Z Specification of Fragmenta

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16/11/2021

Contents

1 Generics

section *Fragmenta_Generics* **parents** *standard_toolkit*

$\mathrm{acyclic}[X] == \{r : X \leftrightarrow X \mid r^{+} \cap \mathrm{id}\, X = \varnothing\}$

$connected[X] == \{r : X \leftrightarrow X \mid \forall x : \mathrm{dom}\, r;\ y : \mathrm{ran}\, r \bullet x \mapsto y \in r^{+}\}$

$\mathrm{tree}[X] == \{r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \land r \in X \rightarrow\!\!\!\!\!+ X\}$

$forest[X] == \{r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \land (\forall s : X \leftrightarrow X \mid s \subseteq r \land s \in connected \bullet s \in tree)\}$

$\mathrm{injrel}[X, Y] == \{r : X \leftrightarrow Y \mid r^{\sim} \in Y \rightarrow\!\!\!\!\!+ X\}$

$\mathrm{antireflexive}[X] == \{r : X \leftrightarrow X \mid r \cap id(\mathrm{dom}\ r) = \varnothing\}$

$$
\begin{array}{l}
[X, Y, Z] \\
\hline
\mathrm{flip} : (X \rightarrow Y \rightarrow\!\!\!\!\!+ Z) \rightarrow (Y \rightarrow X \rightarrow\!\!\!\!\!+ Z) \\
\hline
\forall f : X \rightarrow Y \rightarrow\!\!\!\!\!+ Z \bullet \mathrm{flip}\, f = (\lambda\, y : Y \bullet \lambda\, x : X \bullet f\ x\ y)
\end{array}
$$

$$
\begin{array}{l}
[X, Y, Z, W] \\
\hline
\mathrm{apply} : (X \rightarrow Z) \rightarrow (Y \rightarrow W) \rightarrow (X \times Y) \rightarrow (Z \times W) \\
\hline
\forall f : X \rightarrow Z;\ g : Y \rightarrow W;\ x : X;\ y : Y \bullet \mathrm{apply}\, f\ g\ (x, y) = (f\ x, g\ y)
\end{array}
$$

$$
\begin{array}{l}
[X, Y] \\
\hline
\mathrm{map} : (X \rightarrow Y) \rightarrow \mathbb{P}\, X \rightarrow \mathbb{P}\, Y \\
\mathrm{mapS} : (X \rightarrow Y) \rightarrow \mathrm{seq}\, X \rightarrow \mathrm{seq}\ Y \\
\hline
\forall f : X \rightarrow Y \bullet \mathrm{map}\, f \{\} = \{\} \\
\forall f : X \rightarrow Y;\ x : X;\ xs : \mathbb{P}\, X \bullet \mathrm{map}\, f (\{x\} \cup xs) = \{f\ x\} \cup (\mathrm{map}\, f\ xs) \\
\forall f : X \rightarrow Y \bullet \mathrm{mapS}\, f \langle\rangle = \langle\rangle \\
\forall f : X \rightarrow Y;\ x : X;\ xs : \mathrm{seq}\, X \bullet \mathrm{mapS}\, f (\langle x\rangle \frown xs) = \langle f\ x\rangle \frown (\mathrm{mapS}\, f\ xs)
\end{array}
$$

function 10 **leftassoc** $(\_ \boxdot \_)$

$$
\begin{array}{l}
[X] \\
\hline
\_ \boxdot \_ : ((X \rightarrow X) \times \mathbb{P}\, X) \rightarrow (X \rightarrow\!\!\!\!\!+ X) \\
\hline
\forall f : X \rightarrow X;\ s : \mathbb{P}\, X \bullet f \boxdot s = (\mathrm{id}\ s) \oplus f
\end{array}
$$

function$(\_^{\oplus})$

$$
\begin{array}{l}
[X] \\
\hline
\_^{\oplus} : (X \leftrightarrow X) \rightarrow (X \leftrightarrow X) \\
\hline
\forall r : X \leftrightarrow X \bullet r^{\oplus} = \mathbf{if}\ r \oplus r \mathbin{⨾} r = r\ \mathbf{then}\ r\ \mathbf{else}\ (r \oplus r \mathbin{⨾} r)^{\oplus}
\end{array}
$$

$\mathrm{opt}[X] == \{s : \mathbb{P}\, X \mid \#\, s \leq 1\}$

$$
\begin{array}{l}
[X] \\
\hline
\mathrm{the} : \mathrm{opt}[X] \rightarrow\!\!\!\!\!+ X \\
\hline
\forall x : X \bullet \mathrm{the}\,\{x\} = x
\end{array}
$$

$$
\begin{array}{l}
[X, Y] \\
\hline
\mathit{flatten} : (X \rightarrow\!\!\!\!\!+ \mathbb{P}\, Y) \rightarrow (X \leftrightarrow Y) \\
\hline
\forall f : X \rightarrow\!\!\!\!\!+ \mathbb{P}\, Y \bullet \mathit{flatten}\, f = \{x : \mathrm{dom}\, f;\ y : Y \mid y \in f\, x\}
\end{array}
$$

2 Graphs

section *Fragmenta_Graphs* **parents** *standard_toolkit*, *Fragmenta_Generics*

$[V, E]$

$\mathit{Gr} == \{vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s, t : E \rightarrow\!\!\!\!\!+ V \mid s \in es \rightarrow vs \land t \in es \rightarrow vs\}$

$$
\begin{array}{l}
\mathit{Ns} : \mathit{Gr} \rightarrow \mathbb{P}\, V \\
\mathit{Es} : \mathit{Gr} \rightarrow \mathbb{P}\, E \\
\mathit{src}, \mathit{tgt} : \mathit{Gr} \rightarrow E \rightarrow\!\!\!\!\!+ V \\
\hline
\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!+ V;\ t : E \rightarrow\!\!\!\!\!+ V \bullet \mathit{Ns}(vs, es, s, t) = vs \\
\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!+ V;\ t : E \rightarrow\!\!\!\!\!+ V \bullet \mathit{Es}(vs, es, s, t) = es \\
\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!+ V;\ t : E \rightarrow\!\!\!\!\!+ V \bullet \mathit{src}(vs, es, s, t) = s \\
\forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!+ V;\ t : E \rightarrow\!\!\!\!\!+ V \bullet \mathit{tgt}(vs, es, s, t) = t
\end{array}
$$

$$
\begin{array}{|l}
\varnothing_G : Gr \\
\hline
\varnothing_G = (\varnothing, \varnothing, \varnothing, \varnothing)
\end{array}
$$

$$
\begin{array}{|l}
EsId : Gr \rightarrow \mathbb{P}\, E \\
\hline
\forall\, G : Gr \bullet EsId\ G = \{ e : Es\ G \mid src\ G\ e = tgt\ G\ e \}
\end{array}
$$

relation(adjacent _)

$$
\begin{array}{|l}
\mathrm{adjacent}\_ : \mathbb{P}(Gr \times V \times V) \\
\hline
\forall\, G : Gr;\ v_1, v_2 : V \bullet \mathrm{adjacent}(G, v_1, v_2) \Leftrightarrow \exists\, e : Es\ G \bullet src\ G\ e = v_1 \land tgt\ G\ e = v_2
\end{array}
$$

function 10 **leftassoc** $(\_ \circ\!\!\rightarrow\!\!\circ \_)$

$$
\begin{array}{|l}
\_ \circ\!\!\rightarrow\!\!\circ \_ : Gr \times \mathbb{P}\, V \rightarrow \mathbb{P}\, E \\
\hline
\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet G \circ\!\!\rightarrow\!\!\circ vs = (src\ G)^{\sim} \llparenthesis vs \rrparenthesis \cup (tgt\ G)^{\sim} \llparenthesis vs \rrparenthesis
\end{array}
$$

function 10 **leftassoc** $(\_ \bullet\!\!\leftrightarrow\!\!\bullet \_)$

$$
\begin{array}{|l}
\_ \bullet\!\!\leftrightarrow\!\!\bullet \_ : Gr \times \mathbb{P}\, V \rightarrow \mathbb{P}\, E \\
\hline
\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet G \bullet\!\!\leftrightarrow\!\!\bullet vs = (src\ G)^{\sim} \llparenthesis vs \rrparenthesis \cap (tgt\ G)^{\sim} \llparenthesis vs \rrparenthesis
\end{array}
$$

function 10 **leftassoc** $(\_ \bowtie_{Es} \_)$

$$
\begin{array}{|l}
\_ \bowtie_{Es} \_ : Gr \times \mathbb{P}\, E \rightarrow Gr \\
\hline
\forall\, G : Gr;\ es : \mathbb{P}\, E \bullet G \bowtie_{Es} es = (Ns\ G, Es\ G \cap es, es \lhd src\ G, es \lhd tgt\ G)
\end{array}
$$

function 10 **leftassoc** $(\_ \bowtie_{Ns} \_)$

$$
\begin{array}{l}
\_ \bowtie_{Ns} \_ : Gr \times \mathbb{P}\, V \rightarrow Gr \\
\hline
\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet \\
\quad G \bowtie_{Ns}\ vs = (Ns\ G \cap vs, G\ \bullet\!\!\leftrightarrow\!\!\bullet\ vs, (G\ \bullet\!\!\leftrightarrow\!\!\bullet\ vs) \lhd src\ G, (G\ \bullet\!\!\leftrightarrow\!\!\bullet\ vs) \lhd tgt\ G)
\end{array}
$$

function 10 **leftassoc** $(\_ \ominus_{Ns} \_)$

$$
\begin{array}{l}
\_ \ominus_{Ns} \_ : Gr \times \mathbb{P}\, V \rightarrow Gr \\
\hline
\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet \\
\quad G \ominus_{Ns}\ vs = (Ns\ G \setminus vs, Es\ G \setminus (G\ \circ\!\!\rightarrow\!\!\circ\ vs), (G\ \circ\!\!\rightarrow\!\!\circ\ vs) \ndres src\ G, (G\ \circ\!\!\rightarrow\!\!\circ\ vs) \ndres tgt\ G)
\end{array}
$$

$$
\begin{array}{l}
\mathrm{successors} : V \times Gr \rightarrow \mathbb{P}\, V \\
\hline
\forall\, v : V;\ G : Gr \bullet \mathrm{successors}(v, G) = \{v_1 : Ns\ G \mid \mathrm{adjacent}(G, v, v_1)\}
\end{array}
$$

function$(\_^{\rightleftarrows})$

$$
\begin{array}{l}
\_^{\rightleftarrows} : Gr \rightarrow Gr \\
\hline
\forall\, G : Gr \bullet G^{\rightleftarrows} = (Ns\ G, Es\ G, tgt\ G, src\ G)
\end{array}
$$

function$(\_^{\leftrightarrow})$

$$
\begin{array}{l}
\_^{\leftrightarrow} : Gr \rightarrow V \leftrightarrow V \\
\hline
\forall\, G : Gr \bullet G^{\leftrightarrow} = \{v_1, v_2 : Ns\ G \mid \mathrm{adjacent}(G, v_1, v_2)\}
\end{array}
$$

relation$(\otimes\ \_)$

$$
\begin{array}{l}
\otimes\ \_ : \mathbb{P}\, Gr \\
\hline
\forall\, G : Gr \bullet \otimes\ G \Leftrightarrow G^{\leftrightarrow} \in \mathrm{acyclic}
\end{array}
$$

relation$(\boxminus_{Es}\ \_)$
relation$(\boxminus\ \_)$

$$
\begin{array}{|l}
\boxminus_{Es}\_,\boxminus\_ : \mathbb{P}(Gr \times Gr) \\
\hline
\forall G_1, G_2 : Gr \bullet \boxminus_{Es}(G_1, G_2) \Leftrightarrow Es\ G_1 \cap Es\ G_2 = \varnothing \\
\forall G_1, G_2 : Gr \bullet \boxminus(G_1, G_2) \Leftrightarrow Ns\ G_1 \cap Ns\ G_2 = \varnothing \land \boxminus_{Es}(G_1, G_2)
\end{array}
$$

relation$(\boxplus\ \_)$

$$
\begin{array}{|l|}
\hline
[I] \\
\boxplus\_ : \mathbb{P}(I \nrightarrow Gr) \\
\hline
\forall Gs : I \nrightarrow Gr \bullet \boxplus\ Gs \Leftrightarrow \forall i, j : \mathrm{dom}\ Gs \mid i \neq j \bullet \boxminus(Gs\ i, Gs\ j) \\
\hline
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_G \_)$

$$
\begin{array}{|l}
\_ \cup_G \_ : Gr \times Gr \rightarrow Gr \\
\hline
\forall G_1, G_2 : Gr \bullet G_1 \cup_G G_2 = (Ns\ G_1 \cup Ns\ G_2, Es\ G_1 \cup Es\ G_2, src\ G_1 \cup src\ G_2, tgt\ G_1 \cup tgt\ G_2)
\end{array}
$$

function 10 **leftassoc** $(\_ \odot \_)$

$$
\begin{array}{|l}
\_ \odot \_ : Gr \times (V \leftrightarrow V) \nrightarrow Gr \\
\hline
\forall G : Gr;\ s : V \leftrightarrow V \mid s \in Ns\ G \nrightarrow Ns\ G \land s \in \text{antireflexive} \bullet \\
\quad G \odot s = (Ns\ G \setminus \mathrm{dom}\ s, Es\ G, (s \boxdot Ns\ G) \circ (src\ G), (s \boxdot Ns\ G) \circ (tgt\ G))
\end{array}
$$

$$GrM == (V \nrightarrow V) \times (E \nrightarrow E)$$

$$
\begin{array}{|l}
fV : GrM \rightarrow V \nrightarrow V \\
fE : GrM \rightarrow E \nrightarrow E \\
\hline
\forall fv : V \nrightarrow V;\ fe : E \nrightarrow E \bullet fV(fv, fe) = fv \\
\forall fv : V \nrightarrow V;\ fe : E \nrightarrow E \bullet fE(fv, fe) = fe
\end{array}
$$

$$
\begin{array}{|l}
\mathrm{gid} : Gr \rightarrow GrM \\
\hline
\forall G : Gr \bullet \mathrm{gid}\ G = (id\ (Ns\ G), id\ (Es\ G))
\end{array}
$$

$$\begin{array}{|l} \varnothing_{GM} : GrM \\ \hline \varnothing_{GM} = (\{\}, \{\}) \end{array}$$

$$\begin{array}{|l} \mathrm{domg}, \mathrm{codg} : GrM \rightarrow Gr \\ \hline \forall\, m : GrM;\ G : Gr \bullet \mathrm{domg}\, m = G \Leftrightarrow \mathrm{dom}(fV\ m) = Ns\ G \wedge \mathrm{dom}(fE\ m) = Es\ G \\ \forall\, m : GrM;\ G : Gr \bullet \mathrm{codg}\, m = G \Leftrightarrow \mathrm{ran}(fV\ m) \subseteq Ns\ G \wedge \mathrm{ran}(fE\ m) \subseteq Es\ G \end{array}$$

function 10 **leftassoc** $(\_ \cup_{GM} \_)$

$$\begin{array}{|l} \_ \cup_{GM} \_ : GrM \times GrM \nrightarrow GrM \\ \bigcup_{GM} : \mathbb{P}\, GrM \rightarrow GrM \\ \hline \forall\, f, g : GrM \bullet f \cup_{GM} g = (fV\ f \cup fV\ g, fE\ f \cup fE\ g) \\ \bigcup_{GM} \varnothing = \varnothing_{GM} \\ \forall\, f : GrM;\ fs : \mathbb{P}\, GrM \bullet \bigcup_{GM} (\{f\} \cup fs) = f \cup_{GM} (\bigcup_{GM} fs) \end{array}$$

function 10 **leftassoc** $(\_ \rightarrow_G \_)$

$$\begin{array}{|l} \_ \rightarrow_G \_ : Gr \times Gr \rightarrow \mathbb{P}\, GrM \\ \hline \forall\, G_1, G_2 : Gr \bullet G_1 \rightarrow_G G_2 = \{fv : Ns\ G_1 \rightarrow Ns\ G_2;\ fe : Es\ G_1 \rightarrow Es\ G_2 \mid \\ \quad src\ G_2 \circ fe = fv \circ src\ G_1 \wedge tgt\ G_2 \circ fe = fv \circ tgt\ G_1\} \end{array}$$

function 10 **leftassoc** $(\_ \circ_G \_)$

$$\begin{array}{|l} \_ \circ_G \_ : GrM \times GrM \rightarrow GrM \\ \hline \forall\, g, f : GrM \bullet g \circ_G f = (fV\ g \circ fV\ f, fE\ g \circ fE\ f) \end{array}$$

3 Graphs with typing

section *Fragmenta_GrswT* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$GrwT == \{G : Gr;\ t : GrM \mid \mathrm{domg}\, t = G\}$

$$
\begin{array}{|l}
gOf : GrwT \rightarrow Gr \\
ty : GrwT \rightarrow GrM \\
\hline
\forall G : Gr;\ sm : V \nrightarrow \operatorname{seq} V;\ t : GrM \bullet gOf(G, t) = G \\
\forall G : Gr;\ sm : V \nrightarrow \operatorname{seq} V;\ t : GrM \bullet ty(G, t) = t
\end{array}
$$

$$
\begin{array}{|l}
\varnothing_{GwT} : GrwT \\
\hline
\varnothing_{GwT} = (\varnothing_G, \varnothing_{GM})
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_{GwT} \_)$

$$
\begin{array}{|l}
\_ \cup_{GwT} \_ : GrwT \times GrwT \rightarrow GrwT \\
\hline
\forall G_1, G_2 : GrwT \bullet G_1 \cup_{GwT} G_2 = ((gOf\ G_1) \cup_G (gOf\ G_2), (ty\ G_1) \cup_{GM} (ty\ G_2))
\end{array}
$$

4 SG Element Types

section *Fragmenta_SGElemTys* **parents** *standard_toolkit*, *Fragmenta_Generics*

$SGNT ::= nnrml \mid nabst \mid nprxy \mid nenum \mid nval \mid nvirt \mid nopt$

$SGED ::= dbi \mid duni$

$SGET ::= einh \mid ecomp\langle\!\langle SGED \rangle\!\rangle \mid erel\langle\!\langle SGED \rangle\!\rangle \mid ewander \mid eder$

relation $(\_ \prec_{NT} \_)$

$$
\begin{array}{|l}
\_ \prec_{NT} \_ : SGNT \leftrightarrow SGNT \\
\hline
\forall nt_1, nt_2 : SGNT \bullet nt_1 \prec_{NT} nt_2 \Leftrightarrow (nt_2 = nenum \Leftrightarrow nt_1 = nval) \\
\quad \land (nt_1 = nvirt \Rightarrow nt_2 = nvirt) \land (nt_1 = nabst \Rightarrow nt_2 \in \{nabst, nvirt, nprxy\}) \\
\quad \land nt_1 \notin \{nprxy, nenum\} \land nt_2 \notin \{nopt\}
\end{array}
$$

relation $(\_ \leq_{rNT} \_)$

$$
\begin{array}{|l}
\_ \leq_{rNT} \_ : SGNT \leftrightarrow SGNT \\
\hline
\forall nt_1, nt_2 : SGNT \bullet nt_1 \leq_{rNT} nt_2 \Leftrightarrow nt_1 = nt_2 \lor nt_1 = nprxy \\
\quad \lor nt_2 = nabst \land nt_1 \in \{nnrml, nvirt\} \lor nt_2 \in \{nnrml, nopt\}
\end{array}
$$

relation$(\_ =_{ET} \_)$

$$
\begin{array}{|l}
\_ =_{ET} \_ : SGET \leftrightarrow SGET \\
\hline
\forall et_1, et_2 : SGET \bullet et_1 =_{ET} et_2 \Leftrightarrow et_1 = et_2 \\
\quad \lor (\forall d_1, d_2 : SGED \bullet et_1 = erel\ d_1 \land et_2 = erel\ d_2 \lor et_1 = ecomp\ d_1 \land et_2 = ecomp\ d_2)
\end{array}
$$

relation$(\_ \leq_{ET} \_)$

$$
\begin{array}{|l}
\_ \leq_{ET} \_ : SGET \leftrightarrow SGET \\
\hline
\forall et_1, et_2 : SGET \bullet et_1 \leq_{ET} et_2 \Leftrightarrow einh \notin \{et_1, et_2\} \\
\quad \land (et_1 =_{ET} et_2 \lor et_2 = ewander \\
\quad \lor et_1 = eder \land et_2 \in \mathrm{dom}(erel^{\sim}) \cup \mathrm{dom}(ecomp^{\sim}))
\end{array}
$$

5 Multiplicities

section *Fragmenta_Mult* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_SGElemTys*

$MultVal ::= \mathbf{v}\langle\langle \mathbb{N} \rangle\rangle \mid *$

$MultC ::= mr\langle\langle \mathbb{N} \times MultVal \rangle\rangle \mid ms\langle\langle MultVal \rangle\rangle$

relation$(\_ \leq_{mv} \_)$

$$
\begin{array}{|l}
\_ \leq_{mv} \_ : MultVal \leftrightarrow MultVal \\
\hline
\forall m_1, m_2 : MultVal \bullet m_1 \leq_{mv} m_2 \Leftrightarrow m_2 = * \lor \exists j, k : \mathbb{N} \mid m_1 = \mathbf{v}\ j \land m_2 = \mathbf{v}\ k \bullet j \leq k
\end{array}
$$

$$Mult == \{mc : MultC \mid \exists\, lb : \mathbb{N};\; ub : MultVal \bullet mc = mr(lb, ub) \land \mathbf{v}\ lb \leq_{mv} ub$$
$$\lor \exists\, mv : MultVal \bullet mc = ms\ mv\}$$

$$MultMany == \{ms\, *, mr(0, *)\}$$

$$MultRange == \{m : MultC \mid \exists\, k : \mathbb{N} \mid k > 1 \bullet m = ms\,(\mathbf{v}\ k)$$
$$\lor \exists\, lb : \mathbb{N};\; umv : MultVal \mid \mathbf{v}\ 2 \leq_{mv} umv \bullet m = mr(lb, umv)\}$$

relation$(\_ \between \_)$

$$\_ \between \_ : \mathbb{P}(\mathbb{N} \times (MultVal \times MultVal))$$
$$\forall\, k : \mathbb{N};\; lb, ub : MultVal \bullet k \between (lb, ub) \Leftrightarrow lb \leq_{mv} \mathbf{v}\ k \land \mathbf{v}\ k \leq_{mv} ub$$

$$mlb, mub : MultC \rightarrow\!\!\!\!\!+ MultVal$$
$$mlb(ms\, *) = \mathbf{v}\ 0$$
$$\forall\, k : \mathbb{N} \bullet mlb(ms(\mathbf{v}\ k)) = \mathbf{v}\ k$$
$$\forall\, k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ k$$
$$\forall\, mv : MultVal \bullet mub(ms\ mv) = mv$$
$$\forall\, k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ m$$

relation$(\_ \leq_{\mathcal{M}} \_)$

$$\_ \leq_{\mathcal{M}} \_ : MultC \leftrightarrow MultC$$
$$\forall\, m_1, m_2 : MultC \bullet m_1 \leq_{\mathcal{M}} m_2 \Leftrightarrow mlb\ m_2 \leq_{mv} mlb\ m_1 \land mub\ m_1 \leq_{mv} mub\ m_2$$

relation$(\_ \propto \_)$

$$\_ \propto \_ : \mathbb{P}(SGET \times (MultC \times MultC))$$
$$\forall\, et : SGET;\; m_1, m_2 : MultC \bullet et \propto (m_1, m_2) \Leftrightarrow et = erel\ dbi \lor et = eder$$
$$\lor et = ecomp\ duni \land m_1 = ms(\mathbf{v}\ 1) \lor et = erel\ duni \land m_1 \in MultMany$$
$$\lor et = ecomp\ dbi \land m_1 \in \{ms(\mathbf{v}\ 1), mr(0, \mathbf{v}\ 1)\}$$
$$\lor et = ewander \land (m_1, m_2) \in MultMany \times MultMany$$

relation(rbounded_)

$$
\begin{array}{l}
[X, Y] \\
\hline
\mathrm{rbounded}\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\, X \times \mathit{MultC}) \\
\hline
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ m : \mathit{MultC} \bullet \\
\quad \mathrm{rbounded}(r, s, m) \Leftrightarrow \forall\, x : s \bullet \#(r \,(\!|\, \{x\} \,|\!)) \between (\mathit{mlb}\ m, \mathit{mub}\ m)
\end{array}
$$

relation($r\mathcal{M}Ok\,\_$)

$$
\begin{array}{l}
[X, Y] \\
\hline
r\mathcal{M}Ok\,\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\, X \times \mathbb{P}\, Y \times \mathit{MultC} \times \mathit{MultC}) \\
\hline
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{ms}(\mathbf{v}\ 1), \mathit{ms}(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail\!\!\!\!\twoheadrightarrow t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mr}(0, \mathbf{v}\ 1), \mathit{mr}(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \mathrel{\rightarrowtail\!\!\!\!\!\!+} t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mr}(0, \mathbf{v}\ 1), \mathit{ms}(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{ms}(\mathbf{v}\ 1), \mathit{mr}(0, \mathbf{v}\ 1)) \Leftrightarrow r^{\sim} \in t \rightarrowtail s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mm}, \mathit{ms}(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrow t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{ms}(\mathbf{v}\ 1), \mathit{mm}) \Leftrightarrow r^{\sim} \in t \rightarrow s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm}_1, \mathit{mm}_2 : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mm}_1, \mathit{mm}_2) \Leftrightarrow r \in s \leftrightarrow t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany};\ \mathit{mr} : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mm}, \mathit{mr}) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r, s, \mathit{mr}) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany};\ \mathit{mr} : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mr}, \mathit{mm}) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r^{\sim}, t, \mathit{mr}) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mm}, \mathit{mr}(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \mathrel{\rightarrow\!\!\!\!\!+} t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mm} : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mr}(0, \mathbf{v}\ 1), \mathit{mm}) \Leftrightarrow r^{\sim} \in t \mathrel{\rightarrow\!\!\!\!\!+} s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mr}_1, \mathit{mr}_2 : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mr}_1, \mathit{mr}_2) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r, s, \mathit{mr}_2) \land \mathrm{rbounded}(r^{\sim}, t, \mathit{mr}_1) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mr} : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mr}, \mathit{ms}(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrow t \land \mathrm{rbounded}(r^{\sim}, t, \mathit{mr}) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ \mathit{mr} : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{ms}(\mathbf{v}\ 1), \mathit{mr}) \Leftrightarrow r^{\sim} \in t \rightarrow s \land \mathrm{rbounded}(r, s, \mathit{mr}) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ m : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, m, \mathit{mr}(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \mathrel{\rightarrow\!\!\!\!\!+} t \land \mathrm{rbounded}(r^{\sim}, t, m) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ m : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, \mathit{mr}(0, \mathbf{v}\ 1), m) \Leftrightarrow r^{\sim} \in t \mathrel{\rightarrow\!\!\!\!\!+} s \land \mathrm{rbounded}(r, s, m)
\end{array}
$$

6 Structural Graphs

section $Fragmenta\_SGs$ **parents** $standard\_toolkit, Fragmenta\_Generics, Fragmenta\_Graphs,$ $Fragmenta\_SGElemTys, Fragmenta\_Mult, Fragmenta\_GrswT$

$$SGr_0 == \{G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ db : E \pfun E \mid nt \in Ns\ G \rightarrow SGNT \land et \in Es\ G \rightarrow SGET\}$$

$$\begin{array}{l}
gr : SGr_0 \rightarrow Gr \\
sg_Ns : SGr_0 \rightarrow \mathbb{P}\ V \\
sg_Es : SGr_0 \rightarrow \mathbb{P}\ E \\
sg_src, sg_tgt : SGr_0 \rightarrow E \pfun V \\
nty : SGr_0 \rightarrow V \pfun SGNT \\
ety : SGr_0 \rightarrow E \pfun SGET \\
srcm, tgtm : SGr_0 \rightarrow E \pfun Mult \\
derb : SGr_0 \rightarrow E \pfun E \\
\hline
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ db : E \pfun E \bullet \\
\quad gr\,(G, nt, et, sm, tm, db) = G \\
sg_Ns = Ns \circ gr \\
sg_Es = Es \circ gr \\
sg_src = src \circ gr \\
sg_tgt = tgt \circ gr \\
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ db : E \pfun E \bullet \\
\quad nty(G, nt, et, sm, tm, db) = nt \\
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ db : E \pfun E \bullet \\
\quad ety(G, nt, et, sm, tm, db) = et \\
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ db : E \pfun E \bullet \\
\quad srcm(G, nt, et, sm, tm, db) = sm \\
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ db : E \pfun E \bullet \\
\quad tgtm(G, nt, et, sm, tm, db) = tm \\
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult;\ db : E \pfun E \bullet \\
\quad derb(G, nt, et, sm, tm, db) = db
\end{array}$$

$$\begin{array}{l}
\varnothing_{SG} : SGr_0 \\
\hline
\varnothing_{SG} = (\varnothing_G, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing)
\end{array}$$

$$\begin{array}{l}
NsTy : SGr_0 \rightarrow \mathbb{P}\ SGNT \rightarrow \mathbb{P}\ V \\
EsTy : SGr_0 \rightarrow \mathbb{P}\ SGET \rightarrow \mathbb{P}\ E \\
\hline
\forall SG : SGr_0;\ nts : \mathbb{P}\ SGNT \bullet NsTy\ SG\ nts = (nty\ SG)^{\sim} (\!|\, nts\, |\!) \\
\forall SG : SGr_0;\ ets : \mathbb{P}\ SGET \bullet EsTy\ SG\ ets = (ety\ SG)^{\sim} (\!|\, ets\, |\!)
\end{array}$$

$$EsA, EsW, EsI, EsC, EsD : SGr_0 \to \mathbb{P}\, E$$

$$\forall\, SG : SGr_0 \bullet EsA\; SG = EsTy\; SG\; (erel\; (\!|\; SGED\; |\!) \cup ecomp\; (\!|\; SGED\; |\!))$$

$$EsW = (\text{flip}\; EsTy)\; \{ewander\}$$

$$EsI = (\text{flip}\; EsTy)\; \{einh\}$$

$$EsD = (\text{flip}\; EsTy)\; \{eder\}$$

$$\forall\, SG : SGr_0 \bullet EsC\; SG = EsA\; SG \cup EsW\; SG \cup EsD\; SG$$

$$NsP, NsEther, NsO, NsV : SGr_0 \to \mathbb{P}\, V$$

$$NsP = (\text{flip}\; NsTy)\; \{nprxy\}$$

$$NsEther = (\text{flip}\; NsTy)\; \{nabst, nvirt, nenum\}$$

$$NsO = (\text{flip}\; NsTy)\; \{nopt\}$$

$$NsV = (\text{flip}\; NsTy)\; \{nvirt\}$$

$$\pitchfork : SGr_0 \to Gr$$
$$\prec : SGr_0 \to V \leftrightarrow V$$

$$\forall\, SG : SGr_0 \bullet \pitchfork\; SG = gr\; SG \rhd\!\!\lhd_{Es}\; EsI\; SG$$

$$\prec = (\_^{\leftrightarrow}) \circ \pitchfork$$

$$srcma : SGr_0 \to (E \nrightarrow Mult)$$

$$\forall\, SG : SGr_0 \bullet srcma\; SG = \\ (srcm\; SG) \oplus (EsTy\; SG\; \{ecomp\; duni\} \times \{ms(\mathbf{v}\; 1)\}) \oplus (EsTy\; SG\; \{erel\; duni\} \times \{ms\, *\})$$

relation$(\mathcal{M}etysOk\, \_)$

$$\mathcal{M}etysOk\, \_ : \mathbb{P}\, SGr_0$$

$$\forall\, SG : SGr_0 \bullet \mathcal{M}etysOk\;\; SG \Leftrightarrow \forall\, e : EsC\; SG \bullet (ety\; SG\; e) \propto (srcma\; SG\; e, tgtm\; SG\; e)$$

$$\preceq : SGr_0 \to V \leftrightarrow V$$

$$\forall\, SG : SGr_0 \bullet \preceq\; SG = (\prec\; SG)^*$$

$$
\begin{array}{|l}
srcr, tgtr : SGr_0 \rightarrow E \leftrightarrow V \\
src_0^*, src^*, tgt_0^*, tgt^* : SGr_0 \rightarrow E \leftrightarrow V \\
\hline
\forall SG : SGr_0 \bullet srcr\ SG = sg\_src\ SG \cup (EsW\ SG \lhd sg\_tgt\ SG) \\
\forall SG : SGr_0 \bullet tgtr\ SG = sg\_tgt\ SG \cup (EsW\ SG \lhd sg\_src\ SG) \\
\forall SG : SGr_0 \bullet src_0^*\ SG = EsC\ SG \lhd (srcr\ SG) \\
\forall SG : SGr_0 \bullet src^*\ SG = (src_0^*\ SG) \mathbin{;} (\preceq SG)^{\sim} \\
\forall SG : SGr_0 \bullet tgt_0^*\ SG = EsC\ SG \lhd (tgtr\ SG) \\
\forall SG : SGr_0 \bullet tgt^*\ SG = (tgt_0^*\ SG) \mathbin{;} (\preceq SG)^{\sim}
\end{array}
$$

function 10 **leftassoc** $(\_ \circ\!\!\rightarrow\!\!\circ^* \_)$

$$
\begin{array}{|l}
\_ \circ\!\!\rightarrow\!\!\circ^* \_ : SGr_0 \times \mathbb{P}\ V \rightarrow \mathbb{P}\ E \\
\hline
\forall SG : SGr_0;\ vs : \mathbb{P}\ V \bullet SG \circ\!\!\rightarrow\!\!\circ^*\ vs = (src^*\ SG)^{\sim} (\!|\, vs \,|\!) \cup (tgt^*\ SG)^{\sim} (\!|\, vs \,|\!)
\end{array}
$$

relation(optsVoluntary_)

$$
\begin{array}{|l}
\mathrm{optsVoluntary}\_ : \mathbb{P}\ SGr_0 \\
\hline
\forall SG : SGr_0 \bullet \\
\quad \mathrm{optsVoluntary}\ SG \Leftrightarrow (ety\ SG)\ (\!|\, (SG \circ\!\!\rightarrow\!\!\circ^*\ (NsO\ SG)) \setminus (EsI\ SG) \,|\!) \subseteq \{ewander\}
\end{array}
$$

relation(inhOk_)

$$
\begin{array}{|l}
\mathrm{inhOk}\_ : \mathbb{P}\ SGr_0 \\
\hline
\forall SG : SGr_0 \bullet \mathrm{inhOk}\ SG \\
\quad \Leftrightarrow (\forall v, v' : sg\_Ns\ SG \mid (v, v') \in (\prec SG) \bullet nty\ SG\ v \prec_{NT} nty\ SG\ v') \\
\quad \wedge \oslash(\pitchfork SG) \wedge ((\pitchfork SG) \ominus_{Ns} (NsV\ SG))^{\leftrightarrow} \in sg\_Ns\ SG \rightarrow\!\!\!\!\!\!+\ sg\_Ns\ SG
\end{array}
$$

$$
\begin{array}{l}
SGr == \{SG : SGr_0 \mid \{srcma\ SG, tgtm\ SG\} \subseteq EsC\ SG \rightarrow Mult \wedge \mathrm{dom}(derb\ SG) = EsD\ SG \\
\quad \wedge \mathcal{M}etysOk\ SG \wedge \mathrm{optsVoluntary}\ SG \wedge \mathrm{inhOk}\ SG\}
\end{array}
$$

relation(etherealAreInherited _)

$$
\begin{array}{|l}
\mathrm{etherealAreInherited\_} : \mathbb{P}\, SGr_0 \\
\hline
\forall SG : SGr_0 \bullet \mathrm{etherealAreInherited}\, SG \Leftrightarrow NsEther\, SG \subseteq \mathrm{ran}(\prec SG)
\end{array}
$$

relation(derivedOk _)

$$
\begin{array}{|l}
\mathrm{derivedOk\_} : \mathbb{P}\, SGr_0 \\
\hline
\forall SG : SGr_0 \bullet \mathrm{derivedOk}\, SG \Leftrightarrow \mathrm{ran}(derb\, SG) \subseteq EsA\, SG \\
\quad \wedge (\forall e : EsD\, SG \bullet (sg\_src\, SG\, e, ((sg\_src\, SG) \circ (derb\, SG))\, e) \in (\preceq SG) \\
\qquad \wedge (sg\_tgt\, SG\, e, ((sg\_tgt\, SG) \circ (derb\, SG))\, e) \in (\preceq SG))
\end{array}
$$

$$TSGr == \{ SG : SGr \mid \mathrm{etherealAreInherited}\, SG \wedge \mathrm{derivedOk}\, SG \}$$

relation($\boxminus_{SGs}$ _)

$$
\begin{array}{|l}
\boxminus_{SGs\_} : \mathbb{P}(SGr \times SGr) \\
\hline
\forall SG_1, SG_2 : SGr \bullet \boxminus_{SGs}(SG_1, SG_2) \Leftrightarrow \boxminus(gr\, SG_1, gr\, SG_2)
\end{array}
$$

function 10 **leftassoc** (_ $\cup_{SG}$ _)

$$
\begin{array}{|l}
\_ \cup_{SG} \_ : SGr \times SGr \rightarrow SGr \\
\hline
\forall SG_1, SG_2 : SGr \bullet SG_1 \cup_{SG} SG_2 = (gr\, SG_1 \cup_G gr\, SG_2, nty\, SG_1 \cup nty\, SG_2, \\
\quad ety\, SG_1 \cup ety\, SG_2, srcm\, SG_1 \cup srcm\, SG_2, tgtm\, SG_1 \cup tgtm\, SG_2, derb\, SG_1 \cup derb\, SG_2)
\end{array}
$$

function 10 **leftassoc** (_ $\odot^{SG}$ _)

$$
\begin{array}{|l}
\_ \odot^{SG} \_ : SGr \times (V \rightarrow V) \rightarrow SGr \\
\hline
\forall SG : SGr;\ s : V \rightarrow V \mid s \in NsP\, SG \rightarrow sg\_Ns\, SG \wedge s \in antireflexive \bullet \\
\quad SG \odot^{SG} s = (gr\, SG \odot s, (dom\, s) \lhd nty\, SG, ety\, SG, srcm\, SG, tgtm\, SG, derb\, SG)
\end{array}
$$

function 10 **leftassoc** (_ $\rightarrow_{SG}$ _)

$$\begin{array}{l} \_ \to_{SG} \_ : SGr \times SGr \to \mathbb{P}\ GrM \\ \hline \forall\ SG_s, SG_t : SGr \bullet \\ \quad SG_s \to_{SG} SG_t = \{fv : sg\_Ns\ SG_s \to sg\_Ns\ SG_t;\ fe : EsC\ SG_s \to EsC\ SG_t \mid \\ \qquad fv \circ src^*\ SG_s \subseteq src^*\ SG_t \circ fe \land fv \circ tgt^*\ SG_s \subseteq tgt^*\ SG_t \circ fe \\ \qquad \land\ fv \circ \preceq SG_s \subseteq \preceq SG_t \circ fv\} \end{array}$$

relation$(\_ \Rrightarrow^{SG} \_)$

$$\begin{array}{l} \_ \Rrightarrow^{SG} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\ \hline \forall\ SG_s, SG_t : SGr;\ m : GrM \bullet (SG_s, m) \Rrightarrow^{SG} SG_t \Leftrightarrow m \in SG_s \to_{SG} SG_t \end{array}$$

function 10 **leftassoc** $(\_ \to_{G2SG} \_)$

$$\begin{array}{l} \_ \to_{G2SG} \_ : Gr \times SGr \to \mathbb{P}\ GrM \\ \hline \forall\ G : Gr;\ SG : SGr \bullet G \to_{G2SG} SG = \{fv : Ns\ G \to sg\_Ns\ SG;\ fe : Es\ G \to EsC\ SG \mid \\ \quad fv \circ src\ G \subseteq src^*\ SG \circ fe \land fv \circ tgt\ G \subseteq tgt^*\ SG \circ fe\} \end{array}$$

relation$(\_ \Rrightarrow^{GwT} \_)$

$$\begin{array}{l} \_ \Rrightarrow^{GwT} \_ : (GrwT \leftrightarrow SGr) \\ \hline \forall\ GwT : GrwT;\ SG : SGr \bullet GwT \Rrightarrow^{GwT} SG \Leftrightarrow (ty\ GwT) \in (gOf\ GwT) \to_{G2SG} SG \end{array}$$

$$\begin{array}{l} totaliseForDer : GrM \times SGr \to GrM \\ \hline \forall\ m : GrM;\ SG : SGr \bullet totaliseForDer(m, SG) = (fV\ m, ((derb\ SG) \boxdot (EsC\ SG))\ ⨾\ fE\ m) \end{array}$$

$$\begin{array}{l} insOf : GrM \times SGr \times \mathbb{P}\ V \to \mathbb{P}\ V \\ iesOf : GrM \times \mathbb{P}\ E \to \mathbb{P}\ E \\ igRMEs : GrwT \times \mathbb{P}\ E \to Gr \\ igRMNsEs : GrwT \times SGr \times \mathbb{P}\ V \times \mathbb{P}\ E \to Gr \\ \hline \forall\ m : GrM;\ SG : SGr;\ mns : \mathbb{P}\ V \bullet insOf(m, SG, mns) = (fV\ m)^{\sim} (\!|(\prec SG)^{\sim} (\!| mns |\!) |\!) \\ \forall\ m : GrM;\ mes : \mathbb{P}\ E \bullet iesOf(m, mes) = (fE\ m)^{\sim} (\!| mes |\!) \\ \forall\ GwT : GrwT;\ mes : \mathbb{P}\ E \bullet igRMEs(GwT, mes) = (gOf\ GwT) \bowtie_{Es} iesOf((ty\ GwT), mes) \\ \forall\ GwT : GrwT;\ SG : SGr;\ mns : \mathbb{P}\ V;\ mes : \mathbb{P}\ E \bullet \\ \quad igRMNsEs(GwT, SG, mns, mes) = igRMEs(GwT, mes) \bowtie_{Ns} insOf(ty\ GwT, SG, mns) \end{array}$$

relation(inverted$_{\mathrm{E}}$_)

$$\begin{array}{l}
\mathrm{inverted}_{\mathrm{E}}_ : \mathbb{P}(GrwT \times SGr \times E) \\
gOfwei, igRMEsW : GrwT \times SGr \times E \to Gr \\
gOfweis : GrwT \times SGr \times \mathbb{P}\, E \to Gr \\
\hline
\forall\, G : Gr;\ m : GrM;\ SG : SGr;\ e : E \bullet \\
\quad \mathrm{inverted}_{\mathrm{E}}((G, m), SG, e) \Leftrightarrow ((sg_tgt\ SG) \circ (fE\ m))e = ((fV\ m) \circ (src\ G))e \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E \bullet \\
\quad gOfwei(GwT, SG, e) = \mathbf{if}\ \mathrm{inverted}_{\mathrm{E}}(GwT, SG, e)\ \mathbf{then}\ ((gOf\ GwT) \bowtie_{Es} \{e\})^{\rightleftarrows}\ \mathbf{else}(gOf\ GwT) \bowtie_{Es} \{e\} \\
\forall\, GwT : GrwT;\ SG : SGr \bullet gOfweis(GwT, SG, \{\}) = \varnothing_G \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E;\ es : \mathbb{P}\, E \bullet \\
\quad gOfweis(GwT, SG, \{e\} \cup es) = gOfwei(GwT, SG, e) \cup_G gOfweis(GwT, SG, es) \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E \mid e \in EsD\ SG \bullet igRMEsW(GwT, SG, e) = \\
\quad igRMNsEs(GwT, SG, \{(sg_src\ SG)(derb\ SG\ e), (sg_tgt\ SG)(derb\ SG\ e)\}, \{derb\ SG\ e\}) \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E \mid e \notin EsW\ SG \bullet igRMEsW(GwT, SG, e) = igRMEs(GwT, \{e\}) \\
\forall\, GwT : GrwT;\ SG : SGr;\ e : E \mid e \in EsW\ SG \bullet igRMEsW(GwT, SG, e) = \\
\quad gOfweis(GwT, SG, ((fE \circ ty)\ GwT)^{\sim} (\!| \{e\} |\!))
\end{array}$$

relation($\_ \sqsupseteq^{SG} \_$)
relation($\_ \sqsupseteq^{SG_0} \_$)
relation($\_ \sqsupseteq_{NT} \_$)
relation($\_ \sqsupseteq_{ET} \_$)
relation($\_ \sqsupseteq_{\mathcal{M}} \_$)

$$\begin{array}{l}
_ \sqsupseteq_{NT} _, _ \sqsupseteq_{ET} _ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq_{NT} SG_a \Leftrightarrow \forall\, n : sg_Ns\ SG_c \bullet (nty\ SG_c)\ n \leq_{rNT} ((nty\ SG_a) \circ (fV\ m))\ n \\
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq_{ET} SG_a \Leftrightarrow \forall\, e : EsC\ SG_c \bullet (ety\ SG_c)\ e \leq_{ET} ((ety\ SG_a) \circ (fE\ m))\ e
\end{array}$$

$$\begin{array}{l}
_ \sqsupseteq_{\mathcal{M}} _ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq_{\mathcal{M}} SG_a \Leftrightarrow \forall\, e : EsC\ SG_c \setminus EsD\ SG_c \bullet (srcma\ SG_c)\ e \leq_{\mathcal{M}} ((srcma\ SG_a) \circ (fE\ m))\ e \\
\qquad\qquad \wedge (tgtm\ SG_c)\ e \leq_{\mathcal{M}} ((tgtm\ SG_a) \circ (fE\ m))\ e
\end{array}$$

$$
\begin{array}{|l}
\_ \sqsupseteq^{SG} \_, \_ \sqsupseteq^{SG_0} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq^{SG_0}\ SG_a \Leftrightarrow (SG_c, m) \sqsupseteq_{NT}\ SG_a \wedge (SG_c, m) \sqsupseteq_{ET}\ SG_a \wedge (SG_c, m) \sqsupseteq_{\mathcal{M}}\ SG_a \\
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq^{SG}\ SG_a \Leftrightarrow \exists m' : GrM \mid m' = totaliseForDer(m, SG_c) \bullet \\
\qquad m' \in SG_c \to_{SG} SG_a \wedge (SG_c, m') \sqsupseteq^{SG_0}\ SG_a
\end{array}
$$

relation$(\_ \sqsupset^{SG} \_)$
relation$(\_ \sqsupset^{SG_0} \_)$
relation$(\_ \sqsupset_{AEs} \_)$
relation(_OkRefinedIn_)
relation$(\_ \sqsupset_{ANNs} \_)$

$$
\begin{array}{|l}
\_ \sqsupset_{ANNs} \_ : \mathbb{P}(GrM \times SGr) \\
\hline
\forall SG_a : SGr;\ m : GrM \bullet \\
\quad m \sqsupset_{ANNs} SG_a \Leftrightarrow \forall nn : NsTy\, SG_a\{nnrml\} \bullet (\preceq\ SG_a) \llparenthesis \{nn\} \rrparenthesis \cap \operatorname{ran}(fV\ m) = \varnothing
\end{array}
$$

$$
\begin{array}{|l}
\_\mathrm{OkRefinedIn}\_ : \mathbb{P}((SGr \times E) \times (SGr \times GrM)) \\
\_ \sqsupset_{AEs} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM;\ ae : E \bullet \\
\quad (SG_a, ae)\mathrm{OkRefinedIn}(SG_c, m) \Leftrightarrow \\
\qquad \exists r : V \leftrightarrow V;\ s, t : \mathbb{P}\ V \mid r = (\preceq\ SG_c) \mathbin{;} igRMEsW((gr\ SG_c, m), SG_a, ae)^{\leftrightarrow} \mathbin{;} (\preceq\ SG_c)^{\sim} \\
\qquad\quad \wedge\ s = insOf(m, SG_a, (sg\_src\ SG_a \llparenthesis \{ae\} \rrparenthesis)) \setminus ((NsEther\ SG_c) \setminus \operatorname{dom}\ r) \\
\qquad\quad \wedge\ t = insOf(m, SG_a, (sg\_tgt\ SG_a \llparenthesis \{ae\} \rrparenthesis)) \setminus ((NsEther\ SG_c) \setminus \operatorname{ran}\ r) \\
\qquad\qquad \bullet\ r \in s \leftrightarrow t \wedge r \neq \varnothing \\
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset_{AEs}\ SG_a \Leftrightarrow \forall e : (EsA\ SG_a) \bullet (SG_a, e)\mathrm{OkRefinedIn}(SG_c, m)
\end{array}
$$

$$
\begin{array}{|l}
\_ \sqsupset^{SG} \_, \_ \sqsupset^{SG_0} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset^{SG_0}\ SG_a \Leftrightarrow (SG_c, m) \sqsupseteq^{SG_0}\ SG_a \wedge m \sqsupset_{ANNs} SG_a \wedge (SG_c, m) \sqsupset_{AEs}\ SG_a \\
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset^{SG}\ SG_a \Leftrightarrow \exists m' : GrM \mid m' = totaliseForDer(m, SG_c) \bullet \\
\qquad m' \in SG_c \to_{SG} SG_a \wedge (SG_c, m') \sqsupset^{SG_0}\ SG_a
\end{array}
$$

relation$(\_ \Supset^{SG} \_)$
relation$(\_ \Supset_{\mathcal{M}} \_)$
relation$(\_ \Supset_{FI} \_)$
relation$(\_ \Supset_{PNS} \_)$
relation$(\_\ ME\mathcal{M}Ok\ \_)$

$$\begin{array}{|l}
_\ \mathit{MEMOk}\ _ : \mathbb{P}((\mathit{SGr} \times E) \times \mathit{GrwT}) \\
\hline
\forall\, \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr};\ \mathit{me} : E \bullet (\mathit{SG}, \mathit{me})\ \mathit{ME\mathcal{M}Ok}\ \mathit{GwT} \Leftrightarrow \\
\quad \exists\, r : V \leftrightarrow V;\ s, t : \mathbb{P}\, V \mid r = \mathit{igRMEsW}(\mathit{GwT}, \mathit{SG}, \mathit{me})^{\leftrightarrow} \\
\qquad \wedge\ s = \mathit{insOf}(\mathit{ty}\ \mathit{GwT}, \mathit{SG}, (\mathit{src}^*\ \mathit{SG}) (\!| \{\mathit{me}\} |\!)) \\
\qquad \wedge\ t = \mathit{insOf}(\mathit{ty}\ \mathit{GwT}, \mathit{SG}, (\mathit{tgt}^*\ \mathit{SG}) (\!| \{\mathit{me}\} |\!)) \\
\qquad\quad \bullet\ r\mathcal{M}\mathit{Ok}(r, s, t, \mathit{srcma}\ \mathit{SG}\ \mathit{me}, \mathit{tgtm}\ \mathit{SG}\ \mathit{me})
\end{array}$$

$$\begin{array}{|l}
_ \ni_{\mathcal{M}} _ : \mathit{GrwT} \leftrightarrow \mathit{SGr} \\
_ \ni_{\mathit{FI}} _ : \mathit{GrwT} \leftrightarrow \mathit{SGr} \\
_ \ni_{\mathit{PNS}} _ : \mathit{GrwT} \leftrightarrow \mathit{SGr} \\
\hline
\forall\, \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr} \bullet \mathit{GwT} \ni_{\mathcal{M}} \mathit{SG} \Leftrightarrow \forall\, \mathit{me} : \mathit{EsC}\ \mathit{SG} \bullet (\mathit{SG}, \mathit{me})\ \mathit{ME\mathcal{M}Ok}\ \mathit{GwT} \\
\forall\, \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr} \bullet \mathit{GwT} \ni_{\mathit{FI}} \mathit{SG} \Leftrightarrow ((\mathit{fV} \circ \mathit{ty})\, \mathit{GwT})^{\sim} (\!| \mathit{NsEther}\ \mathit{SG} |\!) = \varnothing \\
\forall\, \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr} \bullet \\
\quad \mathit{GwT} \ni_{\mathit{PNS}} \mathit{SG} \Leftrightarrow \mathit{igRMEs}(\mathit{GwT}, \mathit{EsTy}\ \mathit{SG}\ \{\mathit{ecomp}\ \mathit{dbi}, \mathit{ecomp}\ \mathit{duni}\})^{\leftrightarrow} \in \mathrm{injrel}
\end{array}$$

$$\begin{array}{|l}
_ \ni^{\mathit{SG}} _ : \mathit{GrwT} \leftrightarrow \mathit{SGr} \\
\hline
\forall\, \mathit{GwT} : \mathit{GrwT};\ \mathit{SG} : \mathit{SGr} \bullet \\
\quad \mathit{GwT} \ni^{\mathit{SG}} \mathit{SG} \Leftrightarrow \mathit{GwT} \Rrightarrow^{\mathit{GwT}} \mathit{SG} \wedge \mathit{GwT} \ni_{\mathcal{M}} \mathit{SG} \wedge \mathit{GwT} \ni_{\mathit{FI}} \mathit{SG} \wedge \mathit{GwT} \ni_{\mathit{PNS}} \mathit{SG}
\end{array}$$

7 Fragments

section *Fragmenta_Frs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_SGs*, *Fragmenta_GrswT*

$$\begin{array}{l}
\mathit{Fr}_0 == \{\mathit{SG} : \mathit{SGr};\ \mathit{esr} : \mathbb{P}\, E;\ \mathit{sr}, \mathit{tr} : E \nrightarrow V \mid \mathit{esr} \cap (\mathit{sg_Es}\ \mathit{SG}) = \varnothing \\
\quad \wedge\ \mathit{sr} \in \mathit{esr} \rightarrowtail\!\!\!\!\rightarrow (\mathit{NsP}\ \mathit{SG}) \wedge \mathit{tr} \in \mathit{esr} \rightarrow V\}
\end{array}$$

$$\begin{array}{|l}
\mathit{fSG} : \mathit{Fr}_0 \rightarrow \mathit{SGr} \\
\mathit{EsR} : \mathit{Fr}_0 \rightarrow \mathbb{P}\, E \\
\mathit{srcR}, \mathit{tgtR} : \mathit{Fr}_0 \rightarrow E \nrightarrow V \\
\hline
\forall\, \mathit{SG} : \mathit{SGr};\ \mathit{esr} : \mathbb{P}\, E;\ \mathit{sr}, \mathit{tr} : E \nrightarrow V \bullet \mathit{fSG}(\mathit{SG}, \mathit{esr}, \mathit{sr}, \mathit{tr}) = \mathit{SG} \\
\forall\, \mathit{SG} : \mathit{SGr};\ \mathit{esr} : \mathbb{P}\, E;\ \mathit{sr}, \mathit{tr} : E \nrightarrow V \bullet \mathit{EsR}(\mathit{SG}, \mathit{esr}, \mathit{sr}, \mathit{tr}) = \mathit{esr} \\
\forall\, \mathit{SG} : \mathit{SGr};\ \mathit{esr} : \mathbb{P}\, E;\ \mathit{sr}, \mathit{tr} : E \nrightarrow V \bullet \mathit{srcR}(\mathit{SG}, \mathit{esr}, \mathit{sr}, \mathit{tr}) = \mathit{sr} \\
\forall\, \mathit{SG} : \mathit{SGr};\ \mathit{esr} : \mathbb{P}\, E;\ \mathit{sr}, \mathit{tr} : E \nrightarrow V \bullet \mathit{tgtR}(\mathit{SG}, \mathit{esr}, \mathit{sr}, \mathit{tr}) = \mathit{tr}
\end{array}$$

$$
\begin{array}{|l}
fLEs, fEs, fEsC : Fr_0 \to \mathbb{P}\, E \\
fLNs, fRNs, fNs : Fr_0 \to \mathbb{P}\, V \\
srcF, tgtF : Fr_0 \to E \nrightarrow V \\
\hline
fLEs = (sg\_Es \circ fSG) \\
\forall F : Fr_0 \bullet fEs\, F = fLEs\, F \cup EsR\, F \\
fEsC = EsC \circ fSG \\
fLNs = sg\_Ns \circ fSG \\
fRNs = \operatorname{ran} \circ tgtR \\
\forall F : Fr_0 \bullet fNs\, F = fLNs\, F \cup fRNs\, F \\
\forall F : Fr_0 \bullet srcF\, F = (sg\_src \circ fSG)\, F \cup srcR\, F \\
\forall F : Fr_0 \bullet tgtF\, F = (sg\_tgt \circ fSG)\, F \cup tgtR\, F
\end{array}
$$

$$
\begin{array}{|l}
\overset{G}{\leftrightsquigarrow} : Fr_0 \to Gr \\
\leftrightsquigarrow : Fr_0 \to V \leftrightarrow V \\
\hline
\forall F : Fr_0 \bullet \overset{G}{\leftrightsquigarrow}\ F = ((NsP \circ fSG) F \cup fRNs\, F, EsR\, F, srcR\, F, tgtR\, F) \\
\forall F : Fr_0 \bullet \leftrightsquigarrow\ F = (\overset{G}{\leftrightsquigarrow}\ F)^{\leftrightarrow}
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_F \_)$

$$
\begin{array}{|l}
\varnothing_F : Fr_0 \\
\_ \cup_F \_ : Fr_0 \times Fr_0 \to Fr_0 \\
\bigcup_F : \mathbb{P}\, Fr_0 \to Fr_0 \\
\hline
\varnothing_F = (\varnothing_{SG}, \varnothing, \varnothing, \varnothing) \\
\forall F_1, F_2 : Fr_0 \bullet F_1 \cup_F F_2 = \\
\quad (fSG\, F_1 \cup_{SG} fSG\, F_2, EsR\, F_1 \cup EsR\, F_2, srcR\, F_1 \cup srcR\, F_2, tgtR\, F_1 \cup tgtR\, F_2) \\
\bigcup_F \{\} = \varnothing_F \\
\forall F : Fr_0;\ Fs : \mathbb{P}\, Fr_0 \bullet \bigcup_F (\{F\} \cup Fs) = F \cup_F (\bigcup_F Fs)
\end{array}
$$

$$
\begin{array}{|l}
\rightsquigarrow \,:\, Fr_0 \nrightarrow V \nrightarrow V \\
\circledbullet^{SG} : Fr_0 \nrightarrow SGr \\
rEsR : Fr_0 \nrightarrow \mathbb{P}\, E \\
\circledbullet : Fr_0 \nrightarrow Fr_0 \\
\hline
\forall\, F : Fr_0 \bullet \rightsquigarrow F = (\leftrightsquigarrow F) \rhd (fLNs\, F) \\
\forall\, F : Fr_0 \bullet \circledbullet^{SG} F = (fSG\, F) \odot^{SG} (\rightsquigarrow F) \\
\forall\, F : Fr_0 \bullet rEsR\, F = \mathrm{dom}((srcR\, F) \unrhd \mathrm{dom}(\rightsquigarrow F)) \\
\forall\, F : Fr_0 \bullet \circledbullet F = (\circledbullet^{SG} F, rEsR\, F, (rEsR\, F) \lhd (srcR\, F), (rEsR\, F) \lhd (tgtR\, F))
\end{array}
$$

$$Fr_a == \{F : Fr_0 \mid \otimes(\overset{G}{\leftrightsquigarrow} F)\}$$

$$Fr == \{F : Fr_a \mid \circledbullet^{SG} F \in SGr\}$$

relation(refsLocal_)

$$
\begin{array}{|l}
\mathrm{refsLocal\_} : \mathbb{P}\, Fr_0 \\
\hline
\forall\, F : Fr_0 \bullet \mathrm{refsLocal}\, F \Leftrightarrow fRNs\, F \subseteq fLNs\, F
\end{array}
$$

$$TFr == \{F : Fr_a \mid \mathrm{refsLocal}\, F \wedge \circledbullet^{SG} F \in TSGr\}$$

relation($\boxminus$ _)
relation($\boxplus$ _)

$$
\begin{array}{|l}
\boxminus\_ : Fr \leftrightarrow Fr \\
\hline
\forall\, F_1, F_2 : Fr \bullet \boxminus(F_1, F_2) \Leftrightarrow \boxminus_{SGs}(fSG\, F_1, fSG\, F_2) \wedge EsR\, F_1 \cap EsR\, F_2 = \varnothing
\end{array}
$$

$$
\begin{array}{|l|}
\hline
[I] \\
\hline
\boxplus\_ : \mathbb{P}(I \nrightarrow Fr) \\
\hline
\forall\, Fs : I \nrightarrow Fr \bullet \boxplus\, Fs \Leftrightarrow \forall\, i, j : \mathrm{dom}\, Fs \mid i \neq j \bullet \boxminus(Fs\, i, Fs\, j) \\
\hline
\end{array}
$$

relation(_ $\subseteq^{rs}$ _)
relation(_ $\mapsto$ _)

$$
\begin{array}{|l}
\_ \subseteq^{rs} \_ : Fr \leftrightarrow Fr \\
\_ \mapsto \_ : Fr \leftrightarrow Fr \\
\hline
\forall F_1, F_2 : Fr \bullet F_1 \subseteq^{rs} F_2 \Leftrightarrow \mathrm{ran}(tgtR\ F_1) \subseteq fLNs\ F_2 \\
\forall F_1, F_2 : Fr \bullet F_1 \mapsto F_2 \Leftrightarrow F_1 \subseteq^{rs} F_2 \land \neg\, (F_2 \subseteq^{rs} F_1)
\end{array}
$$

function 10 **leftassoc** $(\_ \rightrightarrows_{\bullet} \_)$

$$
\begin{array}{|l}
\_ \rightrightarrows_{\bullet} \_ : GrM \times (Fr \times Fr) \rightarrow GrM \\
\hline
\forall m : GrM;\ F_s, F_t : Fr_0 \bullet \\
\quad m \rightrightarrows_{\bullet} (F_s, F_t) = (((\rightsquigarrow F_s)^{\oplus} \boxdot (fLNs\ F_s))^{\sim} \fatsemi (fV\ m) \fatsemi ((\rightsquigarrow F_t)^{\oplus} \boxdot (fLNs\ F_t)), fE\ m)
\end{array}
$$

function 1 **leftassoc** $(\_ \rightarrow_F \_)$

$$
\begin{array}{|l}
\_ \rightarrow_F \_ : Fr \times Fr \rightarrow \mathbb{P}\ GrM \\
\hline
\forall F_s, F_t : Fr \bullet F_s \rightarrow_F F_t = \{ fv : fLNs\ F_s \rightarrow fLNs\ F_t;\ fe : fEsC\ F_s \rightarrow fEsC\ F_t \mid \\
\quad (\circledcirc^{SG} F_s, (fv, fe) \rightrightarrows_{\bullet} (F_s, F_t)) \Rrightarrow^{SG} \circledcirc^{SG} F_t \}
\end{array}
$$

relation $(\_ \Rrightarrow^{F} \_)$

$$
\begin{array}{|l}
\_ \Rrightarrow^{F} \_ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall m : GrM;\ F_s, F_t : Fr_0 \bullet (F_s, m) \Rrightarrow^{F} F_t \Leftrightarrow m \in F_s \rightarrow_F F_t
\end{array}
$$

relation $(\_ \sqsupseteq^{F} \_)$

$$
\begin{array}{|l}
\_ \sqsupseteq^{F} \_ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupseteq^{F} F_a \Leftrightarrow (F_c, m) \Rrightarrow^{F} F_a \\
\quad \land (\circledcirc^{SG} F_c, m \rightrightarrows_{\bullet} (F_c, F_a)) \sqsupseteq^{SG_0} (\circledcirc^{SG} F_a)
\end{array}
$$

relation $(\_ \sqsupset^{F} \_)$

$$
\begin{array}{|l}
\_ \sqsupset^{F} \_ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupset^{F} F_a \Leftrightarrow (F_c, m) \Rrightarrow^{F} F_a \\
\quad \wedge (\circledcirc^{SG} F_c, m \rightrightarrows_{\bullet} (F_c, F_a)) \sqsupset^{SG_0} (\circledcirc^{SG} F_a)
\end{array}
$$

relation$(\_ \Supset^{F} \_)$

$$
\begin{array}{|l}
\_ \Supset^{F} \_ : GrwT \leftrightarrow Fr \\
\hline
\forall GwT : GrwT;\ F : Fr \bullet GwT \Supset^{F} F \Leftrightarrow GwT \Supset^{SG} \circledcirc^{SG} F
\end{array}
$$

8 Global Fragment Graphs

section *Fragmenta_GFGs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$$GFGr == \{G : Gr \mid \oslash(G \bowtie_{Es} (Es\ G \setminus EsId\ G))\}$$

function$(\_^{\dashrightarrow})$

$$
\begin{array}{|l}
\_^{\dashrightarrow} : GFGr \rightarrow V \leftrightarrow V \\
\hline
\forall GFG : GFGr \bullet GFG^{\dashrightarrow} = (GFG^{\leftrightarrow})^{+}
\end{array}
$$

9 Models

section *Fragmenta_Mdls* **parents** *standard_toolkit*, *Fragmenta_Frs*, *Fragmenta_GFGs*

$$Mdl_0 == \{GFG : GFGr;\ fd : V \nrightarrow Fr \mid fd \in Ns\ GFG \rightarrow Fr \wedge \boxplus\ fd\}$$

$$
\begin{array}{|l}
mGFG : Mdl_0 \rightarrow GFGr \\
mFD : Mdl_0 \rightarrow V \nrightarrow Fr \\
\hline
\forall GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mGFG(GFG, fd) = GFG \\
\forall GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mFD(GFG, fd) = fd
\end{array}
$$

$$
\begin{array}{|l}
mUFs : Mdl_0 \rightarrow Fr \\
\hline
mUFs = \bigcup_F \circ \operatorname{ran} \circ mFD
\end{array}
$$

$$
\begin{array}{|l}
\text{from} : Mdl_0 \rightarrow V \nrightarrow V \\
\hline
\forall M : Mdl_0;\ v : V \bullet \text{from}\ M\ v = (\mu\, vf : (Ns \circ mGFG) M \mid v \in fLNs(mFD\ M\ vf))
\end{array}
$$

relation$(\Uparrow \_)$

$$
\begin{array}{|l}
\Uparrow \_ : \mathbb{P}\, Mdl_0 \\
\hline
\forall M : Mdl_0 \bullet \Uparrow M \Leftrightarrow \exists\, UF : Fr_0 \mid UF = mUFs\ M \bullet \\
\quad \forall p : (NsP \circ fSG)\, UF \bullet (from\ M\ p, from\ M\ (\leftrightsquigarrow\ UF\ p)) \in ((\_^{\dashrightarrow}) \circ mGFG) M
\end{array}
$$

$Mdl == \{M : Mdl_0 \mid (mUFs\ M) \in TFr \wedge \Uparrow M\}$

$$
\begin{array}{|l}
\circledbullet^M : Mdl \rightarrow Fr \\
\hline
\forall M : Mdl_0 \bullet \circledbullet^M = \circledbullet \circ mUFs
\end{array}
$$

function 1 **leftassoc** $(\_ \rightarrow_M \_)$
relation$(\_ \Rrightarrow^M \_)$

$$
\begin{array}{|l}
\_ \rightarrow_M \_ : Mdl \times Mdl \rightarrow \mathbb{P}\, GrM \\
\_ \Rrightarrow^M \_ : \mathbb{P}((Mdl \times \mathbb{P}\, GrM) \times Mdl) \\
\hline
\forall M_s, M_t : Mdl \bullet M_s \rightarrow_M M_t = \{m : GrM \mid \\
\quad \exists\, UF_s, UF_t : Fr_0 \mid UF_s = mUFs\ M_s \wedge UF_t = mUFs\ M_t \bullet m \in UF_s \rightarrow_F UF_t\} \\
\forall M_s, M_t : Mdl;\ ms : \mathbb{P}\, GrM \bullet (M_s, ms) \Rrightarrow^M M_t \Leftrightarrow \bigcup_{GM} ms \in M_s \rightarrow_M M_t
\end{array}
$$

relation$(\_ \sqsupset^M \_)$

$$
\begin{array}{|l}
\_ \sqsupset^M \_ : (Mdl \times \mathbb{P}\, GrM) \leftrightarrow Mdl \\
\hline
\forall M_c, M_a : Mdl_0;\ ms : \mathbb{P}\, GrM \bullet (M_c, ms) \sqsupset^M M_a \\
\quad \Leftrightarrow \exists\, UF_c, UF_a : Fr_0 \mid UF_c = mUFs\ M_c \wedge UF_a = mUFs\ M_a \bullet (UF_c, \bigcup_{GM} ms) \sqsupset^F UF_a
\end{array}
$$

relation$(\_ \Supset^M \_)$

$$
\begin{array}{|l}
\_ \Supset^M \_ : GrwT \leftrightarrow Mdl \\
\hline
\forall\, GwT : GrwT;\ M : Mdl \bullet GwT \Supset^M M \Leftrightarrow GwT \Supset^F mUFs\, M
\end{array}
$$