCntEsOk says whether the value constraint edges of a SG are well-formed.

relation(VCntEsOk_)

$$VCntEsOk\_ : \mathbb{P}\,SGr_0$$

$$\forall\,SG : SGr_0 \bullet VCntEsOk\ SG \Leftrightarrow vcei\ SG \in EsVCnt\ SG \rightarrow VCI$$
$$\wedge\ \bigcup(map\ second\,((\mathrm{ran} \circ vcei)\ SG)) \subseteq EsA\ SG$$
$$\wedge\ (sg\_tgt\ SG)\ (\!|\ EsVCnt\ SG\ |\!) \subseteq NsVa\ SG \cup NsP\ SG$$

relation($inhOk\_$)

$$inhOk\_ : \mathbb{P}\,SGr_0$$

$$\forall\,SG : SGr_0 \bullet inhOk\ SG$$
$$\Leftrightarrow (\forall\,v, v' : sg\_Ns\ SG \mid (v, v') \in (\prec SG) \bullet nty\ SG\ v \prec_{NT} nty\ SG\ v') \wedge \oslash(\pitchfork SG)$$

$$SGr == \{SG : SGr_0 \mid \{srcma\ SG, tgtm\ SG\} \subseteq EsM\ SG \rightarrow Mult \wedge (\mathrm{dom} \circ pe)\ SG = EsPaCnt\ SG$$
$$\wedge\ ds\ SG \in \mathrm{antireflexive}[EsPa\ SG]$$
$$\wedge\ \mathcal{M}etysOk\ SG \wedge inhOk\ SG \wedge VCntEsOk\ SG\ \}$$

relation(etherealAreInherited $\_$)

$\quad$ etherealAreInherited $\_$: $\mathbb{P}$ SGr_0
$\quad$ ────────────────────────────
$\quad$ $\forall\, SG : SGr_0 \bullet$ etherealAreInherited $SG \Leftrightarrow NsEther\ SG \subseteq \mathrm{ran}(\prec SG)$

relation($derInhOk\ \_$)

$\quad$ $derInhOk\ \_$: $\mathbb{P}$ SGr_0
$\quad$ ────────────────────────────
$\quad$ $\forall\, SG : SGr_0 \bullet derInhOk\ SG \Leftrightarrow \forall\, e : EsD\ SG \bullet$
$\quad\quad$ $(sg\_src\ SG\ e, srcPE(gr\ SG \bowtie_{Es}\ EsA\ SG)(pe\ SG\ e)) \in (\preceq\ SG)$
$\quad\quad$ $\wedge\ (sg\_tgt\ SG\ e, tgtPE(gr\ SG \bowtie_{Es}\ EsA\ SG)(pe\ SG\ e)) \in (\preceq\ SG)$

relation($okPEA\ \_$)
relation($okPEC\ \_$)
relation($okPE\ \_$)
relation($okPEASrc\ \_$)
relation($okPEATgt\ \_$)

$\quad$ $okPEASrc\ \_$: $\mathbb{P}(SGr_0 \times V \times PEA)$
$\quad$ $okPEATgt\ \_$: $\mathbb{P}(SGr_0 \times V \times PEA)$
$\quad$ ────────────────────────────
$\quad$ $\forall\, SG : SGr_0;\ v : V;\ e : E \bullet okPEASrc(SG, v, edg\ e) \Leftrightarrow (e, v) \in src^*\ SG$

$\quad$ $\forall\, SG : SGr_0;\ v : V;\ e : E \bullet okPEASrc(SG, v, einv\ e) \Leftrightarrow (e, v) \in tgt^*\ SG$

$\quad$ $\forall\, SG : SGr_0;\ v : V;\ e : E \bullet okPEATgt(SG, v, edg\ e) \Leftrightarrow (e, v) \in tgt^*\ SG$

$\quad$ $\forall\, SG : SGr_0;\ v : V;\ e : E \bullet okPEATgt(SG, v, einv\ e) \Leftrightarrow (e, v) \in src^*\ SG$

$\quad$ $okPEA\ \_$: $\mathbb{P}(SGr_0 \times PEA)$
$\quad$ ────────────────────────────
$\quad$ $\forall\, SG : SGr_0;\ e : E \bullet okPEA(SG, edg\ e) \Leftrightarrow e \in sg\_Es\ SG$

$\quad$ $\forall\, SG : SGr_0;\ e : E \bullet okPEA(SG, einv\ e) \Leftrightarrow e \in sg\_Es\ SG$

$okPEC\_ : \mathbb{P}(SGr_0 \times PEC)$
$okPE\_ : \mathbb{P}(SGr_0 \times PE)$

$\forall\, SG : SGr_0;\ v : V;\ pea : PEA \bullet okPEC\,(SG, eat\,pea) \Leftrightarrow okPEA(SG, pea)$

$\forall\, SG : SGr_0;\ v : V;\ pea : PEA \bullet$
$\quad okPEC(SG, eresd\,(v, pea)) \Leftrightarrow okPEA(SG, pea) \wedge okPEASrc(SG, v, pea)$

$\forall\, SG : SGr_0;\ v : V;\ pea : PEA \bullet$
$\quad okPEC(SG, eresr\,(pea, v)) \Leftrightarrow okPEA(SG, pea) \wedge okPEATgt(SG, v, pea)$

$\forall\, SG : SGr_0;\ pec : PEC;\ pe : PE \bullet$
$\quad okPE(SG, scmp\,(pec, pe)) \Leftrightarrow okPEC\,(SG, pec) \wedge okPE\,(SG, pe)$
$\quad \wedge\ tgtPEC\,(gr\,SG \bowtie_{Es} EsA\,SG)\,pec = srcPE\,(gr\,SG \bowtie_{Es} EsA\,SG)\,pe$

relation(isVCEECnt$\_$)
relation(isVCENCnt$\_$)

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isVCEECnt$\_$, isVCENCnt$\_$: $\mathbb{P}(SGr \times E)$

$\forall\, SG : SGr;\ vce : E \bullet \mathrm{isVCEECnt}(SG, vce) \Leftrightarrow (second \circ (vcei\,SG))\,vce \neq \varnothing$

$\forall\, SG : SGr;\ vce : E \bullet \mathrm{isVCENCnt}(SG, vce) \Leftrightarrow (second \circ (vcei\,SG))\,vce = \varnothing$

relation(commonAncestor$\_$)
relation(EsVCntsOk$\_$)

---

commonAncestor$\_$: $\mathbb{P}(SGr_0 \times V \times V)$
EsVCntsOk$\_$: $\mathbb{P}\,SGr_0$

$\forall\, SG : SGr_0;\ n_1, n_2 : V \bullet \mathrm{commonAncestor}\,(SG, n_1, n_2) \Leftrightarrow \exists\, nt : sg\_Ns\,SG \bullet (n_1, nt) \in\, \preceq\, SG \wedge (n_2, nt) \in\, \preceq$

$\forall\, SG : SGr_0 \bullet \mathrm{EsVCntsOk}\,SG \Leftrightarrow$
$\quad \forall\, vce : EsVCnt\,SG \bullet \mathrm{isVCEECnt}(SG, vce) \Rightarrow$
$\qquad \mathrm{commonAncestor}\,(SG, sg\_tgt\,SG\,((\mathrm{the} \circ second \circ (vcei\,SG))\,vce), sg\_tgt\,SG\,vce)$
$\qquad \wedge\ \mathrm{isVCENCnt}(SG, vce) \Rightarrow \mathrm{commonAncestor}\,(SG, sg\_src\,SG\,vce, sg\_tgt\,SG\,vce)$

relation($EsCntsOk\_$)

$EsCntsOk\_$: $\mathbb{P}\,SGr_0$

$\forall\, SG : SGr_0 \bullet$
$\quad EsCntsOk\,SG \Leftrightarrow derInhOk\,SG \wedge \forall\, e : EsPaCnt\,SG \bullet okPE(SG, pe\,SG\,e)$
$\qquad \wedge\ \mathrm{EsVCntsOk}\,SG$

relation($inhTree\ \_$)

$inhTree\ \_ : \mathbb{P}\ SGr_0$

$\forall\ SG : SGr_0 \bullet inhTree\ SG \Leftrightarrow ((\pitchfork SG) \ominus_{Ns} (NsVi\ SG))^{\leftrightarrow} \in \text{tree}$

$TSGr == \{\ SG : SGr \mid \text{etherealAreInherited}\ SG \land EsCntsOk\ SG \land inhTree\ SG\}$

relation($\boxminus_{SGs}\ \_$)

$\boxminus_{SGs-} : \mathbb{P}(SGr \times SGr)$

$\forall\ SG_1, SG_2 : SGr \bullet \boxminus_{SGs}(SG_1, SG_2) \Leftrightarrow \boxminus(gr\ SG_1, gr\ SG_2)$

function 10 **leftassoc** ($\_ \cup_{SG}\ \_$)

$\_ \cup_{SG}\ \_ : SGr \times SGr \twoheadrightarrow SGr$

$\forall\ SG_1, SG_2 : SGr \bullet SG_1 \cup_{SG} SG_2 = (gr\ SG_1 \cup_G gr\ SG_2, nty\ SG_1 \cup nty\ SG_2,$
$\quad ety\ SG_1 \cup ety\ SG_2, srcm\ SG_1 \cup srcm\ SG_2, tgtm\ SG_1 \cup tgtm\ SG_2, pe\ SG_1 \cup pe\ SG_2, ds\ SG_1 \cup ds\ SG_2, vcei\ SG_1$

function 10 **leftassoc** ($\_ \odot^{SG}\ \_$)

$\_ \odot^{SG}\ \_ : SGr \times (V \twoheadrightarrow V) \twoheadrightarrow SGr$

$\forall\ SG : SGr;\ s : V \twoheadrightarrow V \mid s \in NsP\ SG \twoheadrightarrow sg\_Ns\ SG \bullet$
$\quad SG \odot^{SG} s = (gr\ SG \odot s, (\text{dom}\ s \setminus \text{ran}\ s) \lhd nty\ SG, ety\ SG, srcm\ SG, tgtm\ SG, pe\ SG, ds\ SG, vcei\ SG)$

function 10 **leftassoc** ($\_ \to_{SG}\ \_$)

$\_ \to_{SG}\ \_ : SGr \times SGr \to \mathbb{P}\ GrM$

$\forall\ SG_s, SG_t : SGr \bullet$
$\quad SG_s \to_{SG} SG_t = \{fv : sg\_Ns\ SG_s \to sg\_Ns\ SG_t;\ fe : EsA\ SG_s \to EsA\ SG_t \mid$
$\quad\quad fv \circ (src_M^*\ SG_s) \subseteq src_M^*\ SG_t \circ fe \land fv \circ (tgt_M^*\ SG_s) \subseteq tgt_M^*\ SG_t \circ fe$
$\quad\quad \land\ fv \circ \preceq SG_s \subseteq \preceq SG_t \circ fv\}$

22

function 10 **leftassoc** $(\_ \twoheadrightarrow_{SG} \_)$

$$
\begin{array}{|l}
\_ \twoheadrightarrow_{SG} \_ : SGr \times SGr \to \mathbb{P}\, GrM \\
\hline
\forall\, SG_s, SG_t : SGr \bullet \\
\quad SG_s \twoheadrightarrow_{SG} SG_t = \{fv : sg\_Ns\, SG_s \twoheadrightarrow sg\_Ns\, SG_t;\ fe : EsA\, SG_s \twoheadrightarrow EsA\, SG_t \mid \\
\quad\quad fv \circ ((\mathrm{dom}\, fe) \lhd (src^*_M\, SG_s)) \subseteq src^*_M\, SG_t \circ fe \wedge fv \circ ((\mathrm{dom}\, fe) \lhd (tgt^*_M\, SG_s)) \subseteq tgt^*_M\, SG_t \circ fe \\
\quad\quad \wedge\, fv \circ \preceq SG_s \subseteq\, \preceq SG_t \circ fv\}
\end{array}
$$

relation$(\_ \Rrightarrow^{SG} \_)$

$$
\begin{array}{|l}
\_ \Rrightarrow^{SG} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_s, SG_t : SGr;\ m : GrM \bullet (SG_s, m) \Rrightarrow^{SG} SG_t \Leftrightarrow m \in SG_s \to_{SG} SG_t
\end{array}
$$

function 10 **leftassoc** $(\_ \to_{G2SG} \_)$

$$
\begin{array}{|l}
\_ \to_{G2SG} \_ : Gr \times SGr \to \mathbb{P}\, GrM \\
\hline
\forall\, G : Gr;\ SG : SGr \bullet G \to_{G2SG} SG = \{fv : Ns\, G \to sg\_Ns\, SG;\ fe : Es\, G \to EsA\, SG \mid \\
\quad fv \circ src\, G \subseteq src^*_M\, SG \circ fe \wedge fv \circ tgt\, G \subseteq tgt^*_M\, SG \circ fe\}
\end{array}
$$

relation$(\_ \Rrightarrow^{GwT} \_)$

$$
\begin{array}{|l}
\_ \Rrightarrow^{GwT} \_ : (GrwT \leftrightarrow SGr) \\
\hline
\forall\, GwT : GrwT;\ SG : SGr \bullet GwT \Rrightarrow^{GwT} SG \Leftrightarrow GwT^T \in GwT^G \to_{G2SG} SG
\end{array}
$$

relation$(\_ \sqsupseteq^{SG} \_)$
relation$(\_ \sqsupseteq^{SG_0} \_)$
relation$(\_ \sqsupseteq^{NT} \_)$
relation$(\_ \sqsupseteq^{ET} \_)$
relation$(\_ \sqsupseteq^{\mathcal{M}} \_)$
relation$(\_ \sqsupseteq_{\mathcal{M}Cnts} \_)$

$$\_ \sqsupseteq^{NT} \_, \_ \sqsupseteq^{ET} \_ : \mathbb{P}((SGr \times GrM) \times SGr)$$

$\forall SG_c, SG_a : SGr; \ m : GrM \bullet$
$\quad (SG_c, m) \sqsupseteq^{NT} SG_a \Leftrightarrow \forall n : sg\_Ns \, SG_c \bullet (nty \, SG_c) \, n \leq_{rNT} ((nty \, SG_a) \circ (fV \, m)) \, n$

$\forall SG_c, SG_a : SGr; \ m : GrM \bullet$
$\quad (SG_c, m) \sqsupseteq^{ET} SG_a \Leftrightarrow \forall e : EsA \, SG_c \bullet (ety \, SG_c) \, e \leq_{ET} ((ety \, SG_a) \circ (fE \, m)) \, e$

$sPEA, tPEA : SGr \rightarrow PEA \rightarrow (E \nrightarrow V)$
$smfPEA, tmfPEA : SGr \rightarrow PEA \rightarrow (E \nrightarrow Mult)$

$\forall SG : SGr; \ e : E \bullet sPEA \, SG \, (edg \, e) = sg\_src \, SG$

$\forall SG : SGr; \ e : E \bullet sPEA \, SG \, (einv \, e) = sg\_tgt \, SG$

$\forall SG : SGr; \ e : E \bullet tPEA \, SG \, (edg \, e) = sg\_tgt \, SG$

$\forall SG : SGr; \ e : E \bullet tPEA \, SG \, (einv \, e) = sg\_src \, SG$

$\forall SG : SGr; \ e : E \bullet smfPEA \, SG \, (edg \, e) = srcma \, SG$

$\forall SG : SGr; \ e : E \bullet smfPEA \, SG \, (einv \, e) = tgtm \, SG$

$\forall SG : SGr; \ e : E \bullet tmfPEA \, SG \, (edg \, e) = tgtm \, SG$

$\forall SG : SGr; \ e : E \bullet tmfPEA \, SG \, (einv \, e) = srcma \, SG$

relation($affectedPE \_$)
relation($affectedPECStart \_$)
relation($affectedPECEnd \_$)
relation($affectedPEAStart \_$)
relation($affectedPEAEnd \_$)
relation($affectedPEStart \_$)
relation($affectedPEEnd \_$)

$affectedPECStart\_, affectedPECEnd\_ : \mathbb{P}(SGr \times GrM \times SGr \times PEC \times E \times E)$
$affectedPEAStart\_, affectedPEAEnd\_ : \mathbb{P}(SGr \times GrM \times SGr \times PEA \times E \times E)$

$\forall\, SG_c, SG_a : SGr;\ m : GrM;\ pea : PEA;\ e, ie : E \bullet affectedPEAStart(SG_c, m, SG_a, pea, e, ie)$
  $\Leftrightarrow ie \in (fE\ m)^{\sim} (\!|\{ePEA\ pea\}|\!) \wedge (fV\ m)(sPEA\ SG_c\ pea\ ie) = sPEA\ SG_a\ pea\ e$

$\forall\, SG_c, SG_a : SGr;\ m : GrM;\ pea : PEA;\ e, ie : E \bullet affectedPEAEnd(SG_c, m, SG_a, pea, e, ie)$
  $\Leftrightarrow ie \in (fE\ m)^{\sim} (\!|\{ePEA\ pea\}|\!) \wedge (fV\ m)(tPEA\ SG_c\ pea\ ie) = tPEA\ SG_a\ pea\ e$

$\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ e, ie : E \bullet affectedPECStart(SG_c, m, SG_a, eat\ pea, e, ie)$
  $\Leftrightarrow affectedPEAStart(SG_c, m, SG_a, pea, e, ie)$

$\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ v : V;\ e, ie : E \bullet$
  $affectedPECStart(SG_c, m, SG_a, eresd\,(v, pea), e, ie)$
  $\Leftrightarrow affectedPEAStart(SG_c, m, SG_a, pea, e, ie)$
    $\wedge\, \exists\, v' : (fV\ m)^{\sim} (\!|\{v\}|\!) \bullet (sPEA\ SG_c\ pea\ ie, v') \in\, \preceq SG_c$

$\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ e, ie : E \bullet affectedPECEnd(SG_c, m, SG_a, eat\ pea, e, ie)$
  $\Leftrightarrow affectedPEAEnd(SG_c, m, SG_a, pea, e, ie)$

$\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ v : V;\ e, ie : E \bullet$
  $affectedPECEnd(SG_c, m, SG_a, eresd\,(v, pea), e, ie)$
  $\Leftrightarrow affectedPEAEnd(SG_c, m, SG_a, pea, e, ie)$
    $\wedge\, \exists\, v' : (fV\ m)^{\sim} (\!|\{v\}|\!) \bullet (sPEA\ SG_c\ pea\ ie, v') \in\, \preceq SG_c$

$\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ v : V;\ e, ie : E \bullet$
  $affectedPECEnd(SG_c, m, SG_a, eresr\,(pea, v), e, ie)$
  $\Leftrightarrow affectedPEAEnd(SG_c, m, SG_a, pea, e, ie)$
    $\wedge\, \exists\, v' : (fV\ m)^{\sim} (\!|\{v\}|\!) \bullet (tPEA\ SG_c\ pea\ ie, v') \in\, \preceq SG_c$

$affectedPEStart\_, affectedPEEnd\_ : \mathbb{P}(SGr \times GrM \times SGr \times PE \times E \times E)$
$affectedPE\_ : \mathbb{P}(SGr \times GrM \times SGr \times PE \times E \times E \times E)$

$\forall SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ e, ie : E \bullet$
$\quad affectedPEStart(SG_c, m, SG_a, ec\ pec, e, ie) \Leftrightarrow affectedPECStart(SG_c, m, SG_a, pec, e, ie)$

$\forall SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ pe : PE;\ e, ie : E \bullet$
$\quad affectedPEStart(SG_c, m, SG_a, scmp\,(pec, pe), e, ie) \Leftrightarrow affectedPECStart(SG_c, m, SG_a, pec, e, ie)$

$\forall SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ e, ie : E \bullet$
$\quad affectedPEEnd(SG_c, m, SG_a, ec\ pec, e, ie) \Leftrightarrow affectedPECEnd(SG_c, m, SG_a, pec, e, ie)$

$\forall SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ pe : PE;\ e, ie : E \bullet$
$\quad affectedPEEnd(SG_c, m, SG_a, scmp\,(pec, pe), e, ie) \Leftrightarrow affectedPEEnd(SG_c, m, SG_a, pe, e, ie)$

$\forall SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ e, ie, ie' : E \bullet$
$\quad affectedPE(SG_c, m, SG_a, ec\ pec, e, ie, ie')$
$\quad \Leftrightarrow ie = ie' \wedge affectedPECStart(SG_c, m, SG_a, pec, e, ie)$
$\quad \wedge affectedPECEnd(SG_c, m, SG_a, pec, e, ie)$

$\forall SG_a, SG_c : SGr;\ m : GrM;\ pec_1, pec_2 : PEC;\ e, ie, ie' : E \bullet$
$\quad affectedPE(SG_c, m, SG_a, scmp\,(pec_1, ec\ pec_2), e, ie, ie')$
$\quad \Leftrightarrow tPEA\ SG_c\,(endEA_C\ pec_1)\ ie = sPEA\ SG_c\,(startEA_C\ pec_2)\ ie'$
$\qquad \wedge affectedPECStart(SG_c, m, SG_a, pec_1, e, ie) \wedge affectedPECEnd(SG_c, m, SG_a, pec_2, e, ie')$

$\forall SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ pe : PE;\ e, ie, ie' : E \bullet$
$\quad affectedPE(SG_c, m, SG_a, scmp\,(pec, pe), e, ie, ie')$
$\quad \Leftrightarrow affectedPECStart(SG_c, m, SG_a, pec, e, ie)$
$\qquad \wedge \exists ie'' : E \mid tPEA\ SG_c\,(endEA_C\ pec)\ ie = sPEA\ SG_c\,(startEA\ pe)\ ie'' \bullet$
$\qquad\quad affectedPE(SG_c, m, SG_a, pe, e, ie'', ie')$

relation($caseMultsOk\_$)
relation($caseMandatoryT\_$)
relation($caseMandatoryS\_$)

$caseMultsOk\_ : \mathbb{P}(SGr \times GrM \times SGr \times E \times PEA \times PEA \times E \times E)$

$\forall SG_c, SG_a : SGr;\ m : GrM;\ peas : PEA;\ peae : PEA;\ e, ie, ie' : E \bullet$
$\quad caseMultsOk(SG_c, m, SG_a, e, peas, peae, ie, ie')$
$\quad \Leftrightarrow affectedPE(SG_c, m, SG_a, pe\ SG_a\ e, e, ie, ie')$
$\qquad \Rightarrow smfPEA\ SG_c\ peas\ ie \leq_{\mathcal{M}} srcma\ SG_a\ e \wedge tmfPEA\ SG_c\ peae\ ie' \leq_{\mathcal{M}} tgtm\ SG_a\ e$

$caseMandatoryT\_ : \mathbb{P}(SGr \times GrM \times SGr \times E \times PEA \times E)$

$\forall SG_c, SG_a : SGr;\ m : GrM;\ peas : PEA;\ peae : PEA;\ e, ie : E \bullet$
$\quad caseMandatoryT(SG_c, m, SG_a, e, peae, ie)$
$\quad \Leftrightarrow (affectedPEStart(SG_c, m, SG_a, pe\ SG_a\ e, e, ie) \wedge mks\ 1 \leq_{\mathcal{M}} (tgtm\ SG_a)\ e)$
$\qquad \Rightarrow (fV\ m)(tPEA\ SG_c\ peae\ ie) = sg\_tgt\ SG_a\ e$

$caseMandatoryS\_ : \mathbb{P}(SGr \times GrM \times SGr \times E \times PEA \times E)$

$\forall\, SG_c, SG_a : SGr;\ m : GrM;\ peas : PEA;\ e, ie : E \bullet$
$\quad caseMandatoryS(SG_c, m, SG_a, e, peas, ie)$
$\quad\quad \Leftrightarrow (affectedPEEnd(SG_c, m, SG_a, pe\, SG_a\, e, e, ie) \wedge mks\, 1 \leq_{\mathcal{M}} (srcma\, SG_a)\, e)$
$\quad\quad\quad \Rightarrow (fV\, m)(sPEA\, SG_c\, peas\, ie) = sg\_src\, SG_a\, e$

$\_ \sqsupseteq_{\mathcal{M}Cnts} \_ : \mathbb{P}((SGr \times GrM) \times SGr)$
$\_ \sqsupseteq^{\mathcal{M}} \_ : \mathbb{P}((SGr \times GrM) \times SGr)$

$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet$
$\quad (SG_c, m) \sqsupseteq^{\mathcal{M}} SG_a \Leftrightarrow \forall\, e : EsA\, SG_c \bullet (srcma\, SG_c)\, e \leq_{\mathcal{M}} ((srcma\, SG_a) \circ (fE\, m))\, e$
$\quad\quad\quad\quad\quad \wedge (tgtm\, SG_c)\, e \leq_{\mathcal{M}} ((tgtm\, SG_a) \circ (fE\, m))\, e$

$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet (SG_c, m) \sqsupseteq_{\mathcal{M}Cnts} SG_a \Leftrightarrow \forall\, e : EsD\, SG_a \bullet$
$\quad \exists\, peas : PEA;\ peae : PEA \mid peas = startEA(pe\, SG_a\, e) \wedge peae = endEA(pe\, SG_a\, e) \bullet$
$\quad\quad \forall\, e' : (fE\, m)^{\sim} (\!|\{ePEA\, peas\}|\!);\ e'' : (fE\, m)^{\sim} (\!|\{ePEA\, peae\}|\!) \bullet$
$\quad\quad\quad caseMultsOk(SG_c, m, SG_a, e, peas, peae, e', e'')$
$\quad\quad\quad \wedge (caseMandatoryT(SG_c, m, SG_a, e, peae, e') \vee caseMandatoryS(SG_c, m, SG_a, e, peas, e''))$

$\_ \sqsupseteq^{SG} \_,\_ \sqsupseteq^{SG_0} \_ : \mathbb{P}((SGr \times GrM) \times SGr)$

$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet$
$\quad (SG_c, m) \sqsupseteq^{SG_0} SG_a \Leftrightarrow (SG_c, m) \sqsupseteq^{NT} SG_a \wedge (SG_c, m) \sqsupseteq^{ET} SG_a \wedge (SG_c, m) \sqsupseteq^{\mathcal{M}} SG_a$
$\quad\quad \wedge (SG_c, m) \sqsupseteq_{\mathcal{M}Cnts} SG_a$

$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet$
$\quad (SG_c, m) \sqsupseteq^{SG} SG_a \Leftrightarrow m \in SG_c \rightarrow_{SG} SG_a \wedge (SG_c, m) \sqsupseteq^{SG_0} SG_a$

$ins : GrM \times SGr \times \mathbb{P}\, V \rightarrow \mathbb{P}\, V$
$ies : GrM \times \mathbb{P}\, E \rightarrow \mathbb{P}\, E$

$\forall\, m : GrM;\ SG : SGr;\ mns : \mathbb{P}\, V \bullet ins\, (m, SG, mns) = (fV\, m)^{\sim} (\!|(\prec SG)^{\sim} (\!|mns|\!)|\!)$

$\forall\, m : GrM;\ mes : \mathbb{P}\, E \bullet ies\, (m, mes) = (fE\, m)^{\sim} (\!|mes|\!)$

$igRMEs : GrwT \times \mathbb{P}\, E \rightarrow Gr$

$\forall\, GwT : GrwT;\ mes : \mathbb{P}\, E \bullet igRMEs(GwT, mes) = GwT^{G} \bowtie_{Es} ies\, (GwT^{T}, mes)$

relation$(\_ \sqsupseteq^{SG} \_)$
relation$(\_ \sqsupseteq^{SG_0} \_)$
relation$(\_ \sqsupseteq_{AEs} \_)$
relation$(\_OkRefinedIn\_)$
relation$(\_ \sqsupseteq_{ANNs} \_)$

$$\underline{\ } \sqsupseteq_{ANNs} \underline{\ } : \mathbb{P}(GrM \times SGr)$$

$$\forall\, SG_a : SGr;\ m : GrM \bullet$$
$$\quad m \sqsupseteq_{ANNs} SG_a \Leftrightarrow \forall\, nn : NsTy\ SG_a\{nnrml\} \bullet (\preceq SG_a)\,(\!|\,\{nn\}\,|\!) \cap \mathrm{ran}(fV\ m) = \varnothing$$

$$\underline{\ }\text{OkRefinedIn}\underline{\ } : \mathbb{P}((SGr \times E) \times (SGr \times GrM))$$
$$\underline{\ } \sqsupseteq_{AEs} \underline{\ } : \mathbb{P}((SGr \times GrM) \times SGr)$$

$$\forall\, SG_c, SG_a : SGr;\ m : GrM;\ ae : E \bullet$$
$$\quad (SG_a, ae)\text{OkRefinedIn}(SG_c, m) \Leftrightarrow$$
$$\qquad \exists\, r : V \leftrightarrow V;\ s, t : \mathbb{P}\,V \mid r = (\preceq SG_c)\,\mathbin{\mathrm{\tiny\begin{smallmatrix}\circ\\[-3pt]\circ\end{smallmatrix}}}\, igRMEs((gr\ SG_c, m), \{ae\}) \xleftrightarrow{}\, \mathbin{\mathrm{\tiny\begin{smallmatrix}\circ\\[-3pt]\circ\end{smallmatrix}}}(\preceq SG_c)^{\sim}$$
$$\qquad \wedge\ s = ins\,(m, SG_a, sg\_src\ SG_a\,(\!|\,\{ae\}\,|\!)) \setminus ((NsEther\ SG_c) \setminus \mathrm{dom}\ r)$$
$$\qquad \wedge\ t = ins\,(m, SG_a, sg\_tgt\ SG_a\,(\!|\,\{ae\}\,|\!)) \setminus ((NsEther\ SG_c) \setminus \mathrm{ran}\ r)$$
$$\qquad\quad \bullet\ r \in s \leftrightarrow t \wedge nty\ SG_a\,(\!|\,(gr\ SG_a) \mathbin{\circ\!\!\!-\!\!\!\twoheadrightarrow}_{Ns} \{ae\}\,|\!) \neq \varnothing \Rightarrow r \neq \varnothing$$

$$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet$$
$$\quad (SG_c, m) \sqsupseteq_{AEs}\ SG_a \Leftrightarrow \forall\, e : (EsA\ SG_a) \bullet (SG_a, e)\text{OkRefinedIn}(SG_c, m)$$

$$\underline{\ } \sqsupseteq^{SG} \underline{\ },\ \underline{\ } \sqsupseteq^{SG_0} \underline{\ } : \mathbb{P}((SGr \times GrM) \times SGr)$$

$$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet$$
$$\quad (SG_c, m) \sqsupseteq^{SG_0}\ SG_a \Leftrightarrow (SG_c, m) \sqsupseteq^{SG_0}\ SG_a \wedge m \sqsupseteq_{ANNs} SG_a \wedge (SG_c, m) \sqsupseteq_{AEs}\ SG_a$$

$$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet$$
$$\quad (SG_c, m) \sqsupseteq^{SG}\ SG_a \Leftrightarrow m \in SG_c \rightarrow_{SG} SG_a \wedge (SG_c, m) \sqsupseteq^{SG_0}\ SG_a$$

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relation$(\underline{\ } \Vdash\!\dashv_{\mathcal{M}} \underline{\ })$
relation$(\underline{\ } \Vdash\!\dashv_{NT} \underline{\ })$
relation$(\underline{\ } \Vdash\!\dashv_{ET} \underline{\ })$
relation$(\underline{\ } \Vdash\!\dashv^{SG} \underline{\ })$

$$\underline{\ } \Vdash\!\dashv_{\mathcal{M}} \underline{\ } : \mathbb{P}((SGr \times GrM) \times SGr)$$

$$\forall\, SG_s, SG_t : SGr;\ m : GrM \bullet$$
$$\quad (SG_s, m) \Vdash\!\dashv_{\mathcal{M}}\ SG_t \Leftrightarrow \forall\, ep : fE\ m \bullet (srcma\ SG_s)\,(first\ ep) \leq_{\mathcal{M}} (srcma\ SG_t)\,(second\ ep)$$
$$\qquad \wedge\ (tgtm\ SG_s)\,(first\ ep) \leq_{\mathcal{M}} (tgtm\ SG_t)\,(second\ ep)$$

$$\underline{\ } \Vdash\!\dashv_{NT} \underline{\ } : \mathbb{P}(GrM \times SGr)$$
$$\underline{\ } \Vdash\!\dashv_{ET} \underline{\ } : \mathbb{P}((SGr \times GrM) \times SGr)$$

$$\forall\, SG_t : SGr;\ m : GrM \bullet$$
$$\quad m \Vdash\!\dashv_{NT}\ SG_t \Leftrightarrow (nty\ SG_t)\,(\!|\,\mathrm{ran}(fV\ m)\,|\!) \subseteq \{nnrml, nabst, nvirt\}$$

$$\forall\, SG_s, SG_t : SGr;\ m : GrM \bullet$$
$$\quad (SG_s, m) \Vdash\!\dashv_{ET}\ SG_t \Leftrightarrow \mathrm{dom}(fE\ m) \lhd (ety\ SG_s) = (ety\ SG_t) \circ (fE\ m)$$

$$\_\dashv\vdash^{SG}\_: \mathbb{P}((SGr \times GrM) \times SGr)$$

$$\forall\, SG_s, SG_t : SGr;\ m : GrM \bullet (SG_s, m) \dashv\vdash^{SG} SG_t \Leftrightarrow m \in SG_s \twoheadrightarrow_{SG} SG_t \wedge (SG_s, m) \dashv\vdash_{\mathcal{M}} SG_t$$
$$\wedge\ m \dashv\vdash_{NT} SG_t \wedge (SG_s, m) \dashv\vdash_{ET} SG_t$$

$$rPEA : GrwT \times SGr \times PEA \to (V \leftrightarrow V)$$

$$\forall\, GwT : GrwT;\ SG : SGr;\ e : E \bullet rPEA\,(GwT, SG, edg\,e) = (GwT^G \bowtie_{Es} ies\,(GwT^T, \{e\}))^{\leftrightarrow}$$

$$\forall\, GwT : GrwT;\ SG : SGr;\ e : E \bullet rPEA\,(GwT, SG, einv\,e) = rPEA\,(GwT, SG, edg\,e)^{\sim}$$

$$rPEC : GrwT \times SGr \times PEC \to (V \leftrightarrow V)$$
$$rPE : GrwT \times SGr \times PE \to (V \leftrightarrow V)$$

$$\forall\, GwT : GrwT;\ SG : SGr;\ pea : PEA \bullet rPEC\,(GwT, SG, eat\,pea) = rPEA\,(GwT, SG, pea)$$

$$\forall\, GwT : GrwT;\ SG : SGr;\ v : V;\ pea : PEA \bullet$$
$$rPEC\,(GwT, SG, eresd\,(v, pea)) = ins\,(GwT^T, SG, \{v\}) \lhd rPEA(GwT, SG, pea)$$

$$\forall\, GwT : GrwT;\ SG : SGr;\ v : V;\ pea : PEA \bullet$$
$$rPEC\,(GwT, SG, eresr\,(pea, v)) = rPEA\,(GwT, SG, pea) \rhd ins\,(GwT^T, SG, \{v\})$$

$$\forall\, GwT : GrwT;\ SG : SGr;\ pec : PEC \bullet$$
$$rPE\,(GwT, SG, ec\,pec) = rPEC\,(GwT, SG, pec)$$

$$\forall\, GwT : GrwT;\ SG : SGr;\ pec : PEC;\ pe : PE \bullet$$
$$rPE\,(GwT, SG, scmp\,(pec, pe)) = rPEC\,(GwT, SG, pec)\,\fatsemi\,rPE\,(GwT, SG, pe)$$

$$ape : SGr \to E \nrightarrow PE$$

$$\forall\, SG : SGr \bullet ape\,SG = (\lambda\, e : E \mid e \in EsM\,SG \bullet (ec \circ eat \circ edg)\,e) \oplus pe\,SG$$

$$src^*_{PEA} : SGr \times PEA \to E \leftrightarrow V$$
$$src^*_{PEC} : SGr \times PEC \to E \leftrightarrow V$$
$$src^*_{PE} : SGr \times PE \to E \leftrightarrow V$$

$$\forall\, SG : SGr;\ e : E \bullet src^*_{PEA}(SG, edg\,e) = src^*\,SG$$

$$\forall\, SG : SGr;\ e : E \bullet src^*_{PEA}(SG, einv\,e) = tgt^*\,SG$$

$$\forall\, SG : SGr;\ pea : PEA \bullet src^*_{PEC}(SG, eat\,pea) = src^*_{PEA}(SG, pea)$$

$$\forall\, SG : SGr;\ v : V;\ pea : PEA \bullet src^*_{PEC}(SG, eresd\,(v, pea)) = src^*_{PEA}(SG, pea) \rhd \{v\}$$

$$\forall\, SG : SGr;\ v : V;\ pea : PEA \bullet src^*_{PEC}(SG, eresr\,(pea, v)) = src^*_{PEA}(SG, pea)$$

$$\forall\, SG : SGr;\ v : V;\ pec : PEC;\ pe : PE \bullet src^*_{PE}(SG, (scmp\,(pec, pe))) = src^*_{PEC}(SG, pec)$$

$tgt^*_{PEA} : SGr \times PEA \to E \leftrightarrow V$
$tgt^*_{PEC} : SGr \times PEC \to E \leftrightarrow V$
$tgt^*_{PE} : SGr \times PE \to E \leftrightarrow V$

$\forall\, SG : SGr;\ e : E \bullet tgt^*_{PEA}(SG, edg\ e) = tgt^*\ SG$

$\forall\, SG : SGr;\ e : E \bullet tgt^*_{PEA}(SG, einv\ e) = src^*\ SG$

$\forall\, SG : SGr;\ pea : PEA \bullet tgt^*_{PEC}(SG, eat\ pea) = tgt^*_{PEA}(SG, pea)$

$\forall\, SG : SGr;\ v : V;\ pea : PEA \bullet tgt^*_{PEC}(SG, eresd\ (v, pea)) = tgt^*_{PEA}(SG, pea)$

$\forall\, SG : SGr;\ v : V;\ pea : PEA \bullet tgt^*_{PEC}(SG, eresr\ (pea, v)) = tgt^*_{PEA}(SG, pea) \rhd \{v\}$

$\forall\, SG : SGr;\ v : V;\ pec : PEC \bullet tgt^*_{PE}(SG, ec\ pec) = tgt^*_{PEC}(SG, pec)$

$\forall\, SG : SGr;\ v : V;\ pec : PEC;\ pe : PE \bullet tgt^*_{PE}(SG, (scmp\ (pec, pe))) = tgt^*_{PE}(SG, pe)$

relation$(\_ \ni^{SG} \_)$
relation$(\_ \ni_{\mathcal{M}} \_)$
relation$(\_ \ni_{FI} \_)$
relation$(\_ \ni_{PNS} \_)$
relation$(\_ \ni_{Cnts} \_)$
relation$(\_ \mathit{MEMOk} \_)$

$rMEMOk : SGr \times E \times GrwT \to V \leftrightarrow V$

$\forall\, SG : SGr;\ me : E;\ GwT : GrwT;\ s, t : \mathbb{P}\ V \mid s = ins(GwT^T, SG, src^*\ SG\ (\!|\{me\}|\!))$
$\quad \wedge\ t = ins(GwT^T, SG, tgt^*\ SG\ (\!|\{me\}|\!)) \bullet$
$\qquad rMEMOk(SG, me, GwT) = s \lhd rPE\ (GwT, SG, ape\ SG\ me) \rhd t$

$src^*MEMOk : SGr \times E \to E \leftrightarrow V$

$\forall\, SG : SGr;\ me : E \bullet src^*MEMOk(SG, me) = src^*_{PE}(SG, ape\ SG\ me) \rhd (src^*\ SG)\ (\!|\{me\}|\!)$

$tgt^*MEMOk : SGr \times E \to E \leftrightarrow V$

$\forall\, SG : SGr;\ me : E \bullet tgt^*MEMOk(SG, me) = tgt^*_{PE}(SG, ape\ SG\ me) \rhd (tgt^*\ SG)\ (\!|\{me\}|\!)$

$$multComp : Mult \times Mult \to Mult$$

$\forall\, m_1, m_2 : Mult \mid m_1 \in MultMany \lor m_2 \in MultMany \bullet multComp(m_1, m_2) = mmanys$

$\forall\, m_1, m_2 : Mult \mid m_1 = mks\, 0 \lor m_2 = mks\, 0 \bullet multComp(m_1, m_2) = mks\, 0$

$\forall\, m_1, m_2 : Mult \mid m_2 = mks\, 1 \bullet multComp(m_1, m_2) = m_1$

$\forall\, m_1, m_2 : Mult \mid m_1 = mks\, 1 \bullet multComp(m_1, m_2) = m_2$

$\forall\, m_1, m_2 : Mult \mid m_1 = mopts \bullet multComp(m_1, m_2) = mks\, 0 \cup m_2$

$\forall\, m_1, m_2 : Mult \mid m_2 = mopts \bullet multComp(m_1, m_2) = mks\, 0 \cup m_1$

$\forall\, m_1, m_2 : MultRange \bullet$
$\quad multComp(m_1, m_2) = \{(\text{the } m_1) *_{mr} (\text{the } m_2)\}$

$\forall\, m_1 : Mult;\ mc : MultC;\ m_2 : MultEither \bullet$
$\quad multComp(m_1, \{mc\} \cup m_2) = multComp(m_1, \{mc\}) \cup multComp(m_1, m_2)$

$\forall\, m_1 : MultEither;\ mc : MultC;\ m_2 : Mult \bullet$
$\quad multComp(\{mc\} \cup m_1, m_2) = multComp(\{mc\}, m_2) \cup multComp(m_1, m_2)$

$$smPEA : SGr \to PEA \to Mult$$
$$smPEC : SGr \to PEC \to Mult$$
$$smPE : SGr \to PE \to Mult$$

$\forall\, SG : SGr;\ e : E \bullet smPEA\, SG\, (edg\, e) = srcma\, SG\, e$

$\forall\, SG : SGr;\ e : E \bullet smPEA\, SG\, (einv\, e) = tgtm\, SG\, e$

$\forall\, SG : SGr;\ pea : PEA \bullet smPEC\, SG\, (eat\, pea) = smPEA\, SG\, pea$

$\forall\, SG : SGr;\ v : V;\ pea : PEA \bullet smPEC\, SG\, (eresd\, (v, pea)) = smPEA\, SG\, pea$

$\forall\, SG : SGr;\ v : V;\ pea : PEA \bullet smPEC\, SG\, (eresr\, (pea, v)) = smPEA\, SG\, pea$

$\forall\, SG : SGr;\ pec : PEC \bullet smPE\, SG\, (ec\, pec) = smPEC\, SG\, pec$

$\forall\, SG : SGr;\ pec : PEC;\ pe : PE \bullet smPE\, SG\, (scmp\, (pec, pe)) = multComp(smPEC\, SG\, pec, smPE\, SG\, pe)$

$$tmPEA : SGr \to PEA \to Mult$$
$$tmPEC : SGr \to PEC \to Mult$$
$$tmPE : SGr \to PE \to Mult$$

$\forall\, SG : SGr;\ e : E \bullet tmPEA\, SG\, (edg\, e) = tgtm\, SG\, e$

$\forall\, SG : SGr;\ e : E \bullet tmPEA\, SG\, (einv\, e) = srcma\, SG\, e$

$\forall\, SG : SGr;\ pea : PEA \bullet tmPEC\, SG\, (eat\, pea) = tmPEA\, SG\, pea$

$\forall\, SG : SGr;\ v : V;\ pea : PEA \bullet tmPEC\, SG\, (eresd\, (v, pea)) = tmPEA\, SG\, pea$

$\forall\, SG : SGr;\ v : V;\ pea : PEA \bullet tmPEC\, SG\, (eresr\, (pea, v)) = tmPEA\, SG\, pea$

$\forall\, SG : SGr;\ pec : PEC \bullet tmPE\, SG\, (ec\, pec) = tmPEC\, SG\, pec$

$\forall\, SG : SGr;\ pec : PEC;\ pe : PE \bullet tmPE\, SG\, (scmp\, (pec, pe)) = multComp(tmPEC\, SG\, pec, tmPE\, SG\, pe)$

$$\underline{} MEMOk \underline{} : \mathbb{P}((SGr \times E) \times GrwT)$$

$$\forall\, GwT : GrwT;\ SG : SGr;\ me : E \bullet (SG, me)\, MEMOk\, GwT \Leftrightarrow$$
$$\exists\, r : V \leftrightarrow V;\ s, t : \mathbb{P}\, V \mid r = rMEMOk\,(SG, me, GwT)$$
$$\wedge\, s = ins(GwT^{T}, SG, src^{*} MEMOk\,(SG, me)\, (\!|\, \{rsrcPE(ape\, SG\, me)\}\, |\!))$$
$$\wedge\, t = ins(GwT^{T}, SG, tgt^{*} MEMOk\,(SG, me)\, (\!|\, \{rtgtPE(ape\, SG\, me)\}\, |\!))$$
$$\bullet\, r\mathcal{M}Ok(r, s, t, smPE\, SG\,(ape\, SG\, me), tmPE\, SG\,(ape\, SG\, me))$$

relation(INumbersOk$\underline{}$)
relation(I$\mathbb{N}$sOk$\underline{}$)
relation(I$\mathbb{Z}$sOk$\underline{}$)
relation(I$\mathbb{R}$sOk$\underline{}$)

$$\text{I}\mathbb{N}\text{sOk}\underline{}, \text{I}\mathbb{Z}\text{sOk}\underline{}, \text{I}\mathbb{R}\text{sOk}\underline{}, \text{INumbersOk}\underline{} : \mathbb{P}\ GrwT$$

$$\forall\, GwT : GrwT \bullet \text{I}\mathbb{N}\text{sOk}\, GwT \Leftrightarrow$$
$$\forall\, nnat : (fV\,(GwT^{T}))^{\sim}\, (\!|\, \{nNatS\}\, |\!) \bullet \exists\, n : \mathbb{N} \bullet n \in \text{to}\mathbb{Z}\, nnat$$

$$\forall\, GwT : GrwT \bullet \text{I}\mathbb{Z}\text{sOk}\, GwT \Leftrightarrow$$
$$\forall\, nint : (fV\,(GwT^{T}))^{\sim}\, (\!|\, \{nIntS\}\, |\!) \bullet \exists\, n : \mathbb{Z} \bullet n \in \text{to}\mathbb{Z}\, nint$$

$$\forall\, GwT : GrwT \bullet \text{I}\mathbb{R}\text{sOk}\, GwT \Leftrightarrow$$
$$\forall\, nreal : (fV\,(GwT^{T}))^{\sim}\, (\!|\, \{nRealS\}\, |\!) \bullet \exists\, x : \mathbb{R} \bullet x \in \text{to}\mathbb{R}\, nreal$$

$$\forall\, SG : SGr;\ GwT : GrwT \bullet \text{INumbersOk}\, GwT \Leftrightarrow$$
$$\text{I}\mathbb{N}\text{sOk}\, GwT \wedge \text{I}\mathbb{Z}\text{sOk}\, GwT \wedge \text{I}\mathbb{R}\text{sOk}\, GwT$$

relation(satisfiesCnt$\underline{}$)
relation(satisfiesVCEECnt$\underline{}$)
relation(satisfiesVCENCnt$\underline{}$)
relation(IVCEsOk$\underline{}$)

$$toNum : SGr \times V \times V \rightarrow \text{opt}[\mathbb{A}]$$

$$\forall\, SG : SGr;\ ns, nt : V \bullet toNum(SG, ns, nt) =$$
$$\mathbf{if}\,(nt, nNatS) \in (\preceq SG) \vee (nt, nIntS) \in (\preceq SG)\, \mathbf{then}\, \text{to}\mathbb{Z}\, ns$$
$$\mathbf{else\ if}\,(nt, nRealS) \in (\preceq SG)\, \mathbf{then}\, \text{to}\mathbb{R}\, ns\, \mathbf{else}\, \varnothing$$

$$rOp : SGVCEOP \rightarrow \mathbb{A} \leftrightarrow \mathbb{A}$$

$$rOp\ eq = \{n_1, n_2 : \mathbb{A} \mid n_1 = n_2\}$$
$$rOp\ neq = (\underline{} \neq \underline{})$$
$$rOp\ leq = (\underline{} \leq \underline{})$$
$$rOp\ geq = (\underline{} \geq \underline{})$$
$$rOp\ lt = (\underline{} < \underline{})$$
$$rOp\ gt = (\underline{} > \underline{})$$

$\text{satisfiesCnt\_} : \mathbb{P}(SGr \times GrM \times V \times SGVCEOP \times V)$
$\text{satisfiesVCEECnt\_} : \mathbb{P}(SGr \times GrwT \times E)$
$\text{satisfiesVCENCnt\_} : \mathbb{P}(SGr \times GrwT \times E)$
$\text{IVCEsOk\_} : \mathbb{P}(SGr \times GrwT)$

$\forall SG : SGr;\ t : GrM;\ op : SGVCEOP;\ ns, nt : V \bullet \text{satisfiesCnt}(SG, t, ns, op, nt) \Leftrightarrow$
$\quad ns \in \text{dom}(fV\ t)$
$\quad \wedge\ (\exists\, n_1, n_2 : \mathbb{A} \mid n_1 \in toNum(SG, ns, (fV\ t)ns) \wedge n_2 \in toNum(SG, nt, nt) \bullet (n_1, n_2) \in rOp\ op$
$\qquad \vee\ (op = eq \wedge (fV\ t)ns = nt \wedge nt \in NsVa\ SG))$

$\forall SG : SGr;\ GwT : GrwT;\ vce : E \bullet \text{satisfiesVCEECnt}(SG, GwT, vce) \Leftrightarrow$
$\quad \forall\, ie : fE\ (GwT^{\,T})^{\sim} (\!(second \circ (vcei\ SG))\ vce)\!) \mid fV\ (GwT^{\,T})(src\ (GwT^{\,G})\ ie) = (sg\_src\ SG\ vce) \bullet$
$\qquad \text{satisfiesCnt}(SG, GwT^{\,T}, tgt\ (GwT^{\,G})\ ie, (first \circ (vcei\ SG))\ vce, sg\_tgt\ SG\ vce)$

$\forall SG : SGr;\ GwT : GrwT;\ vce : E \bullet \text{satisfiesVCENCnt}(SG, GwT, vce) \Leftrightarrow$
$\quad \forall\, in : fV\ (GwT^{\,T})^{\sim} (\!(\{sg\_src\ SG\ vce\})\!) \bullet$
$\qquad \text{satisfiesCnt}(SG, GwT^{\,T}, in, (first \circ (vcei\ SG))\ vce, sg\_tgt\ SG\ vce)$

$\forall SG : SGr;\ GwT : GrwT \bullet \text{IVCEsOk}(SG, GwT) \Leftrightarrow \forall\, vce : EsVCnt\ SG \bullet$
$\quad \text{isVCEECnt}(SG, vce) \Rightarrow \text{satisfiesVCEECnt}(SG, GwT, vce)$
$\quad \wedge\ \text{isVCENCnt}(SG, vce) \Rightarrow \text{satisfiesVCENCnt}(SG, GwT, vce)$

$\_ \unrhd_{\mathcal{M}} \_ : GrwT \leftrightarrow SGr$
$\_ \unrhd_{FI} \_ : GrwT \leftrightarrow SGr$
$\_ \unrhd_{PNS} \_ : GrwT \leftrightarrow SGr$
$\_ \unrhd_{Cnts} \_ : GrwT \leftrightarrow SGr$

$\forall GwT : GrwT;\ SG : SGr \bullet$
$\quad GwT \unrhd_{\mathcal{M}} SG \Leftrightarrow \forall\, me : EsM\ SG \bullet (SG, me)\ \mathcal{MEM}Ok\ GwT$

$\forall GwT : GrwT;\ SG : SGr \bullet$
$\quad GwT \unrhd_{FI} SG \Leftrightarrow (fV\ (GwT^{\,T}))^{\sim} (\!(NsEther\ SG)\!) = \varnothing$

$\forall GwT : GrwT;\ SG : SGr \bullet$
$\quad GwT \unrhd_{PNS} SG \Leftrightarrow igRMEs(GwT, EsTy\ SG\ \{ecomp\ dbi, ecomp\ duni\})^{\leftrightarrow} \in \text{injrel}$

$\forall GwT : GrwT;\ SG : SGr \bullet$
$\quad GwT \unrhd_{Cnts} SG \Leftrightarrow \text{INumbersOk}\ GwT \wedge \text{IVCEsOk}(SG, GwT)$

$\_ \unrhd^{SG} \_ : GrwT \leftrightarrow SGr$

$\forall GwT : GrwT;\ SG : SGr \bullet GwT \unrhd^{SG} SG \Leftrightarrow GwT \Rrightarrow^{GwT} SG \wedge GwT \unrhd_{\mathcal{M}} SG$
$\quad \wedge\ GwT \unrhd_{FI} SG \wedge GwT \unrhd_{PNS} SG \wedge GwT \unrhd_{Cnts} SG$

8 Fragments

section *Fragmenta_Frs* **parents** *standard_toolkit, Fragmenta_Generics, Fragmenta_SGs,*
Fragmenta_GrswT, Fragmenta_GrswET

$Fr_0 == \{SG : SGr_0;\ esr : \mathbb{P}\,E;\ sr, tr : E \nrightarrow V;\ et : GrM \mid esr \cap (sg\_Es\ SG) = \varnothing$
$\quad \wedge\ sr \in esr \rightarrowtail (NsP\ SG) \wedge tr \in esr \rightarrow V \wedge domg\ et \subseteq_p (NsN\ SG, EsA\ SG)\}$

$$
\begin{array}{|l}
fSG : Fr_0 \rightarrow SGr \\
EsR : Fr_0 \rightarrow \mathbb{P}\,E \\
srcR, tgtR : Fr_0 \rightarrow E \nrightarrow V \\
fet : Fr_0 \rightarrow GrM \\
\hline
\forall\,SG : SGr;\ esr : \mathbb{P}\,E;\ sr, tr : E \nrightarrow V;\ et : GrM \bullet fSG(SG, esr, sr, tr, et) = SG \\[4pt]
\forall\,SG : SGr;\ esr : \mathbb{P}\,E;\ sr, tr : E \nrightarrow V;\ et : GrM \bullet EsR(SG, esr, sr, tr, et) = esr \\[4pt]
\forall\,SG : SGr;\ esr : \mathbb{P}\,E;\ sr, tr : E \nrightarrow V;\ et : GrM \bullet srcR(SG, esr, sr, tr, et) = sr \\[4pt]
\forall\,SG : SGr;\ esr : \mathbb{P}\,E;\ sr, tr : E \nrightarrow V;\ et : GrM \bullet tgtR(SG, esr, sr, tr, et) = tr \\[4pt]
\forall\,SG : SGr;\ esr : \mathbb{P}\,E;\ sr, tr : E \nrightarrow V;\ et : GrM \bullet fet(SG, esr, sr, tr, et) = et
\end{array}
$$

$$
\begin{array}{|l}
fLEs, fEs, fEsA : Fr_0 \rightarrow \mathbb{P}\,E \\
fLNs, fRNs, fNs : Fr_0 \rightarrow \mathbb{P}\,V \\
srcF, tgtF : Fr_0 \rightarrow E \nrightarrow V \\
\hline
fLEs = (sg\_Es \circ fSG) \\[4pt]
\forall\,F : Fr_0 \bullet fEs\ F = fLEs\ F \cup EsR\ F \\[4pt]
fEsA = EsA \circ fSG \\[4pt]
fLNs = sg\_Ns \circ fSG \\[4pt]
fRNs = \mathrm{ran} \circ tgtR \\[4pt]
\forall\,F : Fr_0 \bullet fNs\ F = fLNs\ F \cup fRNs\ F \\[4pt]
\forall\,F : Fr_0 \bullet srcF\ F = (sg\_src \circ fSG)\ F \cup srcR\ F \\[4pt]
\forall\,F : Fr_0 \bullet tgtF\ F = (sg\_tgt \circ fSG)\ F \cup tgtR\ F
\end{array}
$$

$$
\begin{array}{|l}
\overset{G}{\leftrightsquigarrow} : Fr_0 \rightarrow Gr \\
\leftrightsquigarrow : Fr_0 \rightarrow V \leftrightarrow V \\
\hline
\forall\,F : Fr_0 \bullet \overset{G}{\leftrightsquigarrow}\ F = ((NsP \circ fSG)F \cup fRNs\ F, EsR\ F, srcR\ F, tgtR\ F) \\[4pt]
\forall\,F : Fr_0 \bullet \leftrightsquigarrow F = (\overset{G}{\leftrightsquigarrow}\ F)^{\leftrightarrow}
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_F \_)$

$$\varnothing_F : Fr_0$$
$$\_ \cup_F \_ : Fr_0 \times Fr_0 \to Fr_0$$
$$\textstyle\bigcup_F : \mathbb{P}\, Fr_0 \to Fr_0$$

$$\varnothing_F = (\varnothing_{SG}, \varnothing, \varnothing, \varnothing, \varnothing_{GM})$$

$$\forall\, F_1, F_2 : Fr_0 \bullet F_1 \cup_F F_2 =$$
$$(fSG\ F_1 \cup_{SG}\ fSG\ F_2,\ EsR\ F_1 \cup\ EsR\ F_2,\ srcR\ F_1 \cup\ srcR\ F_2,\ tgtR\ F_1 \cup\ tgtR\ F_2, fet\ F_1 \cup_{GM}\ fet\ F_2)$$

$$\textstyle\bigcup_F \{\} = \varnothing_F$$

$$\forall\, F : Fr_0;\ Fs : \mathbb{P}\, Fr_0 \bullet \textstyle\bigcup_F (\{F\} \cup Fs) = F \cup_F (\textstyle\bigcup_F Fs)$$

$$\rightsquigarrow\ : Fr_0 \nrightarrow V \nrightarrow V$$
$$\text{\textcircled{\tiny\textbullet}}^{SG}: Fr_0 \nrightarrow SGr$$
$$rEsR : Fr_0 \nrightarrow \mathbb{P}\, E$$
$$\text{\textcircled{\tiny\textbullet}}: Fr_0 \nrightarrow Fr_0$$

$$\forall\, F : Fr_0 \bullet \rightsquigarrow F = (\leftrightsquigarrow F) \rhd (fLNs\ F)$$

$$\forall\, F : Fr_0 \bullet \text{\textcircled{\tiny\textbullet}}^{SG} F = (fSG\ F) \odot^{SG} (\rightsquigarrow F)$$

$$\forall\, F : Fr_0 \bullet rEsR\ F = \mathrm{dom}((srcR\ F) \rhd \mathrm{dom}(\rightsquigarrow F))$$

$$\forall\, F : Fr_0 \bullet \text{\textcircled{\tiny\textbullet}}\, F = (\text{\textcircled{\tiny\textbullet}}^{SG} F, rEsR\ F, (rEsR\ F) \lhd (srcR\ F), (rEsR\ F) \lhd (tgtR\ F), fet\ F)$$

$$Fr_a == \{F : Fr_0 \mid \ominus(\stackrel{G}{\leftrightsquigarrow} F)\}$$
$$Fr == \{F : Fr_a \mid \text{\textcircled{\tiny\textbullet}}^{SG} F \in SGr\}$$

relation(refsLocal_)

$$\text{refsLocal}\_ : \mathbb{P}\, Fr_0$$

$$\forall\, F : Fr_0 \bullet \text{refsLocal}\ F \Leftrightarrow fRNs\ F \subseteq fLNs\ F$$

$$TFr == \{F : Fr_a \mid \text{refsLocal}\ F \wedge \text{\textcircled{\tiny\textbullet}}^{SG} F \in TSGr\}$$

relation($\boxminus\ \_$)
relation($\boxplus\ \_$)

$$\boxminus\_ : Fr \leftrightarrow Fr$$

$$\forall\, F_1, F_2 : Fr \bullet \boxminus(F_1, F_2) \Leftrightarrow fLNs\ F_1 \cap fLNs\ F_2 = \varnothing \wedge fEs\ F_1 \cap fEs\ F_2 = \varnothing$$

$=\ [I]\ $=============================

$\boxplus\,\_ : \mathbb{P}(I \nrightarrow Fr)$

————————————————

$\forall\, Fs : I \nrightarrow Fr \bullet \boxplus\, Fs \Leftrightarrow \forall\, i, j : \operatorname{dom} Fs \mid i \neq j \bullet \boxminus(Fs\ i, Fs\ j)$

—

relation($\_ \subseteq^{rs} \_$)
relation($\_ \mapsto\!\!\!| \_$)

$\_ \subseteq^{rs} \_ : Fr \leftrightarrow Fr$
$\_ \mapsto\!\!\!| \_ : Fr \leftrightarrow Fr$

————————————————

$\forall\, F_1, F_2 : Fr \bullet F_1 \subseteq^{rs} F_2 \Leftrightarrow \operatorname{ran}(tgtR\ F_1) \cap fLNs\ F_2 \neq \varnothing$

$\forall\, F_1, F_2 : Fr \bullet F_1 \mapsto\!\!\!| F_2 \Leftrightarrow F_1 \subseteq^{rs} F_2 \wedge \neg\,(F_2 \subseteq^{rs} F_1)$

function 10 **leftassoc** ($\_ \rightrightarrows_\bullet \_$)

$\_ \rightrightarrows_\bullet \_ : GrM \times (Fr \times Fr) \rightarrow GrM$

————————————————

$\forall\, m : GrM;\ F_s, F_t : Fr_0 \bullet$
  $m \rightrightarrows_\bullet (F_s, F_t) = (((\leadsto F_s)^\oplus \boxdot (fLNs\ F_s))^\sim \,\fatsemi\, (fV\ m) \,\fatsemi\, ((\leadsto F_t)^\oplus \boxdot (fLNs\ F_t)), fE\ m)$

function 1 **leftassoc** ($\_ \rightarrow_F \_$)

$\_ \rightarrow_F \_ : Fr \times Fr \rightarrow \mathbb{P}\ GrM$

————————————————

$\forall\, F_s, F_t : Fr \bullet F_s \rightarrow_F F_t = \{fv : fLNs\ F_s \rightarrow fLNs\ F_t;\ fe : fEsA\ F_s \rightarrow fEsA\ F_t \mid$
  $(\textcircled{\tiny\bullet}^{SG} F_s, (fv, fe) \rightrightarrows_\bullet (F_s, F_t)) \Rightarrow^{SG} \textcircled{\tiny\bullet}^{SG} F_t\}$

relation($\_ \Rightarrow^F \_$)

$\_ \Rightarrow^F \_ : (Fr \times GrM) \leftrightarrow Fr$

————————————————

$\forall\, m : GrM;\ F_s, F_t : Fr_0 \bullet (F_s, m) \Rightarrow^F F_t \Leftrightarrow m \in F_s \rightarrow_F F_t$

relation$(\_ \sqsupseteq^F \_)$

$\_ \sqsupseteq^F \_ : (Fr \times GrM) \leftrightarrow Fr$

$\forall\, F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupseteq^F F_a \Leftrightarrow (F_c, m) \Rrightarrow^F F_a$
$\quad \wedge\, (\textcircled{\bullet}^{SG} F_c, m \rightrightarrows_\bullet (F_c, F_a)) \sqsupseteq^{SG_0} (\textcircled{\bullet}^{SG} F_a)$

relation$(\_ \sqsupset^F \_)$

$\_ \sqsupset^F \_ : (Fr \times GrM) \leftrightarrow Fr$

$\forall\, F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupset^F F_a \Leftrightarrow (F_c, m) \Rrightarrow^F F_a \wedge \{F_a, F_c\} \subseteq Fr$
$\quad \wedge\, (\textcircled{\bullet}^{SG} F_c, m \rightrightarrows_\bullet (F_c, F_a)) \sqsupset^{SG_0} (\textcircled{\bullet}^{SG} F_a)$

relation$(\_ \ni^F \_)$

$\_ \ni^F \_ : GrwT \leftrightarrow Fr$

$\forall\, GwT : GrwT;\ F : Fr \bullet GwT \ni^F F \Leftrightarrow GwT \ni^{SG} \textcircled{\bullet}^{SG} F$

relation$(\_ \dashv\vdash^F \_)$

$\_ \dashv\vdash^F \_ : Fr \leftrightarrow Fr$

$\forall\, m : GrM;\ F_s, F_t : Fr_0 \bullet F_s \dashv\vdash^F F_t \Leftrightarrow fet\, F_s \neq \varnothing_{GM} \wedge (\textcircled{\bullet}^{SG} F_s, fet\, F_s) \dashv\vdash^{SG} (\textcircled{\bullet}^{SG} F_t)$

relation$(\_ \Vdash^F \_)$

$\_ \Vdash^F \_ : \mathbb{P}((GrwET \times Fr_0) \times (GrwT \times Fr_0))$

$\forall\, GwET : GrwET;\ F_s, F_t : Fr_0;\ GwT : GrwT \bullet (GwET, F_s) \Vdash^F (GwT, F_t) \Leftrightarrow GwET^{Gw} \ni^F F_s$
$\quad \wedge\, GwT \ni^F F_t \wedge \mathrm{domg}\, (GwET^{ET}) =_p \mathrm{domg}\, ((fet\, F_s) \circ_G (GwT^T))$
$\quad \wedge\, GwET^{ET} \in (GwET^G) \twoheadrightarrow_G (GwT^G) \wedge F_s \dashv\vdash^F F_t$

9   Global Fragment Graphs

section *Fragmenta_GFGs* **parents** *standard_toolkit, Fragmenta_Generics, Fragmenta_Graphs*

$$GFGr == \{\, G : Gr \mid \oslash(G \bowtie_{Es} (Es\ G \setminus EsId\ G)) \}$$

function($\_ \dashrightarrow$)

$$
\begin{array}{|l}
\_ \dashrightarrow\ :\ GFGr \to V \leftrightarrow V \\
\hline
\forall\, GFG : GFGr \bullet GFG \dashrightarrow\ = (GFG \leftrightarrow)^{+}
\end{array}
$$

10   Models

section *Fragmenta_Mdls* **parents** *standard_toolkit, Fragmenta_Frs, Fragmenta_GFGs*

$$Mdl_0 == \{\, GFG : GFGr;\ fd : V \nrightarrow Fr \mid fd \in Ns\ GFG \to Fr \wedge \boxplus fd \}$$

$$
\begin{array}{|l}
mGFG : Mdl_0 \to GFGr \\
mFD : Mdl_0 \to V \nrightarrow Fr \\
\hline
\forall\, GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mGFG(GFG, fd) = GFG \\
\forall\, GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mFD(GFG, fd) = fd
\end{array}
$$

$$
\begin{array}{|l}
mUFs : Mdl_0 \to Fr \\
\hline
mUFs = \bigcup_F \circ \operatorname{ran} \circ mFD
\end{array}
$$

$$
\begin{array}{|l}
\text{from} : Mdl_0 \to V \nrightarrow V \\
\hline
\forall\, M : Mdl_0;\ v : V \bullet \text{from}\ M\ v = (\mu\ vf : (Ns \circ mGFG)M \mid v \in fLNs(mFD\ M\ vf))
\end{array}
$$

relation($\Uparrow \_$)

$$\boxed{\quad \Uparrow\_ : \mathbb{P}\ Mdl_0 \\[4pt] \forall\ M : Mdl_0 \bullet \Uparrow M \Leftrightarrow \exists\ UF : Fr_0 \mid UF = mUFs\ M\ \bullet \\ \quad \forall\ p : (NsP \circ fSG)\,UF \bullet (from\ M\ p, from\ M\ (\leftrightsquigarrow\ UF\ p)) \in ((\_\dashrightarrow) \circ mGFG)M}$$

$$Mdl == \{M : Mdl_0 \mid (mUFs\ M) \in TFr \wedge \Uparrow M\}$$

$$\boxed{\quad \circledbullet^M : Mdl \to Fr \\[4pt] \forall\ M : Mdl_0 \bullet \circledbullet^M = \circledbullet \circ mUFs}$$

function 1 **leftassoc** $(\_ \to_M \_)$
relation$(\_ \Rightarrow^M \_)$

$$\boxed{\quad \_ \to_M \_ : Mdl \times Mdl \to \mathbb{P}\ GrM \\ \quad \_ \Rightarrow^M \_ : \mathbb{P}((Mdl \times \mathbb{P}\ GrM) \times Mdl) \\[4pt] \forall\ M_s, M_t : Mdl \bullet M_s \to_M M_t = \{m : GrM \mid \\ \quad \exists\ UF_s, UF_t : Fr_0 \mid UF_s = mUFs\ M_s \wedge UF_t = mUFs\ M_t \bullet m \in UF_s \to_F UF_t\} \\ \forall\ M_s, M_t : Mdl;\ ms : \mathbb{P}\ GrM \bullet (M_s, ms) \Rightarrow^M M_t \Leftrightarrow \bigcup_{GM} ms \in M_s \to_M M_t}$$

relation$(\_ \sqsupset^M \_)$

$$\boxed{\quad \_ \sqsupset^M \_ : (Mdl \times \mathbb{P}\ GrM) \leftrightarrow Mdl \\[4pt] \forall\ M_c, M_a : Mdl_0;\ ms : \mathbb{P}\ GrM \bullet (M_c, ms) \sqsupset^M M_a \\ \quad \Leftrightarrow \exists\ UF_c, UF_a : Fr_0 \mid UF_c = mUFs\ M_c \wedge UF_a = mUFs\ M_a \bullet (UF_c, \bigcup_{GM} ms) \sqsupset^F UF_a}$$

relation$(\_ \ni^M \_)$

$$\boxed{\quad \_ \ni^M \_ : GrwT \leftrightarrow Mdl \\[4pt] \forall\ GwT : GrwT;\ M : Mdl \bullet GwT \ni^M M \Leftrightarrow GwT \ni^F mUFs\ M}$$

relation$(\_ \dashv\vdash^F \_)$

$$\_ \dashv\vdash^F \_ : Mdl \leftrightarrow Mdl$$

$$\forall\, m : GrM;\ M_s, M_t : Mdl_0 \bullet M_s \dashv\vdash^F M_t \Leftrightarrow mUFs\ M_s \dashv\vdash^F mUFs\ M_t$$

relation$(\_ \Vdash^M \_)$

$$\_ \Vdash^M \_ : \mathbb{P}((GrwET \times Mdl_0) \times (GrwT \times Mdl_0))$$

$$\forall\, GwET : GrwET;\ M_s, M_t : Mdl_0;\ GwT : GrwT \bullet$$
$$(GwET, M_s) \Vdash^M (GwT, M_t) \Leftrightarrow (GwET, mUFs\ M_s) \Vdash^F (GwT, mUFs\ M_t)$$