# Z Specification of Fragmenta

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## Contents

# 1 Generics

**section** *Fragmenta_Generics* **parents** *standard_toolkit*

$\mathrm{acyclic}[X] == \{r : X \leftrightarrow X \mid r^{+} \cap \mathrm{id}\, X = \varnothing\}$

$connected[X] == \{r : X \leftrightarrow X \mid \forall\, x : \mathrm{dom}\, r;\ y : \mathrm{ran}\, r \bullet x \mapsto y \in r^{+}\}$

$\mathrm{tree}[X] == \{r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \wedge r \in X \nrightarrow X\}$

$forest[X] == \{r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \wedge (\forall\, s : X \leftrightarrow X \mid s \subseteq r \wedge s \in connected \bullet s \in tree)\}$

$\mathrm{injrel}[X, Y] == \{r : X \leftrightarrow Y \mid r^{\sim} \in Y \nrightarrow X\}$

$\mathrm{antireflexive}[X] == \{r : X \leftrightarrow X \mid r \cap id(\mathrm{dom}\ r) = \varnothing\}$

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$[X, Y, Z]$

$\mathrm{flip} : (X \to Y \nrightarrow Z) \to (Y \to X \nrightarrow Z)$

---

$\forall\, f : X \to Y \nrightarrow Z \bullet \mathrm{flip}\, f = (\lambda\, y : Y \bullet \lambda\, x : X \bullet f\, x\, y)$

---

$[X, Y, Z, W]$

$\mathrm{apply} : (X \to Z) \to (Y \to W) \to (X \times Y) \to (Z \times W)$

---

$\forall\, f : X \to Z;\ g : Y \to W;\ x : X;\ y : Y \bullet \mathrm{apply}\, f\, g\, (x, y) = (f\, x, g\, y)$

---

$[X, Y]$

$\mathrm{map} : (X \to Y) \to \mathbb{P}\, X \to \mathbb{P}\, Y$

$\mathrm{mapS} : (X \to Y) \to \mathrm{seq}\, X \to \mathrm{seq}\, Y$

---

$\forall\, f : X \to Y \bullet \mathrm{map}\, f\{\} = \{\}$

$\forall\, f : X \to Y;\ x : X;\ xs : \mathbb{P}\, X \bullet \mathrm{map}\, f(\{x\} \cup xs) = \{f\, x\} \cup (\mathrm{map}\, f\, xs)$

$\forall\, f : X \to Y \bullet \mathrm{mapS}\, f\langle\rangle = \langle\rangle$

$\forall\, f : X \to Y;\ x : X;\ xs : \mathrm{seq}\, X \bullet \mathrm{mapS}\, f(\langle x \rangle \frown xs) = \langle f\, x \rangle \frown (\mathrm{mapS}\, f\, xs)$

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**function** 10 **leftassoc** ($\_ \,\boxdot\, \_$)

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$[X]$

$\_ \,\boxdot\, \_ : ((X \to X) \times \mathbb{P}\, X) \to (X \nrightarrow X)$

---

$\forall\, f : X \to X;\ s : \mathbb{P}\, X \bullet f \boxdot s = (\mathrm{id}\ s) \oplus f$

---

**function($\_^{\oplus}$)**

$$
\begin{array}{|l}
\hline\!\![X]\!\!\rule{8cm}{0pt} \\
\quad \_^{\oplus} : (X \leftrightarrow X) \to (X \leftrightarrow X) \\
\hline
\quad \forall\, r : X \leftrightarrow X \bullet r^{\oplus} = \mathbf{if}\ r \oplus r \,\fatsemi\, r = r\ \mathbf{then}\ r\ \mathbf{else}\ (r \oplus r \,\fatsemi\, r)^{\oplus} \\
\hline
\end{array}
$$

$\mathrm{opt}[X] == \{s : \mathbb{P}\,X \mid \#\,s \le 1\}$

$$
\begin{array}{|l}
\hline\!\![X]\!\!\rule{8cm}{0pt} \\
\quad \mathrm{the} : \mathrm{opt}[X] \twoheadrightarrow X \\
\hline
\quad \forall\, x : X \bullet \mathrm{the}\,\{x\} = x \\
\hline
\end{array}
$$

$$
\begin{array}{|l}
\hline\!\![X, Y]\!\!\rule{7cm}{0pt} \\
\quad \mathit{flatten} : (X \twoheadrightarrow \mathbb{P}\,Y) \to (X \leftrightarrow Y) \\
\hline
\quad \forall\, f : X \twoheadrightarrow \mathbb{P}\,Y \bullet \mathit{flatten}\, f = \{x : \mathrm{dom}\,f;\ y : Y \mid y \in f\,x\} \\
\hline
\end{array}
$$

# 2   Graphs

**section** *Fragmenta_Graphs* **parents** *standard_toolkit, Fragmenta_Generics*

$[V, E]$

$Gr == \{vs : \mathbb{P}\,V;\ es : \mathbb{P}\,E;\ s, t : E \twoheadrightarrow V \mid s \in es \to vs \land t \in es \to vs\}$

$$
\begin{array}{|l}
\hline
\quad Ns : Gr \to \mathbb{P}\,V \\
\quad Es : Gr \to \mathbb{P}\,E \\
\quad src, tgt : Gr \to E \twoheadrightarrow V \\
\hline
\quad \forall\, vs : \mathbb{P}\,V;\ es : \mathbb{P}\,E;\ s : E \twoheadrightarrow V;\ t : E \twoheadrightarrow V \bullet Ns(vs, es, s, t) = vs \\
\quad \forall\, vs : \mathbb{P}\,V;\ es : \mathbb{P}\,E;\ s : E \twoheadrightarrow V;\ t : E \twoheadrightarrow V \bullet Es(vs, es, s, t) = es \\
\quad \forall\, vs : \mathbb{P}\,V;\ es : \mathbb{P}\,E;\ s : E \twoheadrightarrow V;\ t : E \twoheadrightarrow V \bullet src(vs, es, s, t) = s \\
\quad \forall\, vs : \mathbb{P}\,V;\ es : \mathbb{P}\,E;\ s : E \twoheadrightarrow V;\ t : E \twoheadrightarrow V \bullet tgt(vs, es, s, t) = t \\
\hline
\end{array}
$$

$$\varnothing_G : Gr$$
$$\varnothing_G = (\varnothing, \varnothing, \varnothing, \varnothing)$$

$$EsId : Gr \to \mathbb{P}\, E$$
$$\forall\, G : Gr \bullet EsId\ G = \{\, e : Es\ G \mid src\ G\ e = tgt\ G\ e \,\}$$

**relation**(adjacent $_-$ )

$$\text{adjacent}_- : \mathbb{P}(Gr \times V \times V)$$
$$\forall\, G : Gr;\ v_1, v_2 : V \bullet \text{adjacent}(G, v_1, v_2) \Leftrightarrow \exists\, e : Es\ G \bullet src\ G\ e = v_1 \wedge tgt\ G\ e = v_2$$

**function** 10 **leftassoc** ($_- \circ\!\!\!-\!\!\!\multimap\; _-$)

$$_- \circ\!\!\!-\!\!\!\multimap\; _- : Gr \times \mathbb{P}\, V \to \mathbb{P}\, E$$
$$\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet G \circ\!\!\!-\!\!\!\multimap vs = (src\ G)^{\sim} (\!|vs|\!) \cup (tgt\ G)^{\sim} (\!|vs|\!)$$

**function** 10 **leftassoc** ($_- \bullet\!\!\!-\!\!\!\leftrightarrow\!\!\!\bullet\; _-$)

$$_- \bullet\!\!\!\leftrightarrow\!\!\!\bullet\; _- : Gr \times \mathbb{P}\, V \to \mathbb{P}\, E$$
$$\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet G \bullet\!\!\!\leftrightarrow\!\!\!\bullet vs = (src\ G)^{\sim} (\!|vs|\!) \cap (tgt\ G)^{\sim} (\!|vs|\!)$$

**function** 10 **leftassoc** ($_- \bowtie_{Es}\; _-$)

$$_- \bowtie_{Es}\; _- : Gr \times \mathbb{P}\, E \to Gr$$
$$\forall\, G : Gr;\ es : \mathbb{P}\, E \bullet G \bowtie_{Es} es = (Ns\ G, Es\ G \cap es, es \lhd src\ G, es \lhd tgt\ G)$$

**function** 10 **leftassoc** ($_- \bowtie_{Ns}\; _-$)

4

$$\frac{\_\bowtie_{Ns}\_: Gr \times \mathbb{P}\ V \to Gr}{\begin{array}{l}\forall\ G : Gr;\ vs : \mathbb{P}\ V \bullet \\ \quad G \bowtie_{Ns}\ vs = (Ns\ G \cap vs, G \bullet\!\leftrightarrow\!\bullet\ vs, (G \bullet\!\leftrightarrow\!\bullet\ vs) \lhd src\ G, (G \bullet\!\leftrightarrow\!\bullet\ vs) \lhd tgt\ G)\end{array}}$$

**function** $10$ **leftassoc** $(\_\ominus_{Ns}\_)$

$$\frac{\_\ominus_{Ns}\_: Gr \times \mathbb{P}\ V \to Gr}{\begin{array}{l}\forall\ G : Gr;\ vs : \mathbb{P}\ V \bullet \\ \quad G \ominus_{Ns}\ vs = (Ns\ G \setminus vs, Es\ G \setminus (G \circ\!\!\rightarrow\!\!\circ\ vs), (G \circ\!\!\rightarrow\!\!\circ\ vs) \lhd src\ G, (G \circ\!\!\rightarrow\!\!\circ\ vs) \lhd tgt\ G)\end{array}}$$

$$\frac{\text{successors} : V \times Gr \to \mathbb{P}\ V}{\forall\ v : V;\ G : Gr \bullet \text{successors}(v, G) = \{v_1 : Ns\ G \mid \text{adjacent}(G, v, v_1)\}}$$

**function**$(\_\rightleftarrows)$

$$\frac{\_\rightleftarrows : Gr \to Gr}{\forall\ G : Gr \bullet G^{\rightleftarrows} = (Ns\ G, Es\ G, tgt\ G, src\ G)}$$

**function**$(\_\leftrightarrow)$

$$\frac{\_\leftrightarrow : Gr \to V \leftrightarrow V}{\forall\ G : Gr \bullet G^{\leftrightarrow} = \{v_1, v_2 : Ns\ G \mid \text{adjacent}(G, v_1, v_2)\}}$$

**relation**$(\oslash\_)$

$$\frac{\oslash\_: \mathbb{P}\ Gr}{\forall\ G : Gr \bullet \oslash\ G \Leftrightarrow G^{\leftrightarrow} \in \text{acyclic}}$$

**relation**$(\boxminus_{Es}\_)$
**relation**$(\boxminus\_)$

$$\boxminus_{Es}\,\_,\boxminus\,\_ : \mathbb{P}(Gr \times Gr)$$
$$\forall\, G_1, G_2 : Gr \bullet \boxminus_{Es}(G_1, G_2) \Leftrightarrow Es\ G_1 \cap Es\ G_2 = \varnothing$$
$$\forall\, G_1, G_2 : Gr \bullet \boxminus(G_1, G_2) \Leftrightarrow Ns\ G_1 \cap Ns\ G_2 = \varnothing \wedge \boxminus_{Es}(G_1, G_2)$$

**relation**($\boxplus\,\_$)

$$=[I]=$$
$$\boxplus\,\_ : \mathbb{P}(I \nrightarrow Gr)$$
$$\forall\, Gs : I \nrightarrow Gr \bullet \boxplus\ Gs \Leftrightarrow \forall\, i, j : \mathrm{dom}\ Gs \mid i \neq j \bullet \boxminus(Gs\ i, Gs\ j)$$

**function** 10 **leftassoc** ($\_ \cup_G \_$)

$$\_ \cup_G \_ : Gr \times Gr \to Gr$$
$$\forall\, G_1, G_2 : Gr \bullet G_1 \cup_G G_2 = (Ns\ G_1 \cup Ns\ G_2, Es\ G_1 \cup Es\ G_2, src\ G_1 \cup src\ G_2, tgt\ G_1 \cup tgt\ G_2)$$

**function** 10 **leftassoc** ($\_ \odot \_$)

$$\_ \odot \_ : Gr \times (V \leftrightarrow V) \nrightarrow Gr$$
$$\forall\, G : Gr;\ s : V \leftrightarrow V \mid s \in Ns\ G \nrightarrow Ns\ G \wedge s \in \text{antireflexive} \bullet$$
$$G \odot s = (Ns\ G \setminus \mathrm{dom}\ s, Es\ G, (s \mathbin{\boxdot} Ns\ G) \circ (src\ G), (s \mathbin{\boxdot} Ns\ G) \circ (tgt\ G))$$

$$GrM == (V \nrightarrow V) \times (E \nrightarrow E)$$

$$fV : GrM \to V \nrightarrow V$$
$$fE : GrM \to E \nrightarrow E$$
$$\forall\, fv : V \nrightarrow V;\ fe : E \nrightarrow E \bullet fV(fv, fe) = fv$$
$$\forall\, fv : V \nrightarrow V;\ fe : E \nrightarrow E \bullet fE(fv, fe) = fe$$

$$\mathrm{gid} : Gr \to GrM$$
$$\forall\, G : Gr \bullet \mathrm{gid}\ G = (id\ (Ns\ G), id\ (Es\ G))$$

$$\varnothing_{GM} : GrM$$

$$\varnothing_{GM} = (\{\}, \{\})$$

---

$$\text{domg}, \text{codg} : GrM \to Gr$$

$$\forall\, m : GrM;\ G : Gr \bullet \text{domg}\, m = G \Leftrightarrow \text{dom}(fV\ m) = Ns\ G \wedge \text{dom}(fE\ m) = Es\ G$$

$$\forall\, m : GrM;\ G : Gr \bullet \text{codg}\, m = G \Leftrightarrow \text{ran}(fV\ m) \subseteq Ns\ G \wedge \text{ran}(fE\ m) \subseteq Es\ G$$

**function** 10 **leftassoc** $(\_ \cup_{GM} \_)$

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$$\_ \cup_{GM} \_ : GrM \times GrM \twoheadrightarrow GrM$$
$$\bigcup\nolimits_{GM} : \mathbb{P}\, GrM \to GrM$$

$$\forall\, f, g : GrM \bullet f \cup_{GM} g = (fV\ f \cup fV\ g, fE\ f \cup fE\ g)$$

$$\bigcup\nolimits_{GM} \varnothing = \varnothing_{GM}$$

$$\forall\, f : GrM;\ fs : \mathbb{P}\, GrM \bullet \bigcup\nolimits_{GM} (\{f\} \cup fs) = f \cup_{GM} \left( \bigcup\nolimits_{GM} fs \right)$$

**function** 10 **leftassoc** $(\_ \to_{G} \_)$

---

$$\_ \to_{G} \_ : Gr \times Gr \to \mathbb{P}\, GrM$$

$$\forall\, G_1, G_2 : Gr \bullet G_1 \to_{G} G_2 = \{ fv : Ns\ G_1 \to Ns\ G_2;\ fe : Es\ G_1 \to Es\ G_2 \mid$$
$$src\ G_2 \circ fe = fv \circ src\ G_1 \wedge tgt\ G_2 \circ fe = fv \circ tgt\ G_1 \}$$

**function** 10 **leftassoc** $(\_ \circ_{G} \_)$

---

$$\_ \circ_{G} \_ : GrM \times GrM \to GrM$$

$$\forall\, g, f : GrM \bullet g \circ_{G} f = (fV\ g \circ fV\ f, fE\ g \circ fE\ f)$$

# 3 Graphs with typing

**section** *Fragmenta_GrswT* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$$GrwT == \{ G : Gr;\ t : GrM \mid \text{domg}\, t = G \}$$

$$gOf : GrwT \rightarrow Gr$$
$$ty : GrwT \rightarrow GrM$$

$$\forall\, G : Gr;\ sm : V \nrightarrow \text{seq } V;\ t : GrM \bullet gOf(G, t) = G$$

$$\forall\, G : Gr;\ sm : V \nrightarrow \text{seq } V;\ t : GrM \bullet ty(G, t) = t$$

---

$$\varnothing_{GwT} : GrwT$$

$$\varnothing_{GwT} = (\varnothing_G, \varnothing_{GM})$$

---

**function** 10 **leftassoc** $(\_ \cup_{GwT} \_)$

$$\_ \cup_{GwT} \_ : GrwT \times GrwT \rightarrow GrwT$$

$$\forall\, G_1, G_2 : GrwT \bullet G_1 \cup_{GwT} G_2 = ((gOf\ G_1) \cup_G (gOf\ G_2), (ty\ G_1) \cup_{GM} (ty\ G_2))$$

# 4   SG Element Types

**section** *Fragmenta_SGElemTys* **parents** *standard_toolkit, Fragmenta_Generics*

$$SGNT ::= nnrml \mid nabst \mid nprxy \mid nenum \mid nval \mid nvirt \mid nopt$$
$$SGED ::= dbi \mid duni$$
$$SGET ::= einh \mid ecomp\langle\!\langle SGED \rangle\!\rangle \mid erel\langle\!\langle SGED \rangle\!\rangle \mid ewander \mid eder$$

**relation**$(\_ \prec_{NT} \_)$

$$\_ \prec_{NT} \_ : SGNT \leftrightarrow SGNT$$

$$\forall\, nt_1, nt_2 : SGNT \bullet nt_1 \prec_{NT} nt_2 \Leftrightarrow ((nt_2 = nenum \wedge nt_1 = nval)$$
$$\vee\, (nt_1 = nvirt \wedge nt_2 = nvirt) \vee (nt_1 = nabst \wedge nt_2 \in \{nabst, nvirt, nprxy\}))$$
$$\wedge\, nt_1 \notin \{nprxy, nenum, nopt\} \wedge nt_2 \notin \{nopt\}$$

**relation**$(\_ \leq_{rNT} \_)$

$$\begin{array}{|l}
\_ \leq_{rNT} \_ : SGNT \leftrightarrow SGNT \\
\hline
\forall\, nt_1, nt_2 : SGNT \bullet nt_1 \leq_{rNT} nt_2 \Leftrightarrow nt_1 = nt_2 \vee nt_1 = nprxy \\
\quad \vee\, nt_2 = nabst \wedge nt_1 \in \{nnrml, nvirt\} \vee nt_2 \in \{nnrml, nopt\}
\end{array}$$

**relation**$(\_ =_{ET} \_)$

$$\begin{array}{|l}
\_ =_{ET} \_ : SGET \leftrightarrow SGET \\
\hline
\forall\, et_1, et_2 : SGET \bullet et_1 =_{ET} et_2 \Leftrightarrow et_1 = et_2 \\
\quad \vee\, (\forall\, d_1, d_2 : SGED \bullet et_1 = erel\; d_1 \wedge et_2 = erel\; d_2 \vee et_1 = ecomp\; d_1 \wedge et_2 = ecomp\; d_2)
\end{array}$$

**relation**$(\_ \leq_{ET} \_)$

$$\begin{array}{|l}
\_ \leq_{ET} \_ : SGET \leftrightarrow SGET \\
\hline
\forall\, et_1, et_2 : SGET \bullet et_1 \leq_{ET} et_2 \Leftrightarrow einh \notin \{et_1, et_2\} \\
\quad \wedge\, (et_1 =_{ET} et_2 \vee et_2 = ewander \\
\quad \vee\, et_1 = eder \wedge et_2 \in \mathrm{dom}(erel\,^{\sim}) \cup \mathrm{dom}(ecomp\,^{\sim}))
\end{array}$$

# 5  Multiplicities

**section** *Fragmenta_Mult* **parents** *standard_toolkit, Fragmenta_Generics, Fragmenta_SGElemTys*

$MultVal ::= \mathbf{v}\langle\!\langle \mathbb{N} \rangle\!\rangle \mid *$

$MultC ::= mr\langle\!\langle \mathbb{N} \times MultVal \rangle\!\rangle \mid ms\langle\!\langle MultVal \rangle\!\rangle$

**relation**$(\_ \leq_{mv} \_)$

$$\begin{array}{|l}
\_ \leq_{mv} \_ : MultVal \leftrightarrow MultVal \\
\hline
\forall\, m_1, m_2 : MultVal \bullet m_1 \leq_{mv} m_2 \Leftrightarrow m_2 = * \vee \exists\, j, k : \mathbb{N} \mid m_1 = \mathbf{v}\; j \wedge m_2 = \mathbf{v}\; k \bullet j \leq k
\end{array}$$

$Mult == \{mc : MultC \mid \exists\, lb : \mathbb{N};\ ub : MultVal \bullet mc = mr(lb, ub) \wedge \mathbf{v}\ lb \leq_{mv} ub$
$\qquad \vee\ \exists\, mv : MultVal \bullet mc = ms\ mv\}$

$MultMany == \{ms *, mr(0, *)\}$

$MultRange == \{m : MultC \mid \exists\, k : \mathbb{N} \mid k > 1 \bullet m = ms\,(\mathbf{v}\ k)$
$\qquad \vee\ \exists\, lb : \mathbb{N};\ umv : MultVal \mid \mathbf{v}\ 2 \leq_{mv} umv \bullet m = mr(lb, umv)\}$

**relation**$(\_ \between \_)$

$$
\begin{array}{|l}
\_ \between \_ : \mathbb{P}(\mathbb{N} \times (MultVal \times MultVal)) \\
\hline
\forall\, k : \mathbb{N};\ lb, ub : MultVal \bullet k \between (lb, ub) \Leftrightarrow lb \leq_{mv} \mathbf{v}\ k \wedge \mathbf{v}\ k \leq_{mv} ub
\end{array}
$$

$$
\begin{array}{|l}
mlb, mub : MultC \nrightarrow MultVal \\
\hline
mlb(ms *) = \mathbf{v}\ 0 \\
\forall\, k : \mathbb{N} \bullet mlb(ms(\mathbf{v}\ k)) = \mathbf{v}\ k \\
\forall\, k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ k \\
\forall\, mv : MultVal \bullet mub(ms\ mv) = mv \\
\forall\, k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ m
\end{array}
$$

**relation**$(\_ \leq_{\mathcal{M}} \_)$

$$
\begin{array}{|l}
\_ \leq_{\mathcal{M}} \_ : MultC \leftrightarrow MultC \\
\hline
\forall\, m_1, m_2 : MultC \bullet m_1 \leq_{\mathcal{M}} m_2 \Leftrightarrow mlb\ m_2 \leq_{mv} mlb\ m_1 \wedge mub\ m_1 \leq_{mv} mub\ m_2
\end{array}
$$

**relation**$(\_ \propto \_)$

$$
\begin{array}{|l}
\_ \propto \_ : \mathbb{P}(SGET \times (MultC \times MultC)) \\
\hline
\forall\, et : SGET;\ m_1, m_2 : MultC \bullet et \propto (m_1, m_2) \Leftrightarrow et = erel\ dbi \vee et = eder \\
\quad \vee\ et = ecomp\ duni \wedge m_1 = ms(\mathbf{v}\ 1) \vee et = erel\ duni \wedge m_1 \in MultMany \\
\quad \vee\ et = ecomp\ dbi \wedge m_1 \in \{ms(\mathbf{v}\ 1), mr(0, \mathbf{v}\ 1)\} \\
\quad \vee\ et = ewander \wedge (m_1, m_2) \in MultMany \times MultMany
\end{array}
$$

**relation**(rbounded_)

$\boxed{\begin{array}{l} [X, Y] \\ \hline \text{rbounded}\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\,X \times \mathit{MultC}) \\ \hline \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ m : \mathit{MultC} \bullet \\ \quad \text{rbounded}(r, s, m) \Leftrightarrow \forall\, x : s \bullet \#(r \,(\!|\, \{x\} \,|\!)) \between (\mathit{mlb}\ m, \mathit{mub}\ m) \end{array}}$

**relation**($r\mathcal{M}Ok$_)

$\boxed{\begin{array}{l} [X, Y] \\ \hline r\mathcal{M}Ok\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\,X \times \mathbb{P}\,Y \times \mathit{MultC} \times \mathit{MultC}) \\ \hline \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet \\ \quad r\mathcal{M}Ok(r, s, t, ms(\mathbf{v}\ 1), ms(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail t \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet \\ \quad r\mathcal{M}Ok(r, s, t, mr(0, \mathbf{v}\ 1), mr(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail\!\!\!\rightarrow t \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet \\ \quad r\mathcal{M}Ok(r, s, t, mr(0, \mathbf{v}\ 1), ms(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail t \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet \\ \quad r\mathcal{M}Ok(r, s, t, ms(\mathbf{v}\ 1), mr(0, \mathbf{v}\ 1)) \Leftrightarrow r^{\sim} \in t \rightarrowtail s \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : \mathit{MultMany} \bullet \\ \quad r\mathcal{M}Ok(r, s, t, mm, ms(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrow t \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : \mathit{MultMany} \bullet \\ \quad r\mathcal{M}Ok(r, s, t, ms(\mathbf{v}\ 1), mm) \Leftrightarrow r^{\sim} \in t \rightarrow s \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm_1, mm_2 : \mathit{MultMany} \bullet \\ \quad r\mathcal{M}Ok(r, s, t, mm_1, mm_2) \Leftrightarrow r \in s \leftrightarrow t \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : \mathit{MultMany};\ mr : \mathit{MultRange} \bullet \\ \quad r\mathcal{M}Ok(r, s, t, mm, mr) \Leftrightarrow r \in s \leftrightarrow t \wedge \text{rbounded}(r, s, mr) \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : \mathit{MultMany};\ mr : \mathit{MultRange} \bullet \\ \quad r\mathcal{M}Ok(r, s, t, mr, mm) \Leftrightarrow r \in s \leftrightarrow t \wedge \text{rbounded}(r^{\sim}, t, mr) \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : \mathit{MultMany} \bullet \\ \quad r\mathcal{M}Ok(r, s, t, mm, mr(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrow\!\!\!\rightarrow t \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : \mathit{MultMany} \bullet \\ \quad r\mathcal{M}Ok(r, s, t, mr(0, \mathbf{v}\ 1), mm) \Leftrightarrow r^{\sim} \in t \rightarrow\!\!\!\rightarrow s \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mr_1, mr_2 : \mathit{MultRange} \bullet \\ \quad r\mathcal{M}Ok(r, s, t, mr_1, mr_2) \Leftrightarrow r \in s \leftrightarrow t \wedge \text{rbounded}(r, s, mr_2) \wedge \text{rbounded}(r^{\sim}, t, mr_1) \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mr : \mathit{MultRange} \bullet \\ \quad r\mathcal{M}Ok(r, s, t, mr, ms(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrow t \wedge \text{rbounded}(r^{\sim}, t, mr) \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mr : \mathit{MultRange} \bullet \\ \quad r\mathcal{M}Ok(r, s, t, ms(\mathbf{v}\ 1), mr) \Leftrightarrow r^{\sim} \in t \rightarrow s \wedge \text{rbounded}(r, s, mr) \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : \mathit{MultRange} \bullet \\ \quad r\mathcal{M}Ok(r, s, t, m, mr(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrow\!\!\!\rightarrow t \wedge \text{rbounded}(r^{\sim}, t, m) \\ \forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : \mathit{MultRange} \bullet \\ \quad r\mathcal{M}Ok(r, s, t, mr(0, \mathbf{v}\ 1), m) \Leftrightarrow r^{\sim} \in t \rightarrow\!\!\!\rightarrow s \wedge \text{rbounded}(r, s, m) \end{array}}$

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# 6 Structural Graphs

**section** *Fragmenta_SGs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*, *Fragmenta_SGElemTys*, *Fragmenta_Mult*, *Fragmenta_GrswT*

$SGr_0 == \{ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ db : E \nrightarrow E\ |$
  $nt \in Ns\ G \rightarrow SGNT \wedge et \in Es\ G \rightarrow SGET \}$

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$gr : SGr_0 \rightarrow Gr$
$sg\_Ns : SGr_0 \rightarrow \mathbb{P}\ V$
$sg\_Es : SGr_0 \rightarrow \mathbb{P}\ E$
$sg\_src, sg\_tgt : SGr_0 \rightarrow E \nrightarrow V$
$nty : SGr_0 \rightarrow V \nrightarrow SGNT$
$ety : SGr_0 \rightarrow E \nrightarrow SGET$
$srcm, tgtm : SGr_0 \rightarrow E \nrightarrow Mult$
$derb : SGr_0 \rightarrow E \nrightarrow E$

---

$\forall\ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ db : E \nrightarrow E\ \bullet$
  $gr\ (G, nt, et, sm, tm, db) = G$
$sg\_Ns = Ns \circ gr$
$sg\_Es = Es \circ gr$
$sg\_src = src \circ gr$
$sg\_tgt = tgt \circ gr$
$\forall\ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ db : E \nrightarrow E\ \bullet$
  $nty(G, nt, et, sm, tm, db) = nt$
$\forall\ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ db : E \nrightarrow E\ \bullet$
  $ety(G, nt, et, sm, tm, db) = et$
$\forall\ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ db : E \nrightarrow E\ \bullet$
  $srcm(G, nt, et, sm, tm, db) = sm$
$\forall\ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ db : E \nrightarrow E\ \bullet$
  $tgtm(G, nt, et, sm, tm, db) = tm$
$\forall\ G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ db : E \nrightarrow E\ \bullet$
  $derb(G, nt, et, sm, tm, db) = db$

---

$\varnothing_{SG} : SGr_0$

---

$\varnothing_{SG} = (\varnothing_G, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing)$

---

$NsTy : SGr_0 \rightarrow \mathbb{P}\ SGNT \rightarrow \mathbb{P}\ V$
$EsTy : SGr_0 \rightarrow \mathbb{P}\ SGET \rightarrow \mathbb{P}\ E$

---

$\forall\ SG : SGr_0;\ nts : \mathbb{P}\ SGNT \bullet NsTy\ SG\ nts = (nty\ SG)^{\sim} (\!| nts |\!)$

$\forall\ SG : SGr_0;\ ets : \mathbb{P}\ SGET \bullet EsTy\ SG\ ets = (ety\ SG)^{\sim} (\!| ets |\!)$

$EsA, EsW, EsI, EsC, EsD : SGr_0 \rightarrow \mathbb{P} E$

$\forall SG : SGr_0 \bullet EsA\ SG = EsTy\ SG\ (erel\ (\!|\ SGED\ |\!) \cup ecomp\ (\!|\ SGED\ |\!))$

$EsW = (\text{flip } EsTy)\ \{ewander\}$

$EsI = (\text{flip } EsTy)\ \{einh\}$

$EsD = (\text{flip } EsTy)\ \{eder\}$

$\forall SG : SGr_0 \bullet EsC\ SG = EsA\ SG \cup EsW\ SG \cup EsD\ SG$

---

$NsP, NsEther, NsO, NsV : SGr_0 \rightarrow \mathbb{P} V$

$NsP = (\text{flip } NsTy)\ \{nprxy\}$

$NsEther = (\text{flip } NsTy)\ \{nabst, nvirt, nenum\}$

$NsO = (\text{flip } NsTy)\ \{nopt\}$

$NsV = (\text{flip } NsTy)\ \{nvirt\}$

---

$\Cap : SGr_0 \rightarrow Gr$
$\prec : SGr_0 \rightarrow V \leftrightarrow V$

$\forall SG : SGr_0 \bullet \Cap\ SG = gr\ SG \bowtie_{Es} EsI\ SG$

$\prec = (\_\ ^{\leftrightarrow}) \circ \Cap$

---

$srcma : SGr_0 \rightarrow (E \nrightarrow Mult)$

$\forall SG : SGr_0 \bullet srcma\ SG =$
$\quad (srcm\ SG) \oplus (EsTy\ SG\ \{ecomp\ duni\} \times \{ms(\mathbf{v}\ 1)\}) \oplus (EsTy\ SG\ \{erel\ duni\} \times \{ms *\})$

---

**relation**($\mathcal{M}etysOk\ \_$)

---

$\mathcal{M}etysOk\ \_ : \mathbb{P}\ SGr_0$

$\forall SG : SGr_0 \bullet \mathcal{M}etysOk\ SG \Leftrightarrow \forall e : EsC\ SG \bullet (ety\ SG\ e) \propto (srcma\ SG\ e, tgtm\ SG\ e)$

---

$\preceq : SGr_0 \rightarrow V \leftrightarrow V$

$\forall SG : SGr_0 \bullet \preceq SG = (\prec SG)^*$

$$srcr, tgtr : SGr_0 \to E \leftrightarrow V$$
$$src_0^*, src^*, tgt_0^*, tgt^* : SGr_0 \to E \leftrightarrow V$$

$$\forall\, SG : SGr_0 \bullet srcr\; SG = sg\_src\; SG \cup (EsW\; SG \lhd sg\_tgt\; SG)$$

$$\forall\, SG : SGr_0 \bullet tgtr\; SG = sg\_tgt\; SG \cup (EsW\; SG \lhd sg\_src\; SG)$$

$$\forall\, SG : SGr_0 \bullet src_0^*\; SG = EsC\; SG \lhd (srcr\; SG)$$

$$\forall\, SG : SGr_0 \bullet src^*\; SG = (src_0^*\; SG) \,\mathbin{\raise0.3ex\hbox{$\scriptstyle\circ$}}\, (\preceq SG)^{\sim}$$

$$\forall\, SG : SGr_0 \bullet tgt_0^*\; SG = EsC\; SG \lhd (tgtr\; SG)$$

$$\forall\, SG : SGr_0 \bullet tgt^*\; SG = (tgt_0^*\; SG) \,\mathbin{\raise0.3ex\hbox{$\scriptstyle\circ$}}\, (\preceq SG)^{\sim}$$

**function** 10 **leftassoc** $(\_ \mathbin{\circ\!\!\rightarrow\!\!\infty}^* \_)$

$$\_ \mathbin{\circ\!\!\rightarrow\!\!\infty}^* \_ : SGr_0 \times \mathbb{P}\, V \to \mathbb{P}\, E$$

$$\forall\, SG : SGr_0;\; vs : \mathbb{P}\, V \bullet SG \mathbin{\circ\!\!\rightarrow\!\!\infty}^* vs = (src^*\; SG)^{\sim} \,(\!| vs |\!)\, \cup (tgt^*\; SG)^{\sim} \,(\!| vs |\!)$$

**relation**(optsVoluntary$\_$)

$$\text{optsVoluntary}\_ : \mathbb{P}\, SGr_0$$

$$\forall\, SG : SGr_0 \bullet$$
$$\text{optsVoluntary}\; SG \Leftrightarrow (ety\; SG) \,(\!| (SG \mathbin{\circ\!\!\rightarrow\!\!\infty}^* (NsO\; SG)) \setminus (EsI\; SG) |\!)\, \subseteq \{ewander\}$$

**relation**(inhOk$\_$)

$$\text{inhOk}\_ : \mathbb{P}\, SGr_0$$

$$\forall\, SG : SGr_0 \bullet \text{inhOk}\; SG$$
$$\Leftrightarrow (\forall\, v, v' : sg\_Ns\; SG \mid (v, v') \in (\prec SG) \bullet nty\; SG\; v \prec_{NT} nty\; SG\; v')$$
$$\land \oslash(\pitchfork SG) \land ((\pitchfork SG) \ominus_{Ns} (NsV\; SG))^{\leftrightarrow} \in sg\_Ns\; SG \rightarrowtail sg\_Ns\; SG$$

$$SGr == \{SG : SGr_0 \mid \{srcma\; SG, tgtm\; SG\} \subseteq EsC\; SG \to Mult \land \mathrm{dom}(derb\; SG) = EsD\; SG$$
$$\land\; \mathcal{M}etysOk\; SG \land \text{optsVoluntary}\; SG \land \text{inhOk}\; SG\}$$

**relation**(etherealAreInherited$\_$)

$$\begin{array}{|l}
\text{etherealAreInherited}\_ : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \text{etherealAreInherited}\ SG \Leftrightarrow \textit{NsEther}\ SG \subseteq \text{ran}(\prec\ SG)
\end{array}$$

**relation**(derivedOk $\_$)

$$\begin{array}{|l}
\text{derivedOk}\_ : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \text{derivedOk}\ SG \Leftrightarrow \text{ran}(\textit{derb}\ SG) \subseteq \textit{EsA}\ SG \\
\quad \wedge\ (\forall\, e : \textit{EsD}\ SG \bullet (\textit{sg\_src}\ SG\ e, ((\textit{sg\_src}\ SG) \circ (\textit{derb}\ SG))\ e) \in (\preceq\ SG) \\
\qquad \wedge\ (\textit{sg\_tgt}\ SG\ e, ((\textit{sg\_tgt}\ SG) \circ (\textit{derb}\ SG))\ e) \in (\preceq\ SG))
\end{array}$$

$TSGr == \{\, SG : SGr \mid \text{etherealAreInherited}\ SG \wedge \text{derivedOk}\ SG \,\}$

**relation**($\boxminus_{SGs}\,\_$)

$$\begin{array}{|l}
\boxminus_{SGs-} : \mathbb{P}(SGr \times SGr) \\
\hline
\forall\, SG_1, SG_2 : SGr \bullet \boxminus_{SGs}(SG_1, SG_2) \Leftrightarrow \boxminus(gr\ SG_1, gr\ SG_2)
\end{array}$$

**function** 10 **leftassoc** ($\_ \cup_{SG}\, \_$)

$$\begin{array}{|l}
\_ \cup_{SG}\, \_ : SGr \times SGr \nrightarrow SGr \\
\hline
\forall\, SG_1, SG_2 : SGr \bullet SG_1 \cup_{SG} SG_2 = (gr\ SG_1 \cup_G gr\ SG_2,\ nty\ SG_1 \cup nty\ SG_2, \\
\quad ety\ SG_1 \cup ety\ SG_2,\ srcm\ SG_1 \cup srcm\ SG_2,\ tgtm\ SG_1 \cup tgtm\ SG_2,\ derb\ SG_1 \cup derb\ SG_2)
\end{array}$$

**function** 10 **leftassoc** ($\_ \odot^{SG}\, \_$)

$$\begin{array}{|l}
\_ \odot^{SG}\, \_ : SGr \times (V \nrightarrow V) \nrightarrow SGr \\
\hline
\forall\, SG : SGr;\ s : V \nrightarrow V \mid s \in NsP\ SG \nrightarrow sg\_Ns\ SG \wedge s \in \textit{antireflexive} \bullet \\
\quad SG \odot^{SG}\ s = (gr\ SG \odot s, (\text{dom}\ s \setminus \text{ran}\ s) \lhd nty\ SG,\ ety\ SG,\ srcm\ SG,\ tgtm\ SG,\ derb\ SG)
\end{array}$$

**function** 10 **leftassoc** ($\_ \rightarrow_{SG}\, \_$)

$$\_\rightarrow_{SG}\_ : SGr \times SGr \rightarrow \mathbb{P}\ GrM$$

$\forall\ SG_s, SG_t : SGr \bullet$
$\quad SG_s \rightarrow_{SG} SG_t = \{fv : sg\_Ns\ SG_s \rightarrow sg\_Ns\ SG_t;\ fe : EsC\ SG_s \rightarrow EsC\ SG_t\ |$
$\qquad fv \circ src^*\ SG_s \subseteq src^*\ SG_t \circ fe \wedge fv \circ tgt^*\ SG_s \subseteq tgt^*\ SG_t \circ fe$
$\qquad \wedge\ fv \circ \preceq SG_s \subseteq\ \preceq SG_t \circ fv\}$

**relation**$(\_\Rrightarrow^{SG}\_)$

$$\_\Rrightarrow^{SG}\_ : \mathbb{P}((SGr \times GrM) \times SGr)$$

$\forall\ SG_s, SG_t : SGr;\ m : GrM \bullet (SG_s, m) \Rrightarrow^{SG} SG_t \Leftrightarrow m \in SG_s \rightarrow_{SG} SG_t$

**function** $10$ **leftassoc** $(\_\rightarrow_{G2SG}\_)$

$$\_\rightarrow_{G2SG}\_ : Gr \times SGr \rightarrow \mathbb{P}\ GrM$$

$\forall\ G : Gr;\ SG : SGr \bullet G \rightarrow_{G2SG} SG = \{fv : Ns\ G \rightarrow sg\_Ns\ SG;\ fe : Es\ G \rightarrow EsC\ SG\ |$
$\quad fv \circ src\ G \subseteq src^*\ SG \circ fe \wedge fv \circ tgt\ G \subseteq tgt^*\ SG \circ fe\}$

**relation**$(\_\Rrightarrow^{GwT}\_)$

$$\_\Rrightarrow^{GwT}\_ : (GrwT \leftrightarrow SGr)$$

$\forall\ GwT : GrwT;\ SG : SGr \bullet GwT \Rrightarrow^{GwT} SG \Leftrightarrow (ty\ GwT) \in (gOf\ GwT) \rightarrow_{G2SG} SG$

$$totaliseForDer : GrM \times SGr \rightarrow GrM$$

$\forall\ m : GrM;\ SG : SGr \bullet totaliseForDer(m, SG) = (fV\ m, ((derb\ SG) \boxdot (EsC\ SG)) \mathbin{\mathring{,}} fE\ m)$

$$insOf : GrM \times SGr \times \mathbb{P}\ V \rightarrow \mathbb{P}\ V$$
$$iesOf : GrM \times \mathbb{P}\ E \rightarrow \mathbb{P}\ E$$
$$igRMEs : GrwT \times \mathbb{P}\ E \rightarrow Gr$$
$$igRMNsEs : GrwT \times SGr \times \mathbb{P}\ V \times \mathbb{P}\ E \rightarrow Gr$$

$\forall\ m : GrM;\ SG : SGr;\ mns : \mathbb{P}\ V \bullet insOf(m, SG, mns) = (fV\ m)^\sim (\!|(\prec SG)^\sim (\!|mns|\!)|\!)$

$\forall\ m : GrM;\ mes : \mathbb{P}\ E \bullet iesOf(m, mes) = (fE\ m)^\sim (\!|mes|\!)$

$\forall\ GwT : GrwT;\ mes : \mathbb{P}\ E \bullet igRMEs(GwT, mes) = (gOf\ GwT) \bowtie_{Es} iesOf((ty\ GwT), mes)$

$\forall\ GwT : GrwT;\ SG : SGr;\ mns : \mathbb{P}\ V;\ mes : \mathbb{P}\ E \bullet$
$\quad igRMNsEs(GwT, SG, mns, mes) = igRMEs(GwT, mes) \bowtie_{Ns} insOf(ty\ GwT, SG, mns)$

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**relation**(inverted$_{E-}$)

---

inverted$_{E-}$ : $\mathbb{P}(GrwT \times SGr \times E)$
$gOfwei, igRMEsW : GrwT \times SGr \times E \to Gr$
$gOfweis : GrwT \times SGr \times \mathbb{P}\,E \to Gr$

---

$\forall\, G : Gr;\ m : GrM;\ SG : SGr;\ e : E \bullet$
  $\text{inverted}_{E}((G, m), SG, e) \Leftrightarrow ((sg\_tgt\ SG) \circ (fE\ m))e = ((fV\ m) \circ (src\ G))e$

$\forall\, GwT : GrwT;\ SG : SGr;\ e : E \bullet$
  $gOfwei(GwT, SG, e) = \textbf{if}\ \text{inverted}_{E}(GwT, SG, e)\ \textbf{then}\ ((gOf\ GwT) \bowtie_{Es} \{e\})^{\rightleftarrows} \textbf{else}(gOf\ GwT) \bowtie_{Es} \{e\}$

$\forall\, GwT : GrwT;\ SG : SGr \bullet gOfweis(GwT, SG, \{\}) = \varnothing_G$

$\forall\, GwT : GrwT;\ SG : SGr;\ e : E;\ es : \mathbb{P}\,E \bullet$
  $gOfweis(GwT, SG, \{e\} \cup es) = gOfwei(GwT, SG, e) \cup_G gOfweis(GwT, SG, es)$

$\forall\, GwT : GrwT;\ SG : SGr;\ e : E \mid e \in EsD\ SG \bullet igRMEsW(GwT, SG, e) =$
  $igRMNsEs(GwT, SG, \{(sg\_src\ SG)(derb\ SG\ e), (sg\_tgt\ SG)(derb\ SG\ e)\}, \{derb\ SG\ e\})$

$\forall\, GwT : GrwT;\ SG : SGr;\ e : E \mid e \notin EsW\ SG \bullet igRMEsW(GwT, SG, e) = igRMEs(GwT, \{e\})$

$\forall\, GwT : GrwT;\ SG : SGr;\ e : E \mid e \in EsW\ SG \bullet igRMEsW(GwT, SG, e) =$
  $gOfweis(GwT, SG, ((fE \circ ty)\ GwT)^{\sim}(\!(\{e\})\!))$

---

**relation**($_- \sqsupseteq^{SG} {_-}$)
**relation**($_- \sqsupseteq^{SG_0} {_-}$)
**relation**($_- \sqsupseteq_{NT} {_-}$)
**relation**($_- \sqsupseteq_{ET} {_-}$)
**relation**($_- \sqsupseteq_{\mathcal{M}} {_-}$)

---

$_- \sqsupseteq_{NT} {_-}, _- \sqsupseteq_{ET} {_-} : \mathbb{P}((SGr \times GrM) \times SGr)$

---

$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet$
  $(SG_c, m) \sqsupseteq_{NT}\ SG_a \Leftrightarrow \forall\, n : sg\_Ns\ SG_c \bullet (nty\ SG_c)\ n \leq_{rNT} ((nty\ SG_a) \circ (fV\ m))\ n$

$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet$
  $(SG_c, m) \sqsupseteq_{ET}\ SG_a \Leftrightarrow \forall\, e : EsC\ SG_c \bullet (ety\ SG_c)\ e \leq_{ET} ((ety\ SG_a) \circ (fE\ m))\ e$

---

$_- \sqsupseteq_{\mathcal{M}} {_-} : \mathbb{P}((SGr \times GrM) \times SGr)$

---

$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet$
  $(SG_c, m) \sqsupseteq_{\mathcal{M}}\ SG_a \Leftrightarrow \forall\, e : EsC\ SG_c \setminus EsD\ SG_c \bullet (srcma\ SG_c)\ e \leq_{\mathcal{M}} ((srcma\ SG_a) \circ (fE\ m))\ e$
      $\wedge\ (tgtm\ SG_c)\ e \leq_{\mathcal{M}} ((tgtm\ SG_a) \circ (fE\ m))\ e$

$$\_\sqsupseteq^{SG}\_,\_\sqsupseteq^{SG_0}\_ : \mathbb{P}((SGr \times GrM) \times SGr)$$

$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet$
$\quad (SG_c, m)\sqsupseteq^{SG_0} SG_a \Leftrightarrow (SG_c, m)\sqsupseteq_{NT} SG_a \land (SG_c, m)\sqsupseteq_{ET} SG_a \land (SG_c, m)\sqsupseteq_{\mathcal{M}} SG_a$

$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet$
$\quad (SG_c, m)\sqsupseteq^{SG} SG_a \Leftrightarrow \exists\, m' : GrM \mid m' = totaliseForDer(m, SG_c) \bullet$
$\qquad m' \in SG_c \rightarrow_{SG} SG_a \land (SG_c, m')\sqsupseteq^{SG_0} SG_a$

**relation**$(\_\sqsupseteq^{SG}\_)$
**relation**$(\_\sqsupseteq^{SG_0}\_)$
**relation**$(\_\sqsupseteq_{AEs}\_)$
**relation**$(\_\mathrm{OkRefinedIn}\_)$
**relation**$(\_\sqsupseteq_{ANNs}\_)$

$$\_\sqsupseteq_{ANNs}\_ : \mathbb{P}(GrM \times SGr)$$

$\forall\, SG_a : SGr;\ m : GrM \bullet$
$\quad m\sqsupseteq_{ANNs} SG_a \Leftrightarrow \forall\, nn : NsTy\, SG_a\{nnrml\} \bullet (\preceq SG_a)\,(\!|\,\{nn\}\,|\!)\cap \mathrm{ran}(fV\,m) = \varnothing$

$$\_\mathrm{OkRefinedIn}\_ : \mathbb{P}((SGr \times E) \times (SGr \times GrM))$$
$$\_\sqsupseteq_{AEs}\_ : \mathbb{P}((SGr \times GrM) \times SGr)$$

$\forall\, SG_c, SG_a : SGr;\ m : GrM;\ ae : E \bullet$
$\quad (SG_a, ae)\mathrm{OkRefinedIn}(SG_c, m) \Leftrightarrow$
$\qquad \exists\, r : V \leftrightarrow V;\ s, t : \mathbb{P}\,V \mid r = (\preceq SG_c)\,\mathbin{\raise.3ex\hbox{$_9^\circ$}}\, igRMEsW((gr\, SG_c, m), SG_a, ae)^{\leftrightarrow}\mathbin{\raise.3ex\hbox{$_9^\circ$}}(\preceq SG_c)^{\sim}$
$\qquad\quad \land\ s = insOf(m, SG_a, (sg\_src\, SG_a\,(\!|\,\{ae\}\,|\!))) \setminus ((NsEther\, SG_c) \setminus \mathrm{dom}\, r)$
$\qquad\quad \land\ t = insOf(m, SG_a, (sg\_tgt\, SG_a\,(\!|\,\{ae\}\,|\!))) \setminus ((NsEther\, SG_c) \setminus \mathrm{ran}\, r)$
$\qquad\qquad \bullet\ r \in s \leftrightarrow t \land r \neq \varnothing$

$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet$
$\quad (SG_c, m)\sqsupseteq_{AEs} SG_a \Leftrightarrow \forall\, e : (EsA\, SG_a) \bullet (SG_a, e)\mathrm{OkRefinedIn}(SG_c, m)$

$$\_\sqsupset^{SG}\_,\_\sqsupset^{SG_0}\_ : \mathbb{P}((SGr \times GrM) \times SGr)$$

$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet$
$\quad (SG_c, m)\sqsupset^{SG_0} SG_a \Leftrightarrow (SG_c, m)\sqsupseteq^{SG_0} SG_a \land m\sqsupseteq_{ANNs} SG_a \land (SG_c, m)\sqsupset_{AEs} SG_a$

$\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet$
$\quad (SG_c, m)\sqsupset^{SG} SG_a \Leftrightarrow \exists\, m' : GrM \mid m' = totaliseForDer(m, SG_c) \bullet$
$\qquad m' \in SG_c \rightarrow_{SG} SG_a \land (SG_c, m')\sqsupset^{SG_0} SG_a$

**relation**$(\_\sqsupset^{SG}\_)$
**relation**$(\_\sqsupset_{\mathcal{M}}\_)$
**relation**$(\_\sqsupset_{FI}\_)$
**relation**$(\_\sqsupset_{PNS}\_)$
**relation**$(\_\mathcal{MEM}Ok\_)$

$\_\,\mathcal{MEMOk}\,\_ : \mathbb{P}((SGr \times E) \times GrwT)$

---

$\forall\, GwT : GrwT;\ SG : SGr;\ me : E \bullet (SG, me)\, \mathcal{MEMOk}\, GwT \Leftrightarrow$
$\quad \exists\, r : V \leftrightarrow V;\ s, t : \mathbb{P}\, V \mid r = igRMEsW(GwT, SG, me) \leftrightarrow$
$\qquad \wedge\, s = insOf(ty\, GwT, SG, (src^*\, SG)\, (\!|\,\{me\}\,|\!))$
$\qquad \wedge\, t = insOf(ty\, GwT, SG, (tgt^*\, SG)\, (\!|\,\{me\}\,|\!))$
$\qquad\quad \bullet\, r\mathcal{MOk}(r, s, t, srcma\, SG\, me, tgtm\, SG\, me)$

<br>

$\_ \ni_{\mathcal{M}} \_ : GrwT \leftrightarrow SGr$
$\_ \ni_{FI} \_ : GrwT \leftrightarrow SGr$
$\_ \ni_{PNS} \_ : GrwT \leftrightarrow SGr$

---

$\forall\, GwT : GrwT;\ SG : SGr \bullet GwT \ni_{\mathcal{M}} SG \Leftrightarrow \forall\, me : EsC\, SG \bullet (SG, me)\, \mathcal{MEMOk}\, GwT$

$\forall\, GwT : GrwT;\ SG : SGr \bullet GwT \ni_{FI} SG \Leftrightarrow ((fV \circ ty)\, GwT)^{\sim}\, (\!|\, NsEther\, SG\, |\!) = \varnothing$

$\forall\, GwT : GrwT;\ SG : SGr \bullet$
$\quad GwT \ni_{PNS} SG \Leftrightarrow igRMEs(GwT, EsTy\, SG\, \{ecomp\, dbi, ecomp\, duni\})^{\leftrightarrow} \in \mathrm{injrel}$

<br>

$\_ \ni^{SG} \_ : GrwT \leftrightarrow SGr$

---

$\forall\, GwT : GrwT;\ SG : SGr \bullet$
$\quad GwT \ni^{SG} SG \Leftrightarrow GwT \Rrightarrow^{GwT} SG \wedge GwT \ni_{\mathcal{M}} SG \wedge GwT \ni_{FI} SG \wedge GwT \ni_{PNS} SG$

# 7 Fragments

**section** $Fragmenta\_Frs$ **parents** $standard\_toolkit, Fragmenta\_Generics, Fragmenta\_SGs, Fragmenta\_GrswT$

<br>

$Fr_0 == \{SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V \mid esr \cap (sg\_Es\, SG) = \varnothing$
$\quad \wedge\, sr \in esr \rightarrowtail (NsP\, SG) \wedge tr \in esr \rightarrow (V \setminus (NsO\, SG))\}$

<br>

$fSG : Fr_0 \rightarrow SGr$
$EsR : Fr_0 \rightarrow \mathbb{P}\, E$
$srcR, tgtR : Fr_0 \rightarrow E \nrightarrow V$

---

$\forall\, SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V \bullet fSG(SG, esr, sr, tr) = SG$
$\forall\, SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V \bullet EsR(SG, esr, sr, tr) = esr$
$\forall\, SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V \bullet srcR(SG, esr, sr, tr) = sr$
$\forall\, SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V \bullet tgtR(SG, esr, sr, tr) = tr$

$fLEs, fEs, fEsC : Fr_0 \to \mathbb{P} \, E$
$fLNs, fRNs, fNs : Fr_0 \to \mathbb{P} \, V$
$srcF, tgtF : Fr_0 \to E \nrightarrow V$

$fLEs = (sg\_Es \circ fSG)$

$\forall \, F : Fr_0 \bullet fEs \, F = fLEs \, F \cup EsR \, F$

$fEsC = EsC \circ fSG$

$fLNs = sg\_Ns \circ fSG$

$fRNs = \mathrm{ran} \circ tgtR$

$\forall \, F : Fr_0 \bullet fNs \, F = fLNs \, F \cup fRNs \, F$

$\forall \, F : Fr_0 \bullet srcF \, F = (sg\_src \circ fSG) \, F \cup srcR \, F$

$\forall \, F : Fr_0 \bullet tgtF \, F = (sg\_tgt \circ fSG) \, F \cup tgtR \, F$

---

$\overset{G}{\leftrightsquigarrow} \; : Fr_0 \to Gr$
$\leftrightsquigarrow \; : Fr_0 \to V \leftrightarrow V$

$\forall \, F : Fr_0 \bullet \overset{G}{\leftrightsquigarrow} F = ((NsP \circ fSG)F \cup fRNs \, F, EsR \, F, srcR \, F, tgtR \, F)$

$\forall \, F : Fr_0 \bullet \leftrightsquigarrow F = (\overset{G}{\leftrightsquigarrow} F)^{\leftrightarrow}$

**function** $10$ **leftassoc** $(\_ \cup_F \_)$

---

$\varnothing_F : Fr_0$
$\_ \cup_F \_ : Fr_0 \times Fr_0 \to Fr_0$
$\bigcup_F : \mathbb{P} \, Fr_0 \to Fr_0$

$\varnothing_F = (\varnothing_{SG}, \varnothing, \varnothing, \varnothing)$

$\forall \, F_1, F_2 : Fr_0 \bullet F_1 \cup_F \, F_2 =$
$\quad (fSG \, F_1 \cup_{SG} \, fSG \, F_2, EsR \, F_1 \cup \, EsR \, F_2, srcR \, F_1 \cup \, srcR \, F_2, tgtR \, F_1 \cup \, tgtR \, F_2)$

$\bigcup_F \{\} = \varnothing_F$

$\forall \, F : Fr_0; \; Fs : \mathbb{P} \, Fr_0 \bullet \bigcup_F (\{F\} \cup Fs) = F \cup_F (\bigcup_F Fs)$

$$\begin{array}{|l}
\rightsquigarrow : Fr_0 \nrightarrow V \nrightarrow V \\
\bigodot^{SG} : Fr_0 \nrightarrow SGr \\
rEsR : Fr_0 \nrightarrow \mathbb{P}\,E \\
\bigodot : Fr_0 \nrightarrow Fr_0 \\
\hline
\forall\, F : Fr_0 \bullet \rightsquigarrow F = (\leftrightsquigarrow F) \rhd (fLNs\ F) \\[4pt]
\forall\, F : Fr_0 \bullet \bigodot^{SG} F = (fSG\ F) \odot^{SG} (\rightsquigarrow F) \\[4pt]
\forall\, F : Fr_0 \bullet rEsR\ F = \mathrm{dom}((srcR\ F) \rhd \mathrm{dom}(\rightsquigarrow F)) \\[4pt]
\forall\, F : Fr_0 \bullet \bigodot F = (\bigodot^{SG}F, rEsR\ F, (rEsR\ F) \lhd (srcR\ F), (rEsR\ F) \lhd (tgtR\ F))
\end{array}$$

$$Fr_a == \{F : Fr_0 \mid \oslash(\overset{G}{\leftrightsquigarrow} F)\}$$
$$Fr == \{F : Fr_a \mid \bigodot^{SG} F \in SGr\}$$

**relation**(refsLocal_)

$$\begin{array}{|l}
\mathrm{refsLocal\_} : \mathbb{P}\,Fr_0 \\
\hline
\forall\, F : Fr_0 \bullet \mathrm{refsLocal}\ F \Leftrightarrow fRNs\ F \subseteq fLNs\ F
\end{array}$$

$$TFr == \{F : Fr_a \mid \mathrm{refsLocal}\ F \wedge \bigodot^{SG} F \in TSGr\}$$

**relation**($\boxminus$ _)
**relation**($\boxplus$ _)

$$\begin{array}{|l}
\boxminus\_ : Fr \leftrightarrow Fr \\
\hline
\forall\, F_1, F_2 : Fr \bullet \boxminus(F_1, F_2) \Leftrightarrow fLNs\ F_1 \cap fLNs\ F_2 = \varnothing \wedge fEs\ F_1 \cap fEs\ F_2 = \varnothing
\end{array}$$

$$\begin{array}{|l}
[I] \\
\hline
\boxplus\_ : \mathbb{P}(I \nrightarrow Fr) \\
\hline
\forall\, Fs : I \nrightarrow Fr \bullet \boxplus Fs \Leftrightarrow \forall\, i, j : \mathrm{dom}\ Fs \mid i \neq j \bullet \boxminus(Fs\ i, Fs\ j)
\end{array}$$

**relation**(_ $\subseteq^{rs}$ _)
**relation**(_ $\mapsto$ _)

$$\_ \subseteq^{rs} \_ : Fr \leftrightarrow Fr$$
$$\_ \mapsto \_ : Fr \leftrightarrow Fr$$

$$\forall\, F_1, F_2 : Fr \bullet F_1 \subseteq^{rs} F_2 \Leftrightarrow \mathrm{ran}(tgtR\ F_1) \cap fLNs\ F_2 \neq \varnothing$$

$$\forall\, F_1, F_2 : Fr \bullet F_1 \mapsto F_2 \Leftrightarrow F_1 \subseteq^{rs} F_2 \wedge \neg\, (F_2 \subseteq^{rs} F_1)$$

**function** 10 **leftassoc** $(\_ \rightrightarrows_{\bullet} \_)$

$$\_ \rightrightarrows_{\bullet} \_ : GrM \times (Fr \times Fr) \to GrM$$

$$\forall\, m : GrM;\ F_s, F_t : Fr_0 \bullet$$
$$m \rightrightarrows_{\bullet} (F_s, F_t) = (((\rightsquigarrow F_s)^{\oplus} \boxdot (fLNs\ F_s))^{\sim} \,\fatsemi\, (fV\ m) \,\fatsemi\, ((\rightsquigarrow F_t)^{\oplus} \boxdot (fLNs\ F_t)), fE\ m)$$

**function** 1 **leftassoc** $(\_ \to_F \_)$

$$\_ \to_F \_ : Fr \times Fr \to \mathbb{P}\ GrM$$
$$\forall\, F_s, F_t : Fr \bullet F_s \to_F F_t = \{fv : fLNs\ F_s \to fLNs\ F_t;\ fe : fEsC\ F_s \to fEsC\ F_t \mid$$
$$(\textcircled{\tiny\bullet}^{SG} F_s, (fv, fe) \rightrightarrows_{\bullet} (F_s, F_t)) \Rrightarrow^{SG} \textcircled{\tiny\bullet}^{SG} F_t\}$$

**relation**$(\_ \Rrightarrow^F \_)$

$$\_ \Rrightarrow^F \_ : (Fr \times GrM) \leftrightarrow Fr$$

$$\forall\, m : GrM;\ F_s, F_t : Fr_0 \bullet (F_s, m) \Rrightarrow^F F_t \Leftrightarrow m \in F_s \to_F F_t$$

**relation**$(\_ \sqsupseteq^F \_)$

$$\_ \sqsupseteq^F \_ : (Fr \times GrM) \leftrightarrow Fr$$
$$\forall\, F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupseteq^F F_a \Leftrightarrow (F_c, m) \Rrightarrow^F F_a$$
$$\wedge\ (\textcircled{\tiny\bullet}^{SG} F_c, m \rightrightarrows_{\bullet} (F_c, F_a)) \sqsupseteq^{SG_0} (\textcircled{\tiny\bullet}^{SG} F_a)$$

**relation**$(\_ \sqsupset^F \_)$

$$\underline{\ } \sqsupset^F \underline{\ } : (Fr \times GrM) \leftrightarrow Fr$$

$$\forall F_c, F_a : Fr_0; \ m : GrM \bullet (F_c, m) \sqsupset^F F_a \Leftrightarrow (F_c, m) \Rrightarrow^F F_a$$
$$\wedge (\textcircled{\bullet}^{SG} F_c, m \rightrightarrows_\bullet (F_c, F_a)) \sqsupset^{SG_0} (\textcircled{\bullet}^{SG} F_a)$$

**relation**$(\underline{\ } \ni^F \underline{\ })$

$$\underline{\ } \ni^F \underline{\ } : GrwT \leftrightarrow Fr$$

$$\forall GwT : GrwT; \ F : Fr \bullet GwT \ni^F F \Leftrightarrow GwT \ni^{SG} \textcircled{\bullet}^{SG} F$$

# 8 Global Fragment Graphs

**section** *Fragmenta_GFGs* **parents** *standard_toolkit, Fragmenta_Generics, Fragmenta_Graphs*

$$GFGr == \{ G : Gr \mid \oslash(G \bowtie_{Es} (Es \ G \setminus EsId \ G)) \}$$

**function**$(\underline{\ } {}^{\dashrightarrow})$

$$\underline{\ } {}^{\dashrightarrow} : GFGr \rightarrow V \leftrightarrow V$$

$$\forall GFG : GFGr \bullet GFG {}^{\dashrightarrow} = (GFG {}^{\leftrightarrow})^+$$

# 9 Models

**section** *Fragmenta_Mdls* **parents** *standard_toolkit, Fragmenta_Frs, Fragmenta_GFGs*

$$Mdl_0 == \{ GFG : GFGr; \ fd : V \nrightarrow Fr \mid fd \in Ns \ GFG \rightarrow Fr \wedge \boxplus fd \}$$

$$mGFG : Mdl_0 \rightarrow GFGr$$
$$mFD : Mdl_0 \rightarrow V \nrightarrow Fr$$

$$\forall GFG : GFGr; \ fd : V \nrightarrow Fr \bullet mGFG(GFG, fd) = GFG$$

$$\forall GFG : GFGr; \ fd : V \nrightarrow Fr \bullet mFD(GFG, fd) = fd$$

$$\begin{array}{|l}
mUFs : Mdl_0 \to Fr \\
\hline
mUFs = \bigcup_F \circ \mathrm{ran} \circ mFD
\end{array}$$

$$\begin{array}{|l}
\mathrm{from} : Mdl_0 \to V \nrightarrow V \\
\hline
\forall\, M : Mdl_0;\ v : V \bullet \mathrm{from}\, M\, v = (\mu\, vf : (Ns \circ mGFG)M \mid v \in fLNs(mFD\, M\, vf))
\end{array}$$

**relation**($\Uparrow$ _)

$$\begin{array}{|l}
\Uparrow\,_- : \mathbb{P}\, Mdl_0 \\
\hline
\forall\, M : Mdl_0 \bullet\, \Uparrow M \Leftrightarrow \exists\, UF : Fr_0 \mid UF = mUFs\, M \bullet \\
\quad \forall\, p : (NsP \circ fSG)UF \bullet (\mathrm{from}\, M\, p, \mathrm{from}\, M\, (\leftrightsquigarrow UF\, p)) \in ((\_ \dashrightarrow) \circ mGFG)M
\end{array}$$

$$Mdl == \{M : Mdl_0 \mid (mUFs\, M) \in TFr \wedge\, \Uparrow M\}$$

$$\begin{array}{|l}
\textcircled{\bullet}^{M} : Mdl \to Fr \\
\hline
\forall\, M : Mdl_0 \bullet \textcircled{\bullet}^{M} = \textcircled{\bullet} \circ mUFs
\end{array}$$

**function** 1 **leftassoc** $(\_ \to_M \_)$
**relation**$(\_ \Rrightarrow^{M} \_)$

$$\begin{array}{|l}
\_ \to_M \_ : Mdl \times Mdl \to \mathbb{P}\, GrM \\
\_ \Rrightarrow^{M} \_ : \mathbb{P}((Mdl \times \mathbb{P}\, GrM) \times Mdl) \\
\hline
\forall\, M_s, M_t : Mdl \bullet M_s \to_M M_t = \{m : GrM \mid \\
\quad \exists\, UF_s, UF_t : Fr_0 \mid UF_s = mUFs\, M_s \wedge UF_t = mUFs\, M_t \bullet m \in UF_s \to_F UF_t\} \\
\forall\, M_s, M_t : Mdl;\ ms : \mathbb{P}\, GrM \bullet (M_s, ms) \Rrightarrow^{M} M_t \Leftrightarrow \bigcup_{GM} ms \in M_s \to_M M_t
\end{array}$$

**relation**$(\_ \sqsupset^{M} \_)$

$$\begin{array}{|l}
\_ \sqsupset^{M} \_ : (Mdl \times \mathbb{P}\, GrM) \leftrightarrow Mdl \\
\hline
\forall\, M_c, M_a : Mdl_0;\ ms : \mathbb{P}\, GrM \bullet (M_c, ms) \sqsupset^{M} M_a \\
\quad \Leftrightarrow \exists\, UF_c, UF_a : Fr_0 \mid UF_c = mUFs\, M_c \wedge UF_a = mUFs\, M_a \bullet (UF_c, \bigcup_{GM} ms) \sqsupset^{F} UF_a
\end{array}$$

**relation**$(\_ \ni^M \_)$

$\quad \_ \ni^M \_ : GrwT \leftrightarrow Mdl$

$\quad \forall\ GwT : GrwT;\ M : Mdl \bullet GwT \ni^M M \Leftrightarrow GwT \ni^F mUFs\ M$