Z Specification of Fragmenta

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November 18, 2023

Contents

1 Generics

section *Fragmenta_Generics* **parents** *standard_toolkit*

$$\mathbb{R} : \mathbb{P}\,\mathbb{A}$$

$\mathrm{acyclic}[X] == \{r : X \leftrightarrow X \mid r^{+} \cap \mathrm{id}\, X = \varnothing\}$

$\mathit{connected}[X] == \{r : X \leftrightarrow X \mid \forall x : \mathrm{dom}\, r;\ y : \mathrm{ran}\, r \bullet x \mapsto y \in r^{+}\}$

$\mathrm{tree}[X] == \{r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \land r \in X \nrightarrow X\}$

$\mathit{forest}[X] == \{r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \land (\forall s : X \leftrightarrow X \mid s \subseteq r \land s \in \mathit{connected} \bullet s \in \mathit{tree})\}$

$\mathrm{injrel}[X, Y] == \{r : X \leftrightarrow Y \mid r^{\sim} \in Y \nrightarrow X\}$

$\mathrm{antireflexive}[X] == \{r : X \leftrightarrow X \mid r \cap (\mathit{id}\, X) = \varnothing\}$

$[X, Y, Z]$

$\mathrm{flip} : (X \rightarrow Y \nrightarrow Z) \rightarrow (Y \rightarrow X \nrightarrow Z)$

$\forall f : X \rightarrow Y \nrightarrow Z \bullet \mathrm{flip}\, f = (\lambda\, y : Y \bullet \lambda\, x : X \bullet f\, x\, y)$

$[X, Y, Z, W]$

$\mathrm{apply} : (X \rightarrow Z) \rightarrow (Y \rightarrow W) \rightarrow (X \times Y) \rightarrow (Z \times W)$

$\forall f : X \rightarrow Z;\ g : Y \rightarrow W;\ x : X;\ y : Y \bullet \mathrm{apply}\, f\, g\, (x, y) = (f\, x, g\, y)$

relation$(\_ =_p \_)$
relation$(\_ \subseteq_p \_)$

$[X, Y]$

$\_ =_p \_ : \mathbb{P}((\mathbb{P}\, X \times \mathbb{P}\, Y) \times (\mathbb{P}\, X \times \mathbb{P}\, Y))$
$\_ \subseteq_p \_ : \mathbb{P}((\mathbb{P}\, X \times \mathbb{P}\, Y) \times (\mathbb{P}\, X \times \mathbb{P}\, Y))$

$\forall xs, zs : \mathbb{P}\, X;\ ys, ws : \mathbb{P}\, Y \bullet (xs, ys) =_p (zs, ws) \Leftrightarrow xs = zs \land ys = ws$

$\forall xs, zs : \mathbb{P}\, X;\ ys, ws : \mathbb{P}\, Y \bullet (xs, ys) \subseteq_p (zs, ws) \Leftrightarrow xs \subseteq zs \land ys \subseteq ws$

$$[X, Y]$$

$$
\begin{array}{l}
\mathrm{map} : (X \rightarrow Y) \rightarrow \mathbb{P}\, X \rightarrow \mathbb{P}\, Y \\
\mathrm{mapS} : (X \rightarrow Y) \rightarrow \mathrm{seq}\, X \rightarrow \mathrm{seq}\, Y \\
\hline
\forall f : X \rightarrow Y \bullet \mathrm{map}\, f\, \{\} = \{\} \\
\forall f : X \rightarrow Y;\ x : X;\ xs : \mathbb{P}\, X \bullet \mathrm{map}\, f\, (\{x\} \cup xs) = \{f\ x\} \cup (\mathrm{map}\, f\ xs) \\
\forall f : X \rightarrow Y \bullet \mathrm{mapS}\, f\, \langle\rangle = \langle\rangle \\
\forall f : X \rightarrow Y;\ x : X;\ xs : \mathrm{seq}\, X \bullet \mathrm{mapS}\, f\, (\langle x\rangle \frown xs) = \langle f\ x\rangle \frown (\mathrm{mapS}\, f\ xs)
\end{array}
$$

function 10 **leftassoc** $(\_ \boxdot \_)$

$$[X]$$

$$
\begin{array}{l}
\_ \boxdot \_ : ((X \rightarrow X) \times \mathbb{P}\, X) \rightarrow (X \mathrel{+\!\!\!\!\rightarrow} X) \\
\hline
\forall f : X \rightarrow X;\ s : \mathbb{P}\, X \bullet f \boxdot s = (\mathrm{id}\ s) \oplus f
\end{array}
$$

function $(\_^{\oplus})$

$$[X]$$

$$
\begin{array}{l}
\_^{\oplus} : (X \leftrightarrow X) \rightarrow (X \leftrightarrow X) \\
\hline
\forall r : X \leftrightarrow X \bullet r^{\oplus} = \mathbf{if}\ r \oplus r \mathbin{;} r = r\ \mathbf{then}\ r\ \mathbf{else}\ (r \oplus r \mathbin{;} r)^{\oplus}
\end{array}
$$

$\mathrm{opt}[X] == \{s : \mathbb{P}\, X \mid \#\, s \leq 1\}$

$$[X]$$

$$
\begin{array}{l}
\mathrm{the} : \mathrm{opt}[X] \mathrel{+\!\!\!\!\rightarrow} X \\
\hline
\forall x : X \bullet \mathrm{the}\, \{x\} = x
\end{array}
$$

$$[X, Y]$$

$$
\begin{array}{l}
\mathit{flatten} : (X \mathrel{+\!\!\!\!\rightarrow} \mathbb{P}\, Y) \rightarrow (X \leftrightarrow Y) \\
\hline
\forall f : X \mathrel{+\!\!\!\!\rightarrow} \mathbb{P}\, Y \bullet \mathit{flatten}\, f = \{x : \mathrm{dom}\, f;\ y : Y \mid y \in f\ x\}
\end{array}
$$

2 Graphs

section *Fragmenta_Graphs* **parents** *standard_toolkit*, *Fragmenta_Generics*

$[V, E]$

$Gr == \{vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s, t : E \nrightarrow V \mid s \in es \to vs \land t \in es \to vs\}$

$$
\begin{array}{|l}
Ns : Gr \to \mathbb{P}\, V \\
Es : Gr \to \mathbb{P}\, E \\
src, tgt : Gr \to E \nrightarrow V \\
\hline
\forall\, vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \nrightarrow V;\ t : E \nrightarrow V \bullet Ns(vs, es, s, t) = vs \\
\forall\, vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \nrightarrow V;\ t : E \nrightarrow V \bullet Es(vs, es, s, t) = es \\
\forall\, vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \nrightarrow V;\ t : E \nrightarrow V \bullet src(vs, es, s, t) = s \\
\forall\, vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \nrightarrow V;\ t : E \nrightarrow V \bullet tgt(vs, es, s, t) = t
\end{array}
$$

$$
\begin{array}{|l}
nNatS, nIntS, nRealS : V \\
\mathrm{to}\mathbb{Z} : V \to \mathrm{opt}[\mathbb{Z}] \\
\mathrm{to}\mathbb{R} : V \to \mathrm{opt}[\mathbb{R}]
\end{array}
$$

$$
\begin{array}{|l}
\varnothing_G : Gr \\
\hline
\varnothing_G = (\varnothing, \varnothing, \varnothing, \varnothing)
\end{array}
$$

$$
\begin{array}{|l}
Els : Gr \to (\mathbb{P}\, V \times \mathbb{P}\, E) \\
EsId : Gr \to \mathbb{P}\, E \\
\hline
\forall\, G : Gr \bullet Els\ G = (Ns\ G, Es\ G) \\
\forall\, G : Gr \bullet EsId\ G = \{e : Es\ G \mid src\ G\ e = tgt\ G\ e\}
\end{array}
$$

relation$(adjacent\ \_)$

$$
\begin{array}{|l}
adjacent\ \_ : \mathbb{P}(Gr \times V \times V) \\
\hline
\forall\, G : Gr;\ v_1, v_2 : V \bullet adjacent(G, v_1, v_2) \Leftrightarrow \exists\, e : Es\ G \bullet src\ G\ e = v_1 \land tgt\ G\ e = v_2
\end{array}
$$

$\mathbf{relation}(adjacent_E \_)$

$$\begin{array}{|l} adjacent_E \_ : \mathbb{P}(Gr \times E \times E) \\ \hline \forall e_1, e_2 : E;\ G : Gr \bullet adjacent_E(G, e_1, e_2) \Leftrightarrow tgt\ G\ e_1 = src\ G\ e_2 \end{array}$$

$\mathbf{function}\ 10\ \mathbf{leftassoc}\ (\_ \circ\!\!\rightarrow\!\!\circ_{Es} \_)$

$$\begin{array}{|l} \_ \circ\!\!\rightarrow\!\!\circ_{Es} \_ : Gr \times \mathbb{P}\ V \rightarrow \mathbb{P}\ E \\ \hline \forall G : Gr;\ vs : \mathbb{P}\ V \bullet G \circ\!\!\rightarrow\!\!\circ_{Es}\ vs = (src\ G)^{\sim} (\!| vs |\!) \cup (tgt\ G)^{\sim} (\!| vs |\!) \end{array}$$

$\mathbf{function}\ 10\ \mathbf{leftassoc}\ (\_ \bullet\!\!\leftrightarrow\!\!\bullet \_)$

$$\begin{array}{|l} \_ \bullet\!\!\leftrightarrow\!\!\bullet \_ : Gr \times \mathbb{P}\ V \rightarrow \mathbb{P}\ E \\ \hline \forall G : Gr;\ vs : \mathbb{P}\ V \bullet G \bullet\!\!\leftrightarrow\!\!\bullet\ vs = (src\ G)^{\sim} (\!| vs |\!) \cap (tgt\ G)^{\sim} (\!| vs |\!) \end{array}$$

$\mathbf{function}\ 10\ \mathbf{leftassoc}\ (\_ \circ\!\!\rightarrow\!\!\circ_{Ns} \_)$

$$\begin{array}{|l} \_ \circ\!\!\rightarrow\!\!\circ_{Ns} \_ : Gr \times \mathbb{P}\ E \rightarrow \mathbb{P}\ V \\ \hline \forall G : Gr;\ es : \mathbb{P}\ E \bullet G \circ\!\!\rightarrow\!\!\circ_{Ns}\ es = \mathrm{ran}(es \lhd src\ G) \cup \mathrm{ran}(es \lhd tgt\ G) \end{array}$$

$\mathbf{function}\ 10\ \mathbf{leftassoc}\ (\_ \bowtie_{Es} \_)$

$$\begin{array}{|l} \_ \bowtie_{Es} \_ : Gr \times \mathbb{P}\ E \rightarrow Gr \\ \hline \forall G : Gr;\ es : \mathbb{P}\ E \bullet G \bowtie_{Es}\ es = (G \circ\!\!\rightarrow\!\!\circ_{Ns}\ es, Es\ G \cap es, es \lhd src\ G, es \lhd tgt\ G) \end{array}$$

$\mathbf{function}\ 10\ \mathbf{leftassoc}\ (\_ \bowtie_{Ns} \_)$

$$
\begin{array}{|l}
\_ \bowtie_{Ns} \_ : Gr \times \mathbb{P}\, V \rightarrow Gr \\
\hline
\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet \\
\quad G \bowtie_{Ns} vs = (Ns\ G \cap vs, G \bullet\!\!\leftrightarrow\!\!\bullet\ vs, (G \bullet\!\!\leftrightarrow\!\!\bullet\ vs) \lhd src\ G, (G \bullet\!\!\leftrightarrow\!\!\bullet\ vs) \lhd tgt\ G)
\end{array}
$$

function 10 **leftassoc** $(\_ \ominus_{Ns} \_)$

$$
\begin{array}{|l}
\_ \ominus_{Ns} \_ : Gr \times \mathbb{P}\, V \rightarrow Gr \\
\hline
\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet \\
\quad G \ominus_{Ns} vs = (Ns\ G \setminus vs, Es\ G \setminus (G \circ\!\!\rightarrow\!\!\circ_{Es}\ vs), (G \circ\!\!\rightarrow\!\!\circ_{Es}\ vs) \ndres src\ G, (G \circ\!\!\rightarrow\!\!\circ_{Es}\ vs) \ndres tgt\ G)
\end{array}
$$

$$
\begin{array}{|l}
\mathrm{successors} : V \times Gr \rightarrow \mathbb{P}\, V \\
\hline
\forall\, v : V;\ G : Gr \bullet \mathrm{successors}(v, G) = \{v_1 : Ns\ G \mid adjacent(G, v, v_1)\}
\end{array}
$$

function$(\_^{\rightleftarrows})$

$$
\begin{array}{|l}
\_^{\rightleftarrows} : Gr \rightarrow Gr \\
\hline
\forall\, G : Gr \bullet G^{\rightleftarrows} = (Ns\ G, Es\ G, tgt\ G, src\ G)
\end{array}
$$

function$(\_^{\leftrightarrow})$

$$
\begin{array}{|l}
\_^{\leftrightarrow} : Gr \rightarrow V \leftrightarrow V \\
\hline
\forall\, G : Gr \bullet G^{\leftrightarrow} = \{v_1, v_2 : Ns\ G \mid adjacent(G, v_1, v_2)\}
\end{array}
$$

function$(\_^{\leftrightarrow_E})$

$$
\begin{array}{|l}
\_^{\leftrightarrow_E} : Gr \rightarrow E \leftrightarrow E \\
\hline
\forall\, G : Gr \bullet G^{\leftrightarrow_E} = \{e_1, e_2 : Es\ G \mid adjacent_E(G, e_1, e_2)\}
\end{array}
$$

relation$(\otimes\ \_)$

$$\begin{array}{|l} \circledcirc\_ : \mathbb{P}\ Gr \\ \hline \forall\ G : Gr \bullet \circledcirc\ G \Leftrightarrow G^{\leftrightarrow_E} \in \text{acyclic} \end{array}$$

relation$(\boxminus_{Es}\ \_)$
relation$(\boxminus\ \_)$

$$\begin{array}{|l} \boxminus_{Es}\ \_, \boxminus\ \_ : \mathbb{P}(Gr \times Gr) \\ \hline \forall\ G_1, G_2 : Gr \bullet \boxminus_{Es}(G_1, G_2) \Leftrightarrow Es\ G_1 \cap Es\ G_2 = \varnothing \\ \forall\ G_1, G_2 : Gr \bullet \boxminus(G_1, G_2) \Leftrightarrow Ns\ G_1 \cap Ns\ G_2 = \varnothing \land \boxminus_{Es}(G_1, G_2) \end{array}$$

relation$(\boxplus\ \_)$

$$\begin{array}{|l} [I] \\ \boxplus\ \_ : \mathbb{P}(I \rightarrowtail\!\!\!\!\!+ Gr) \\ \hline \forall\ Gs : I \rightarrowtail\!\!\!\!\!+ Gr \bullet \boxplus\ Gs \Leftrightarrow \forall\ i, j : \mathrm{dom}\ Gs \mid i \neq j \bullet \boxminus(Gs\ i, Gs\ j) \\ \hline \end{array}$$

function 10 **leftassoc** $(\_ \cup_G \_)$

$$\begin{array}{|l} \_ \cup_G \_ : Gr \times Gr \rightarrow Gr \\ \hline \forall\ G_1, G_2 : Gr \bullet G_1 \cup_G G_2 = (Ns\ G_1 \cup Ns\ G_2, Es\ G_1 \cup Es\ G_2, src\ G_1 \cup src\ G_2, tgt\ G_1 \cup tgt\ G_2) \end{array}$$

function 10 **leftassoc** $(\_ \odot \_)$

$$\begin{array}{|l} \_ \odot \_ : Gr \times (V \leftrightarrow V) \nrightarrow Gr \\ \hline \forall\ G : Gr;\ s : V \leftrightarrow V \mid s \in Ns\ G \nrightarrow Ns\ G \land s \in \text{antireflexive} \bullet \\ \quad G \odot s = (Ns\ G \setminus \mathrm{dom}\ s, Es\ G, (s \boxdot Ns\ G) \circ (src\ G), (s \boxdot Ns\ G) \circ (tgt\ G)) \end{array}$$

$GrM == (V \nrightarrow V) \times (E \nrightarrow E)$

$$
\begin{array}{|l}
fV : GrM \to V \nrightarrow V \\
fE : GrM \to E \nrightarrow E \\
\hline
\forall fv : V \nrightarrow V;\ fe : E \nrightarrow E \bullet fV(fv, fe) = fv \\
\forall fv : V \nrightarrow V;\ fe : E \nrightarrow E \bullet fE(fv, fe) = fe
\end{array}
$$

$$
\begin{array}{|l}
\mathrm{gid} : Gr \to GrM \\
\hline
\forall G : Gr \bullet \mathrm{gid}\ G = (id\ (Ns\ G), id\ (Es\ G))
\end{array}
$$

$$
\begin{array}{|l}
\varnothing_{GM} : GrM \\
\hline
\varnothing_{GM} = (\{\}, \{\})
\end{array}
$$

$$
\begin{array}{|l}
\mathrm{domg}, \mathrm{codg} : GrM \to (\mathbb{P}\ V \times \mathbb{P}\ E) \\
\hline
\forall m : GrM \bullet \mathrm{domg}\ m = (\mathrm{dom}(fV\ m), \mathrm{dom}(fE\ m)) \\
\forall m : GrM \bullet \mathrm{codg}\ m = (\mathrm{ran}(fV\ m), \mathrm{ran}(fE\ m))
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_{GM} \_)$

$$
\begin{array}{|l}
\_ \cup_{GM} \_ : GrM \times GrM \nrightarrow GrM \\
\bigcup_{GM} : \mathbb{P}\ GrM \to GrM \\
\hline
\forall f, g : GrM \bullet f \cup_{GM} g = (fV\ f \cup fV\ g, fE\ f \cup fE\ g) \\
\bigcup_{GM} \varnothing = \varnothing_{GM} \\
\forall f : GrM;\ fs : \mathbb{P}\ GrM \bullet \bigcup_{GM} (\{f\} \cup fs) = f \cup_{GM} (\bigcup_{GM} fs)
\end{array}
$$

function 10 **leftassoc** $(\_ \to_G \_)$

$$
\begin{array}{|l}
\_ \to_G \_ : Gr \times Gr \to \mathbb{P}\ GrM \\
\hline
\forall G_1, G_2 : Gr \bullet G_1 \to_G G_2 = \{fv : Ns\ G_1 \to Ns\ G_2;\ fe : Es\ G_1 \to Es\ G_2\ | \\
\quad src\ G_2 \circ fe = fv \circ src\ G_1 \land tgt\ G_2 \circ fe = fv \circ tgt\ G_1\}
\end{array}
$$

function 10 **leftassoc** $(\_ \nrightarrow_G \_)$

$$
\begin{array}{|l}
\_ \rightarrowtail_G \_ : Gr \times Gr \rightarrow \mathbb{P}\, GrM \\
\hline
\forall\, G_1, G_2 : Gr \bullet G_1 \rightarrow_G G_2 = \{fv : Ns\ G_1 \nrightarrow Ns\ G_2;\ fe : Es\ G_1 \nrightarrow Es\ G_2 \mid \\
\quad src\ G_2 \circ fe = fv \circ ((\mathrm{dom}\, fe) \lhd (src\ G_1)) \land tgt\ G_2 \circ fe = fv \circ ((\mathrm{dom}\, fe) \lhd (tgt\ G_1))\}
\end{array}
$$

function 10 **leftassoc** $(\_ \circ_G \_)$

$$
\begin{array}{|l}
\_ \circ_G \_ : GrM \times GrM \rightarrow GrM \\
\hline
\forall\, g, f : GrM \bullet g \circ_G f = (fV\ g \circ fV\ f, fE\ g \circ fE\ f)
\end{array}
$$

3 Graphs with typing

section *Fragmenta_GrswT* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$GrwT == \{G : Gr;\ t : GrM \mid \mathrm{domg}\, t =_p Els\ G\}$

function$(\_^G)$
function$(\_^T)$

$$
\begin{array}{|l}
\_^G : GrwT \rightarrow Gr \\
\_^T : GrwT \rightarrow GrM \\
\hline
\forall\, G : Gr;\ t : GrM \bullet (G, t)^G = G \\
\forall\, G : Gr;\ t : GrM \bullet (G, t)^T = t
\end{array}
$$

$$
\begin{array}{|l}
\varnothing_{GwT} : GrwT \\
\hline
\varnothing_{GwT} = (\varnothing_G, \varnothing_{GM})
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_{GwT} \_)$

$$
\begin{array}{|l}
\_ \cup_{GwT} \_ : GrwT \times GrwT \rightarrow GrwT \\
\hline
\forall\, G_1, G_2 : GrwT \bullet G_1 \cup_{GwT} G_2 = ({G_1}^G \cup_G {G_2}^G, {G_1}^T \cup_{GM} {G_2}^T)
\end{array}
$$

4 Graphs with Extra typing

section *Fragmenta_GrswET* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*, *Fragmenta_GrswT*

$$GrwET == \{G : GrwT;\ et : GrM \mid \operatorname{domg} et \subseteq_p Els\,(G^{G})\}$$

$$\begin{array}{|l}
\varnothing_{GwET} : GrwET \\
\hline
\varnothing_{GwET} = (\varnothing_{GwT}, \varnothing_{GM})
\end{array}$$

function($\_^{Gw}$)
function($\_^{G}$)
function($\_^{T}$)
function($\_^{ET}$)

$$\begin{array}{|l}
_^{Gw} : GrwET \rightarrow GrwT \\
_^{G} : GrwET \rightarrow Gr \\
_^{T} : GrwET \rightarrow GrM \\
_^{ET} : GrwET \rightarrow GrM \\
\hline
\forall G : GrwT;\ et : GrM \bullet (G, et)^{Gw} = G \\
\forall G : GrwT;\ et : GrM \bullet (G, et)^{G} = G^{G} \\
\forall G : GrwT;\ et : GrM \bullet (G, et)^{T} = G^{T} \\
\forall G : GrwT;\ et : GrM \bullet (G, et)^{ET} = et
\end{array}$$

function 10 **leftassoc** ($\_ \cup_{GwET} \_$)

$$\begin{array}{|l}
_ \cup_{GwET} _ : GrwET \times GrwET \rightarrow GrwET \\
\hline
\forall G_1, G_2 : GrwET \bullet G_1 \cup_{GwET} G_2 = ({G_1}^{Gw} \cup_{GwT} {G_2}^{Gw}, {G_1}^{ET} \cup_{GM} {G_2}^{ET})
\end{array}$$

5 SG Element Types

section *Fragmenta_SGElemTys* **parents** *standard_toolkit*, *Fragmenta_Generics*

$$SGNT ::= nnrml \mid nabst \mid nprxy \mid nenum \mid nval \mid nvirt$$

$$SGED ::= dbi \mid duni$$

$$SGET ::= einh \mid ecomp\langle\langle SGED\rangle\rangle \mid erel\langle\langle SGED\rangle\rangle \mid eder \mid epath \mid evcnt$$

relation$(\_ \prec_{NT} \_)$

$\prec_{NT}$, an ordering relation on set $SGNT$, indicates the node types that can be inheritance-related:

$$\begin{array}{|l}
_ \prec_{NT} _ : SGNT \leftrightarrow SGNT \\
\hline
\forall\, nt_1, nt_2 : SGNT \bullet nt_1 \prec_{NT} nt_2 \Leftrightarrow nt_1 \neq nprxy \land nt_2 \neq nval \\
\quad \land\, nt_1 \in \{nabst, nvirt\} \Rightarrow nt_2 \in \{nabst, nvirt\} \land (nt_1 = nenum \Rightarrow nt_2 = nvirt)
\end{array}$$

$\leq_{rNT}$ and $\leq_{ET}$ are ordering relations on sets $SGNT$ and $SGET$, respectively, indicating node and edge types that can be refinement-related, respectively:

relation$(\_ \leq_{rNT} \_)$

$$\begin{array}{|l}
_ \leq_{rNT} _ : SGNT \leftrightarrow SGNT \\
\hline
\forall\, nt_1, nt_2 : SGNT \bullet nt_1 \leq_{rNT} nt_2 \Leftrightarrow nt_1 = nt_2 \lor nt_1 = nprxy \lor nt_1 = nnrml \\
\quad \land\, nt_2 = nprxy \lor nt_2 \in \{nabst, nvirt\} \land nt_1 \in \{nnrml, nvirt, nabst\} \lor nt_2 = nnrml
\end{array}$$

relation$(\_ =_{ET} \_)$

$$\begin{array}{|l}
_ =_{ET} _ : SGET \leftrightarrow SGET \\
\hline
\forall\, et_1, et_2 : SGET \bullet et_1 =_{ET} et_2 \Leftrightarrow et_1 = et_2 \\
\quad \lor\, (\forall\, d_1, d_2 : SGED \bullet et_1 = erel\ d_1 \land et_2 = erel\ d_2 \lor et_1 = ecomp\ d_1 \land et_2 = ecomp\ d_2)
\end{array}$$

relation$(\_ \leq_{ET} \_)$

$$\begin{array}{|l}
_ \leq_{ET} _ : SGET \leftrightarrow SGET \\
\hline
\forall\, et_1, et_2 : SGET \bullet et_1 \leq_{ET} et_2 \Leftrightarrow \neg\, \{einh, eder, epath, evcnt\} \subseteq \{et_1, et_2\} \\
\quad \land\, (et_1 =_{ET} et_2 \lor et_1 \in \mathrm{dom}(ecomp^{\sim}) \land et_2 = erel\ dbi \\
\qquad \lor\, et_1 = ecomp\ duni \land et_2 = erel\ duni)
\end{array}$$

6 Multiplicities

section *Fragmenta_Mult* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_SGElemTys*

$MultVal ::= \mathbf{v}\langle\langle \mathbb{N} \rangle\rangle \mid *$

$MultC ::= mr\langle\langle \mathbb{N} \times MultVal \rangle\rangle \mid ms\langle\langle MultVal \rangle\rangle$

relation$(\_ =_{mv} \_)$

$$\begin{array}{|l}
_ =_{mv} _ : MultVal \leftrightarrow MultVal \\
\hline
\forall m_1, m_2 : MultVal \bullet m_1 =_{mv} m_2 \Leftrightarrow \{m_1, m_2\} \subseteq \{*\} \lor \exists n : \mathbb{N} \bullet m_1 = \mathbf{v}\ n \land m_2 = \mathbf{v}\ n
\end{array}$$

function 10 **leftassoc** $(\_ *_{mv} \_)$

$$\begin{array}{|l}
_ *_{mv} _ : MultVal \times MultVal \to MultVal \\
\hline
\forall m : MultVal \bullet * *_{mv} m = * \\
\forall m : MultVal \bullet m *_{mv} * = * \\
\forall n_1, n_2 : \mathbb{N} \bullet (\mathbf{v}\ n_1) *_{mv} (\mathbf{v}\ n_2) = \mathbf{v}(n_1 * n_2)
\end{array}$$

$$\begin{array}{|l}
mlbn : MultC \to \mathbb{N} \\
mlb, mub : MultC \to MultVal \\
\hline
mlbn(ms\, *) = 0 \\
\forall k : \mathbb{N} \bullet mlbn(ms(\mathbf{v}\ k)) = k \\
\forall k, m : \mathbb{N} \bullet mlbn(mr(k, \mathbf{v}\ m)) = k \\
\forall mc : MultC \bullet mlb\ mc = \mathbf{v}(mlbn\ mc) \\
\forall mv : MultVal \bullet mub(ms\ mv) = mv \\
\forall k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ m
\end{array}$$

function 10 **leftassoc** $(\_ *_{mr} \_)$

$$\begin{array}{|l}
_ *_{mr} _ : \mathit{MultC} \times \mathit{MultC} \pfun \mathit{MultC} \\
\hline
\forall n_1, n_2 : \mathbb{N};\ mv_1, mv_2 : \mathit{MultVal} \bullet mr(n_1, mv_1) *_{mr} mr(n_2, mv_2) = mr(n_1 * n_2, mv_1 *_{mv} mv_2)
\end{array}$$

$\mathbf{relation}(\_ \leq_{mv} \_)$

$$\begin{array}{|l}
_ \leq_{mv} _ : \mathit{MultVal} \leftrightarrow \mathit{MultVal} \\
\hline
\forall m_1, m_2 : \mathit{MultVal} \bullet m_1 =_{mv} m_2 \Leftrightarrow \{m_1, m_2\} \subseteq \{*\} \\
\quad \lor ((mlb \circ ms)\, m_1 = (mlb \circ ms)\, m_2 \land (mub \circ ms)\, m_1 = (mub \circ ms)\, m_2) \\
\forall m_1, m_2 : \mathit{MultVal} \bullet m_1 \leq_{mv} m_2 \Leftrightarrow m_2 = * \lor \exists j, k : \mathbb{N} \mid m_1 = \mathbf{v}\ j \land m_2 = \mathbf{v}\ k \bullet j \leq k
\end{array}$$

$$\begin{array}{l}
\mathit{MultCOk} == \{m : \mathit{MultC} \mid \exists lb : \mathbb{N};\ ub : \mathit{MultVal} \bullet m = mr(lb, ub) \land \mathbf{v}\ lb \leq_{mv} ub \\
\quad \lor \exists mv : \mathit{MultVal} \bullet m = ms\ mv\,\}
\end{array}$$

$$\mathit{MultCMany} == \{ms\,*, mr(0, *)\}$$

$$\mathit{Mult} == \{\, ms : \mathbb{P}_1\, \mathit{MultCOk} \mid \#\, ms > 1 \Rightarrow (\forall m : ms \bullet m \notin \mathit{MultCMany})\,\}$$

$$\begin{array}{|l}
mk : \mathbb{N} \to \mathit{MultC} \\
mks : \mathbb{N} \to \mathit{Mult} \\
mrs : \mathbb{N} \times \mathit{MultVal} \to \mathit{Mult} \\
mopt : \mathit{MultC} \\
mopts, mmanys : \mathit{Mult} \\
\hline
\forall k : \mathbb{N} \bullet mk\ k = ms(\mathbf{v}\ k) \land mks\ k = \{mk\ k\} \\
mopt = mr(0, \mathbf{v}\ 1) \land mopts = \{mopt\} \land mmanys = \{ms\,*\} \\
\forall lb : \mathbb{N};\ ub : \mathit{MultVal} \bullet mrs(lb, ub) = \{mr(lb, ub)\}
\end{array}$$

$$\mathit{MultMany} == \{\, s : \mathit{Mult} \mid \exists m : \mathit{MultC} \bullet s = \{m\} \land m \in \mathit{MultCMany}\,\}$$

$$\begin{array}{l}
\mathit{MultRange} == \{s : \mathit{Mult} \mid \exists m : \mathit{MultC} \bullet s = \{m\} \land (\exists k : \mathbb{N} \mid k > 1 \bullet m = ms(\mathbf{v}\ k) \\
\quad \lor \exists lb : \mathbb{N};\ umv : \mathit{MultVal} \mid \mathbf{v}\ 2 \leq_{mv} umv \bullet m = mr(lb, umv))\}
\end{array}$$

$$\mathit{MultEither} == \{s : \mathit{Mult} \mid \#\, s > 1\}$$

$$\mathit{MultLBZ} == \{ms : \mathit{Mult} \mid \exists m : ms \bullet mlbn\ m = 0\}$$

$\mathbf{relation}(\_ \between \_)$
$\mathbf{relation}(\_ \cdots \_)$

$$\_ \between \_ : \mathbb{P}(\mathbb{N} \times MultC)$$
$$\forall k : \mathbb{N};\ m : MultC \bullet k \between m \Leftrightarrow mlb\ m \leq_{mv} \mathbf{v}\ k \land \mathbf{v}\ k \leq_{mv} mub\ m$$

$$\_ \cdots \_ : \mathbb{P}(\mathbb{N} \times Mult)$$
$$\forall k : \mathbb{N};\ m : MultC \bullet k \cdots \{m\} \Leftrightarrow k \between m$$
$$\forall k : \mathbb{N};\ m : MultC;\ sms : Mult \bullet k \cdots (\{ m \} \cup sms) \Leftrightarrow k \between m \lor k \cdots sms$$

relation$(\_ \leq_{\mathcal{M}c} \_)$

$$\_ \leq_{\mathcal{M}c} \_ : MultC \leftrightarrow MultC$$
$$\forall m_1, m_2 : MultC \bullet m_1 \leq_{\mathcal{M}c} m_2 \Leftrightarrow mlb\ m_2 \leq_{mv} mlb\ m_1 \land mub\ m_1 \leq_{mv} mub\ m_2$$

relation$(\_ \leq_{\mathcal{M}} \_)$

$$\_ \leq_{\mathcal{M}} \_ : Mult \leftrightarrow Mult$$
$$\forall m_1, m_2 : Mult \bullet m_1 \leq_{\mathcal{M}} m_2 \Leftrightarrow \forall mc_1 : m_1 \bullet \exists mc_2 : m_2 \bullet mc_1 \leq_{\mathcal{M}c} mc_2$$

relation$(\_ \propto \_)$

$$\_ \propto \_ : \mathbb{P}(SGET \times (Mult \times Mult))$$
$$\begin{aligned}&\forall et : SGET;\ m_1, m_2 : Mult \bullet et \propto (m_1, m_2) \Leftrightarrow et = erel\ dbi \lor et = eder\\ &\quad \lor et = ecomp\ duni \land m_1 = mks\ 1\\ &\quad \lor et = erel\ duni \land m_1 \in MultMany\\ &\quad \lor et = ecomp\ dbi \land m_1 \in \{mks\ 1, mopts\}\end{aligned}$$

relation(rbounded_)
relation(eitherbounded_)

$$[X, Y]$$
$$\mathrm{rbounded}\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\, X \times MultC)$$
$$\mathrm{eitherbounded}\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\, X \times Mult)$$
$$\begin{aligned}&\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ m : MultC \bullet\\ &\quad \mathrm{rbounded}(r, s, m) \Leftrightarrow \forall x : s \bullet \#(r \llparenthesis \{x\} \rrparenthesis) \between m\\ &\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ ms : Mult \bullet\\ &\quad \mathrm{eitherbounded}(r, s, ms) \Leftrightarrow \forall x : s \bullet \#(r \llparenthesis \{x\} \rrparenthesis) \cdots ms\end{aligned}$$

relation$(r\mathcal{M}Ok\,\_)$

$$
\begin{array}{l}
[X, Y] \\
\hline
r\mathcal{M}Ok\,\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\,X \times \mathbb{P}\,Y \times \mathit{Mult} \times \mathit{Mult}) \\
\hline
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : \mathit{Mult} \bullet r\mathcal{M}Ok(r, s, t, \mathit{mks}\,0, m) \Leftrightarrow s \lhd r = \{\} \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : \mathit{Mult} \bullet r\mathcal{M}Ok(r, s, t, m, \mathit{mks}\,0) \Leftrightarrow r \rhd t = \{\} \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet r\mathcal{M}Ok(r, s, t, \mathit{mks}\,1, \mathit{mks}\,1) \Leftrightarrow r \in s \mathrel{⤖} t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet r\mathcal{M}Ok(r, s, t, \mathit{mopts}, \mathit{mks}\,1) \Leftrightarrow r \in s \rightarrowtail t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet r\mathcal{M}Ok(r, s, t, \mathit{mks}\,1, \mathit{mopts}) \Leftrightarrow r^{\sim} \in t \rightarrowtail s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : \mathit{MultMany} \bullet r\mathcal{M}Ok(r, s, t, mm, \mathit{mks}\,1) \Leftrightarrow r \in s \to t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : \mathit{MultMany} \bullet r\mathcal{M}Ok(r, s, t, \mathit{mks}\,1, mm) \Leftrightarrow r^{\sim} \in t \to s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mr : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr, \mathit{mks}\,1) \Leftrightarrow r \in s \to t \land \mathrm{rbounded}(r^{\sim}, t, \mathrm{the}\, mr) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mr : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mks}\,1, mr) \Leftrightarrow r^{\sim} \in t \to s \land \mathrm{rbounded}(r, s, \mathrm{the}\, mr) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ me : \mathit{MultEither} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, me, \mathit{mks}\,1) \Leftrightarrow r \in s \to t \land \mathrm{eitherbounded}(r^{\sim}, t, me) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ me : \mathit{MultEither} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mks}\,1, me) \Leftrightarrow r^{\sim} \in t \to s \land \mathrm{eitherbounded}(r, s, me) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet r\mathcal{M}Ok(r, s, t, \mathit{mopts}, \mathit{mopts}) \Leftrightarrow r \in s \mathrel{⤔} t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : \mathit{MultMany} \bullet r\mathcal{M}Ok(r, s, t, mm, \mathit{mopts}) \Leftrightarrow r \in s \mathrel{⇸} t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mopts}, mm) \Leftrightarrow r^{\sim} \in t \mathrel{⇸} s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, m, \mathit{mopts}) \Leftrightarrow r \in s \mathrel{⇸} t \land \mathrm{rbounded}(r^{\sim}, t, \mathrm{the}\, m) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mopts}, m) \Leftrightarrow r^{\sim} \in t \mathrel{⇸} s \land \mathrm{rbounded}(r, s, \mathrm{the}\, m) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : \mathit{MultEither} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, m, \mathit{mopts}) \Leftrightarrow r \in s \mathrel{⇸} t \land \mathrm{eitherbounded}(r^{\sim}, t, m) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : \mathit{MultEither} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mopts}, m) \Leftrightarrow r^{\sim} \in t \mathrel{⇸} s \land \mathrm{eitherbounded}(r, s, m) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm_1, mm_2 : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mm_1, mm_2) \Leftrightarrow r \in s \leftrightarrow t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : \mathit{MultMany};\ mr : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mm, mr) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r, s, \mathrm{the}\, mr) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : \mathit{MultMany};\ mr : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr, mm) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r^{\sim}, t, \mathrm{the}\, mr) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : \mathit{MultMany};\ me : \mathit{MultEither} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mm, me) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{eitherbounded}(r, s, me) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : \mathit{MultMany};\ me : \mathit{MultEither} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, me, mm) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{eitherbounded}(r^{\sim}, t, me) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mr_1, mr_2 : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr_1, mr_2) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r, s, \mathrm{the}\, mr_2) \land \mathrm{rbounded}(r^{\sim}, t, \mathrm{the}\, mr_1) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ me_1, me_2 : \mathit{MultEither} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, me_1, me_2) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{eitherbounded}(r, s, me_2) \land \mathrm{eitherbounded}(r^{\sim}, t, me_1)
\end{array}
$$

7 Path Expressions

section $Fragmenta\_PEs$ **parents** $standard\_toolkit, Fragmenta\_Generics, Fragmenta\_Graphs$

$PEA ::= edg \langle\!\langle E \rangle\!\rangle \mid einv \langle\!\langle E \rangle\!\rangle$

$PEC ::= eat \langle\!\langle PEA \rangle\!\rangle \mid eresd \langle\!\langle V \times PEA \rangle\!\rangle \mid eresr \langle\!\langle PEA \times V \rangle\!\rangle$

$PE ::= ec \langle\!\langle PEC \rangle\!\rangle \mid scmp \langle\!\langle PEC \times PE \rangle\!\rangle$

$ePEA : PEA \rightarrow E$

$\forall e : E \bullet ePEA(edg\ e) = e$

$\forall e : E \bullet ePEA(einv\ e) = e$

$startEA_C : PEC \rightarrow PEA$
$startEA : PE \rightarrow PEA$

$\forall pea : PEA \bullet startEA_C(eat\ pea) = pea$

$\forall v : V;\ pea : PEA \bullet startEA_C(eresd\ (v, pea)) = pea$

$\forall v : V;\ pea : PEA \bullet startEA_C(eresr\ (pea, v)) = pea$

$\forall pec : PEC \bullet startEA(ec\ pec) = startEA_C\ pec$

$\forall pec : PEC;\ pe : PE \bullet startEA(scmp\ (pec, pe)) = startEA_C\ pec$

$endEA_C : PEC \rightarrow PEA$
$endEA : PE \rightarrow PEA$

$\forall pea : PEA \bullet endEA_C(eat\ pea) = pea$

$\forall v : V;\ pea : PEA \bullet endEA_C(eresd\ (v, pea)) = pea$

$\forall v : V;\ pea : PEA \bullet endEA_C(eresr\ (pea, v)) = pea$

$\forall pec : PEC \bullet endEA(ec\ pec) = endEA_C(pec)$

$\forall pec : PEC;\ pe : PE \bullet endEA(scmp\ (pec, pe)) = endEA\ pe$

$$
\begin{array}{|l}
srcPEA : Gr \to PEA \to V \\
srcPEC : Gr \to PEC \to V \\
srcPE : Gr \to PE \to V \\
\hline
\forall\, G : Gr;\ e : E \bullet srcPEA\ G\ (edg\ e) = src\ G\ e \\
\forall\, G : Gr;\ e : E \bullet srcPEA\ G\ (einv\ e) = tgt\ G\ e \\
\forall\, G : Gr;\ v : V;\ pea : PEA \bullet srcPEC\ G\ (eat\ pea) = srcPEA\ G\ pea \\
\forall\, G : Gr;\ v : V;\ pea : PEA \bullet srcPEC\ G\ (eresd\ (v, pea)) = srcPEA\ G\ pea \\
\forall\, G : Gr;\ v : V;\ pea : PEA \bullet srcPEC\ G\ (eresr\ (pea, v)) = srcPEA\ G\ pea \\
\forall\, G : Gr;\ pec : PEC \bullet srcPE\ G\ (ec\ pec) = srcPEC\ G\ pec \\
\forall\, G : Gr;\ pec : PEC;\ pe : PE \bullet srcPE\ G\ (scmp\ (pec, pe)) = srcPEC\ G\ pec
\end{array}
$$

$$
\begin{array}{|l}
tgtPEA : Gr \to PEA \to V \\
tgtPEC : Gr \to PEC \to V \\
tgtPE : Gr \to PE \to V \\
\hline
\forall\, G : Gr;\ e : E \bullet tgtPEA\ G\ (edg\ e) = tgt\ G\ e \\
\forall\, G : Gr;\ e : E \bullet tgtPEA\ G\ (einv\ e) = src\ G\ e \\
\forall\, G : Gr;\ v : V;\ pea : PEA \bullet tgtPEC\ G\ (eat\ pea) = tgtPEA\ G\ pea \\
\forall\, G : Gr;\ v : V;\ pea : PEA \bullet tgtPEC\ G\ (eresd\ (v, pea)) = tgtPEA\ G\ pea \\
\forall\, G : Gr;\ v : V;\ pea : PEA \bullet tgtPEC\ G\ (eresr\ (pea, v)) = tgtPEA\ G\ pea \\
\forall\, G : Gr;\ pec : PEC \bullet tgtPE\ G\ (ec\ pec) = tgtPEC\ G\ pec \\
\forall\, G : Gr;\ pec : PEC;\ pe : PE \bullet tgtPE\ G\ (scmp\ (pec, pe)) = tgtPE\ G\ pe
\end{array}
$$

$$
\begin{array}{|l}
rsrcPE : PE \to E \\
rtgtPE : PE \to E \\
\hline
\forall\, pe : PE \bullet rsrcPE\ pe = ePEA\ (startEA\ pe) \\
\forall\, pe : PE \bullet rtgtPE\ pe = ePEA\ (endEA\ pe)
\end{array}
$$

8 Structural Graphs

section *Fragmenta_SGs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*, *Fragmenta_SGElemTys*, *Fragmenta_Mult*, *Fragmenta_PEs*, *Fragmenta_GrswT*

Z Type VCI represents the information associated with a value constraint edge: an operator such as equality ($SGVCEOP$) and an optional edge, which is either one of the edges or the node or none if the constraint edge refers to the values of the node itself.

$SGVCEOP ::= eq \mid neq \mid leq \mid geq \mid lt \mid gt$

$VCI == SGVCEOP \times opt[E]$

$$SGr_0 == \{G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ p : E \pfun PE;$$
$$d : E \leftrightarrow E;\ vci : E \pfun VCI \mid nt \in Ns\ G \rightarrow SGNT \land et \in Es\ G \rightarrow SGET \land d \in Es\ G \leftrightarrow Es\ G\}$$

$$gr : SGr_0 \rightarrow Gr$$
$$sg\_Ns : SGr_0 \rightarrow \mathbb{P}\ V$$
$$sg\_Es : SGr_0 \rightarrow \mathbb{P}\ E$$
$$sg\_src, sg\_tgt : SGr_0 \rightarrow E \pfun V$$
$$nty : SGr_0 \rightarrow V \pfun SGNT$$
$$ety : SGr_0 \rightarrow E \pfun SGET$$
$$srcm, tgtm : SGr_0 \rightarrow E \pfun Mult$$
$$pe : SGr_0 \rightarrow E \pfun PE$$
$$ds : SGr_0 \rightarrow E \leftrightarrow E$$
$$vcei : SGr_0 \rightarrow E \pfun SGVCEOP \times opt[E]$$

$$\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ p : E \pfun PE;\ d : E \leftrightarrow E;$$
$$vci : E \pfun VCI \bullet gr\,(G, nt, et, sm, tm, p, d, vci) = G$$
$$sg\_Ns = Ns \circ gr$$
$$sg\_Es = Es \circ gr$$
$$sg\_src = src \circ gr$$
$$sg\_tgt = tgt \circ gr$$
$$\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ p : E \pfun PE;\ d : E \leftrightarrow E;$$
$$vci : E \pfun VCI \bullet nty(G, nt, et, sm, tm, p, d, vci) = nt$$
$$\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ p : E \pfun PE;\ d : E \leftrightarrow E;$$
$$vci : E \pfun VCI \bullet ety(G, nt, et, sm, tm, p, d, vci) = et$$
$$\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ p : E \pfun PE;\ d : E \leftrightarrow E;$$
$$vci : E \pfun VCI \bullet srcm(G, nt, et, sm, tm, p, d, vci) = sm$$
$$\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ p : E \pfun PE;\ d : E \leftrightarrow E;$$
$$vci : E \pfun VCI \bullet tgtm(G, nt, et, sm, tm, p, d, vci) = tm$$
$$\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ p : E \pfun PE;\ d : E \leftrightarrow E;$$
$$vci : E \pfun VCI \bullet pe(G, nt, et, sm, tm, p, d, vci) = p$$
$$\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ p : E \pfun PE;\ d : E \leftrightarrow E;$$
$$vci : E \pfun VCI \bullet ds(G, nt, et, sm, tm, p, d, vci) = d$$
$$\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ p : E \pfun PE;\ d : E \leftrightarrow E;$$
$$vci : E \pfun VCI \bullet vcei(G, nt, et, sm, tm, p, d, vci) = vci$$

$$\emptyset_{SG} : SGr_0$$

$$\emptyset_{SG} = (\emptyset_G, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset)$$

$$NsTy : SGr_0 \rightarrow \mathbb{P}\,SGNT \rightarrow \mathbb{P}\ V$$
$$EsTy : SGr_0 \rightarrow \mathbb{P}\,SGET \rightarrow \mathbb{P}\ E$$

$$\forall SG : SGr_0;\ nts : \mathbb{P}\,SGNT \bullet NsTy\ SG\ nts = (nty\ SG)^\sim (\!|\, nts \,|\!)$$
$$\forall SG : SGr_0;\ ets : \mathbb{P}\,SGET \bullet EsTy\ SG\ ets = (ety\ SG)^\sim (\!|\, ets \,|\!)$$

$$EsA, EsI, EsM, EsD, EsVCnt, EsPa, EsPaCnt : SGr_0 \to \mathbb{P}\ E$$

$$EsA = (\text{flip}\ EsTy)\ (erel \mathbin{(\!|}\ SGED\ \mathbin{|\!)} \cup ecomp \mathbin{(\!|}\ SGED\ \mathbin{|\!)})$$
$$EsI = (\text{flip}\ EsTy)\ \{einh\}$$
$$EsD = (\text{flip}\ EsTy)\ \{eder\}$$
$$EsPa = (\text{flip}\ EsTy)\ \{epath\}$$
$$EsVCnt = (\text{flip}\ EsTy)\ \{evcnt\}$$
$$\forall\ SG : SGr_0 \bullet EsM\ SG = EsA\ SG \cup EsD\ SG$$
$$\forall\ SG : SGr_0 \bullet EsPaCnt\ SG = EsD\ SG \cup EsPa\ SG$$

$$NsN, NsP, NsEther, NsVi, NsVa : SGr_0 \to \mathbb{P}\ V$$

$$NsN = (\text{flip}\ NsTy)\ \{nnrml\}$$
$$NsP = (\text{flip}\ NsTy)\ \{nprxy\}$$
$$NsEther = (\text{flip}\ NsTy)\ \{nabst, nvirt, nenum\}$$
$$NsVi = (\text{flip}\ NsTy)\ \{nvirt\}$$
$$NsVa = (\text{flip}\ NsTy)\ \{nval\}$$

$$tpe : SGr_0 \to E \nrightarrow PE$$

$$\forall\ SG : SGr_0 \bullet tpe\ SG = \{e : sg\_Es\ SG \bullet (e, (ec \circ eat \circ edg)\ e)\} \oplus pe\ SG$$

$$\pitchfork : SGr_0 \to Gr$$
$$\prec : SGr_0 \to V \leftrightarrow V$$

$$\forall\ SG : SGr_0 \bullet \pitchfork\ SG = gr\ SG \bowtie_{Es} EsI\ SG$$
$$\prec = (\_^{\leftrightarrow}) \circ \pitchfork$$

$$srcma : SGr_0 \to (E \nrightarrow Mult)$$

$$\forall\ SG : SGr_0 \bullet srcma\ SG = (srcm\ SG) \oplus (EsTy\ SG\ \{ecomp\ duni\} \times \{mks\ 1\}) \oplus (EsTy\ SG\ \{erel\ duni\} \times \{\{ms\ *\}\})$$

relation$(\mathcal{M}etysOk\ \_)$

$$\begin{array}{|l}
\mathcal{M}etysOk\,_ : \mathbb{P}\,SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \mathcal{M}etysOk\ SG \Leftrightarrow \forall\, e : EsM\ SG \bullet (ety\ SG\ e) \propto (srcma\ SG\ e, tgtm\ SG\ e)
\end{array}$$

$$\begin{array}{|l}
\preceq\, : SGr_0 \to V \leftrightarrow V \\
\hline
\forall\, SG : SGr_0 \bullet \preceq SG = (\prec SG)^{*}
\end{array}$$

$$\begin{array}{|l}
rsrc, rtgt : SGr_0 \to \mathbb{P}\,E \to E \leftrightarrow V \\
src_M^*, src^*, tgt_M^*, tgt^* : SGr_0 \to E \leftrightarrow V \\
\hline
\forall\, SG : SGr_0;\ es : \mathbb{P}\,E \bullet rsrc\,SG\,es = es \lhd (sg_src\,SG) \\
\forall\, SG : SGr_0;\ es : \mathbb{P}\,E \bullet rtgt\,SG\,es = es \lhd (sg_tgt\,SG) \\
\forall\, SG : SGr_0 \bullet src_M^*\ SG = (rsrc\,SG\,(EsA\,SG)) \mathbin{;} (\preceq SG)^{\sim} \\
\forall\, SG : SGr_0 \bullet src^*\ SG = (rsrc\,SG\,(EsA\,SG \cup EsPaCnt\,SG)) \mathbin{;} (\preceq SG)^{\sim} \\
\forall\, SG : SGr_0 \bullet tgt_M^*\ SG = (rtgt\,SG\,(EsA\,SG)) \mathbin{;} (\preceq SG)^{\sim} \\
\forall\, SG : SGr_0 \bullet tgt^*\ SG = (rtgt\,SG\,(EsA\,SG \cup EsPaCnt\,SG)) \mathbin{;} (\preceq SG)^{\sim}
\end{array}$$

Predicate VCntEsOk says whether the value constraint edges of a SG are well-formed.

relation(VCntEsOk_)

$$\begin{array}{|l}
\mathrm{VCntEsOk_} : \mathbb{P}\,SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \mathrm{VCntEsOk}\,SG \Leftrightarrow vcei\,SG \in EsVCnt\,SG \to VCI \\
\quad \land \bigcup(map\ second\ ((\mathrm{ran} \circ vcei)\ SG)) \subseteq EsA\,SG \\
\quad \land (sg_tgt\,SG)\,(\!|\,EsVCnt\,SG\,|\!) \subseteq NsVa\,SG \cup NsP\,SG
\end{array}$$

relation(*inhOk* _)

$$\begin{array}{|l}
inhOk\,_ : \mathbb{P}\,SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet inhOk\ SG \\
\quad \Leftrightarrow (\forall\, v, v' : sg_Ns\,SG \mid (v, v') \in (\prec SG) \bullet nty\,SG\,v \prec_{NT} nty\,SG\,v') \land \otimes(\pitchfork SG)
\end{array}$$

$$\begin{array}{l}
SGr == \{SG : SGr_0 \mid \{srcma\,SG, tgtm\,SG\} \subseteq EsM\ SG \to Mult \land (\mathrm{dom} \circ pe)\,SG = EsPaCnt\,SG \\
\quad \land ds\,SG \in \mathrm{antireflexive}[EsPa\,SG] \\
\quad \land \mathcal{M}etysOk\ SG \land inhOk\ SG \land \mathrm{VCntEsOk}\,SG\,\}
\end{array}$$

relation(etherealAreInherited _)

$$\begin{array}{l}
\text{etherealAreInherited}_ : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \text{etherealAreInherited}\, SG \Leftrightarrow NsEther\, SG \subseteq \operatorname{ran}(\prec SG)
\end{array}$$

relation(*derInhOk* _)

$$\begin{array}{l}
derInhOk\,_ : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet derInhOk\ \ SG \Leftrightarrow \forall\, e : EsD\ SG \bullet \\
\quad (sg_src\, SG\, e, srcPE(gr\ SG \bowtie_{Es}\ EsA\, SG)(pe\, SG\, e)) \in (\preceq SG) \\
\quad \land (sg_tgt\, SG\, e, tgtPE(gr\ SG \bowtie_{Es}\ EsA\, SG)(pe\, SG\, e)) \in (\preceq SG)
\end{array}$$

relation(*okPEA* _)
relation(*okPEC* _)
relation(*okPE* _)
relation(*okPEASrc* _)
relation(*okPEATgt* _)

$$\begin{array}{l}
okPEASrc\,_ : \mathbb{P}(SGr_0 \times V \times PEA) \\
okPEATgt\,_ : \mathbb{P}(SGr_0 \times V \times PEA) \\
\hline
\forall\, SG : SGr_0;\ v : V;\ e : E \bullet okPEASrc(SG, v, edg\ e) \Leftrightarrow (e, v) \in src^*\ SG \\
\forall\, SG : SGr_0;\ v : V;\ e : E \bullet okPEASrc(SG, v, einv\ e) \Leftrightarrow (e, v) \in tgt^*\ SG \\
\forall\, SG : SGr_0;\ v : V;\ e : E \bullet okPEATgt(SG, v, edg\ e) \Leftrightarrow (e, v) \in tgt^*\ SG \\
\forall\, SG : SGr_0;\ v : V;\ e : E \bullet okPEATgt(SG, v, einv\ e) \Leftrightarrow (e, v) \in src^*\ SG
\end{array}$$

$$\begin{array}{l}
okPEA\,_ : \mathbb{P}(SGr_0 \times PEA) \\
\hline
\forall\, SG : SGr_0;\ e : E \bullet okPEA(SG, edg\ e) \Leftrightarrow e \in sg_Es\, SG \\
\forall\, SG : SGr_0;\ e : E \bullet okPEA(SG, einv\ e) \Leftrightarrow e \in sg_Es\, SG
\end{array}$$

$$
\begin{array}{|l}
okPEC\,\_ : \mathbb{P}(SGr_0 \times PEC) \\
okPE\,\_ : \mathbb{P}(SGr_0 \times PE) \\
\hline
\forall SG : SGr_0;\ v : V;\ pea : PEA \bullet okPEC\,(SG, eat\,pea) \Leftrightarrow okPEA(SG, pea) \\
\forall SG : SGr_0;\ v : V;\ pea : PEA \bullet \\
\quad okPEC(SG, eresd\,(v, pea)) \Leftrightarrow okPEA(SG, pea) \land okPEASrc(SG, v, pea) \\
\forall SG : SGr_0;\ v : V;\ pea : PEA \bullet \\
\quad okPEC(SG, eresr\,(pea, v)) \Leftrightarrow okPEA(SG, pea) \land okPEATgt(SG, v, pea) \\
\forall SG : SGr_0;\ pec : PEC;\ pe : PE \bullet \\
\quad okPE(SG, scmp\,(pec, pe)) \Leftrightarrow okPEC\,(SG, pec) \land okPE\,(SG, pe) \\
\quad \land\ tgtPEC\,(gr\ SG \bowtie_{Es}\ EsA\ SG)\ pec = srcPE\,(gr\ SG \bowtie_{Es}\ EsA\ SG)\ pe
\end{array}
$$

relation(isVCEECnt_)
relation(isVCENCnt_)

$$
\begin{array}{|l}
\mathrm{isVCEECnt\_}, \mathrm{isVCENCnt\_} : \mathbb{P}(SGr \times E) \\
\hline
\forall SG : SGr;\ vce : E \bullet \mathrm{isVCEECnt}(SG, vce) \Leftrightarrow (second \circ (vcei\ SG))\ vce \neq \varnothing \\
\forall SG : SGr;\ vce : E \bullet \mathrm{isVCENCnt}(SG, vce) \Leftrightarrow (second \circ (vcei\ SG))\ vce = \varnothing
\end{array}
$$

relation(commonAncestor_)
relation(EsVCntsOk_)

$$
\begin{array}{|l}
\mathrm{commonAncestor\_} : \mathbb{P}(SGr_0 \times V \times V) \\
\mathrm{EsVCntsOk\_} : \mathbb{P}\,SGr_0 \\
\hline
\forall SG : SGr_0;\ n_1, n_2 : V \bullet \mathrm{commonAncestor}\,(SG, n_1, n_2) \Leftrightarrow \exists\, nt : sg\_Ns\ SG \bullet \\
\quad (n_1, nt) \in \preceq SG \land (n_2, nt) \in \preceq SG \\
\forall SG : SGr_0 \bullet \mathrm{EsVCntsOk}\,SG \Leftrightarrow \\
\quad \forall\, vce : EsVCnt\ SG \bullet \mathrm{isVCEECnt}(SG, vce) \Rightarrow \\
\qquad\quad \mathrm{commonAncestor}\,(SG, sg\_tgt\ SG\,((\mathrm{the} \circ second \circ (vcei\ SG))\ vce), sg\_tgt\ SG\ vce) \\
\qquad \land\ \mathrm{isVCENCnt}(SG, vce) \Rightarrow \mathrm{commonAncestor}\,(SG, sg\_src\ SG\ vce, sg\_tgt\ SG\ vce)
\end{array}
$$

relation($EsCntsOk\,\_$)

$$
\begin{array}{|l}
EsCntsOk\,\_ : \mathbb{P}\,SGr_0 \\
\hline
\forall SG : SGr_0 \bullet \\
\quad EsCntsOk\ SG \Leftrightarrow derInhOk\ SG \land \forall\, e : EsPaCnt\ SG \bullet okPE(SG, pe\ SG\ e) \\
\qquad \land\ \mathrm{EsVCntsOk}\,SG
\end{array}
$$

relation$(inhTree\ \_)$

$$\begin{array}{|l}
inhTree\ _ : \mathbb{P}\ SGr_0 \\
\hline
\forall\ SG : SGr_0 \bullet inhTree\ SG \Leftrightarrow ((\pitchfork\ SG) \ominus_{Ns} (NsVi\ SG))^{\leftrightarrow} \in \mathrm{tree}
\end{array}$$

$TSGr == \{\ SG : SGr \mid \mathrm{etherealAreInherited}\ SG \wedge EsCntsOk\ SG\ \wedge\ inhTree\ SG\}$

relation$(\boxminus_{SGs}\ \_)$

$$\begin{array}{|l}
\boxminus_{SGs_} : \mathbb{P}(SGr \times SGr) \\
\hline
\forall\ SG_1, SG_2 : SGr \bullet \boxminus_{SGs}(SG_1, SG_2) \Leftrightarrow \boxminus(gr\ SG_1, gr\ SG_2)
\end{array}$$

function 10 **leftassoc** $(\_ \cup_{SG} \_)$

$$\begin{array}{|l}
_ \cup_{SG} _ : SGr \times SGr \rightarrow\!\!\!\!\!\!+\ SGr \\
\hline
\forall\ SG_1, SG_2 : SGr \bullet SG_1 \cup_{SG} SG_2 = (gr\ SG_1 \cup_G gr\ SG_2, nty\ SG_1 \cup nty\ SG_2, \\
\quad ety\ SG_1 \cup ety\ SG_2, srcm\ SG_1 \cup srcm\ SG_2, tgtm\ SG_1 \cup tgtm\ SG_2, pe\ SG_1 \cup pe\ SG_2, ds\ SG_1 \cup ds\ SG_2, vcei\ SG_1
\end{array}$$

function 10 **leftassoc** $(\_ \odot^{SG} \_)$

$$\begin{array}{|l}
_ \odot^{SG} _ : SGr \times (V \rightarrow\!\!\!\!\!\!+\ V) \rightarrow\!\!\!\!\!\!+\ SGr \\
\hline
\forall\ SG : SGr;\ s : V \rightarrow\!\!\!\!\!\!+\ V \mid s \in NsP\ SG \rightarrow\!\!\!\!\!\!+\ sg_Ns\ SG \bullet \\
\quad SG\ \odot^{SG}\ s = (gr\ SG \odot s, (\mathrm{dom}\ s \setminus \mathrm{ran}\ s) \lhd nty\ SG, ety\ SG, srcm\ SG, tgtm\ SG, pe\ SG, ds\ SG, vcei\ SG)
\end{array}$$

function 10 **leftassoc** $(\_ \rightarrow_{SG} \_)$

$$\begin{array}{|l}
_ \rightarrow_{SG} _ : SGr \times SGr \rightarrow \mathbb{P}\ GrM \\
\hline
\forall\ SG_s, SG_t : SGr \bullet \\
\quad SG_s \rightarrow_{SG} SG_t = \{fv : sg_Ns\ SG_s \rightarrow sg_Ns\ SG_t;\ fe : EsA\ SG_s \rightarrow EsA\ SG_t \mid \\
\quad\quad fv \circ (src^*_M\ SG_s) \subseteq src^*_M\ SG_t \circ fe \wedge fv \circ (tgt^*_M\ SG_s) \subseteq tgt^*_M\ SG_t \circ fe \\
\quad\quad \wedge fv \circ \preceq SG_s \subseteq \preceq SG_t \circ fv\}
\end{array}$$

function 10 **leftassoc** $(\_ \nrightarrow_{SG} \_)$

$$\begin{array}{|l}
_ \nrightarrow_{SG} _ : SGr \times SGr \rightarrow \mathbb{P}\, GrM \\
\hline
\forall\, SG_s, SG_t : SGr \bullet \\
\quad SG_s \nrightarrow_{SG} SG_t = \{ fv : sg_Ns\, SG_s \nrightarrow sg_Ns\, SG_t;\ fe : EsA\, SG_s \nrightarrow EsA\, SG_t \mid \\
\qquad fv \circ ((\operatorname{dom} fe) \lhd (src^*_M\ SG_s)) \subseteq src^*_M\ SG_t \circ fe \wedge fv \circ ((\operatorname{dom} fe) \lhd (tgt^*_M\ SG_s)) \subseteq tgt^*_M\ SG_t \circ fe \\
\qquad \wedge fv \circ \preceq SG_s \subseteq \preceq SG_t \circ fv \}
\end{array}$$

relation$(\_ \Rrightarrow^{SG} \_)$

$$\begin{array}{|l}
_ \Rrightarrow^{SG} _ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_s, SG_t : SGr;\ m : GrM \bullet (SG_s, m) \Rrightarrow^{SG} SG_t \Leftrightarrow m \in SG_s \rightarrow_{SG} SG_t
\end{array}$$

function 10 **leftassoc** $(\_ \rightarrow_{G2SG} \_)$

$$\begin{array}{|l}
_ \rightarrow_{G2SG} _ : Gr \times SGr \rightarrow \mathbb{P}\, GrM \\
\hline
\forall\, G : Gr;\ SG : SGr \bullet G \rightarrow_{G2SG} SG = \{ fv : Ns\, G \rightarrow sg_Ns\, SG;\ fe : Es\, G \rightarrow EsA\, SG \mid \\
\quad fv \circ src\, G \subseteq src^*_M\ SG \circ fe \wedge fv \circ tgt\, G \subseteq tgt^*_M\ SG \circ fe \}
\end{array}$$

relation$(\_ \Rrightarrow^{GwT} \_)$

$$\begin{array}{|l}
_ \Rrightarrow^{GwT} _ : (GrwT \leftrightarrow SGr) \\
\hline
\forall\, GwT : GrwT;\ SG : SGr \bullet GwT \Rrightarrow^{GwT} SG \Leftrightarrow GwT^{\,T} \in GwT^{\,G} \rightarrow_{G2SG} SG
\end{array}$$

relation$(\_ \sqsupseteq^{SG} \_)$
relation$(\_ \sqsupseteq^{SG_0} \_)$
relation$(\_ \sqsupseteq^{NT} \_)$
relation$(\_ \sqsupseteq^{ET} \_)$
relation$(\_ \sqsupseteq^{\mathcal{M}} \_)$
relation$(\_ \sqsupseteq_{\mathcal{M}Cnts} \_)$

$$\_ \sqsupseteq^{NT} \_, \_ \sqsupseteq^{ET} \_ : \mathbb{P}((SGr \times GrM) \times SGr)$$

$$\forall SG_c, SG_a : SGr;\ m : GrM \bullet$$
$$(SG_c, m) \sqsupseteq^{NT}\ SG_a \Leftrightarrow \forall n : sg\_Ns\, SG_c \bullet (nty\, SG_c)\, n \leq_{rNT} ((nty\, SG_a) \circ (fV\ m))\, n$$

$$\forall SG_c, SG_a : SGr;\ m : GrM \bullet$$
$$(SG_c, m) \sqsupseteq^{ET}\ SG_a \Leftrightarrow \forall e : EsA\, SG_c \bullet (ety\, SG_c)\, e \leq_{ET} ((ety\, SG_a) \circ (fE\ m))\, e$$

$$sPEA, tPEA : SGr \rightarrow PEA \rightarrow (E \nrightarrow V)$$
$$smfPEA, tmfPEA : SGr \rightarrow PEA \rightarrow (E \nrightarrow Mult)$$

$$\forall SG : SGr;\ e : E \bullet sPEA\, SG\, (edg\ e) = sg\_src\, SG$$

$$\forall SG : SGr;\ e : E \bullet sPEA\, SG\, (einv\ e) = sg\_tgt\, SG$$

$$\forall SG : SGr;\ e : E \bullet tPEA\, SG\, (edg\ e) = sg\_tgt\, SG$$

$$\forall SG : SGr;\ e : E \bullet tPEA\, SG\, (einv\ e) = sg\_src\, SG$$

$$\forall SG : SGr;\ e : E \bullet smfPEA\, SG\, (edg\ e) = srcma\, SG$$

$$\forall SG : SGr;\ e : E \bullet smfPEA\, SG\, (einv\ e) = tgtm\, SG$$

$$\forall SG : SGr;\ e : E \bullet tmfPEA\, SG\, (edg\ e) = tgtm\, SG$$

$$\forall SG : SGr;\ e : E \bullet tmfPEA\, SG\, (einv\ e) = srcma\, SG$$

relation(*affectedPE* _)
relation(*affectedPECStart* _)
relation(*affectedPECEnd* _)
relation(*affectedPEAStart* _)
relation(*affectedPEAEnd* _)
relation(*affectedPEStart* _)
relation(*affectedPEEnd* _)

$$
\begin{array}{l}
\mathit{affectedPECStart}\,\_, \mathit{affectedPECEnd}\,\_ : \mathbb{P}(SGr \times GrM \times SGr \times PEC \times E \times E) \\
\mathit{affectedPEAStart}\,\_, \mathit{affectedPEAEnd}\,\_ : \mathbb{P}(SGr \times GrM \times SGr \times PEA \times E \times E)
\end{array}
$$

$$
\begin{array}{l}
\forall\, SG_c, SG_a : SGr;\ m : GrM;\ pea : PEA;\ e, ie : E \bullet \mathit{affectedPEAStart}(SG_c, m, SG_a, pea, e, ie) \\
\quad \Leftrightarrow ie \in (fE\ m)^\sim \,(\!|\{ePEA\ pea\}|\!) \wedge (fV\ m)(sPEA\ SG_c\ pea\ ie) = sPEA\ SG_a\ pea\ e \\[1ex]
\forall\, SG_c, SG_a : SGr;\ m : GrM;\ pea : PEA;\ e, ie : E \bullet \mathit{affectedPEAEnd}(SG_c, m, SG_a, pea, e, ie) \\
\quad \Leftrightarrow ie \in (fE\ m)^\sim \,(\!|\{ePEA\ pea\}|\!) \wedge (fV\ m)(tPEA\ SG_c\ pea\ ie) = tPEA\ SG_a\ pea\ e \\[1ex]
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ e, ie : E \bullet \mathit{affectedPECStart}(SG_c, m, SG_a, eat\ pea, e, ie) \\
\quad \Leftrightarrow \mathit{affectedPEAStart}(SG_c, m, SG_a, pea, e, ie) \\[1ex]
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ v : V;\ e, ie : E \bullet \\
\quad \mathit{affectedPECStart}(SG_c, m, SG_a, eresd\ (v, pea), e, ie) \\
\quad \Leftrightarrow \mathit{affectedPEAStart}(SG_c, m, SG_a, pea, e, ie) \\
\qquad \wedge \exists\, v' : (fV\ m)^\sim \,(\!|\{v\}|\!) \bullet (sPEA\ SG_c\ pea\ ie, v') \in\ \preceq SG_c \\[1ex]
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ e, ie : E \bullet \mathit{affectedPECEnd}(SG_c, m, SG_a, eat\ pea, e, ie) \\
\quad \Leftrightarrow \mathit{affectedPEAEnd}(SG_c, m, SG_a, pea, e, ie) \\[1ex]
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ v : V;\ e, ie : E \bullet \\
\quad \mathit{affectedPECEnd}(SG_c, m, SG_a, eresd\ (v, pea), e, ie) \\
\quad \Leftrightarrow \mathit{affectedPEAEnd}(SG_c, m, SG_a, pea, e, ie) \\
\qquad \wedge \exists\, v' : (fV\ m)^\sim \,(\!|\{v\}|\!) \bullet (sPEA\ SG_c\ pea\ ie, v') \in\ \preceq SG_c \\[1ex]
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ v : V;\ e, ie : E \bullet \\
\quad \mathit{affectedPECEnd}(SG_c, m, SG_a, eresr\ (pea, v), e, ie) \\
\quad \Leftrightarrow \mathit{affectedPEAEnd}(SG_c, m, SG_a, pea, e, ie) \\
\qquad \wedge \exists\, v' : (fV\ m)^\sim \,(\!|\{v\}|\!) \bullet (tPEA\ SG_c\ pea\ ie, v') \in\ \preceq SG_c
\end{array}
$$

$$
\begin{array}{|l}
affectedPEStart\,\_,\ affectedPEEnd\,\_ : \mathbb{P}(SGr \times GrM \times SGr \times PE \times E \times E) \\
affectedPE\,\_ : \mathbb{P}(SGr \times GrM \times SGr \times PE \times E \times E \times E) \\
\hline
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ e, ie : E \bullet \\
\quad affectedPEStart(SG_c, m, SG_a, ec\, pec, e, ie) \Leftrightarrow affectedPECStart(SG_c, m, SG_a, pec, e, ie) \\
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ pe : PE;\ e, ie : E \bullet \\
\quad affectedPEStart(SG_c, m, SG_a, scmp\,(pec, pe), e, ie) \Leftrightarrow affectedPECStart(SG_c, m, SG_a, pec, e, ie) \\
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ e, ie : E \bullet \\
\quad affectedPEEnd(SG_c, m, SG_a, ec\, pec, e, ie) \Leftrightarrow affectedPECEnd(SG_c, m, SG_a, pec, e, ie) \\
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ pe : PE;\ e, ie : E \bullet \\
\quad affectedPEEnd(SG_c, m, SG_a, scmp\,(pec, pe), e, ie) \Leftrightarrow affectedPEEnd(SG_c, m, SG_a, pe, e, ie) \\
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ e, ie, ie' : E \bullet \\
\quad affectedPE(SG_c, m, SG_a, ec\, pec, e, ie, ie') \\
\quad \Leftrightarrow ie = ie' \land affectedPECStart(SG_c, m, SG_a, pec, e, ie) \\
\quad \land\ affectedPECEnd(SG_c, m, SG_a, pec, e, ie) \\
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pec_1, pec_2 : PEC;\ e, ie, ie' : E \bullet \\
\quad affectedPE(SG_c, m, SG_a, scmp\,(pec_1, ec\, pec_2), e, ie, ie') \\
\quad \Leftrightarrow tPEA\, SG_c\, (endEA_C\, pec_1)\, ie = sPEA\, SG_c\, (startEA_C\, pec_2)\, ie' \\
\qquad \land\ affectedPECStart(SG_c, m, SG_a, pec_1, e, ie) \land affectedPECEnd(SG_c, m, SG_a, pec_2, e, ie') \\
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ pe : PE;\ e, ie, ie' : E \bullet \\
\quad affectedPE(SG_c, m, SG_a, scmp\,(pec, pe), e, ie, ie') \\
\quad \Leftrightarrow affectedPECStart(SG_c, m, SG_a, pec, e, ie) \\
\qquad \land\ \exists\, ie'' : E \mid tPEA\, SG_c\, (endEA_C\, pec)\, ie = sPEA\, SG_c\, (startEA\, pe)\, ie'' \bullet \\
\qquad\quad affectedPE(SG_c, m, SG_a, pe, e, ie'', ie')
\end{array}
$$

relation$(caseMultsOk\,\_)$
relation$(caseMandatoryT\,\_)$
relation$(caseMandatoryS\,\_)$

$$
\begin{array}{|l}
caseMultsOk\,\_ : \mathbb{P}(SGr \times GrM \times SGr \times E \times PEA \times PEA \times E \times E) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM;\ peas : PEA;\ peae : PEA;\ e, ie, ie' : E \bullet \\
\quad caseMultsOk(SG_c, m, SG_a, e, peas, peae, ie, ie') \\
\quad \Leftrightarrow affectedPE(SG_c, m, SG_a, pe\, SG_a\, e, e, ie, ie') \\
\qquad \Rightarrow smfPEA\, SG_c\, peas\, ie \leq_{\mathcal{M}} srcma\, SG_a\, e \land tmfPEA\, SG_c\, peae\, ie' \leq_{\mathcal{M}} tgtm\, SG_a\, e
\end{array}
$$

$$
\begin{array}{|l}
caseMandatoryT\,\_ : \mathbb{P}(SGr \times GrM \times SGr \times E \times PEA \times E) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM;\ peas : PEA;\ peae : PEA;\ e, ie : E \bullet \\
\quad caseMandatoryT(SG_c, m, SG_a, e, peae, ie) \\
\quad \Leftrightarrow (affectedPEStart(SG_c, m, SG_a, pe\, SG_a\, e, e, ie) \land mks\, 1 \leq_{\mathcal{M}} (tgtm\, SG_a)\, e) \\
\qquad \Rightarrow (fV\ m)(tPEA\, SG_c\, peae\, ie) = sg\_tgt\, SG_a\, e
\end{array}
$$

$$
\begin{array}{|l}
caseMandatoryS\,\_ : \mathbb{P}(SGr \times GrM \times SGr \times E \times PEA \times E) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM;\ peas : PEA;\ e, ie : E \bullet \\
\quad caseMandatoryS(SG_c, m, SG_a, e, peas, ie) \\
\quad \Leftrightarrow (affectedPEEnd(SG_c, m, SG_a, pe\, SG_a\, e, e, ie) \wedge mks\, 1 \leq_{\mathcal{M}} (srcma\, SG_a)\, e) \\
\qquad \Rightarrow (fV\ m)(sPEA\, SG_c\, peas\ ie) = sg\_src\, SG_a\, e
\end{array}
$$

$$
\begin{array}{|l}
\_ \sqsupseteq_{\mathcal{M}Cnts} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\_ \sqsupseteq^{\mathcal{M}} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq^{\mathcal{M}} SG_a \Leftrightarrow \forall\, e : EsA\, SG_c \bullet (srcma\, SG_c)\, e \leq_{\mathcal{M}} ((srcma\, SG_a) \circ (fE\ m))\, e \\
\qquad\qquad\qquad \wedge\ (tgtm\, SG_c)\, e \leq_{\mathcal{M}} ((tgtm\, SG_a) \circ (fE\ m))\, e \\
\\
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet (SG_c, m) \sqsupseteq_{\mathcal{M}Cnts} SG_a \Leftrightarrow \forall\, e : EsD\, SG_a \bullet \\
\quad \exists\, peas : PEA;\ peae : PEA \mid peas = startEA(pe\, SG_a\, e) \wedge peae = endEA(pe\, SG_a\, e) \bullet \\
\qquad \forall\, e' : (fE\ m)^{\sim} (\!|\{ePEA\, peas\}|\!);\ e'' : (fE\ m)^{\sim} (\!|\{ePEA\, peae\}|\!) \bullet \\
\qquad\quad caseMultsOk(SG_c, m, SG_a, e, peas, peae, e', e'') \\
\qquad\quad \wedge\ (caseMandatoryT(SG_c, m, SG_a, e, peae, e') \vee caseMandatoryS(SG_c, m, SG_a, e, peas, e''))
\end{array}
$$

$$
\begin{array}{|l}
\_ \sqsupseteq^{SG} \_, \_ \sqsupseteq^{SG_0} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq^{SG_0} SG_a \Leftrightarrow (SG_c, m) \sqsupseteq^{NT} SG_a \wedge (SG_c, m) \sqsupseteq^{ET} SG_a \wedge (SG_c, m) \sqsupseteq^{\mathcal{M}} SG_a \\
\qquad \wedge\ (SG_c, m) \sqsupseteq_{\mathcal{M}Cnts} SG_a \\
\\
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq^{SG} SG_a \Leftrightarrow m \in SG_c \to_{SG} SG_a \wedge (SG_c, m) \sqsupseteq^{SG_0} SG_a
\end{array}
$$

$$
\begin{array}{|l}
ins : GrM \times SGr \times \mathbb{P}\, V \to \mathbb{P}\, V \\
ies : GrM \times \mathbb{P}\, E \to \mathbb{P}\, E \\
\hline
\forall\, m : GrM;\ SG : SGr;\ mns : \mathbb{P}\, V \bullet ins\,(m, SG, mns) = (fV\ m)^{\sim} (\!|(\prec SG)^{\sim} (\!|mns|\!)|\!) \\
\\
\forall\, m : GrM;\ mes : \mathbb{P}\, E \bullet ies\,(m, mes) = (fE\ m)^{\sim} (\!|mes|\!)
\end{array}
$$

$$
\begin{array}{|l}
igRMEs : GrwT \times \mathbb{P}\, E \to Gr \\
\hline
\forall\, GwT : GrwT;\ mes : \mathbb{P}\, E \bullet igRMEs(GwT, mes) = GwT^{G} \bowtie_{Es} ies\,(GwT^{T},\ mes)
\end{array}
$$

relation$(\_ \sqsupset^{SG} \_)$
relation$(\_ \sqsupset^{SG_0} \_)$
relation$(\_ \sqsupset_{AEs} \_)$
relation(_OkRefinedIn_)
relation$(\_ \sqsupset_{ANNs} \_)$

$$
\begin{array}{|l}
\_ \sqsupset_{ANNs} \_ : \mathbb{P}(GrM \times SGr) \\
\hline
\forall SG_a : SGr;\ m : GrM \bullet \\
\quad m \sqsupset_{ANNs} SG_a \Leftrightarrow \forall nn : NsTy\, SG_a\{nnrml\} \bullet (\preceq SG_a) \,(\!|\, \{nn\} \,|\!)\, \cap \mathrm{ran}(fV\ m) = \varnothing
\end{array}
$$

$$
\begin{array}{|l}
\_\mathrm{OkRefinedIn}\_ : \mathbb{P}((SGr \times E) \times (SGr \times GrM)) \\
\_ \sqsupset_{AEs} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM;\ ae : E \bullet \\
\quad (SG_a, ae)\mathrm{OkRefinedIn}(SG_c, m) \Leftrightarrow \\
\quad\quad \exists r : V \leftrightarrow V;\ s, t : \mathbb{P}\ V \mid r = (\preceq SG_c) \fatsemi igRMEs((gr\ SG_c, m), \{ae\})^{\leftrightarrow} \fatsemi (\preceq SG_c)^{\sim} \\
\quad\quad\quad \wedge\ s = ins\,(m, SG_a, sg\_src\ SG_a \,(\!|\, \{ae\} \,|\!)) \setminus ((NsEther\ SG_c) \setminus \mathrm{dom}\ r) \\
\quad\quad\quad \wedge\ t = ins\,(m, SG_a, sg\_tgt\ SG_a \,(\!|\, \{ae\} \,|\!)) \setminus ((NsEther\ SG_c) \setminus \mathrm{ran}\ r) \\
\quad\quad\quad\quad \bullet\ r \in s \leftrightarrow t \wedge nty\ SG_a \,(\!|\, (gr\ SG_a) \circ\!\!\rightarrow\!\!\circ_{Ns} \{ae\} \,|\!)\, \neq \varnothing \Rightarrow r \neq \varnothing \\
\\
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset_{AEs}\ SG_a \Leftrightarrow \forall e : (EsA\ SG_a) \bullet (SG_a, e)\mathrm{OkRefinedIn}(SG_c, m)
\end{array}
$$

$$
\begin{array}{|l}
\_ \sqsupset^{SG} \_, \_ \sqsupset^{SG_0} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset^{SG_0}\ SG_a \Leftrightarrow (SG_c, m) \sqsupseteq^{SG_0}\ SG_a \wedge m \sqsupset_{ANNs} SG_a \wedge (SG_c, m) \sqsupset_{AEs}\ SG_a \\
\\
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset^{SG}\ SG_a \Leftrightarrow m \in SG_c \rightarrow_{SG} SG_a \wedge (SG_c, m) \sqsupset^{SG_0}\ SG_a
\end{array}
$$

relation$(\_ \dashv\vdash_{\mathcal{M}} \_)$
relation$(\_ \dashv\vdash_{NT} \_)$
relation$(\_ \dashv\vdash_{ET} \_)$
relation$(\_ \dashv\vdash^{SG} \_)$

$$
\begin{array}{|l}
\_ \dashv\vdash_{\mathcal{M}} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_s, SG_t : SGr;\ m : GrM \bullet \\
\quad (SG_s, m) \dashv\vdash_{\mathcal{M}}\ SG_t \Leftrightarrow \forall ep : fE\ m \bullet (srcma\ SG_s)\,(first\ ep) \leq_{\mathcal{M}} (srcma\ SG_t)\,(second\ ep) \\
\quad\quad\quad\quad \wedge\ (tgtm\ SG_s)\,(first\ ep) \leq_{\mathcal{M}} (tgtm\ SG_t)\,(second\ ep)
\end{array}
$$

$$
\begin{array}{|l}
\_ \dashv\vdash_{NT} \_ : \mathbb{P}(GrM \times SGr) \\
\_ \dashv\vdash_{ET} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_t : SGr;\ m : GrM \bullet \\
\quad m \dashv\vdash_{NT}\ SG_t \Leftrightarrow (nty\ SG_t) \,(\!|\, \mathrm{ran}(fV\ m) \,|\!)\, \subseteq \{nnrml, nabst, nvirt\} \\
\\
\forall SG_s, SG_t : SGr;\ m : GrM \bullet \\
\quad (SG_s, m) \dashv\vdash_{ET}\ SG_t \Leftrightarrow \mathrm{dom}(fE\ m) \lhd (ety\ SG_s) = (ety\ SG_t) \circ (fE\ m)
\end{array}
$$

$$\_ \dashv\vdash^{SG} \_ : \mathbb{P}((SGr \times GrM) \times SGr)$$

$$\forall SG_s, SG_t : SGr;\ m : GrM \bullet (SG_s, m) \dashv\vdash^{SG} SG_t \Leftrightarrow m \in SG_s \twoheadrightarrow_{SG} SG_t \land (SG_s, m) \dashv\vdash_{\mathcal{M}} SG_t \land m \dashv\vdash_{NT} SG_t \land (SG_s, m) \dashv\vdash_{ET} SG_t$$

$$rPEA : GrwT \times SGr \times PEA \rightarrow (V \leftrightarrow V)$$

$$\forall GwT : GrwT;\ SG : SGr;\ e : E \bullet rPEA\,(GwT, SG, edg\ e) = (GwT^{G} \bowtie_{Es} ies\,(GwT^{T}, \{e\}))^{\leftrightarrow}$$

$$\forall GwT : GrwT;\ SG : SGr;\ e : E \bullet rPEA\,(GwT, SG, einv\ e) = rPEA\,(GwT, SG, edg\ e)^{\sim}$$

$$rPEC : GrwT \times SGr \times PEC \rightarrow (V \leftrightarrow V)$$
$$rPE : GrwT \times SGr \times PE \rightarrow (V \leftrightarrow V)$$

$$\forall GwT : GrwT;\ SG : SGr;\ pea : PEA \bullet rPEC\,(GwT, SG, eat\ pea) = rPEA\,(GwT, SG, pea)$$

$$\forall GwT : GrwT;\ SG : SGr;\ v : V;\ pea : PEA \bullet rPEC\,(GwT, SG, eresd\,(v, pea)) = ins\,(GwT^{T},\ SG, \{v\}) \lhd rPEA(GwT, SG, pea)$$

$$\forall GwT : GrwT;\ SG : SGr;\ v : V;\ pea : PEA \bullet rPEC\,(GwT, SG, eresr\,(pea, v)) = rPEA\,(GwT, SG, pea) \rhd ins\,(GwT^{T},\ SG,\ \{v\})$$

$$\forall GwT : GrwT;\ SG : SGr;\ pec : PEC \bullet rPE\,(GwT, SG, ec\ pec) = rPEC\,(GwT, SG, pec)$$

$$\forall GwT : GrwT;\ SG : SGr;\ pec : PEC;\ pe : PE \bullet rPE\,(GwT, SG, scmp\,(pec, pe)) = rPEC\,(GwT, SG, pec) \mathbin{⨾} rPE\,(GwT, SG, pe)$$

$$ape : SGr \rightarrow E \pfun PE$$

$$\forall SG : SGr \bullet ape\ SG = (\lambda\, e : E \mid e \in EsM\ SG \bullet (ec \circ eat \circ edg)\ e) \oplus pe\ SG$$

$$src^{*}_{PEA} : SGr \times PEA \rightarrow E \leftrightarrow V$$
$$src^{*}_{PEC} : SGr \times PEC \rightarrow E \leftrightarrow V$$
$$src^{*}_{PE} : SGr \times PE \rightarrow E \leftrightarrow V$$

$$\forall SG : SGr;\ e : E \bullet src^{*}_{PEA}(SG, edg\ e) = src^{*}\ SG$$

$$\forall SG : SGr;\ e : E \bullet src^{*}_{PEA}(SG, einv\ e) = tgt^{*}\ SG$$

$$\forall SG : SGr;\ pea : PEA \bullet src^{*}_{PEC}(SG, eat\ pea) = src^{*}_{PEA}(SG, pea)$$

$$\forall SG : SGr;\ v : V;\ pea : PEA \bullet src^{*}_{PEC}(SG, eresd\,(v, pea)) = src^{*}_{PEA}(SG, pea) \rhd \{v\}$$

$$\forall SG : SGr;\ v : V;\ pea : PEA \bullet src^{*}_{PEC}(SG, eresr\,(pea, v)) = src^{*}_{PEA}(SG, pea)$$

$$\forall SG : SGr;\ v : V;\ pec : PEC;\ pe : PE \bullet src^{*}_{PE}(SG, (scmp\,(pec, pe))) = src^{*}_{PEC}(SG, pec)$$

$$
\begin{array}{|l}
tgt^*_{PEA} : SGr \times PEA \rightarrow E \leftrightarrow V \\
tgt^*_{PEC} : SGr \times PEC \rightarrow E \leftrightarrow V \\
tgt^*_{PE} : SGr \times PE \rightarrow E \leftrightarrow V \\
\hline
\forall SG : SGr;\ e : E \bullet tgt^*_{PEA}(SG, edg\ e) = tgt^*\ SG \\
\forall SG : SGr;\ e : E \bullet tgt^*_{PEA}(SG, einv\ e) = src^*\ SG \\
\forall SG : SGr;\ pea : PEA \bullet tgt^*_{PEC}(SG, eat\ pea) = tgt^*_{PEA}(SG, pea) \\
\forall SG : SGr;\ v : V;\ pea : PEA \bullet tgt^*_{PEC}(SG, eresd\ (v, pea)) = tgt^*_{PEA}(SG, pea) \\
\forall SG : SGr;\ v : V;\ pea : PEA \bullet tgt^*_{PEC}(SG, eresr\ (pea, v)) = tgt^*_{PEA}(SG, pea) \rhd \{v\} \\
\forall SG : SGr;\ v : V;\ pec : PEC \bullet tgt^*_{PE}(SG, ec\ pec) = tgt^*_{PEC}(SG, pec) \\
\forall SG : SGr;\ v : V;\ pec : PEC;\ pe : PE \bullet tgt^*_{PE}(SG, (scmp\ (pec, pe))) = tgt^*_{PE}(SG, pe)
\end{array}
$$

relation$(\_ \Supset^{SG} \_)$
relation$(\_ \Supset_{\mathcal{M}} \_)$
relation$(\_ \Supset_{FI} \_)$
relation$(\_ \Supset_{PNS} \_)$
relation$(\_ \Supset_{Cnts} \_)$
relation$(\_\ ME\mathcal{M}Ok\ \_)$

$$
\begin{array}{|l}
rME\mathcal{M}Ok : SGr \times E \times GrwT \rightarrow V \leftrightarrow V \\
\hline
\forall SG : SGr;\ me : E;\ GwT : GrwT;\ s, t : \mathbb{P}\ V \mid s = ins(GwT^{\,T}, SG, src^*\ SG\ (\!|\ \{me\}\ |\!)) \\
\quad \wedge\ t = ins(GwT^{\,T}, SG, tgt^*\ SG\ (\!|\ \{me\}\ |\!)) \bullet \\
\qquad rME\mathcal{M}Ok(SG, me, GwT) = s \lhd rPE\ (GwT, SG, ape\ SG\ me) \rhd\ t
\end{array}
$$

$$
\begin{array}{|l}
src^*ME\mathcal{M}Ok : SGr \times E \rightarrow E \leftrightarrow V \\
\hline
\forall SG : SGr;\ me : E \bullet src^*ME\mathcal{M}Ok(SG, me) = src^*_{PE}(SG, ape\ SG\ me) \rhd (src^*\ SG)\ (\!|\ \{me\}\ |\!)
\end{array}
$$

$$
\begin{array}{|l}
tgt^*ME\mathcal{M}Ok : SGr \times E \rightarrow E \leftrightarrow V \\
\hline
\forall SG : SGr;\ me : E \bullet tgt^*ME\mathcal{M}Ok(SG, me) = tgt^*_{PE}(SG, ape\ SG\ me) \rhd (tgt^*\ SG)\ (\!|\ \{me\}\ |\!)
\end{array}
$$

$$\begin{array}{|l} multComp : Mult \times Mult \rightarrow Mult \\ \hline \forall m_1, m_2 : Mult \mid m_1 \in MultMany \lor m_2 \in MultMany \bullet multComp(m_1, m_2) = mmanys \\ \forall m_1, m_2 : Mult \mid m_1 = mks\,0 \lor m_2 = mks\,0 \bullet multComp(m_1, m_2) = mks\,0 \\ \forall m_1, m_2 : Mult \mid m_2 = mks\,1 \bullet multComp(m_1, m_2) = m_1 \\ \forall m_1, m_2 : Mult \mid m_1 = mks\,1 \bullet multComp(m_1, m_2) = m_2 \\ \forall m_1, m_2 : Mult \mid m_1 = mopts \bullet multComp(m_1, m_2) = mks\,0 \cup m_2 \\ \forall m_1, m_2 : Mult \mid m_2 = mopts \bullet multComp(m_1, m_2) = mks\,0 \cup m_1 \\ \forall m_1, m_2 : MultRange \bullet \\ \quad multComp(m_1, m_2) = \{(\mathrm{the}\, m_1) *_{mr} (\mathrm{the}\, m_2)\} \\ \forall m_1 : Mult;\ mc : MultC;\ m_2 : MultEither \bullet \\ \quad multComp(m_1, \{mc\} \cup m_2) = multComp(m_1, \{mc\}) \cup multComp(m_1, m_2) \\ \forall m_1 : MultEither;\ mc : MultC;\ m_2 : Mult \bullet \\ \quad multComp(\{mc\} \cup m_1, m_2) = multComp(\{mc\}, m_2) \cup multComp(m_1, m_2) \end{array}$$

$$\begin{array}{|l} smPEA : SGr \rightarrow PEA \rightarrow Mult \\ smPEC : SGr \rightarrow PEC \rightarrow Mult \\ smPE : SGr \rightarrow PE \rightarrow Mult \\ \hline \forall SG : SGr;\ e : E \bullet smPEA\ SG\ (edg\ e) = srcma\ SG\ e \\ \forall SG : SGr;\ e : E \bullet smPEA\ SG\ (einv\ e) = tgtm\ SG\ e \\ \forall SG : SGr;\ pea : PEA \bullet smPEC\ SG\ (eat\ pea) = smPEA\ SG\ pea \\ \forall SG : SGr;\ v : V;\ pea : PEA \bullet smPEC\ SG\ (eresd\ (v, pea)) = smPEA\ SG\ pea \\ \forall SG : SGr;\ v : V;\ pea : PEA \bullet smPEC\ SG\ (eresr\ (pea, v)) = smPEA\ SG\ pea \\ \forall SG : SGr;\ pec : PEC \bullet smPE\ SG\ (ec\ pec) = smPEC\ SG\ pec \\ \forall SG : SGr;\ pec : PEC;\ pe : PE \bullet smPE\ SG\ (scmp\ (pec, pe)) = multComp(smPEC\ SG\ pec, smPE\ SG\ pe) \end{array}$$

$$\begin{array}{|l} tmPEA : SGr \rightarrow PEA \rightarrow Mult \\ tmPEC : SGr \rightarrow PEC \rightarrow Mult \\ tmPE : SGr \rightarrow PE \rightarrow Mult \\ \hline \forall SG : SGr;\ e : E \bullet tmPEA\ SG\ (edg\ e) = tgtm\ SG\ e \\ \forall SG : SGr;\ e : E \bullet tmPEA\ SG\ (einv\ e) = srcma\ SG\ e \\ \forall SG : SGr;\ pea : PEA \bullet tmPEC\ SG\ (eat\ pea) = tmPEA\ SG\ pea \\ \forall SG : SGr;\ v : V;\ pea : PEA \bullet tmPEC\ SG\ (eresd\ (v, pea)) = tmPEA\ SG\ pea \\ \forall SG : SGr;\ v : V;\ pea : PEA \bullet tmPEC\ SG\ (eresr\ (pea, v)) = tmPEA\ SG\ pea \\ \forall SG : SGr;\ pec : PEC \bullet tmPE\ SG\ (ec\ pec) = tmPEC\ SG\ pec \\ \forall SG : SGr;\ pec : PEC;\ pe : PE \bullet tmPE\ SG\ (scmp\ (pec, pe)) = multComp(tmPEC\ SG\ pec, tmPE\ SG\ pe) \end{array}$$

$$\begin{array}{|l}
\,\mathit{MEMOk}\, : \mathbb{P}((\mathit{SGr} \times E) \times \mathit{GrwT}) \\
\hline
\forall\, \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr};\ \mathit{me} : E \bullet (\mathit{SG}, \mathit{me})\ \mathit{MEMOk}\ \mathit{GwT} \Leftrightarrow \\
\quad \exists\, r : V \leftrightarrow V;\ s, t : \mathbb{P}\ V \mid r = \mathit{rMEMOk}\,(\mathit{SG}, \mathit{me}, \mathit{GwT}) \\
\qquad \wedge\ s = \mathit{ins}(\mathit{GwT}^{\,T}, \mathit{SG}, \mathit{src}^{*}\,\mathit{MEMOk}\,(\mathit{SG}, \mathit{me})\ (\!|\ \{\mathit{rsrcPE}(\mathit{ape}\ \mathit{SG}\ \mathit{me})\}\ |\!)) \\
\qquad \wedge\ t = \mathit{ins}(\mathit{GwT}^{\,T}, \mathit{SG}, \mathit{tgt}^{*}\,\mathit{MEMOk}\,(\mathit{SG}, \mathit{me})\ (\!|\ \{\mathit{rtgtPE}(\mathit{ape}\ \mathit{SG}\ \mathit{me})\}\ |\!)) \\
\qquad\quad \bullet\ r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{smPE}\ \mathit{SG}\ (\mathit{ape}\ \mathit{SG}\ \mathit{me}), \mathit{tmPE}\ \mathit{SG}\ (\mathit{ape}\ \mathit{SG}\ \mathit{me}))
\end{array}$$

relation(INumbersOk_)
relation(I$\mathbb{N}$sOk_)
relation(I$\mathbb{Z}$sOk_)
relation(I$\mathbb{R}$sOk_)

$$\begin{array}{|l}
\mathrm{I}\mathbb{N}\mathrm{sOk}_, \mathrm{I}\mathbb{Z}\mathrm{sOk}_, \mathrm{I}\mathbb{R}\mathrm{sOk}_, \mathrm{INumbersOk}_ : \mathbb{P}\ \mathit{GrwT} \\
\hline
\forall\, \mathit{GwT} : \mathit{GrwT} \bullet \mathrm{I}\mathbb{N}\mathrm{sOk}\ \mathit{GwT} \Leftrightarrow \\
\quad \forall\, \mathit{nnat} : (\mathit{fV}\ (\mathit{GwT}^{\,T}))^{\sim}\ (\!|\{\mathit{nNatS}\}\ |\!) \bullet \exists\, n : \mathbb{N} \bullet n \in \mathrm{to}\mathbb{Z}\ \mathit{nnat} \\
\forall\, \mathit{GwT} : \mathit{GrwT} \bullet \mathrm{I}\mathbb{Z}\mathrm{sOk}\ \mathit{GwT} \Leftrightarrow \\
\quad \forall\, \mathit{nint} : (\mathit{fV}\ (\mathit{GwT}^{\,T}))^{\sim}\ (\!|\{\mathit{nIntS}\}\ |\!) \bullet \exists\, n : \mathbb{Z} \bullet n \in \mathrm{to}\mathbb{Z}\ \mathit{nint} \\
\forall\, \mathit{GwT} : \mathit{GrwT} \bullet \mathrm{I}\mathbb{R}\mathrm{sOk}\ \mathit{GwT} \Leftrightarrow \\
\quad \forall\, \mathit{nreal} : (\mathit{fV}\ (\mathit{GwT}^{\,T}))^{\sim}\ (\!|\{\mathit{nRealS}\}\ |\!) \bullet \exists\, x : \mathbb{R} \bullet x \in \mathrm{to}\mathbb{R}\ \mathit{nreal} \\
\forall\, \mathit{SG} : \mathit{SGr};\ \mathit{GwT} : \mathit{GrwT} \bullet \mathrm{INumbersOk}\ \mathit{GwT} \Leftrightarrow \\
\quad \mathrm{I}\mathbb{N}\mathrm{sOk}\ \mathit{GwT} \wedge \mathrm{I}\mathbb{Z}\mathrm{sOk}\ \mathit{GwT} \wedge \mathrm{I}\mathbb{R}\mathrm{sOk}\ \mathit{GwT}
\end{array}$$

relation(satisfiesACnt_)
relation(satisfiesVCEECnt_)
relation(satisfiesVCENCnt_)
relation(IVCEsOk_)

$$\begin{array}{|l}
\mathit{toNum} : \mathit{SGr} \times V \times V \to \mathrm{opt}[\mathbb{A}] \\
\hline
\forall\ \mathit{SG} : \mathit{SGr};\ \mathit{ns}, \mathit{nt} : V \bullet \mathit{toNum}(\mathit{SG}, \mathit{ns}, \mathit{nt}) = \\
\quad \mathbf{if}\,(\mathit{nt}, \mathit{nNatS}) \in (\preceq\ \mathit{SG}) \vee (\mathit{nt}, \mathit{nIntS}) \in (\preceq\ \mathit{SG})\ \mathbf{then}\ \mathrm{to}\mathbb{Z}\ \mathit{ns} \\
\quad \mathbf{else\,if}\,(\mathit{nt}, \mathit{nRealS}) \in (\preceq\ \mathit{SG})\ \mathbf{then}\ \mathrm{to}\mathbb{R}\ \mathit{ns}\ \mathbf{else}\ \varnothing
\end{array}$$

$$\begin{array}{|l}
\mathit{rOp} : \mathit{SGVCEOP} \to \mathbb{A} \leftrightarrow \mathbb{A} \\
\hline
\mathit{rOp}\ \mathit{eq} = \{n_1, n_2 : \mathbb{A} \mid n_1 = n_2\} \\
\mathit{rOp}\ \mathit{neq} = (_ \neq _) \\
\mathit{rOp}\ \mathit{leq} = (_ \leq _) \\
\mathit{rOp}\ \mathit{geq} = (_ \geq _) \\
\mathit{rOp}\ \mathit{lt} = (_ < _) \\
\mathit{rOp}\ \mathit{gt} = (_ > _)
\end{array}$$

$$
\begin{array}{|l}
\mathrm{satisfiesACnt\_} : \mathbb{P}(SGr \times GrM \times V \times SGVCEOP \times V)\\
\mathrm{satisfiesVCEECnt\_} : \mathbb{P}(SGr \times GrwT \times E)\\
\mathrm{satisfiesVCENCnt\_} : \mathbb{P}(SGr \times GrwT \times E)\\
\mathrm{IVCEsOk\_} : \mathbb{P}(SGr \times GrwT)\\
\hline
\forall SG : SGr;\ t : GrM;\ op : SGVCEOP;\ ns, nt : V \bullet \mathrm{satisfiesACnt}(SG, t, ns, op, nt) \Leftrightarrow\\
\quad ns \in \mathrm{dom}(fV\ t)\\
\quad \land \exists n_1, n_2 : \mathbb{A} \mid n_1 \in toNum(SG, ns, (fV\ t)ns) \land n_2 \in toNum(SG, nt, nt) \bullet (n_1, n_2) \in rOp\ op\\
\forall SG : SGr;\ GwT : GrwT;\ vce : E \bullet \mathrm{satisfiesVCEECnt}(SG, GwT, vce) \Leftrightarrow\\
\quad \forall ie : fE\ (GwT^{T})^{\sim} (\!|(second \circ (vcei\ SG))\ vce|\!) \mid fV\ (GwT^{T})(src\ (GwT^{G})\ ie) = (sg\_src\ SG\ vce) \bullet\\
\qquad \mathrm{satisfiesACnt}(SG, GwT^{T}, tgt\ (GwT^{G})\ ie, (first \circ (vcei\ SG))\ vce, sg\_tgt\ SG\ vce)\\
\forall SG : SGr;\ GwT : GrwT;\ vce : E \bullet \mathrm{satisfiesVCENCnt}(SG, GwT, vce) \Leftrightarrow\\
\quad \forall in : fV\ (GwT^{T})^{\sim} (\!|\{sg\_src\ SG\ vce\}|\!) \bullet\\
\qquad \mathrm{satisfiesACnt}(SG, GwT^{T}, in, (first \circ (vcei\ SG))\ vce, sg\_tgt\ SG\ vce)\\
\forall SG : SGr;\ GwT : GrwT \bullet \mathrm{IVCEsOk}(SG, GwT) \Leftrightarrow \forall vce : EsVCnt\ SG \bullet\\
\quad \mathrm{isVCEECnt}(SG, vce) \Rightarrow \mathrm{satisfiesVCEECnt}(SG, GwT, vce)\\
\quad \land \mathrm{isVCENCnt}(SG, vce) \Rightarrow \mathrm{satisfiesVCENCnt}(SG, GwT, vce)
\end{array}
$$

$$
\begin{array}{|l}
\_ \Supset_{\mathcal{M}} \_ : GrwT \leftrightarrow SGr\\
\_ \Supset_{FI} \_ : GrwT \leftrightarrow SGr\\
\_ \Supset_{PNS} \_ : GrwT \leftrightarrow SGr\\
\_ \Supset_{Cnts} \_ : GrwT \leftrightarrow SGr\\
\hline
\forall GwT : GrwT;\ SG : SGr \bullet\\
\quad GwT \Supset_{\mathcal{M}} SG \Leftrightarrow \forall me : EsM\ SG \bullet (SG, me)\ ME\mathcal{M}Ok\ GwT\\
\forall GwT : GrwT;\ SG : SGr \bullet\\
\quad GwT \Supset_{FI} SG \Leftrightarrow (fV\ (GwT^{T}))^{\sim} (\!|NsEther\ SG|\!) = \varnothing\\
\forall GwT : GrwT;\ SG : SGr \bullet\\
\quad GwT \Supset_{PNS} SG \Leftrightarrow igRMEs(GwT, EsTy\ SG\ \{ecomp\ dbi, ecomp\ duni\})^{\leftrightarrow} \in \mathrm{injrel}\\
\forall GwT : GrwT;\ SG : SGr \bullet\\
\quad GwT \Supset_{Cnts} SG \Leftrightarrow \mathrm{INumbersOk}\ GwT \land \mathrm{IVCEsOk}(SG, GwT)
\end{array}
$$

$$
\begin{array}{|l}
\_ \Supset^{SG} \_ : GrwT \leftrightarrow SGr\\
\hline
\forall GwT : GrwT;\ SG : SGr \bullet GwT \Supset^{SG} SG \Leftrightarrow GwT \Rrightarrow^{GwT} SG \land GwT \Supset_{\mathcal{M}} SG\\
\qquad \land GwT \Supset_{FI} SG \land GwT \Supset_{PNS} SG \land GwT \Supset_{Cnts} SG
\end{array}
$$

9 Fragments

section *Fragmenta_Frs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_SGs*, *Fragmenta_GrswT*, *Fragmenta_GrswET*

$$Fr_0 == \{SG : SGr_0;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V;\ et : GrM \mid esr \cap (sg\_Es\ SG) = \varnothing \\ \land sr \in esr \rightarrowtail\!\!\!\rightarrow (NsP\ SG) \land tr \in esr \rightarrow V \land domg\ et \subseteq_p (NsN\ SG, EsA\ SG)\}$$

$$\begin{array}{|l}
fSG : Fr_0 \rightarrow SGr \\
EsR : Fr_0 \rightarrow \mathbb{P}\, E \\
srcR, tgtR : Fr_0 \rightarrow E \nrightarrow V \\
fet : Fr_0 \rightarrow GrM \\
\hline
\forall\, SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V;\ et : GrM \bullet fSG(SG, esr, sr, tr, et) = SG \\
\forall\, SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V;\ et : GrM \bullet EsR(SG, esr, sr, tr, et) = esr \\
\forall\, SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V;\ et : GrM \bullet srcR(SG, esr, sr, tr, et) = sr \\
\forall\, SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V;\ et : GrM \bullet tgtR(SG, esr, sr, tr, et) = tr \\
\forall\, SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V;\ et : GrM \bullet fet(SG, esr, sr, tr, et) = et
\end{array}$$

$$\begin{array}{|l}
fLEs, fEs, fEsA : Fr_0 \rightarrow \mathbb{P}\, E \\
fLNs, fRNs, fNs : Fr_0 \rightarrow \mathbb{P}\, V \\
srcF, tgtF : Fr_0 \rightarrow E \nrightarrow V \\
\hline
fLEs = (sg_Es \circ fSG) \\
\forall\, F : Fr_0 \bullet fEs\ F = fLEs\ F \cup EsR\ F \\
fEsA = EsA \circ fSG \\
fLNs = sg_Ns \circ fSG \\
fRNs = \mathrm{ran} \circ tgtR \\
\forall\, F : Fr_0 \bullet fNs\ F = fLNs\ F \cup fRNs\ F \\
\forall\, F : Fr_0 \bullet srcF\ F = (sg_src \circ fSG)\ F \cup srcR\ F \\
\forall\, F : Fr_0 \bullet tgtF\ F = (sg_tgt \circ fSG)\ F \cup tgtR\ F
\end{array}$$

$$\begin{array}{|l}
\overset{G}{\leftrightsquigarrow} : Fr_0 \rightarrow Gr \\
\leftrightsquigarrow : Fr_0 \rightarrow V \leftrightarrow V \\
\hline
\forall\, F : Fr_0 \bullet \overset{G}{\leftrightsquigarrow}\ F = ((NsP \circ fSG)F \cup fRNs\ F, EsR\ F, srcR\ F, tgtR\ F) \\
\forall\, F : Fr_0 \bullet \leftrightsquigarrow\ F = (\overset{G}{\leftrightsquigarrow}\ F)^{\leftrightarrow}
\end{array}$$

function 10 **leftassoc** $(\_ \cup_F \_)$

$$\begin{array}{|l}
\varnothing_F : Fr_0 \\
_ \cup_F _ : Fr_0 \times Fr_0 \to Fr_0 \\
\bigcup_F : \mathbb{P}\, Fr_0 \to Fr_0 \\
\hline
\varnothing_F = (\varnothing_{SG}, \varnothing, \varnothing, \varnothing, \varnothing_{GM}) \\
\forall\, F_1, F_2 : Fr_0 \bullet F_1 \cup_F F_2 = \\
\quad (fSG\, F_1 \cup_{SG} fSG\, F_2, EsR\, F_1 \cup EsR\, F_2, srcR\, F_1 \cup srcR\, F_2, tgtR\, F_1 \cup tgtR\, F_2, fet\, F_1 \cup_{GM} fet\, F_2) \\
\bigcup_F \{\} = \varnothing_F \\
\forall\, F : Fr_0;\ Fs : \mathbb{P}\, Fr_0 \bullet \bigcup_F (\{F\} \cup Fs) = F \cup_F (\bigcup_F Fs)
\end{array}$$

$$\begin{array}{|l}
\rightsquigarrow : Fr_0 \nrightarrow V \nrightarrow V \\
\circledbullet^{SG} : Fr_0 \nrightarrow SGr \\
rEsR : Fr_0 \nrightarrow \mathbb{P}\, E \\
\circledbullet : Fr_0 \nrightarrow Fr_0 \\
\hline
\forall\, F : Fr_0 \bullet \rightsquigarrow F = (\leftrightsquigarrow F) \rhd (fLNs\, F) \\
\forall\, F : Fr_0 \bullet \circledbullet^{SG} F = (fSG\, F) \odot^{SG} (\rightsquigarrow F) \\
\forall\, F : Fr_0 \bullet rEsR\, F = \mathrm{dom}((srcR\, F) \unrhd \mathrm{dom}(\rightsquigarrow F)) \\
\forall\, F : Fr_0 \bullet \circledbullet F = (\circledbullet^{SG} F, rEsR\, F, (rEsR\, F) \lhd (srcR\, F), (rEsR\, F) \lhd (tgtR\, F), fet\, F)
\end{array}$$

$$Fr_a == \{F : Fr_0 \mid \otimes(\overset{G}{\leftrightsquigarrow} F)\}$$
$$Fr == \{F : Fr_a \mid \circledbullet^{SG} F \in SGr\}$$

relation(refsLocal_)

$$\begin{array}{|l}
\mathrm{refsLocal}_ : \mathbb{P}\, Fr_0 \\
\hline
\forall\, F : Fr_0 \bullet \mathrm{refsLocal}\, F \Leftrightarrow fRNs\, F \subseteq fLNs\, F
\end{array}$$

$$TFr == \{F : Fr_a \mid \mathrm{refsLocal}\, F \wedge \circledbullet^{SG} F \in TSGr\}$$

relation($\boxminus\ \_$)
relation($\boxplus\ \_$)

$$\begin{array}{|l}
\boxminus_ : Fr \leftrightarrow Fr \\
\hline
\forall\, F_1, F_2 : Fr \bullet \boxminus(F_1, F_2) \Leftrightarrow fLNs\, F_1 \cap fLNs\, F_2 = \varnothing \wedge fEs\, F_1 \cap fEs\, F_2 = \varnothing
\end{array}$$

$[I]$

$\boxplus\_ : \mathbb{P}(I \nrightarrow Fr)$

$\forall Fs : I \nrightarrow Fr \bullet \boxplus\ Fs \Leftrightarrow \forall i, j : \mathrm{dom}\, Fs \mid i \neq j \bullet \boxminus(Fs\, i, Fs\, j)$

relation$(\_ \subseteq^{rs} \_)$
relation$(\_ \mapsto\!\!\!\Rightarrow \_)$

$\_ \subseteq^{rs} \_ : Fr \leftrightarrow Fr$
$\_ \mapsto\!\!\!\Rightarrow \_ : Fr \leftrightarrow Fr$

$\forall F_1, F_2 : Fr \bullet F_1 \subseteq^{rs} F_2 \Leftrightarrow \mathrm{ran}(tgtR\, F_1) \cap fLNs\, F_2 \neq \varnothing$

$\forall F_1, F_2 : Fr \bullet F_1 \mapsto\!\!\!\Rightarrow F_2 \Leftrightarrow F_1 \subseteq^{rs} F_2 \wedge \neg\,(F_2 \subseteq^{rs} F_1)$

function 10 **leftassoc** $(\_ \rightrightarrows_{\bullet} \_)$

$\_ \rightrightarrows_{\bullet} \_ : GrM \times (Fr \times Fr) \to GrM$

$\forall m : GrM;\ F_s, F_t : Fr_0 \bullet$
$\quad m \rightrightarrows_{\bullet} (F_s, F_t) = (((\rightsquigarrow F_s)^{\oplus} \boxdot (fLNs\, F_s))^{\sim} \circ (fV\ m) \circ ((\rightsquigarrow F_t)^{\oplus} \boxdot (fLNs\, F_t)), fE\ m)$

function 1 **leftassoc** $(\_ \to_F \_)$

$\_ \to_F \_ : Fr \times Fr \to \mathbb{P}\, GrM$

$\forall F_s, F_t : Fr \bullet F_s \to_F F_t = \{fv : fLNs\, F_s \to fLNs\, F_t;\ fe : fEsA\, F_s \to fEsA\, F_t \mid$
$\quad (\circledbullet^{SG} F_s, (fv, fe) \rightrightarrows_{\bullet} (F_s, F_t)) \Rrightarrow^{SG} \circledbullet^{SG} F_t\}$

relation$(\_ \Rrightarrow^{F} \_)$

$\_ \Rrightarrow^{F} \_ : (Fr \times GrM) \leftrightarrow Fr$

$\forall m : GrM;\ F_s, F_t : Fr_0 \bullet (F_s, m) \Rrightarrow^{F} F_t \Leftrightarrow m \in F_s \to_F F_t$

relation$(\_ \sqsupseteq^F \_)$

$$
\begin{array}{|l}
\_ \sqsupseteq^F \_ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupseteq^F F_a \Leftrightarrow (F_c, m) \Rrightarrow^F F_a \\
\quad \wedge (\circledbullet^{SG} F_c, m \rightrightarrows_{\bullet} (F_c, F_a)) \sqsupseteq^{SG_0} (\circledbullet^{SG} F_a)
\end{array}
$$

relation$(\_ \sqsupset^F \_)$

$$
\begin{array}{|l}
\_ \sqsupset^F \_ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupset^F F_a \Leftrightarrow (F_c, m) \Rrightarrow^F F_a \wedge \{F_a, F_c\} \subseteq Fr \\
\quad \wedge (\circledbullet^{SG} F_c, m \rightrightarrows_{\bullet} (F_c, F_a)) \sqsupset^{SG_0} (\circledbullet^{SG} F_a)
\end{array}
$$

relation$(\_ \Supset^F \_)$

$$
\begin{array}{|l}
\_ \Supset^F \_ : GrwT \leftrightarrow Fr \\
\hline
\forall GwT : GrwT;\ F : Fr \bullet GwT \Supset^F F \Leftrightarrow GwT \Supset^{SG} \circledbullet^{SG} F
\end{array}
$$

relation$(\_ \dashv\vdash^F \_)$

$$
\begin{array}{|l}
\_ \dashv\vdash^F \_ : Fr \leftrightarrow Fr \\
\hline
\forall m : GrM;\ F_s, F_t : Fr_0 \bullet F_s \dashv\vdash^F F_t \Leftrightarrow fet\, F_s \neq \varnothing_{GM} \wedge (\circledbullet^{SG} F_s, fet\, F_s) \dashv\vdash^{SG} (\circledbullet^{SG} F_t)
\end{array}
$$

relation$(\_ \Vvdash^F \_)$

$$
\begin{array}{|l}
\_ \Vvdash^F \_ : \mathbb{P}((GrwET \times Fr_0) \times (GrwT \times Fr_0)) \\
\hline
\forall GwET : GrwET;\ F_s, F_t : Fr_0;\ GwT : GrwT \bullet (GwET, F_s) \Vvdash^F (GwT, F_t) \Leftrightarrow GwET^{Gw} \Supset^F F_s \\
\quad \wedge GwT \Supset^F F_t \wedge \mathrm{domg}\,(GwET^{ET}) =_p \mathrm{domg}\,((fet\, F_s) \circ_G (GwET^{T})) \\
\quad \wedge GwET^{ET} \in (GwET^{G}) \nrightarrow_G (GwT^{G}) \wedge F_s \dashv\vdash^F F_t
\end{array}
$$

10 Global Fragment Graphs

section *Fragmenta_GFGs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$$GFGr == \{G : Gr \mid \circledcirc(G \bowtie_{Es} (Es\ G \setminus EsId\ G))\}$$

function$(\_ ^{\dashrightarrow})$

$$\begin{array}{|l}
_ ^{\dashrightarrow} : GFGr \to V \leftrightarrow V \\
\hline
\forall GFG : GFGr \bullet GFG ^{\dashrightarrow} = (GFG ^{\leftrightarrow})^{+}
\end{array}$$

11 Models

section *Fragmenta_Mdls* **parents** *standard_toolkit*, *Fragmenta_Frs*, *Fragmenta_GFGs*

$$Mdl_0 == \{GFG : GFGr;\ fd : V \nrightarrow Fr \mid fd \in Ns\ GFG \to Fr \land \boxplus\ fd\}$$

$$\begin{array}{|l}
mGFG : Mdl_0 \to GFGr \\
mFD : Mdl_0 \to V \nrightarrow Fr \\
\hline
\forall GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mGFG(GFG, fd) = GFG \\
\forall GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mFD(GFG, fd) = fd
\end{array}$$

$$\begin{array}{|l}
mUFs : Mdl_0 \to Fr \\
\hline
mUFs = \bigcup_F \circ \operatorname{ran} \circ mFD
\end{array}$$

$$\begin{array}{|l}
\mathrm{from} : Mdl_0 \to V \nrightarrow V \\
\hline
\forall M : Mdl_0;\ v : V \bullet \mathrm{from}\ M\ v = (\mu\ vf : (Ns \circ mGFG)M \mid v \in fLNs(mFD\ M\ vf))
\end{array}$$

relation$(\Uparrow \_)$

$$\Uparrow \_ : \mathbb{P}\, Mdl_0$$
$$\forall M : Mdl_0 \bullet \Uparrow M \Leftrightarrow \exists UF : Fr_0 \mid UF = mUFs\, M \bullet$$
$$\quad \forall p : (NsP \circ fSG)\, UF \bullet (from\, M\, p, from\, M\, (\leftrightsquigarrow\, UF\, p)) \in ((\_^{\dashrightarrow}) \circ mGFG) M$$

$$Mdl == \{M : Mdl_0 \mid (mUFs\, M) \in TFr \wedge \Uparrow M\}$$

$$\circledbullet^M : Mdl \to Fr$$
$$\forall M : Mdl_0 \bullet \circledbullet^M = \circledbullet \circ mUFs$$

function 1 **leftassoc** $(\_ \to_M \_)$
relation$(\_ \Rrightarrow^M \_)$

$$\_ \to_M \_ : Mdl \times Mdl \to \mathbb{P}\, GrM$$
$$\_ \Rrightarrow^M \_ : \mathbb{P}((Mdl \times \mathbb{P}\, GrM) \times Mdl)$$
$$\forall M_s, M_t : Mdl \bullet M_s \to_M M_t = \{m : GrM \mid$$
$$\quad \exists UF_s, UF_t : Fr_0 \mid UF_s = mUFs\, M_s \wedge UF_t = mUFs\, M_t \bullet m \in UF_s \to_F UF_t\}$$
$$\forall M_s, M_t : Mdl;\ ms : \mathbb{P}\, GrM \bullet (M_s, ms) \Rrightarrow^M M_t \Leftrightarrow \bigcup_{GM} ms \in M_s \to_M M_t$$

relation$(\_ \sqsupset^M \_)$

$$\_ \sqsupset^M \_ : (Mdl \times \mathbb{P}\, GrM) \leftrightarrow Mdl$$
$$\forall M_c, M_a : Mdl_0;\ ms : \mathbb{P}\, GrM \bullet (M_c, ms) \sqsupset^M M_a$$
$$\quad \Leftrightarrow \exists UF_c, UF_a : Fr_0 \mid UF_c = mUFs\, M_c \wedge UF_a = mUFs\, M_a \bullet (UF_c, \bigcup_{GM} ms) \sqsupset^F UF_a$$

relation$(\_ \Supset^M \_)$

$$\_ \Supset^M \_ : GrwT \leftrightarrow Mdl$$
$$\forall GwT : GrwT;\ M : Mdl \bullet GwT \Supset^M M \Leftrightarrow GwT \Supset^F mUFs\, M$$

relation$(\_ \dashv\vdash^F \_)$

$$
\begin{array}{|l}
\_ \dashv\vdash^{F} \_ : Mdl \leftrightarrow Mdl \\
\hline
\forall\, m : GrM;\ M_s, M_t : Mdl_0 \bullet M_s \dashv\vdash^{F} M_t \Leftrightarrow mUFs\, M_s \dashv\vdash^{F} mUFs\, M_t
\end{array}
$$

relation($\_ \Vdash\!\vdash^{M} \_$)

$$
\begin{array}{|l}
\_ \Vdash\!\vdash^{M} \_ : \mathbb{P}((GrwET \times Mdl_0) \times (GrwT \times Mdl_0)) \\
\hline
\forall\, GwET : GrwET;\ M_s, M_t : Mdl_0;\ GwT : GrwT \bullet \\
\quad (GwET, M_s) \Vdash\!\vdash^{M} (GwT, M_t) \Leftrightarrow (GwET, mUFs\, M_s) \Vdash\!\vdash^{F} (GwT, mUFs\, M_t)
\end{array}
$$