Z Specification of Fragmenta

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Contents

1 Generics

section *Fragmenta_Generics* **parents** *standard_toolkit*

$$\mathbb{R} : \mathbb{P}\,\mathbb{A}$$

$$\mathrm{acyclic}[X] == \{r : X \leftrightarrow X \mid r^{+} \cap \mathrm{id}\, X = \varnothing\}$$

$$\mathit{connected}[X] == \{r : X \leftrightarrow X \mid \forall x : \mathrm{dom}\, r;\ y : \mathrm{ran}\, r \bullet x \mapsto y \in r^{+}\}$$

$$\mathrm{tree}[X] == \{r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \land r \in X \nrightarrow X\}$$

$$\mathit{forest}[X] == \{r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \land (\forall s : X \leftrightarrow X \mid s \subseteq r \land s \in \mathit{connected} \bullet s \in \mathit{tree})\}$$

$$\mathrm{injrel}[X, Y] == \{r : X \leftrightarrow Y \mid r^{\sim} \in Y \nrightarrow X\}$$

$$\mathrm{antireflexive}[X] == \{r : X \leftrightarrow X \mid r \cap (\mathit{id}\, X) = \varnothing\}$$

$[X, Y, Z]$

$$\mathrm{flip} : (X \rightarrow Y \nrightarrow Z) \rightarrow (Y \rightarrow X \nrightarrow Z)$$

$$\forall f : X \rightarrow Y \nrightarrow Z \bullet \mathrm{flip}\, f = (\lambda\, y : Y \bullet \lambda\, x : X \bullet f\ x\ y)$$

$[X, Y, Z, W]$

$$\mathrm{apply} : (X \rightarrow Z) \rightarrow (Y \rightarrow W) \rightarrow (X \times Y) \rightarrow (Z \times W)$$

$$\forall f : X \rightarrow Z;\ g : Y \rightarrow W;\ x : X;\ y : Y \bullet \mathrm{apply}\, f\ g\ (x, y) = (f\ x, g\ y)$$

relation$(\_ =_p \_)$
relation$(\_ \subseteq_p \_)$

$[X, Y]$

$$\_ =_p \_ : \mathbb{P}((\mathbb{P}\, X \times \mathbb{P}\, Y) \times (\mathbb{P}\, X \times \mathbb{P}\, Y))$$
$$\_ \subseteq_p \_ : \mathbb{P}((\mathbb{P}\, X \times \mathbb{P}\, Y) \times (\mathbb{P}\, X \times \mathbb{P}\, Y))$$

$$\forall xs, zs : \mathbb{P}\, X;\ ys, ws : \mathbb{P}\, Y \bullet (xs, ys) =_p (zs, ws) \Leftrightarrow xs = zs \land ys = ws$$

$$\forall xs, zs : \mathbb{P}\, X;\ ys, ws : \mathbb{P}\, Y \bullet (xs, ys) \subseteq_p (zs, ws) \Leftrightarrow xs \subseteq zs \land ys \subseteq ws$$

$$
\begin{array}{|l}
\hline
[X, Y] \\
\quad \mathrm{map} : (X \to Y) \to \mathbb{P}\, X \to \mathbb{P}\, Y \\
\quad \mathrm{mapS} : (X \to Y) \to \mathrm{seq}\, X \to \mathrm{seq}\, Y \\
\hline
\quad \forall f : X \to Y \bullet \mathrm{map}\, f\, \{\} = \{\} \\
\quad \forall f : X \to Y;\ x : X;\ xs : \mathbb{P}\, X \bullet \mathrm{map}\, f\, (\{x\} \cup xs) = \{f\ x\} \cup (\mathrm{map}\, f\ xs) \\
\quad \forall f : X \to Y \bullet \mathrm{mapS}\, f\, \langle\rangle = \langle\rangle \\
\quad \forall f : X \to Y;\ x : X;\ xs : \mathrm{seq}\, X \bullet \mathrm{mapS}\, f\, (\langle x\rangle \frown xs) = \langle f\ x\rangle \frown (\mathrm{mapS}\, f\ xs) \\
\hline
\end{array}
$$

function 10 **leftassoc** $(\_ \boxdot \_)$

$$
\begin{array}{|l}
\hline
[X] \\
\quad \_ \boxdot \_ : ((X \to X) \times \mathbb{P}\, X) \to (X \mathrel{+\!\!\!\!\to} X) \\
\hline
\quad \forall f : X \to X;\ s : \mathbb{P}\, X \bullet f \boxdot s = (\mathrm{id}\ s) \oplus f \\
\hline
\end{array}
$$

function $(\_^{\oplus})$

$$
\begin{array}{|l}
\hline
[X] \\
\quad \_^{\oplus} : (X \leftrightarrow X) \to (X \leftrightarrow X) \\
\hline
\quad \forall r : X \leftrightarrow X \bullet r^{\oplus} = \mathbf{if}\ r \oplus r \mathbin{;} r = r\ \mathbf{then}\ r\ \mathbf{else}\ (r \oplus r \mathbin{;} r)^{\oplus} \\
\hline
\end{array}
$$

$$\mathrm{opt}[X] == \{s : \mathbb{P}\, X \mid \#\, s \leq 1\}$$

$$
\begin{array}{|l}
\hline
[X] \\
\quad \mathrm{the} : \mathrm{opt}[X] \mathrel{+\!\!\!\!\to} X \\
\hline
\quad \forall x : X \bullet \mathrm{the}\, \{x\} = x \\
\hline
\end{array}
$$

$$
\begin{array}{|l}
\hline
[X, Y] \\
\quad \mathit{flatten} : (X \mathrel{+\!\!\!\!\to} \mathbb{P}\, Y) \to (X \leftrightarrow Y) \\
\hline
\quad \forall f : X \mathrel{+\!\!\!\!\to} \mathbb{P}\, Y \bullet \mathit{flatten}\, f = \{x : \mathrm{dom}\, f;\ y : Y \mid y \in f\ x\} \\
\hline
\end{array}
$$

2 Graphs

section *Fragmenta_Graphs* **parents** *standard_toolkit*, *Fragmenta_Generics*

$[V, E]$

$$Gr == \{vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s, t : E \pfun V \mid s \in es \rightarrow vs \land t \in es \rightarrow vs\}$$

$$\begin{array}{l} Ns : Gr \rightarrow \mathbb{P}\, V \\ Es : Gr \rightarrow \mathbb{P}\, E \\ src, tgt : Gr \rightarrow E \pfun V \\ \hline \forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \pfun V;\ t : E \pfun V \bullet Ns(vs, es, s, t) = vs \\ \forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \pfun V;\ t : E \pfun V \bullet Es(vs, es, s, t) = es \\ \forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \pfun V;\ t : E \pfun V \bullet src(vs, es, s, t) = s \\ \forall vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \pfun V;\ t : E \pfun V \bullet tgt(vs, es, s, t) = t \end{array}$$

$$\begin{array}{l} nNatS, nIntS, nRealS : V \\ \mathrm{to}\mathbb{Z} : V \rightarrow \mathrm{opt}[\mathbb{Z}] \\ \mathrm{to}\mathbb{R} : V \rightarrow \mathrm{opt}[\mathbb{R}] \end{array}$$

$$\begin{array}{l} \varnothing_G : Gr \\ \hline \varnothing_G = (\varnothing, \varnothing, \varnothing, \varnothing) \end{array}$$

$$\begin{array}{l} Els : Gr \rightarrow (\mathbb{P}\, V \times \mathbb{P}\, E) \\ EsId : Gr \rightarrow \mathbb{P}\, E \\ \hline \forall G : Gr \bullet Els\ G = (Ns\ G, Es\ G) \\ \forall G : Gr \bullet EsId\ G = \{e : Es\ G \mid src\ G\ e = tgt\ G\ e\} \end{array}$$

relation$(adjacent\ \_)$

$$\begin{array}{l} adjacent\ \_ : \mathbb{P}(Gr \times V \times V) \\ \hline \forall G : Gr;\ v_1, v_2 : V \bullet adjacent(G, v_1, v_2) \Leftrightarrow \exists e : Es\ G \bullet src\ G\ e = v_1 \land tgt\ G\ e = v_2 \end{array}$$

relation$(adjacent_E \_)$

$$\begin{array}{|l} adjacent_E \_ : \mathbb{P}(Gr \times E \times E) \\ \hline \forall e_1, e_2 : E;\ G : Gr \bullet adjacent_E(G, e_1, e_2) \Leftrightarrow tgt\ G\ e_1 = src\ G\ e_2 \end{array}$$

function 10 **leftassoc** $(\_ \circ\!\!\rightarrow\!\!\circ_{Es} \_)$

$$\begin{array}{|l} \_ \circ\!\!\rightarrow\!\!\circ_{Es} \_ : Gr \times \mathbb{P}\ V \rightarrow \mathbb{P}\ E \\ \hline \forall G : Gr;\ vs : \mathbb{P}\ V \bullet G \circ\!\!\rightarrow\!\!\circ_{Es}\ vs = (src\ G)^{\sim} (\!| vs |\!) \cup (tgt\ G)^{\sim} (\!| vs |\!) \end{array}$$

function 10 **leftassoc** $(\_ \bullet\!\!\leftrightarrow\!\!\bullet \_)$

$$\begin{array}{|l} \_ \bullet\!\!\leftrightarrow\!\!\bullet \_ : Gr \times \mathbb{P}\ V \rightarrow \mathbb{P}\ E \\ \hline \forall G : Gr;\ vs : \mathbb{P}\ V \bullet G \bullet\!\!\leftrightarrow\!\!\bullet\ vs = (src\ G)^{\sim} (\!| vs |\!) \cap (tgt\ G)^{\sim} (\!| vs |\!) \end{array}$$

function 10 **leftassoc** $(\_ \circ\!\!\rightarrow\!\!\circ_{Ns} \_)$

$$\begin{array}{|l} \_ \circ\!\!\rightarrow\!\!\circ_{Ns} \_ : Gr \times \mathbb{P}\ E \rightarrow \mathbb{P}\ V \\ \hline \forall G : Gr;\ es : \mathbb{P}\ E \bullet G \circ\!\!\rightarrow\!\!\circ_{Ns}\ es = \mathrm{ran}(es \lhd src\ G) \cup \mathrm{ran}(es \lhd tgt\ G) \end{array}$$

function 10 **leftassoc** $(\_ \bowtie_{Es} \_)$

$$\begin{array}{|l} \_ \bowtie_{Es} \_ : Gr \times \mathbb{P}\ E \rightarrow Gr \\ \hline \forall G : Gr;\ es : \mathbb{P}\ E \bullet G \bowtie_{Es}\ es = (G \circ\!\!\rightarrow\!\!\circ_{Ns}\ es, Es\ G \cap es, es \lhd src\ G, es \lhd tgt\ G) \end{array}$$

function 10 **leftassoc** $(\_ \bowtie_{Ns} \_)$

$$\_ \bowtie_{Ns} \_ : Gr \times \mathbb{P}\, V \to Gr$$

$$\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet$$
$$\quad G \bowtie_{Ns} vs = (Ns\ G \cap vs, G \mathrel{\bullet\!\!\leftrightarrow\!\!\bullet} vs, (G \mathrel{\bullet\!\!\leftrightarrow\!\!\bullet} vs) \lhd src\ G, (G \mathrel{\bullet\!\!\leftrightarrow\!\!\bullet} vs) \lhd tgt\ G)$$

function 10 **leftassoc** $(\_ \ominus_{Ns} \_)$

$$\_ \ominus_{Ns} \_ : Gr \times \mathbb{P}\, V \to Gr$$

$$\forall\, G : Gr;\ vs : \mathbb{P}\, V \bullet$$
$$\quad G \ominus_{Ns} vs = (Ns\ G \setminus vs, Es\ G \setminus (G \mathrel{\circ\!\!\leftrightarrow\!\!\circ}_{Es} vs), (G \mathrel{\circ\!\!\leftrightarrow\!\!\circ}_{Es} vs) \ntriangleleft src\ G, (G \mathrel{\circ\!\!\leftrightarrow\!\!\circ}_{Es} vs) \ntriangleleft tgt\ G)$$

$$\text{successors} : V \times Gr \to \mathbb{P}\, V$$

$$\forall\, v : V;\ G : Gr \bullet \text{successors}(v, G) = \{v_1 : Ns\ G \mid adjacent(G, v, v_1)\}$$

function$(\_^{\rightleftarrows})$

$$\_^{\rightleftarrows} : Gr \to Gr$$

$$\forall\, G : Gr \bullet G^{\rightleftarrows} = (Ns\ G, Es\ G, tgt\ G, src\ G)$$

function$(\_^{\leftrightarrow})$

$$\_^{\leftrightarrow} : Gr \to V \leftrightarrow V$$

$$\forall\, G : Gr \bullet G^{\leftrightarrow} = \{v_1, v_2 : Ns\ G \mid adjacent(G, v_1, v_2)\}$$

function$(\_^{\leftrightarrow_E})$

$$\_^{\leftrightarrow_E} : Gr \to E \leftrightarrow E$$

$$\forall\, G : Gr \bullet G^{\leftrightarrow_E} = \{e_1, e_2 : Es\ G \mid adjacent_E(G, e_1, e_2)\}$$

relation$(\otimes \_)$

$$\begin{array}{|l} \otimes\_ : \mathbb{P}\, Gr \\ \hline \forall\, G : Gr \bullet \otimes\, G \Leftrightarrow G^{\leftrightarrow_E} \in \mathrm{acyclic} \end{array}$$

relation$(\boxminus_{Es}\ \_)$
relation$(\boxminus\ \_)$

$$\begin{array}{|l} \boxminus_{Es}\ \_, \boxminus\ \_ : \mathbb{P}(Gr \times Gr) \\ \hline \forall\, G_1, G_2 : Gr \bullet \boxminus_{Es}(G_1, G_2) \Leftrightarrow Es\ G_1 \cap Es\ G_2 = \varnothing \\ \forall\, G_1, G_2 : Gr \bullet \boxminus(G_1, G_2) \Leftrightarrow Ns\ G_1 \cap Ns\ G_2 = \varnothing \wedge \boxminus_{Es}(G_1, G_2) \end{array}$$

relation$(\boxplus\ \_)$

$$\begin{array}{|l} [I] \\ \boxplus\ \_ : \mathbb{P}(I \nrightarrow Gr) \\ \hline \forall\, Gs : I \nrightarrow Gr \bullet \boxplus\ Gs \Leftrightarrow \forall\, i, j : \mathrm{dom}\ Gs \mid i \neq j \bullet \boxminus\,(Gs\, i, Gs\, j) \\ \hline \end{array}$$

function 10 **leftassoc** $(\_ \cup_G \_)$

$$\begin{array}{|l} \_ \cup_G \_ : Gr \times Gr \rightarrow Gr \\ \hline \forall\, G_1, G_2 : Gr \bullet G_1 \cup_G G_2 = (Ns\ G_1 \cup Ns\ G_2, Es\ G_1 \cup Es\ G_2, src\ G_1 \cup src\ G_2, tgt\ G_1 \cup tgt\ G_2) \end{array}$$

function 10 **leftassoc** $(\_ \odot \_)$

$$\begin{array}{|l} \_ \odot \_ : Gr \times (V \leftrightarrow V) \nrightarrow Gr \\ \hline \forall\, G : Gr;\ s : V \leftrightarrow V \mid s \in Ns\ G \nrightarrow Ns\ G \wedge s \in \mathrm{antireflexive} \bullet \\ \quad G \odot s = (Ns\ G \setminus \mathrm{dom}\, s, Es\ G, (s \boxdot Ns\ G) \circ (src\ G), (s \boxdot Ns\ G) \circ (tgt\ G)) \end{array}$$

$$GrM == (V \nrightarrow V) \times (E \nrightarrow E)$$

$$
\begin{array}{|l}
fV : GrM \rightarrow V \nrightarrow V \\
fE : GrM \rightarrow E \nrightarrow E \\
\hline
\forall fv : V \nrightarrow V;\ fe : E \nrightarrow E \bullet fV(fv, fe) = fv \\
\forall fv : V \nrightarrow V;\ fe : E \nrightarrow E \bullet fE(fv, fe) = fe
\end{array}
$$

$$
\begin{array}{|l}
\mathrm{gid} : Gr \rightarrow GrM \\
\hline
\forall G : Gr \bullet \mathrm{gid}\ G = (id\ (Ns\ G), id\ (Es\ G))
\end{array}
$$

$$
\begin{array}{|l}
\varnothing_{GM} : GrM \\
\hline
\varnothing_{GM} = (\{\}, \{\})
\end{array}
$$

$$
\begin{array}{|l}
\mathrm{domg}, \mathrm{codg} : GrM \rightarrow (\mathbb{P}\ V \times \mathbb{P}\ E) \\
\hline
\forall m : GrM \bullet \mathrm{domg}\ m = (\mathrm{dom}(fV\ m), \mathrm{dom}(fE\ m)) \\
\forall m : GrM \bullet \mathrm{codg}\ m = (\mathrm{ran}(fV\ m), \mathrm{ran}(fE\ m))
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_{GM} \_)$

$$
\begin{array}{|l}
\_ \cup_{GM} \_ : GrM \times GrM \nrightarrow GrM \\
\bigcup_{GM} : \mathbb{P}\ GrM \rightarrow GrM \\
\hline
\forall f, g : GrM \bullet f \cup_{GM} g = (fV\ f \cup fV\ g, fE\ f \cup fE\ g) \\
\bigcup_{GM} \varnothing = \varnothing_{GM} \\
\forall f : GrM;\ fs : \mathbb{P}\ GrM \bullet \bigcup_{GM} (\{f\} \cup fs) = f \cup_{GM} (\bigcup_{GM} fs)
\end{array}
$$

function 10 **leftassoc** $(\_ \rightarrow_G \_)$

$$
\begin{array}{|l}
\_ \rightarrow_G \_ : Gr \times Gr \rightarrow \mathbb{P}\ GrM \\
\hline
\forall G_1, G_2 : Gr \bullet G_1 \rightarrow_G G_2 = \{fv : Ns\ G_1 \rightarrow Ns\ G_2;\ fe : Es\ G_1 \rightarrow Es\ G_2 \mid \\
\quad src\ G_2 \circ fe = fv \circ src\ G_1 \wedge tgt\ G_2 \circ fe = fv \circ tgt\ G_1\}
\end{array}
$$

function 10 **leftassoc** $(\_ \nrightarrow_G \_)$

$$\begin{array}{l} \_ \nrightarrow_G \_ : Gr \times Gr \rightarrow \mathbb{P}\, GrM \\ \hline \forall G_1, G_2 : Gr \bullet G_1 \rightarrow_G G_2 = \{fv : Ns\ G_1 \nrightarrow Ns\ G_2;\ fe : Es\ G_1 \nrightarrow Es\ G_2 \mid \\ \quad src\ G_2 \circ fe = fv \circ ((\mathrm{dom}\, fe) \lhd (src\ G_1)) \land tgt\ G_2 \circ fe = fv \circ ((\mathrm{dom}\, fe) \lhd (tgt\ G_1))\} \end{array}$$

function 10 **leftassoc** $(\_ \circ_G \_)$

$$\begin{array}{l} \_ \circ_G \_ : GrM \times GrM \rightarrow GrM \\ \hline \forall g, f : GrM \bullet g \circ_G f = (fV\ g \circ fV\ f, fE\ g \circ fE\ f) \end{array}$$

3 Graphs with typing

section *Fragmenta_GrswT* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$GrwT == \{G : Gr;\ t : GrM \mid \mathrm{domg}\, t =_p Els\ G\}$

function$(\_^G)$
function$(\_^T)$

$$\begin{array}{l} \_^G : GrwT \rightarrow Gr \\ \_^T : GrwT \rightarrow GrM \\ \hline \forall G : Gr;\ t : GrM \bullet (G, t)^G = G \\ \forall G : Gr;\ t : GrM \bullet (G, t)^T = t \end{array}$$

$$\begin{array}{l} \varnothing_{GwT} : GrwT \\ \hline \varnothing_{GwT} = (\varnothing_G, \varnothing_{GM}) \end{array}$$

function 10 **leftassoc** $(\_ \cup_{GwT} \_)$

$$\begin{array}{l} \_ \cup_{GwT} \_ : GrwT \times GrwT \rightarrow GrwT \\ \hline \forall G_1, G_2 : GrwT \bullet G_1 \cup_{GwT} G_2 = ({G_1}^G \cup_G {G_2}^G, {G_1}^T \cup_{GM} {G_2}^T) \end{array}$$

4 SG Element Types

section *Fragmenta_SGElemTys* **parents** *standard_toolkit*, *Fragmenta_Generics*

$SGNT ::= nnrml \mid nabst \mid nprxy \mid nenum \mid nval \mid nvirt$
$SGED ::= dbi \mid duni$
$SGET ::= einh \mid ecomp\langle\langle SGED\rangle\rangle \mid erel\langle\langle SGED\rangle\rangle \mid eder \mid epath \mid evcnt$

relation$(\_ \prec_{NT} \_)$

$\prec_{NT}$, an ordering relation on $SGNT$, indicates the node types that may be in an inheritance relation:

$$\begin{array}{|l}
_ \prec_{NT} _ : SGNT \leftrightarrow SGNT \\
\hline
\forall nt_1, nt_2 : SGNT \bullet nt_1 \prec_{NT} nt_2 \Leftrightarrow nt_1 \neq nprxy \land nt_2 \neq nval \\
\quad \land nt_1 = nabst \Rightarrow nt_2 \in \{nabst, nvirt, nprxy\} \land (nt_1 = nenum \Rightarrow nt_2 \in \{nvirt, nprxy\})
\end{array}$$

Above, $\prec_{NT}$ stipulates that (i) proxies must not inherit, (ii) values must not be inherited, (iii) abstract nodes may inherit from abstract and virtuals only, (iv) enumerations may only inherit virtuals or proxies

$\leq_{rNT}$ and $\leq_{ET}$ are ordering relations on sets $SGNT$ and $SGET$, respectively, indicating node and edge types that can be refinement-related, respectively:

relation$(\_ \leq_{rNT} \_)$

$$\begin{array}{|l}
_ \leq_{rNT} _ : SGNT \leftrightarrow SGNT \\
\hline
\forall nt_1, nt_2 : SGNT \bullet nt_1 \leq_{rNT} nt_2 \Leftrightarrow nt_1 = nt_2 \lor nt_1 = nprxy \lor nt_1 = nnrml \land nt_2 = nprxy \\
\quad \lor nt_2 \in \{nabst, nvirt\} \land nt_1 \in \{nnrml, nvirt, nabst\} \lor nt_2 = nnrml
\end{array}$$

relation$(\_ =_{ET} \_)$

$$\begin{array}{|l}
_ =_{ET} _ : SGET \leftrightarrow SGET \\
\hline
\forall et_1, et_2 : SGET \bullet et_1 =_{ET} et_2 \Leftrightarrow et_1 = et_2 \\
\quad \lor (\forall d_1, d_2 : SGED \bullet et_1 = erel\ d_1 \land et_2 = erel\ d_2 \lor et_1 = ecomp\ d_1 \land et_2 = ecomp\ d_2)
\end{array}$$

relation$(\_ \leq_{ET} \_)$

$$\begin{array}{|l}
_ \leq_{ET} _ : SGET \leftrightarrow SGET \\
\hline
\forall et_1, et_2 : SGET \bullet et_1 \leq_{ET} et_2 \Leftrightarrow \neg \{einh, eder, epath, evcnt\} \subseteq \{et_1, et_2\} \\
\quad \land (et_1 =_{ET} et_2 \lor et_1 \in \mathrm{dom}(ecomp^{\sim}) \land et_2 = erel\ dbi \\
\qquad \lor et_1 = ecomp\ duni \land et_2 = erel\ duni)
\end{array}$$

5 Multiplicities

section *Fragmenta_Mult* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_SGElemTys*

$MultVal ::= \mathbf{v}\langle\langle \mathbb{N} \rangle\rangle \mid *$

$MultC ::= mr\langle\langle \mathbb{N} \times MultVal \rangle\rangle \mid ms\langle\langle MultVal \rangle\rangle$

relation$(\_ =_{mv} \_)$

$$\begin{array}{|l}
_ =_{mv} _ : MultVal \leftrightarrow MultVal \\
\hline
\forall m_1, m_2 : MultVal \bullet m_1 =_{mv} m_2 \Leftrightarrow \{m_1, m_2\} \subseteq \{*\} \lor \exists n : \mathbb{N} \bullet m_1 = \mathbf{v}\ n \land m_2 = \mathbf{v}\ n
\end{array}$$

function 10 **leftassoc** $(\_ *_{mv} \_)$

$$\begin{array}{|l}
_ *_{mv} _ : MultVal \times MultVal \to MultVal \\
\hline
\forall m : MultVal \bullet * *_{mv} m = * \\
\forall m : MultVal \bullet m *_{mv} * = * \\
\forall n_1, n_2 : \mathbb{N} \bullet (\mathbf{v}\ n_1) *_{mv} (\mathbf{v}\ n_2) = \mathbf{v}(n_1 * n_2)
\end{array}$$

$$\begin{array}{|l}
mlbn : MultC \to \mathbb{N} \\
mlb, mub : MultC \to MultVal \\
\hline
mlbn(ms *) = 0 \\
\forall k : \mathbb{N} \bullet mlbn(ms(\mathbf{v}\ k)) = k \\
\forall k, m : \mathbb{N} \bullet mlbn(mr(k, \mathbf{v}\ m)) = k \\
\forall mc : MultC \bullet mlb\ mc = \mathbf{v}(mlbn\ mc) \\
\forall mv : MultVal \bullet mub(ms\ mv) = mv \\
\forall k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ m
\end{array}$$

function 10 **leftassoc** $(\_ *_{mr} \_)$

$$
\begin{array}{|l}
\_ *_{mr} \_ : MultC \times MultC \pfun MultC \\
\hline
\forall n_1, n_2 : \mathbb{N};\ mv_1, mv_2 : MultVal \bullet mr(n_1, mv_1) *_{mr} mr(n_2, mv_2) = mr(n_1 * n_2, mv_1 *_{mv} mv_2)
\end{array}
$$

relation$(\_ \leq_{mv} \_)$

$$
\begin{array}{|l}
\_ \leq_{mv} \_ : MultVal \leftrightarrow MultVal \\
\hline
\forall m_1, m_2 : MultVal \bullet m_1 =_{mv} m_2 \Leftrightarrow \{m_1, m_2\} \subseteq \{*\} \\
\quad \lor ((mlb \circ ms)\ m_1 = (mlb \circ ms)\ m_2 \land (mub \circ ms)\ m_1 = (mub \circ ms)\ m_2) \\
\forall m_1, m_2 : MultVal \bullet m_1 \leq_{mv} m_2 \Leftrightarrow m_2 = * \lor \exists j, k : \mathbb{N} \mid m_1 = \mathbf{v}\ j \land m_2 = \mathbf{v}\ k \bullet j \leq k
\end{array}
$$

$MultCOk == \{m : MultC \mid \exists lb : \mathbb{N};\ ub : MultVal \bullet m = mr(lb, ub) \land \mathbf{v}\ lb \leq_{mv} ub$
$\quad \lor \exists mv : MultVal \bullet m = ms\ mv\}$

$MultCMany == \{ms\ *, mr(0, *)\}$

$Mult == \{ ms : \mathbb{P}_1\ MultCOk \mid \#\ ms > 1 \Rightarrow (\forall m : ms \bullet m \notin MultCMany) \}$

$$
\begin{array}{|l}
mk : \mathbb{N} \to MultC \\
mks : \mathbb{N} \to Mult \\
mrs : \mathbb{N} \times MultVal \to Mult \\
mopt : MultC \\
mopts, mmanys : Mult \\
\hline
\forall k : \mathbb{N} \bullet mk\ k = ms(\mathbf{v}\ k) \land mks\ k = \{mk\ k\} \\
mopt = mr(0, \mathbf{v}\ 1) \land mopts = \{mopt\} \land mmanys = \{ms\ *\} \\
\forall lb : \mathbb{N};\ ub : MultVal \bullet mrs(lb, ub) = \{mr(lb, ub)\}
\end{array}
$$

$MultMany == \{ s : Mult \mid \exists m : MultC \bullet s = \{m\} \land m \in MultCMany \}$

$MultRange == \{s : Mult \mid \exists m : MultC \bullet s = \{m\} \land (\exists k : \mathbb{N} \mid k > 1 \bullet m = ms(\mathbf{v}\ k)$
$\quad \lor \exists lb : \mathbb{N};\ umv : MultVal \mid \mathbf{v}\ 2 \leq_{mv} umv \bullet m = mr(lb, umv))\}$

$MultEither == \{s : Mult \mid \#\ s > 1\}$

$MultLBZ == \{ms : Mult \mid \exists m : ms \bullet mlbn\ m = 0\}$

relation$(\_ \between \_)$
relation$(\_ \cdots \_)$

$$
\begin{array}{|l}
\_ \between \_ : \mathbb{P}(\mathbb{N} \times \mathit{MultC}) \\
\hline
\forall k : \mathbb{N};\ m : \mathit{MultC} \bullet k \between m \Leftrightarrow \mathit{mlb}\ m \leq_{mv} \mathbf{v}\ k \land \mathbf{v}\ k \leq_{mv} \mathit{mub}\ m
\end{array}
$$

$$
\begin{array}{|l}
\_ \cdots \_ : \mathbb{P}(\mathbb{N} \times \mathit{Mult}) \\
\hline
\forall k : \mathbb{N};\ m : \mathit{MultC} \bullet k \cdots \{m\} \Leftrightarrow k \between m \\
\forall k : \mathbb{N};\ m : \mathit{MultC};\ \mathit{sms} : \mathit{Mult} \bullet k \cdots (\{\, m \,\} \cup \mathit{sms}) \Leftrightarrow k \between m \lor k \cdots \mathit{sms}
\end{array}
$$

relation$(\_ \leq_{\mathcal{M}c} \_)$

$$
\begin{array}{|l}
\_ \leq_{\mathcal{M}c} \_ : \mathit{MultC} \leftrightarrow \mathit{MultC} \\
\hline
\forall m_1, m_2 : \mathit{MultC} \bullet m_1 \leq_{\mathcal{M}c} m_2 \Leftrightarrow \mathit{mlb}\ m_2 \leq_{mv} \mathit{mlb}\ m_1 \land \mathit{mub}\ m_1 \leq_{mv} \mathit{mub}\ m_2
\end{array}
$$

relation$(\_ \leq_{\mathcal{M}} \_)$

$$
\begin{array}{|l}
\_ \leq_{\mathcal{M}} \_ : \mathit{Mult} \leftrightarrow \mathit{Mult} \\
\hline
\forall m_1, m_2 : \mathit{Mult} \bullet m_1 \leq_{\mathcal{M}} m_2 \Leftrightarrow \forall \mathit{mc}_1 : m_1 \bullet \exists \mathit{mc}_2 : m_2 \bullet \mathit{mc}_1 \leq_{\mathcal{M}c} \mathit{mc}_2
\end{array}
$$

relation$(\_ \propto \_)$

$$
\begin{array}{|l}
\_ \propto \_ : \mathbb{P}(\mathit{SGET} \times (\mathit{Mult} \times \mathit{Mult})) \\
\hline
\forall \mathit{et} : \mathit{SGET};\ m_1, m_2 : \mathit{Mult} \bullet \mathit{et} \propto (m_1, m_2) \Leftrightarrow \mathit{et} = \mathit{erel}\ \mathit{dbi} \lor \mathit{et} = \mathit{eder} \\
\quad \lor\ \mathit{et} = \mathit{ecomp}\ \mathit{duni} \land m_1 = \mathit{mks}\, 1 \\
\quad \lor\ \mathit{et} = \mathit{erel}\ \mathit{duni} \land m_1 \in \mathit{MultMany} \\
\quad \lor\ \mathit{et} = \mathit{ecomp}\ \mathit{dbi} \land m_1 \in \{\mathit{mks}\, 1, \mathit{mopts}\}
\end{array}
$$

relation(rbounded_)
relation(eitherbounded_)

$$
\begin{array}{|l}
[X, Y] \\
\hline
\mathrm{rbounded}\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\, X \times \mathit{MultC}) \\
\mathrm{eitherbounded}\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\, X \times \mathit{Mult}) \\
\hline
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ m : \mathit{MultC} \bullet \\
\quad \mathrm{rbounded}(r, s, m) \Leftrightarrow \forall x : s \bullet \#(r \,(\!|\, \{x\} \,|\!)\,) \between m \\
\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ \mathit{ms} : \mathit{Mult} \bullet \\
\quad \mathrm{eitherbounded}(r, s, \mathit{ms}) \Leftrightarrow \forall x : s \bullet \#(r \,(\!|\, \{x\} \,|\!)\,) \cdots \mathit{ms} \\
\hline
\end{array}
$$

relation$(r\mathcal{M}Ok\,\_)$

$[X, Y]$

$r\mathcal{M}Ok\,\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\,X \times \mathbb{P}\,Y \times Mult \times Mult)$

$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : Mult \bullet r\mathcal{M}Ok(r, s, t, mks\,0, m) \Leftrightarrow s \lhd r = \{\}$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : Mult \bullet r\mathcal{M}Ok(r, s, t, m, mks\,0) \Leftrightarrow r \rhd t = \{\}$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet r\mathcal{M}Ok(r, s, t, mks\,1, mks\,1) \Leftrightarrow r \in s \rightarrowtail\!\!\!\!\twoheadrightarrow t$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet r\mathcal{M}Ok(r, s, t, mopts, mks\,1) \Leftrightarrow r \in s \rightarrowtail t$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet r\mathcal{M}Ok(r, s, t, mks\,1, mopts) \Leftrightarrow r^{\sim} \in t \rightarrowtail s$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany \bullet r\mathcal{M}Ok(r, s, t, mm, mks\,1) \Leftrightarrow r \in s \rightarrow t$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany \bullet r\mathcal{M}Ok(r, s, t, mks\,1, mm) \Leftrightarrow r^{\sim} \in t \rightarrow s$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mr : MultRange \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mr, mks\,1) \Leftrightarrow r \in s \rightarrow t \land \mathrm{rbounded}(r^{\sim}, t, \mathrm{the}\,mr)$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mr : MultRange \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mks\,1, mr) \Leftrightarrow r^{\sim} \in t \rightarrow s \land \mathrm{rbounded}(r, s, \mathrm{the}\,mr)$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ me : MultEither \bullet$
$\quad r\mathcal{M}Ok(r, s, t, me, mks\,1) \Leftrightarrow r \in s \rightarrow t \land \mathrm{eitherbounded}(r^{\sim}, t, me)$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ me : MultEither \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mks\,1, me) \Leftrightarrow r^{\sim} \in t \rightarrow s \land \mathrm{eitherbounded}(r, s, me)$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y \bullet r\mathcal{M}Ok(r, s, t, mopts, mopts) \Leftrightarrow r \in s \rightarrowtail\!\!\!\!\!+\, t$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany \bullet r\mathcal{M}Ok(r, s, t, mm, mopts) \Leftrightarrow r \in s \nrightarrow t$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mopts, mm) \Leftrightarrow r^{\sim} \in t \nrightarrow s$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : MultRange \bullet$
$\quad r\mathcal{M}Ok(r, s, t, m, mopts) \Leftrightarrow r \in s \nrightarrow t \land \mathrm{rbounded}(r^{\sim}, t, \mathrm{the}\,m)$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : MultRange \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mopts, m) \Leftrightarrow r^{\sim} \in t \nrightarrow s \land \mathrm{rbounded}(r, s, \mathrm{the}\,m)$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : MultEither \bullet$
$\quad r\mathcal{M}Ok(r, s, t, m, mopts) \Leftrightarrow r \in s \nrightarrow t \land \mathrm{eitherbounded}(r^{\sim}, t, m)$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ m : MultEither \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mopts, m) \Leftrightarrow r^{\sim} \in t \nrightarrow s \land \mathrm{eitherbounded}(r, s, m)$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm_1, mm_2 : MultMany \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mm_1, mm_2) \Leftrightarrow r \in s \leftrightarrow t$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany;\ mr : MultRange \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mm, mr) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r, s, \mathrm{the}\,mr)$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany;\ mr : MultRange \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mr, mm) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r^{\sim}, t, \mathrm{the}\,mr)$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany;\ me : MultEither \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mm, me) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{eitherbounded}(r, s, me)$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mm : MultMany;\ me : MultEither \bullet$
$\quad r\mathcal{M}Ok(r, s, t, me, mm) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{eitherbounded}(r^{\sim}, t, me)$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ mr_1, mr_2 : MultRange \bullet$
$\quad r\mathcal{M}Ok(r, s, t, mr_1, mr_2) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r, s, \mathrm{the}\,mr_2) \land \mathrm{rbounded}(r^{\sim}, t, \mathrm{the}\,mr_1)$
$\forall r : X \leftrightarrow Y;\ s : \mathbb{P}\,X;\ t : \mathbb{P}\,Y;\ me_1, me_2 : MultEither \bullet$
$\quad r\mathcal{M}Ok(r, s, t, me_1, me_2) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{eitherbounded}(r, s, me_2) \land \mathrm{eitherbounded}(r^{\sim}, t, me_1)$

6 Path Expressions

section $Fragmenta\_PEs$ **parents** $standard\_toolkit, Fragmenta\_Generics, Fragmenta\_Graphs$

$PEA ::= edg\langle\langle E\rangle\rangle \mid einv\langle\langle E\rangle\rangle$

$PEC ::= eat\langle\langle PEA\rangle\rangle \mid eresd\langle\langle V \times PEA\rangle\rangle \mid eresr\langle\langle PEA \times V\rangle\rangle$

$PE ::= ec\langle\langle PEC\rangle\rangle \mid scmp\langle\langle PEC \times PE\rangle\rangle$

$ePEA : PEA \to E$

$\forall e : E \bullet ePEA(edg\ e) = e$

$\forall e : E \bullet ePEA(einv\ e) = e$

$startEA_C : PEC \to PEA$
$startEA : PE \to PEA$

$\forall pea : PEA \bullet startEA_C(eat\ pea) = pea$

$\forall v : V;\ pea : PEA \bullet startEA_C(eresd\ (v, pea)) = pea$

$\forall v : V;\ pea : PEA \bullet startEA_C(eresr\ (pea, v)) = pea$

$\forall pec : PEC \bullet startEA(ec\ pec) = startEA_C\ pec$

$\forall pec : PEC;\ pe : PE \bullet startEA(scmp\ (pec, pe)) = startEA_C\ pec$

$endEA_C : PEC \to PEA$
$endEA : PE \to PEA$

$\forall pea : PEA \bullet endEA_C(eat\ pea) = pea$

$\forall v : V;\ pea : PEA \bullet endEA_C(eresd\ (v, pea)) = pea$

$\forall v : V;\ pea : PEA \bullet endEA_C(eresr\ (pea, v)) = pea$

$\forall pec : PEC \bullet endEA(ec\ pec) = endEA_C(pec)$

$\forall pec : PEC;\ pe : PE \bullet endEA(scmp\ (pec, pe)) = endEA\ pe$

$$
\begin{array}{|l}
srcPEA : Gr \to PEA \to V \\
srcPEC : Gr \to PEC \to V \\
srcPE : Gr \to PE \to V \\
\hline
\forall G : Gr;\ e : E \bullet srcPEA\ G\ (edg\ e) = src\ G\ e \\
\forall G : Gr;\ e : E \bullet srcPEA\ G\ (einv\ e) = tgt\ G\ e \\
\forall G : Gr;\ v : V;\ pea : PEA \bullet srcPEC\ G\ (eat\ pea) = srcPEA\ G\ pea \\
\forall G : Gr;\ v : V;\ pea : PEA \bullet srcPEC\ G\ (eresd\ (v, pea)) = srcPEA\ G\ pea \\
\forall G : Gr;\ v : V;\ pea : PEA \bullet srcPEC\ G\ (eresr\ (pea, v)) = srcPEA\ G\ pea \\
\forall G : Gr;\ pec : PEC \bullet srcPE\ G\ (ec\ pec) = srcPEC\ G\ pec \\
\forall G : Gr;\ pec : PEC;\ pe : PE \bullet srcPE\ G\ (scmp\ (pec, pe)) = srcPEC\ G\ pec
\end{array}
$$

$$
\begin{array}{|l}
tgtPEA : Gr \to PEA \to V \\
tgtPEC : Gr \to PEC \to V \\
tgtPE : Gr \to PE \to V \\
\hline
\forall G : Gr;\ e : E \bullet tgtPEA\ G\ (edg\ e) = tgt\ G\ e \\
\forall G : Gr;\ e : E \bullet tgtPEA\ G\ (einv\ e) = src\ G\ e \\
\forall G : Gr;\ v : V;\ pea : PEA \bullet tgtPEC\ G\ (eat\ pea) = tgtPEA\ G\ pea \\
\forall G : Gr;\ v : V;\ pea : PEA \bullet tgtPEC\ G\ (eresd\ (v, pea)) = tgtPEA\ G\ pea \\
\forall G : Gr;\ v : V;\ pea : PEA \bullet tgtPEC\ G\ (eresr\ (pea, v)) = tgtPEA\ G\ pea \\
\forall G : Gr;\ pec : PEC \bullet tgtPE\ G\ (ec\ pec) = tgtPEC\ G\ pec \\
\forall G : Gr;\ pec : PEC;\ pe : PE \bullet tgtPE\ G\ (scmp\ (pec, pe)) = tgtPE\ G\ pe
\end{array}
$$

$$
\begin{array}{|l}
rsrcPE : PE \to E \\
rtgtPE : PE \to E \\
\hline
\forall pe : PE \bullet rsrcPE\ pe = ePEA\ (startEA\ pe) \\
\forall pe : PE \bullet rtgtPE\ pe = ePEA\ (endEA\ pe)
\end{array}
$$

7 Structural Graphs

section *Fragmenta_SGs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*, *Fragmenta_SGElemTys*, *Fragmenta_Mult*, *Fragmenta_PEs*, *Fragmenta_GrswT*

Z Type *VCI* represents the information associated with a value constraint edge: an operator such as equality (*SGVCEOP*) and an optional edge, which is either one of the edges or the node or none if the constraint edge refers to the values of the node itself.

$$SGVCEOP ::= eq \mid neq \mid leq \mid geq \mid lt \mid gt$$

$$VCI == SGVCEOP \times opt[E]$$

$$SGr_0 == \{G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E;\ vci : E \nrightarrow VCI \mid nt \in Ns\ G \rightarrow SGNT \land et \in Es\ G \rightarrow SGET \land d \in Es\ G \leftrightarrow Es\ G\}$$

$$
\begin{array}{|l}
gr : SGr_0 \rightarrow Gr \\
sg\_Ns : SGr_0 \rightarrow \mathbb{P}\ V \\
sg\_Es : SGr_0 \rightarrow \mathbb{P}\ E \\
sg\_src, sg\_tgt : SGr_0 \rightarrow E \nrightarrow V \\
nty : SGr_0 \rightarrow V \nrightarrow SGNT \\
ety : SGr_0 \rightarrow E \nrightarrow SGET \\
srcm, tgtm : SGr_0 \rightarrow E \nrightarrow Mult \\
pe : SGr_0 \rightarrow E \nrightarrow PE \\
ds : SGr_0 \rightarrow E \leftrightarrow E \\
vcei : SGr_0 \rightarrow E \nrightarrow SGVCEOP \times opt[E] \\
\hline
\forall G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E; \\
\quad vci : E \nrightarrow VCI \bullet gr\ (G, nt, et, sm, tm, p, d, vci) = G \\
sg\_Ns = Ns \circ gr \\
sg\_Es = Es \circ gr \\
sg\_src = src \circ gr \\
sg\_tgt = tgt \circ gr \\
\forall G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E; \\
\quad vci : E \nrightarrow VCI \bullet nty(G, nt, et, sm, tm, p, d, vci) = nt \\
\forall G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E; \\
\quad vci : E \nrightarrow VCI \bullet ety(G, nt, et, sm, tm, p, d, vci) = et \\
\forall G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E; \\
\quad vci : E \nrightarrow VCI \bullet srcm(G, nt, et, sm, tm, p, d, vci) = sm \\
\forall G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E; \\
\quad vci : E \nrightarrow VCI \bullet tgtm(G, nt, et, sm, tm, p, d, vci) = tm \\
\forall G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E; \\
\quad vci : E \nrightarrow VCI \bullet pe(G, nt, et, sm, tm, p, d, vci) = p \\
\forall G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E; \\
\quad vci : E \nrightarrow VCI \bullet ds(G, nt, et, sm, tm, p, d, vci) = d \\
\forall G : Gr;\ nt : V \nrightarrow SGNT;\ et : E \nrightarrow SGET;\ sm, tm : E \nrightarrow Mult;\ p : E \nrightarrow PE;\ d : E \leftrightarrow E; \\
\quad vci : E \nrightarrow VCI \bullet vcei(G, nt, et, sm, tm, p, d, vci) = vci
\end{array}
$$

$$
\begin{array}{|l}
\varnothing_{SG} : SGr_0 \\
\hline
\varnothing_{SG} = (\varnothing_G, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing)
\end{array}
$$

$$
\begin{array}{|l}
NsTy : SGr_0 \rightarrow \mathbb{P}\ SGNT \rightarrow \mathbb{P}\ V \\
EsTy : SGr_0 \rightarrow \mathbb{P}\ SGET \rightarrow \mathbb{P}\ E \\
\hline
\forall SG : SGr_0;\ nts : \mathbb{P}\ SGNT \bullet NsTy\ SG\ nts = (nty\ SG)^\sim (\!|\ nts\ |\!) \\
\forall SG : SGr_0;\ ets : \mathbb{P}\ SGET \bullet EsTy\ SG\ ets = (ety\ SG)^\sim (\!|\ ets\ |\!)
\end{array}
$$

$$EsA, EsI, EsM, EsD, EsVCnt, EsPa, EsPaCnt : SGr_0 \to \mathbb{P}\, E$$

$$EsA = (\mathrm{flip}\ EsTy)\ (erel\ (\!|\ SGED\ |\!) \cup ecomp\ (\!|\ SGED\ |\!))$$
$$EsI = (\mathrm{flip}\ EsTy)\ \{einh\}$$
$$EsD = (\mathrm{flip}\ EsTy)\ \{eder\}$$
$$EsPa = (\mathrm{flip}\ EsTy)\ \{epath\}$$
$$EsVCnt = (\mathrm{flip}\ EsTy)\ \{evcnt\}$$
$$\forall\, SG : SGr_0 \bullet EsM\ SG = EsA\ SG \cup EsD\ SG$$
$$\forall\, SG : SGr_0 \bullet EsPaCnt\ SG = EsD\ SG \cup EsPa\ SG$$

$$NsN, NsP, NsEther, NsVi, NsVa : SGr_0 \to \mathbb{P}\, V$$

$$NsN = (\mathrm{flip}\ NsTy)\ \{nnrml\}$$
$$NsP = (\mathrm{flip}\ NsTy)\ \{nprxy\}$$
$$NsEther = (\mathrm{flip}\ NsTy)\ \{nabst, nvirt, nenum\}$$
$$NsVi = (\mathrm{flip}\ NsTy)\ \{nvirt\}$$
$$NsVa = (\mathrm{flip}\ NsTy)\ \{nval\}$$

$$tpe : SGr_0 \to E \nrightarrow PE$$

$$\forall\, SG : SGr_0 \bullet tpe\ SG = \{e : sg\_Es\ SG \bullet (e, (ec \circ eat \circ edg)\ e)\} \oplus pe\ SG$$

$$\pitchfork : SGr_0 \to Gr$$
$$\prec : SGr_0 \to V \leftrightarrow V$$

$$\forall\, SG : SGr_0 \bullet \pitchfork\ SG = gr\ SG \bowtie_{Es} EsI\ SG$$
$$\prec = (\_^{\leftrightarrow}) \circ \pitchfork$$

$$srcma : SGr_0 \to (E \nrightarrow Mult)$$

$$\forall\, SG : SGr_0 \bullet srcma\ SG =$$
$$(srcm\ SG) \oplus (EsTy\ SG\ \{ecomp\ duni\} \times \{mks\ 1\}) \oplus (EsTy\ SG\ \{erel\ duni\} \times \{\{ms\ *\}\})$$

relation($\mathcal{M}etysOk\ \_$)

$$\mathcal{M}etysOk\_ : \mathbb{P}\, SGr_0$$
$$\forall SG : SGr_0 \bullet \mathcal{M}etysOk\ SG \Leftrightarrow \forall e : EsM\ SG \bullet (ety\ SG\ e) \propto (srcma\ SG\ e, tgtm\ SG\ e)$$

$$\preceq : SGr_0 \to V \leftrightarrow V$$
$$\forall SG : SGr_0 \bullet \preceq SG = (\prec SG)^*$$

$$rsrc, rtgt : SGr_0 \to \mathbb{P}\, E \to E \leftrightarrow V$$
$$src_M^*, src^*, tgt_M^*, tgt^* : SGr_0 \to E \leftrightarrow V$$
$$\forall SG : SGr_0;\ es : \mathbb{P}\, E \bullet rsrc\ SG\ es = es \lhd (sg\_src\ SG)$$
$$\forall SG : SGr_0;\ es : \mathbb{P}\, E \bullet rtgt\ SG\ es = es \lhd (sg\_tgt\ SG)$$
$$\forall SG : SGr_0 \bullet src_M^*\ SG = (rsrc\ SG\ (EsA\ SG)) \fatsemi (\preceq SG)^\sim$$
$$\forall SG : SGr_0 \bullet src^*\ SG = (rsrc\ SG\ (EsA\ SG \cup EsPaCnt\ SG)) \fatsemi (\preceq SG)^\sim$$
$$\forall SG : SGr_0 \bullet tgt_M^*\ SG = (rtgt\ SG\ (EsA\ SG)) \fatsemi (\preceq SG)^\sim$$
$$\forall SG : SGr_0 \bullet tgt^*\ SG = (rtgt\ SG\ (EsA\ SG \cup EsPaCnt\ SG)) \fatsemi (\preceq SG)^\sim$$

Predicate VCntEsOk says whether the value constraint edges of a SG are well-formed.

relation(VCntEsOk_)

$$\text{VCntEsOk\_} : \mathbb{P}\, SGr_0$$
$$\begin{aligned}\forall SG : SGr_0 \bullet \text{VCntEsOk}\, SG \Leftrightarrow vcei\ SG \in EsVCnt\ SG \to VCI\\ \wedge \bigcup(map\ second\ ((\mathrm{ran} \circ vcei)\ SG)) \subseteq EsA\ SG\\ \wedge (sg\_tgt\ SG)\ (\!|\ EsVCnt\ SG\ |\!) \subseteq NsVa\ SG \cup NsP\ SG\end{aligned}$$

relation($inhOk\_$)

$$inhOk\_ : \mathbb{P}\, SGr_0$$
$$\begin{aligned}\forall SG : SGr_0 \bullet inhOk\ SG\\ \Leftrightarrow (\forall v, v' : sg\_Ns\ SG \mid (v, v') \in (\prec SG) \bullet nty\ SG\ v \prec_{NT} nty\ SG\ v') \wedge \otimes(\pitchfork SG)\end{aligned}$$

$$\begin{aligned}SGr == \{SG : SGr_0 \mid \{srcma\ SG, tgtm\ SG\} \subseteq EsM\ SG \to Mult \wedge (\mathrm{dom} \circ pe)\ SG = EsPaCnt\ SG\\ \wedge ds\ SG \in \text{antireflexive}[EsPa\ SG]\\ \wedge \mathcal{M}etysOk\ SG \wedge inhOk\ SG \wedge \text{VCntEsOk}\, SG\}\end{aligned}$$

relation(etherealAreInherited _)

$$\begin{array}{|l} \text{etherealAreInherited\_} : \mathbb{P}\, SGr_0 \\ \hline \forall\, SG : SGr_0 \bullet \text{etherealAreInherited}\, SG \Leftrightarrow NsEther\, SG \subseteq \operatorname{ran}(\prec SG) \end{array}$$

relation($derInhOk$ _)

$$\begin{array}{|l} derInhOk\,\_ : \mathbb{P}\, SGr_0 \\ \hline \forall\, SG : SGr_0 \bullet derInhOk\ SG \Leftrightarrow \forall\, e : EsD\ SG \bullet \\ \quad (sg\_src\, SG\, e, srcPE(gr\ SG \bowtie_{Es}\ EsA\, SG)(pe\, SG\, e)) \in (\preceq SG) \\ \quad \wedge\ (sg\_tgt\, SG\, e, tgtPE(gr\ SG \bowtie_{Es}\ EsA\, SG)(pe\, SG\, e)) \in (\preceq SG) \end{array}$$

relation($okPEA$ _)
relation($okPEC$ _)
relation($okPE$ _)
relation($okPEASrc$ _)
relation($okPEATgt$ _)

$$\begin{array}{|l} okPEASrc\,\_ : \mathbb{P}(SGr_0 \times V \times PEA) \\ okPEATgt\,\_ : \mathbb{P}(SGr_0 \times V \times PEA) \\ \hline \forall\, SG : SGr_0;\ v : V;\ e : E \bullet okPEASrc(SG, v, edg\ e) \Leftrightarrow (e, v) \in src^*\ SG \\ \forall\, SG : SGr_0;\ v : V;\ e : E \bullet okPEASrc(SG, v, einv\ e) \Leftrightarrow (e, v) \in tgt^*\ SG \\ \forall\, SG : SGr_0;\ v : V;\ e : E \bullet okPEATgt(SG, v, edg\ e) \Leftrightarrow (e, v) \in tgt^*\ SG \\ \forall\, SG : SGr_0;\ v : V;\ e : E \bullet okPEATgt(SG, v, einv\ e) \Leftrightarrow (e, v) \in src^*\ SG \end{array}$$

$$\begin{array}{|l} okPEA\,\_ : \mathbb{P}(SGr_0 \times PEA) \\ \hline \forall\, SG : SGr_0;\ e : E \bullet okPEA(SG, edg\ e) \Leftrightarrow e \in sg\_Es\, SG \\ \forall\, SG : SGr_0;\ e : E \bullet okPEA(SG, einv\ e) \Leftrightarrow e \in sg\_Es\, SG \end{array}$$

$$
\begin{array}{|l}
okPEC\,\_ : \mathbb{P}(SGr_0 \times PEC) \\
okPE\,\_ : \mathbb{P}(SGr_0 \times PE) \\
\hline
\forall\, SG : SGr_0;\ v : V;\ pea : PEA \bullet okPEC\,(SG, eat\ pea) \Leftrightarrow okPEA(SG, pea) \\
\forall\, SG : SGr_0;\ v : V;\ pea : PEA \bullet \\
\quad okPEC(SG, eresd\,(v, pea)) \Leftrightarrow okPEA(SG, pea) \land okPEASrc(SG, v, pea) \\
\forall\, SG : SGr_0;\ v : V;\ pea : PEA \bullet \\
\quad okPEC(SG, eresr\,(pea, v)) \Leftrightarrow okPEA(SG, pea) \land okPEATgt(SG, v, pea) \\
\forall\, SG : SGr_0;\ pec : PEC;\ pe : PE \bullet \\
\quad okPE(SG, scmp\,(pec, pe)) \Leftrightarrow okPEC\,(SG, pec) \land okPE\,(SG, pe) \\
\quad \land\ tgtPEC\,(gr\ SG \bowtie_{Es}\ EsA\ SG)\ pec = srcPE\,(gr\ SG \bowtie_{Es}\ EsA\ SG)\ pe
\end{array}
$$

relation(isVCEECnt_)
relation(isVCENCnt_)

$$
\begin{array}{|l}
\mathrm{isVCEECnt\_}, \mathrm{isVCENCnt\_} : \mathbb{P}(SGr \times E) \\
\hline
\forall\, SG : SGr;\ vce : E \bullet \mathrm{isVCEECnt}(SG, vce) \Leftrightarrow (second \circ (vcei\ SG))\ vce \neq \varnothing \\
\forall\, SG : SGr;\ vce : E \bullet \mathrm{isVCENCnt}(SG, vce) \Leftrightarrow (second \circ (vcei\ SG))\ vce = \varnothing
\end{array}
$$

relation(commonAncestor_)
relation(EsVCntsOk_)

$$
\begin{array}{|l}
\mathrm{commonAncestor\_} : \mathbb{P}(SGr_0 \times V \times V) \\
\mathrm{EsVCntsOk\_} : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0;\ n_1, n_2 : V \bullet \mathrm{commonAncestor}\,(SG, n_1, n_2) \Leftrightarrow \exists\, nt : sg\_Ns\ SG \bullet (n_1, nt) \in \preceq SG \land (n_2, nt) \in \preceq \\
\forall\, SG : SGr_0 \bullet \mathrm{EsVCntsOk}\ SG \Leftrightarrow \\
\quad \forall\, vce : EsVCnt\ SG \bullet \mathrm{isVCEECnt}(SG, vce) \Rightarrow \\
\qquad \mathrm{commonAncestor}\,(SG, sg\_tgt\ SG\,((\mathrm{the} \circ second \circ (vcei\ SG))\ vce), sg\_tgt\ SG\ vce) \\
\quad\ \land\ \mathrm{isVCENCnt}(SG, vce) \Rightarrow \mathrm{commonAncestor}\,(SG, sg\_src\ SG\ vce, sg\_tgt\ SG\ vce)
\end{array}
$$

relation($EsCntsOk\,\_$)

$$
\begin{array}{|l}
EsCntsOk\,\_ : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \\
\quad EsCntsOk\ SG \Leftrightarrow derInhOk\ SG \land \forall\, e : EsPaCnt\ SG \bullet okPE(SG, pe\ SG\ e) \\
\quad\ \land\ \mathrm{EsVCntsOk}\ SG
\end{array}
$$

relation$(inhTree\ \_)$

$$
\begin{array}{|l}
inhTree\ \_ : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet inhTree\ SG \Leftrightarrow ((\pitchfork\ SG) \ominus_{Ns} (NsVi\ SG))^{\leftrightarrow} \in \mathrm{tree}
\end{array}
$$

$$TSGr == \{\, SG : SGr \mid \mathrm{etherealAreInherited}\, SG \wedge EsCntsOk\ SG\ \wedge\ inhTree\ SG\}$$

relation$(\boxminus_{SGs}\ \_)$

$$
\begin{array}{|l}
\boxminus_{SGs}\_ : \mathbb{P}(SGr \times SGr) \\
\hline
\forall\, SG_1, SG_2 : SGr \bullet \boxminus_{SGs}(SG_1, SG_2) \Leftrightarrow \boxminus(gr\ SG_1, gr\ SG_2)
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_{SG} \_)$

$$
\begin{array}{|l}
\_ \cup_{SG} \_ : SGr \times SGr \nrightarrow SGr \\
\hline
\forall\, SG_1, SG_2 : SGr \bullet SG_1 \cup_{SG} SG_2 = (gr\ SG_1 \cup_G gr\ SG_2, nty\ SG_1 \cup nty\ SG_2, \\
\quad ety\ SG_1 \cup ety\ SG_2, srcm\ SG_1 \cup srcm\ SG_2, tgtm\ SG_1 \cup tgtm\ SG_2, pe\ SG_1 \cup pe\ SG_2, ds\ SG_1 \cup ds\ SG_2, vcei\ SG_1
\end{array}
$$

function 10 **leftassoc** $(\_ \odot^{SG} \_)$

$$
\begin{array}{|l}
\_ \odot^{SG} \_ : SGr \times (V \nrightarrow V) \nrightarrow SGr \\
\hline
\forall\, SG : SGr;\ s : V \nrightarrow V \mid s \in NsP\ SG \nrightarrow sg\_Ns\ SG \bullet \\
\quad SG\ \odot^{SG}\ s = (gr\ SG \odot s, (\mathrm{dom}\ s \setminus \mathrm{ran}\ s) \lhd nty\ SG, ety\ SG, srcm\ SG, tgtm\ SG, pe\ SG, ds\ SG, vcei\ SG)
\end{array}
$$

function 10 **leftassoc** $(\_ \rightarrow_{SG} \_)$

$$
\begin{array}{|l}
\_ \rightarrow_{SG} \_ : SGr \times SGr \rightarrow \mathbb{P}\, GrM \\
\hline
\forall\, SG_s, SG_t : SGr \bullet \\
\quad SG_s \rightarrow_{SG} SG_t = \{fv : sg\_Ns\ SG_s \rightarrow sg\_Ns\ SG_t;\ fe : EsA\ SG_s \rightarrow EsA\ SG_t \mid \\
\qquad fv \circ (src^*_M\ SG_s) \subseteq src^*_M\ SG_t \circ fe \wedge fv \circ (tgt^*_M\ SG_s) \subseteq tgt^*_M\ SG_t \circ fe \\
\qquad \wedge fv \circ \preceq SG_s \subseteq \preceq SG_t \circ fv\}
\end{array}
$$

function 10 **leftassoc** $(\_ \nrightarrow_{SG} \_)$

$$
\begin{array}{|l}
\_ \nrightarrow_{SG} \_ : SGr \times SGr \rightarrow \mathbb{P}\, GrM \\
\hline
\forall\, SG_s, SG_t : SGr \bullet \\
\quad SG_s \nrightarrow_{SG} SG_t = \{ fv : sg\_Ns\, SG_s \nrightarrow sg\_Ns\, SG_t;\ fe : EsA\, SG_s \nrightarrow EsA\, SG_t \mid \\
\qquad fv \circ ((\mathrm{dom}\, fe) \lhd (src_M^*\ SG_s)) \subseteq src_M^*\ SG_t \circ fe \land fv \circ ((\mathrm{dom}\, fe) \lhd (tgt_M^*\ SG_s)) \subseteq tgt_M^*\ SG_t \circ fe \\
\qquad \land fv \circ \preceq SG_s \subseteq \preceq SG_t \circ fv \}
\end{array}
$$

relation $(\_ \Rrightarrow^{SG} \_)$

$$
\begin{array}{|l}
\_ \Rrightarrow^{SG} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_s, SG_t : SGr;\ m : GrM \bullet (SG_s, m) \Rrightarrow^{SG} SG_t \Leftrightarrow m \in SG_s \rightarrow_{SG} SG_t
\end{array}
$$

function 10 **leftassoc** $(\_ \rightarrow_{G2SG} \_)$

$$
\begin{array}{|l}
\_ \rightarrow_{G2SG} \_ : Gr \times SGr \rightarrow \mathbb{P}\, GrM \\
\hline
\forall\, G : Gr;\ SG : SGr \bullet G \rightarrow_{G2SG} SG = \{ fv : Ns\ G \rightarrow sg\_Ns\, SG;\ fe : Es\ G \rightarrow EsA\, SG \mid \\
\quad fv \circ src\ G \subseteq src_M^*\ SG \circ fe \land fv \circ tgt\ G \subseteq tgt_M^*\ SG \circ fe \}
\end{array}
$$

relation $(\_ \Rrightarrow^{GwT} \_)$

$$
\begin{array}{|l}
\_ \Rrightarrow^{GwT} \_ : (GrwT \leftrightarrow SGr) \\
\hline
\forall\, GwT : GrwT;\ SG : SGr \bullet GwT \Rrightarrow^{GwT} SG \Leftrightarrow GwT^{\,T} \in GwT^{\,G} \rightarrow_{G2SG} SG
\end{array}
$$

relation $(\_ \sqsupseteq^{SG} \_)$
relation $(\_ \sqsupseteq^{SG_0} \_)$
relation $(\_ \sqsupseteq^{NT} \_)$
relation $(\_ \sqsupseteq^{ET} \_)$
relation $(\_ \sqsupseteq^{\mathcal{M}} \_)$
relation $(\_ \sqsupseteq_{\mathcal{M}Cnts} \_)$

$$\_ \sqsupseteq^{NT} \_, \_ \sqsupseteq^{ET} \_ : \mathbb{P}((SGr \times GrM) \times SGr)$$

$$\forall SG_c, SG_a : SGr;\ m : GrM \bullet$$
$$(SG_c, m) \sqsupseteq^{NT} SG_a \Leftrightarrow \forall n : sg\_Ns\, SG_c \bullet (nty\, SG_c)\, n \leq_{rNT} ((nty\, SG_a) \circ (fV\ m))\, n$$

$$\forall SG_c, SG_a : SGr;\ m : GrM \bullet$$
$$(SG_c, m) \sqsupseteq^{ET} SG_a \Leftrightarrow \forall e : EsA\, SG_c \bullet (ety\, SG_c)\, e \leq_{ET} ((ety\, SG_a) \circ (fE\ m))\, e$$

$$sPEA, tPEA : SGr \rightarrow PEA \rightarrow (E \nrightarrow V)$$
$$smfPEA, tmfPEA : SGr \rightarrow PEA \rightarrow (E \nrightarrow Mult)$$

$$\forall SG : SGr;\ e : E \bullet sPEA\, SG\, (edg\ e) = sg\_src\, SG$$

$$\forall SG : SGr;\ e : E \bullet sPEA\, SG\, (einv\ e) = sg\_tgt\, SG$$

$$\forall SG : SGr;\ e : E \bullet tPEA\, SG\, (edg\ e) = sg\_tgt\, SG$$

$$\forall SG : SGr;\ e : E \bullet tPEA\, SG\, (einv\ e) = sg\_src\, SG$$

$$\forall SG : SGr;\ e : E \bullet smfPEA\, SG\, (edg\ e) = srcma\, SG$$

$$\forall SG : SGr;\ e : E \bullet smfPEA\, SG\, (einv\ e) = tgtm\, SG$$

$$\forall SG : SGr;\ e : E \bullet tmfPEA\, SG\, (edg\ e) = tgtm\, SG$$

$$\forall SG : SGr;\ e : E \bullet tmfPEA\, SG\, (einv\ e) = srcma\, SG$$

relation($affectedPE\ \_$)
relation($affectedPECStart\ \_$)
relation($affectedPECEnd\ \_$)
relation($affectedPEAStart\ \_$)
relation($affectedPEAEnd\ \_$)
relation($affectedPEStart\ \_$)
relation($affectedPEEnd\ \_$)

$$
\begin{array}{l}
affectedPECStart\,\_, affectedPECEnd\,\_ : \mathbb{P}(SGr \times GrM \times SGr \times PEC \times E \times E) \\
affectedPEAStart\,\_, affectedPEAEnd\,\_ : \mathbb{P}(SGr \times GrM \times SGr \times PEA \times E \times E)
\end{array}
$$

$$
\begin{array}{l}
\forall\, SG_c, SG_a : SGr;\ m : GrM;\ pea : PEA;\ e, ie : E \bullet affectedPEAStart(SG_c, m, SG_a, pea, e, ie) \\
\quad \Leftrightarrow ie \in (fE\ m)^{\sim} \ (\!|\{ ePEA\ pea\}|\!) \wedge (fV\ m)(sPEA\ SG_c\ pea\ ie) = sPEA\ SG_a\ pea\ e \\[1ex]
\forall\, SG_c, SG_a : SGr;\ m : GrM;\ pea : PEA;\ e, ie : E \bullet affectedPEAEnd(SG_c, m, SG_a, pea, e, ie) \\
\quad \Leftrightarrow ie \in (fE\ m)^{\sim} \ (\!|\{ ePEA\ pea\}|\!) \wedge (fV\ m)(tPEA\ SG_c\ pea\ ie) = tPEA\ SG_a\ pea\ e \\[1ex]
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ e, ie : E \bullet affectedPECStart(SG_c, m, SG_a, eat\ pea, e, ie) \\
\quad \Leftrightarrow affectedPEAStart(SG_c, m, SG_a, pea, e, ie) \\[1ex]
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ v : V;\ e, ie : E \bullet \\
\quad affectedPECStart(SG_c, m, SG_a, eresd\ (v, pea), e, ie) \\
\quad \Leftrightarrow affectedPEAStart(SG_c, m, SG_a, pea, e, ie) \\
\qquad \wedge \exists\, v' : (fV\ m)^{\sim} \ (\!|\{v\}|\!) \bullet (sPEA\ SG_c\ pea\ ie, v') \in \preceq SG_c \\[1ex]
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ e, ie : E \bullet affectedPECEnd(SG_c, m, SG_a, eat\ pea, e, ie) \\
\quad \Leftrightarrow affectedPEAEnd(SG_c, m, SG_a, pea, e, ie) \\[1ex]
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ v : V;\ e, ie : E \bullet \\
\quad affectedPECEnd(SG_c, m, SG_a, eresd\ (v, pea), e, ie) \\
\quad \Leftrightarrow affectedPEAEnd(SG_c, m, SG_a, pea, e, ie) \\
\qquad \wedge \exists\, v' : (fV\ m)^{\sim} \ (\!|\{v\}|\!) \bullet (sPEA\ SG_c\ pea\ ie, v') \in \preceq SG_c \\[1ex]
\forall\, SG_a, SG_c : SGr;\ m : GrM;\ pea : PEA;\ v : V;\ e, ie : E \bullet \\
\quad affectedPECEnd(SG_c, m, SG_a, eresr\ (pea, v), e, ie) \\
\quad \Leftrightarrow affectedPEAEnd(SG_c, m, SG_a, pea, e, ie) \\
\qquad \wedge \exists\, v' : (fV\ m)^{\sim} \ (\!|\{v\}|\!) \bullet (tPEA\ SG_c\ pea\ ie, v') \in \preceq SG_c
\end{array}
$$

$$
\begin{array}{l}
affectedPEStart\,\_, affectedPEEnd\,\_ : \mathbb{P}(SGr \times GrM \times SGr \times PE \times E \times E) \\
affectedPE\,\_ : \mathbb{P}(SGr \times GrM \times SGr \times PE \times E \times E \times E) \\
\hline
\forall SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ e, ie : E \bullet \\
\quad affectedPEStart(SG_c, m, SG_a, ec\, pec, e, ie) \Leftrightarrow affectedPECStart(SG_c, m, SG_a, pec, e, ie) \\
\forall SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ pe : PE;\ e, ie : E \bullet \\
\quad affectedPEStart(SG_c, m, SG_a, scmp\,(pec, pe), e, ie) \Leftrightarrow affectedPECStart(SG_c, m, SG_a, pec, e, ie) \\
\forall SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ e, ie : E \bullet \\
\quad affectedPEEnd(SG_c, m, SG_a, ec\, pec, e, ie) \Leftrightarrow affectedPECEnd(SG_c, m, SG_a, pec, e, ie) \\
\forall SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ pe : PE;\ e, ie : E \bullet \\
\quad affectedPEEnd(SG_c, m, SG_a, scmp\,(pec, pe), e, ie) \Leftrightarrow affectedPEEnd(SG_c, m, SG_a, pe, e, ie) \\
\forall SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ e, ie, ie' : E \bullet \\
\quad affectedPE(SG_c, m, SG_a, ec\, pec, e, ie, ie') \\
\quad \Leftrightarrow ie = ie' \land affectedPECStart(SG_c, m, SG_a, pec, e, ie) \\
\quad \land\ affectedPECEnd(SG_c, m, SG_a, pec, e, ie) \\
\forall SG_a, SG_c : SGr;\ m : GrM;\ pec_1, pec_2 : PEC;\ e, ie, ie' : E \bullet \\
\quad affectedPE(SG_c, m, SG_a, scmp\,(pec_1, ec\, pec_2), e, ie, ie') \\
\quad \Leftrightarrow tPEA\, SG_c\,(endEA_C\, pec_1)\, ie = sPEA\, SG_c\,(startEA_C\, pec_2)\, ie' \\
\qquad \land\ affectedPECStart(SG_c, m, SG_a, pec_1, e, ie) \land affectedPECEnd(SG_c, m, SG_a, pec_2, e, ie') \\
\forall SG_a, SG_c : SGr;\ m : GrM;\ pec : PEC;\ pe : PE;\ e, ie, ie' : E \bullet \\
\quad affectedPE(SG_c, m, SG_a, scmp\,(pec, pe), e, ie, ie') \\
\quad \Leftrightarrow affectedPECStart(SG_c, m, SG_a, pec, e, ie) \\
\qquad \land\ \exists\, ie'' : E \mid tPEA\, SG_c\,(endEA_C\, pec)\, ie = sPEA\, SG_c\,(startEA\, pe)\, ie'' \bullet \\
\qquad\quad affectedPE(SG_c, m, SG_a, pe, e, ie'', ie')
\end{array}
$$

relation$(caseMultsOk\,\_)$
relation$(caseMandatoryT\,\_)$
relation$(caseMandatoryS\,\_)$

$$
\begin{array}{l}
caseMultsOk\,\_ : \mathbb{P}(SGr \times GrM \times SGr \times E \times PEA \times PEA \times E \times E) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM;\ peas : PEA;\ peae : PEA;\ e, ie, ie' : E \bullet \\
\quad caseMultsOk(SG_c, m, SG_a, e, peas, peae, ie, ie') \\
\quad \Leftrightarrow affectedPE(SG_c, m, SG_a, pe\, SG_a\, e, e, ie, ie') \\
\qquad \Rightarrow smfPEA\, SG_c\, peas\, ie \leq_{\mathcal{M}} srcma\, SG_a\, e \land tmfPEA\, SG_c\, peae\, ie' \leq_{\mathcal{M}} tgtm\, SG_a\, e
\end{array}
$$

$$
\begin{array}{l}
caseMandatoryT\,\_ : \mathbb{P}(SGr \times GrM \times SGr \times E \times PEA \times E) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM;\ peas : PEA;\ peae : PEA;\ e, ie : E \bullet \\
\quad caseMandatoryT(SG_c, m, SG_a, e, peae, ie) \\
\quad \Leftrightarrow (affectedPEStart(SG_c, m, SG_a, pe\, SG_a\, e, e, ie) \land mks\, 1 \leq_{\mathcal{M}} (tgtm\, SG_a)\, e) \\
\qquad \Rightarrow (fV\ m)(tPEA\, SG_c\, peae\, ie) = sg\_tgt\, SG_a\, e
\end{array}
$$

$$
\begin{array}{|l}
\mathit{caseMandatoryS}\,\_ : \mathbb{P}(SGr \times GrM \times SGr \times E \times PEA \times E) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM;\ peas : PEA;\ e, ie : E \bullet \\
\quad \mathit{caseMandatoryS}(SG_c, m, SG_a, e, peas, ie) \\
\quad \Leftrightarrow (\mathit{affectedPEEnd}(SG_c, m, SG_a, pe\, SG_a\, e, e, ie) \land mks\, 1 \leq_{\mathcal{M}} (srcma\, SG_a)\, e) \\
\qquad \Rightarrow (fV\ m)(sPEA\, SG_c\, peas\ ie) = sg\_src\, SG_a\, e
\end{array}
$$

$$
\begin{array}{|l}
\_ \sqsupseteq_{\mathcal{M}Cnts} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\_ \sqsupseteq^{\mathcal{M}} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq^{\mathcal{M}}\ SG_a \Leftrightarrow \forall\, e : EsA\, SG_c \bullet (srcma\, SG_c)\, e \leq_{\mathcal{M}} ((srcma\, SG_a) \circ (fE\ m))\, e \\
\qquad\qquad\qquad \land\, (tgtm\, SG_c)\, e \leq_{\mathcal{M}} ((tgtm\, SG_a) \circ (fE\ m))\, e \\
\\
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet (SG_c, m) \sqsupseteq_{\mathcal{M}Cnts}\ SG_a \Leftrightarrow \forall\, e : EsD\, SG_a \bullet \\
\quad \exists\, peas : PEA;\ peae : PEA \mid peas = startEA(pe\, SG_a\, e) \land peae = endEA(pe\, SG_a\, e) \bullet \\
\qquad \forall\, e' : (fE\ m)^\sim \,(\!|\{ePEA\, peas\}|\!);\ e'' : (fE\ m)^\sim \,(\!|\{ePEA\, peae\}|\!) \bullet \\
\qquad\quad \mathit{caseMultsOk}(SG_c, m, SG_a, e, peas, peae, e', e'') \\
\qquad\quad \land\, (\mathit{caseMandatoryT}(SG_c, m, SG_a, e, peae, e') \lor \mathit{caseMandatoryS}(SG_c, m, SG_a, e, peas, e''))
\end{array}
$$

$$
\begin{array}{|l}
\_ \sqsupseteq^{SG} \_, \_ \sqsupseteq^{SG_0} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq^{SG_0}\ SG_a \Leftrightarrow (SG_c, m) \sqsupseteq^{NT}\ SG_a \land (SG_c, m) \sqsupseteq^{ET}\ SG_a \land (SG_c, m) \sqsupseteq^{\mathcal{M}}\ SG_a \\
\qquad \land\, (SG_c, m) \sqsupseteq_{\mathcal{M}Cnts}\ SG_a \\
\\
\forall\, SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq^{SG}\ SG_a \Leftrightarrow m \in SG_c \to_{SG} SG_a \land (SG_c, m) \sqsupseteq^{SG_0}\ SG_a
\end{array}
$$

$$
\begin{array}{|l}
ins : GrM \times SGr \times \mathbb{P}\ V \to \mathbb{P}\ V \\
ies : GrM \times \mathbb{P}\, E \to \mathbb{P}\, E \\
\hline
\forall\, m : GrM;\ SG : SGr;\ mns : \mathbb{P}\ V \bullet ins\,(m, SG, mns) = (fV\ m)^\sim \,(\!|(\prec\ SG)^\sim \,(\!|mns|\!)\,|\!) \\
\\
\forall\, m : GrM;\ mes : \mathbb{P}\, E \bullet ies\,(m, mes) = (fE\ m)^\sim \,(\!|mes|\!)
\end{array}
$$

$$
\begin{array}{|l}
igRMEs : GrwT \times \mathbb{P}\, E \to Gr \\
\hline
\forall\, GwT : GrwT;\ mes : \mathbb{P}\, E \bullet igRMEs(GwT, mes) = GwT^G \rtimes_{Es} ies\,(GwT^T,\ mes)
\end{array}
$$

relation$(\_ \sqsupseteq^{SG} \_)$
relation$(\_ \sqsupseteq^{SG_0} \_)$
relation$(\_ \sqsupseteq_{AEs} \_)$
relation(_OkRefinedIn_)
relation$(\_ \sqsupseteq_{ANNs} \_)$

$$
\begin{array}{|l}
\_ \sqsupset_{ANNs} \_ : \mathbb{P}(GrM \times SGr) \\
\hline
\forall SG_a : SGr;\ m : GrM \bullet \\
\quad m \sqsupset_{ANNs} SG_a \Leftrightarrow \forall nn : NsTy\, SG_a\{nnrml\} \bullet (\preceq SG_a) (\!|\, \{nn\} \,|\!) \cap \operatorname{ran}(fV\ m) = \varnothing
\end{array}
$$

$$
\begin{array}{|l}
\_\mathrm{OkRefinedIn}\_ : \mathbb{P}((SGr \times E) \times (SGr \times GrM)) \\
\_ \sqsupset_{AEs} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM;\ ae : E \bullet \\
\quad (SG_a, ae)\mathrm{OkRefinedIn}(SG_c, m) \Leftrightarrow \\
\qquad \exists r : V \leftrightarrow V;\ s, t : \mathbb{P}\, V \mid r = (\preceq SG_c) \fatsemi igRMEs((gr\ SG_c, m), \{ae\})^{\leftrightarrow} \fatsemi (\preceq SG_c)^{\sim} \\
\qquad\quad \wedge\ s = ins\,(m, SG_a, sg\_src\ SG_a (\!|\, \{ae\} \,|\!)) \setminus ((NsEther\ SG_c) \setminus \operatorname{dom} r) \\
\qquad\quad \wedge\ t = ins\,(m, SG_a, sg\_tgt\ SG_a (\!|\, \{ae\} \,|\!)) \setminus ((NsEther\ SG_c) \setminus \operatorname{ran} r) \\
\qquad\qquad \bullet\ r \in s \leftrightarrow t \wedge nty\ SG_a (\!|\, (gr\ SG_a) \circ\!\!\rightarrow\!\!\circ_{Ns} \{ae\} \,|\!) \neq \varnothing \Rightarrow r \neq \varnothing \\
\\
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset_{AEs} SG_a \Leftrightarrow \forall e : (EsA\ SG_a) \bullet (SG_a, e)\mathrm{OkRefinedIn}(SG_c, m)
\end{array}
$$

$$
\begin{array}{|l}
\_ \sqsupset^{SG} \_, \_ \sqsupset^{SG_0} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset^{SG_0} SG_a \Leftrightarrow (SG_c, m) \sqsupseteq^{SG_0} SG_a \wedge m \sqsupset_{ANNs} SG_a \wedge (SG_c, m) \sqsupset_{AEs} SG_a \\
\\
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset^{SG} SG_a \Leftrightarrow m \in SG_c \to_{SG} SG_a \wedge (SG_c, m) \sqsupset^{SG_0} SG_a
\end{array}
$$

relation$(\_ \dashv\vdash_{\mathcal{M}} \_)$
relation$(\_ \dashv\vdash_{NT} \_)$
relation$(\_ \dashv\vdash_{ET} \_)$
relation$(\_ \dashv\vdash^{SG} \_)$

$$
\begin{array}{|l}
\_ \dashv\vdash_{\mathcal{M}} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_s, SG_t : SGr;\ m : GrM \bullet \\
\quad (SG_s, m) \dashv\vdash_{\mathcal{M}} SG_t \Leftrightarrow \forall ep : fE\ m \bullet (srcma\ SG_s)\,(first\ ep) \leq_{\mathcal{M}} (srcma\ SG_t)\,(second\ ep) \\
\qquad\qquad\qquad \wedge\ (tgtm\ SG_s)\,(first\ ep) \leq_{\mathcal{M}} (tgtm\ SG_t)\,(second\ ep)
\end{array}
$$

$$
\begin{array}{|l}
\_ \dashv\vdash_{NT} \_ : \mathbb{P}(GrM \times SGr) \\
\_ \dashv\vdash_{ET} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_t : SGr;\ m : GrM \bullet \\
\quad m \dashv\vdash_{NT} SG_t \Leftrightarrow (nty\ SG_t) (\!|\, \operatorname{ran}(fV\ m) \,|\!) \subseteq \{nnrml, nabst, nvirt\} \\
\\
\forall SG_s, SG_t : SGr;\ m : GrM \bullet \\
\quad (SG_s, m) \dashv\vdash_{ET} SG_t \Leftrightarrow \operatorname{dom}(fE\ m) \lhd (ety\ SG_s) = (ety\ SG_t) \circ (fE\ m)
\end{array}
$$

$$\_ \dashv\vdash^{SG} \_ : \mathbb{P}((SGr \times GrM) \times SGr)$$

$$\forall SG_s, SG_t : SGr;\ m : GrM \bullet (SG_s, m) \dashv\vdash^{SG} SG_t \Leftrightarrow m \in SG_s \twoheadrightarrow_{SG} SG_t \land (SG_s, m) \dashv\vdash_{\mathcal{M}} SG_t \land m \dashv\vdash_{NT} SG_t \land (SG_s, m) \dashv\vdash_{ET} SG_t$$

$$rPEA : GrwT \times SGr \times PEA \to (V \leftrightarrow V)$$

$$\forall GwT : GrwT;\ SG : SGr;\ e : E \bullet rPEA\,(GwT, SG, edg\ e) = (GwT^{G} \bowtie_{Es} ies\,(GwT^{T}, \{e\}))^{\leftrightarrow}$$

$$\forall GwT : GrwT;\ SG : SGr;\ e : E \bullet rPEA\,(GwT, SG, einv\ e) = rPEA\,(GwT, SG, edg\ e)^{\sim}$$

$$rPEC : GrwT \times SGr \times PEC \to (V \leftrightarrow V)$$
$$rPE : GrwT \times SGr \times PE \to (V \leftrightarrow V)$$

$$\forall GwT : GrwT;\ SG : SGr;\ pea : PEA \bullet rPEC\,(GwT, SG, eat\ pea) = rPEA\,(GwT, SG, pea)$$

$$\forall GwT : GrwT;\ SG : SGr;\ v : V;\ pea : PEA \bullet rPEC\,(GwT, SG, eresd\,(v, pea)) = ins\,(GwT^{T},\ SG, \{v\}) \lhd rPEA(GwT, SG, pea)$$

$$\forall GwT : GrwT;\ SG : SGr;\ v : V;\ pea : PEA \bullet rPEC\,(GwT, SG, eresr\,(pea, v)) = rPEA\,(GwT, SG, pea) \rhd ins\,(GwT^{T},\ SG,\ \{v\})$$

$$\forall GwT : GrwT;\ SG : SGr;\ pec : PEC \bullet rPE\,(GwT, SG, ec\ pec) = rPEC\,(GwT, SG, pec)$$

$$\forall GwT : GrwT;\ SG : SGr;\ pec : PEC;\ pe : PE \bullet rPE\,(GwT, SG, scmp\,(pec, pe)) = rPEC\,(GwT, SG, pec) \fatsemi rPE\,(GwT, SG, pe)$$

$$ape : SGr \to E \twoheadrightarrow PE$$

$$\forall SG : SGr \bullet ape\ SG = (\lambda\, e : E \mid e \in EsM\ SG \bullet (ec \circ eat \circ edg)\ e) \oplus pe\ SG$$

$$src^{*}_{PEA} : SGr \times PEA \to E \leftrightarrow V$$
$$src^{*}_{PEC} : SGr \times PEC \to E \leftrightarrow V$$
$$src^{*}_{PE} : SGr \times PE \to E \leftrightarrow V$$

$$\forall SG : SGr;\ e : E \bullet src^{*}_{PEA}(SG, edg\ e) = src^{*}\ SG$$

$$\forall SG : SGr;\ e : E \bullet src^{*}_{PEA}(SG, einv\ e) = tgt^{*}\ SG$$

$$\forall SG : SGr;\ pea : PEA \bullet src^{*}_{PEC}(SG, eat\ pea) = src^{*}_{PEA}(SG, pea)$$

$$\forall SG : SGr;\ v : V;\ pea : PEA \bullet src^{*}_{PEC}(SG, eresd\,(v, pea)) = src^{*}_{PEA}(SG, pea) \rhd \{v\}$$

$$\forall SG : SGr;\ v : V;\ pea : PEA \bullet src^{*}_{PEC}(SG, eresr\,(pea, v)) = src^{*}_{PEA}(SG, pea)$$

$$\forall SG : SGr;\ v : V;\ pec : PEC;\ pe : PE \bullet src^{*}_{PE}(SG, (scmp\,(pec, pe))) = src^{*}_{PEC}(SG, pec)$$

$$
\begin{array}{|l}
tgt^*_{PEA} : SGr \times PEA \rightarrow E \leftrightarrow V \\
tgt^*_{PEC} : SGr \times PEC \rightarrow E \leftrightarrow V \\
tgt^*_{PE} : SGr \times PE \rightarrow E \leftrightarrow V \\
\hline
\forall SG : SGr;\ e : E \bullet tgt^*_{PEA}(SG, edg\ e) = tgt^*\ SG \\
\forall SG : SGr;\ e : E \bullet tgt^*_{PEA}(SG, einv\ e) = src^*\ SG \\
\forall SG : SGr;\ pea : PEA \bullet tgt^*_{PEC}(SG, eat\ pea) = tgt^*_{PEA}(SG, pea) \\
\forall SG : SGr;\ v : V;\ pea : PEA \bullet tgt^*_{PEC}(SG, eresd\ (v, pea)) = tgt^*_{PEA}(SG, pea) \\
\forall SG : SGr;\ v : V;\ pea : PEA \bullet tgt^*_{PEC}(SG, eresr\ (pea, v)) = tgt^*_{PEA}(SG, pea) \rhd \{v\} \\
\forall SG : SGr;\ v : V;\ pec : PEC \bullet tgt^*_{PE}(SG, ec\ pec) = tgt^*_{PEC}(SG, pec) \\
\forall SG : SGr;\ v : V;\ pec : PEC;\ pe : PE \bullet tgt^*_{PE}(SG, (scmp\ (pec, pe))) = tgt^*_{PE}(SG, pe)
\end{array}
$$

relation$(\_ \Supset^{SG} \_)$
relation$(\_ \Supset_{\mathcal{M}} \_)$
relation$(\_ \Supset_{FI} \_)$
relation$(\_ \Supset_{PNS} \_)$
relation$(\_ \Supset_{Cnts} \_)$
relation$(\_\ MEMOk\ \_)$

$$
\begin{array}{|l}
rME\mathcal{M}Ok : SGr \times E \times GrwT \rightarrow V \leftrightarrow V \\
\hline
\forall SG : SGr;\ me : E;\ GwT : GrwT;\ s, t : \mathbb{P}\ V \mid s = ins(GwT^T, SG, src^*\ SG\ (\!|\ \{me\}\ |\!)) \\
\quad \wedge\ t = ins(GwT^T, SG, tgt^*\ SG\ (\!|\ \{me\}\ |\!)) \bullet \\
\qquad rME\mathcal{M}Ok(SG, me, GwT) = s \lhd rPE\ (GwT, SG, ape\ SG\ me) \rhd\ t
\end{array}
$$

$$
\begin{array}{|l}
src^*ME\mathcal{M}Ok : SGr \times E \rightarrow E \leftrightarrow V \\
\hline
\forall SG : SGr;\ me : E \bullet src^*ME\mathcal{M}Ok(SG, me) = src^*_{PE}(SG, ape\ SG\ me) \rhd (src^*\ SG)\ (\!|\ \{me\}\ |\!)
\end{array}
$$

$$
\begin{array}{|l}
tgt^*ME\mathcal{M}Ok : SGr \times E \rightarrow E \leftrightarrow V \\
\hline
\forall SG : SGr;\ me : E \bullet tgt^*ME\mathcal{M}Ok(SG, me) = tgt^*_{PE}(SG, ape\ SG\ me) \rhd (tgt^*\ SG)\ (\!|\ \{me\}\ |\!)
\end{array}
$$

$$multComp : Mult \times Mult \rightarrow Mult$$

$$\forall m_1, m_2 : Mult \mid m_1 \in MultMany \lor m_2 \in MultMany \bullet multComp(m_1, m_2) = mmanys$$

$$\forall m_1, m_2 : Mult \mid m_1 = mks\,0 \lor m_2 = mks\,0 \bullet multComp(m_1, m_2) = mks\,0$$

$$\forall m_1, m_2 : Mult \mid m_2 = mks\,1 \bullet multComp(m_1, m_2) = m_1$$

$$\forall m_1, m_2 : Mult \mid m_1 = mks\,1 \bullet multComp(m_1, m_2) = m_2$$

$$\forall m_1, m_2 : Mult \mid m_1 = mopts \bullet multComp(m_1, m_2) = mks\,0 \cup m_2$$

$$\forall m_1, m_2 : Mult \mid m_2 = mopts \bullet multComp(m_1, m_2) = mks\,0 \cup m_1$$

$$\forall m_1, m_2 : MultRange \bullet \\ multComp(m_1, m_2) = \{(\mathrm{the}\, m_1) *_{mr} (\mathrm{the}\, m_2)\}$$

$$\forall m_1 : Mult;\ mc : MultC;\ m_2 : MultEither \bullet \\ multComp(m_1, \{mc\} \cup m_2) = multComp(m_1, \{mc\}) \cup multComp(m_1, m_2)$$

$$\forall m_1 : MultEither;\ mc : MultC;\ m_2 : Mult \bullet \\ multComp(\{mc\} \cup m_1, m_2) = multComp(\{mc\}, m_2) \cup multComp(m_1, m_2)$$

$$smPEA : SGr \rightarrow PEA \rightarrow Mult \\ smPEC : SGr \rightarrow PEC \rightarrow Mult \\ smPE : SGr \rightarrow PE \rightarrow Mult$$

$$\forall SG : SGr;\ e : E \bullet smPEA\, SG\, (edg\, e) = srcma\, SG\, e$$

$$\forall SG : SGr;\ e : E \bullet smPEA\, SG\, (einv\, e) = tgtm\, SG\, e$$

$$\forall SG : SGr;\ pea : PEA \bullet smPEC\, SG\, (eat\, pea) = smPEA\, SG\, pea$$

$$\forall SG : SGr;\ v : V;\ pea : PEA \bullet smPEC\, SG\, (eresd\, (v, pea)) = smPEA\, SG\, pea$$

$$\forall SG : SGr;\ v : V;\ pea : PEA \bullet smPEC\, SG\, (eresr\, (pea, v)) = smPEA\, SG\, pea$$

$$\forall SG : SGr;\ pec : PEC \bullet smPE\, SG\, (ec\, pec) = smPEC\, SG\, pec$$

$$\forall SG : SGr;\ pec : PEC;\ pe : PE \bullet smPE\, SG\, (scmp\, (pec, pe)) = multComp(smPEC\, SG\, pec, smPE\, SG\, pe)$$

$$tmPEA : SGr \rightarrow PEA \rightarrow Mult \\ tmPEC : SGr \rightarrow PEC \rightarrow Mult \\ tmPE : SGr \rightarrow PE \rightarrow Mult$$

$$\forall SG : SGr;\ e : E \bullet tmPEA\, SG\, (edg\, e) = tgtm\, SG\, e$$

$$\forall SG : SGr;\ e : E \bullet tmPEA\, SG\, (einv\, e) = srcma\, SG\, e$$

$$\forall SG : SGr;\ pea : PEA \bullet tmPEC\, SG\, (eat\, pea) = tmPEA\, SG\, pea$$

$$\forall SG : SGr;\ v : V;\ pea : PEA \bullet tmPEC\, SG\, (eresd\, (v, pea)) = tmPEA\, SG\, pea$$

$$\forall SG : SGr;\ v : V;\ pea : PEA \bullet tmPEC\, SG\, (eresr\, (pea, v)) = tmPEA\, SG\, pea$$

$$\forall SG : SGr;\ pec : PEC \bullet tmPE\, SG\, (ec\, pec) = tmPEC\, SG\, pec$$

$$\forall SG : SGr;\ pec : PEC;\ pe : PE \bullet tmPE\, SG\, (scmp\, (pec, pe)) = multComp(tmPEC\, SG\, pec, tmPE\, SG\, pe)$$

$$
\begin{array}{|l}
\_\,\mathit{MEMOk}\,\_ : \mathbb{P}((SGr \times E) \times GrwT) \\
\hline
\forall\, GwT : GrwT;\ SG : SGr;\ me : E \bullet (SG, me)\ \mathit{ME\mathcal{M}Ok}\ GwT \Leftrightarrow \\
\quad \exists\, r : V \leftrightarrow V;\ s, t : \mathbb{P}\ V \mid r = rMEMOk\,(SG, me, GwT) \\
\quad\quad \land\ s = ins(GwT^{T}, SG, src^{*}\,\mathit{ME\mathcal{M}Ok}\,(SG, me)\ (\!|\ \{rsrcPE(ape\ SG\ me)\}\ |\!)) \\
\quad\quad \land\ t = ins(GwT^{T}, SG, tgt^{*}\,\mathit{ME\mathcal{M}Ok}\,(SG, me)\ (\!|\ \{rtgtPE(ape\ SG\ me)\}\ |\!)) \\
\quad\quad\quad \bullet\ r\mathcal{M}Ok(r, s, t, smPE\ SG\ (ape\ SG\ me), tmPE\ SG\ (ape\ SG\ me))
\end{array}
$$

relation(INumbersOk_)
relation(I$\mathbb{N}$sOk_)
relation(I$\mathbb{Z}$sOk_)
relation(I$\mathbb{R}$sOk_)

$$
\begin{array}{|l}
\mathrm{I\mathbb{N}sOk\_}, \mathrm{I\mathbb{Z}sOk\_}, \mathrm{I\mathbb{R}sOk\_}, \mathrm{INumbersOk\_} : \mathbb{P}\ GrwT \\
\hline
\forall\, GwT : GrwT \bullet \mathrm{I\mathbb{N}sOk}\ GwT \Leftrightarrow \\
\quad \forall\, nnat : (fV\ (GwT^{T}))^{\sim}\ (\!|\{nNatS\}\ |\!) \bullet \exists\, n : \mathbb{N} \bullet n \in \mathrm{to\mathbb{Z}}\ nnat \\
\forall\, GwT : GrwT \bullet \mathrm{I\mathbb{Z}sOk}\ GwT \Leftrightarrow \\
\quad \forall\, nint : (fV\ (GwT^{T}))^{\sim}\ (\!|\{nIntS\}\ |\!) \bullet \exists\, n : \mathbb{Z} \bullet n \in \mathrm{to\mathbb{Z}}\ nint \\
\forall\, GwT : GrwT \bullet \mathrm{I\mathbb{R}sOk}\ GwT \Leftrightarrow \\
\quad \forall\, nreal : (fV\ (GwT^{T}))^{\sim}\ (\!|\{nRealS\}\ |\!) \bullet \exists\, x : \mathbb{R} \bullet x \in \mathrm{to\mathbb{R}}\ nreal \\
\forall\, SG : SGr;\ GwT : GrwT \bullet \mathrm{INumbersOk}\ GwT \Leftrightarrow \\
\quad \mathrm{I\mathbb{N}sOk}\ GwT \land \mathrm{I\mathbb{Z}sOk}\ GwT \land \mathrm{I\mathbb{R}sOk}\ GwT
\end{array}
$$

relation(satisfiesCnt_)
relation(satisfiesVCEECnt_)
relation(satisfiesVCENCnt_)
relation(IVCEsOk_)

$$
\begin{array}{|l}
toNum : SGr \times V \times V \rightarrow \mathrm{opt}[\mathbb{A}] \\
\hline
\forall\ SG : SGr;\ ns, nt : V \bullet toNum(SG, ns, nt) = \\
\quad \mathbf{if}(nt, nNatS) \in (\preceq\ SG) \lor (nt, nIntS) \in (\preceq\ SG)\ \mathbf{then}\ \mathrm{to\mathbb{Z}}\ ns \\
\quad \mathbf{else\,if}(nt, nRealS) \in (\preceq\ SG)\ \mathbf{then}\ \mathrm{to\mathbb{R}}\ ns\ \mathbf{else}\ \varnothing
\end{array}
$$

$$
\begin{array}{|l}
rOp : SGVCEOP \rightarrow \mathbb{A} \leftrightarrow \mathbb{A} \\
\hline
rOp\ eq = \{n_1, n_2 : \mathbb{A} \mid n_1 = n_2\} \\
rOp\ neq = (\_ \neq \_) \\
rOp\ leq = (\_ \leq \_) \\
rOp\ geq = (\_ \geq \_) \\
rOp\ lt = (\_ < \_) \\
rOp\ gt = (\_ > \_)
\end{array}
$$

$$
\begin{array}{|l}
\text{satisfiesCnt\_} : \mathbb{P}(SGr \times GrM \times V \times SGVCEOP \times V) \\
\text{satisfiesVCEECnt\_} : \mathbb{P}(SGr \times GrwT \times E) \\
\text{satisfiesVCENCnt\_} : \mathbb{P}(SGr \times GrwT \times E) \\
\text{IVCEsOk\_} : \mathbb{P}(SGr \times GrwT) \\
\hline
\forall SG : SGr;\ t : GrM;\ op : SGVCEOP;\ ns, nt : V \bullet \text{satisfiesCnt}(SG, t, ns, op, nt) \Leftrightarrow \\
\quad ns \in \operatorname{dom}(fV\ t) \\
\quad \land (\exists n_1, n_2 : \mathbb{A} \mid n_1 \in toNum(SG, ns, (fV\ t)ns) \land n_2 \in toNum(SG, nt, nt) \bullet (n_1, n_2) \in rOp\ op \\
\qquad \lor (op = eq \land (fV\ t)ns = nt \land nt \in NsVa\ SG)) \\
\\
\forall SG : SGr;\ GwT : GrwT;\ vce : E \bullet \text{satisfiesVCEECnt}(SG, GwT, vce) \Leftrightarrow \\
\quad \forall ie : fE\ (GwT^{T})^{\sim} (\!| (second \circ (vcei\ SG))\ vce |\!) \mid fV\ (GwT^{T})(src\ (GwT^{G})\ ie) = (sg\_src\ SG\ vce) \bullet \\
\qquad \text{satisfiesCnt}(SG, GwT^{T}, tgt\ (GwT^{G})\ ie, (first \circ (vcei\ SG))\ vce, sg\_tgt\ SG\ vce) \\
\\
\forall SG : SGr;\ GwT : GrwT;\ vce : E \bullet \text{satisfiesVCENCnt}(SG, GwT, vce) \Leftrightarrow \\
\quad \forall in : fV\ (GwT^{T})^{\sim} (\!| \{sg\_src\ SG\ vce\} |\!) \bullet \\
\qquad \text{satisfiesCnt}(SG, GwT^{T}, in, (first \circ (vcei\ SG))\ vce, sg\_tgt\ SG\ vce) \\
\\
\forall SG : SGr;\ GwT : GrwT \bullet \text{IVCEsOk}(SG, GwT) \Leftrightarrow \forall vce : EsVCnt\ SG \bullet \\
\quad \text{isVCEECnt}(SG, vce) \Rightarrow \text{satisfiesVCEECnt}(SG, GwT, vce) \\
\quad \land \text{isVCENCnt}(SG, vce) \Rightarrow \text{satisfiesVCENCnt}(SG, GwT, vce)
\end{array}
$$

$$
\begin{array}{|l}
\_ \ni_{\mathcal{M}} \_ : GrwT \leftrightarrow SGr \\
\_ \ni_{FI} \_ : GrwT \leftrightarrow SGr \\
\_ \ni_{PNS} \_ : GrwT \leftrightarrow SGr \\
\_ \ni_{Cnts} \_ : GrwT \leftrightarrow SGr \\
\hline
\forall GwT : GrwT;\ SG : SGr \bullet \\
\quad GwT \ni_{\mathcal{M}} SG \Leftrightarrow \forall me : EsM\ SG \bullet (SG, me)\ ME\mathcal{M}Ok\ GwT \\
\\
\forall GwT : GrwT;\ SG : SGr \bullet \\
\quad GwT \ni_{FI} SG \Leftrightarrow (fV\ (GwT^{T}))^{\sim} (\!| NsEther\ SG |\!) = \varnothing \\
\\
\forall GwT : GrwT;\ SG : SGr \bullet \\
\quad GwT \ni_{PNS} SG \Leftrightarrow igRMEs(GwT, EsTy\ SG\ \{ecomp\ dbi, ecomp\ duni\})^{\leftrightarrow} \in \text{injrel} \\
\\
\forall GwT : GrwT;\ SG : SGr \bullet \\
\quad GwT \ni_{Cnts} SG \Leftrightarrow \text{INumbersOk}\ GwT \land \text{IVCEsOk}(SG, GwT)
\end{array}
$$

$$
\begin{array}{|l}
\_ \ni^{SG} \_ : GrwT \leftrightarrow SGr \\
\hline
\forall GwT : GrwT;\ SG : SGr \bullet GwT \ni^{SG} SG \Leftrightarrow GwT \Rrightarrow^{GwT} SG \land GwT \ni_{\mathcal{M}} SG \\
\qquad \land GwT \ni_{FI} SG \land GwT \ni_{PNS} SG \land GwT \ni_{Cnts} SG
\end{array}
$$

8 Fragments

section *Fragmenta_Frs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_SGs*, *Fragmenta_GrswT*, *Fragmenta_GrswET*

$$Fr_0 == \{SG : SGr_0;\ esr : \mathbb{P}\, E;\ sr, tr : E \pfun V;\ et : GrM \mid esr \cap (sg\_Es\ SG) = \varnothing$$
$$\land\ sr \in esr \pinj (NsP\ SG) \land tr \in esr \to V \land domg\ et \subseteq_p (NsN\ SG, EsA\ SG)\}$$

$$\begin{array}{|l}
fSG : Fr_0 \to SGr \\
EsR : Fr_0 \to \mathbb{P}\, E \\
srcR, tgtR : Fr_0 \to E \pfun V \\
fet : Fr_0 \to GrM \\
\hline
\forall SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \pfun V;\ et : GrM \bullet fSG(SG, esr, sr, tr, et) = SG \\
\forall SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \pfun V;\ et : GrM \bullet EsR(SG, esr, sr, tr, et) = esr \\
\forall SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \pfun V;\ et : GrM \bullet srcR(SG, esr, sr, tr, et) = sr \\
\forall SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \pfun V;\ et : GrM \bullet tgtR(SG, esr, sr, tr, et) = tr \\
\forall SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \pfun V;\ et : GrM \bullet fet(SG, esr, sr, tr, et) = et
\end{array}$$

$$\begin{array}{|l}
fLEs, fEs, fEsA : Fr_0 \to \mathbb{P}\, E \\
fLNs, fRNs, fNs : Fr_0 \to \mathbb{P}\, V \\
srcF, tgtF : Fr_0 \to E \pfun V \\
\hline
fLEs = (sg_Es \circ fSG) \\
\forall F : Fr_0 \bullet fEs\ F = fLEs\ F \cup EsR\ F \\
fEsA = EsA \circ fSG \\
fLNs = sg_Ns \circ fSG \\
fRNs = \mathrm{ran} \circ tgtR \\
\forall F : Fr_0 \bullet fNs\ F = fLNs\ F \cup fRNs\ F \\
\forall F : Fr_0 \bullet srcF\ F = (sg_src \circ fSG)\ F \cup srcR\ F \\
\forall F : Fr_0 \bullet tgtF\ F = (sg_tgt \circ fSG)\ F \cup tgtR\ F
\end{array}$$

$$\begin{array}{|l}
\overset{G}{\leftrightsquigarrow} : Fr_0 \to Gr \\
\leftrightsquigarrow : Fr_0 \to V \leftrightarrow V \\
\hline
\forall F : Fr_0 \bullet \overset{G}{\leftrightsquigarrow}\ F = ((NsP \circ fSG)F \cup fRNs\ F, EsR\ F, srcR\ F, tgtR\ F) \\
\forall F : Fr_0 \bullet \leftrightsquigarrow\ F = (\overset{G}{\leftrightsquigarrow}\ F)^{\leftrightarrow}
\end{array}$$

function 10 **leftassoc** $(\_ \cup_F \_)$

$$
\begin{array}{|l}
\varnothing_F : Fr_0 \\
\_ \cup_F \_ : Fr_0 \times Fr_0 \rightarrow Fr_0 \\
\bigcup_F : \mathbb{P}\, Fr_0 \rightarrow Fr_0 \\
\hline
\varnothing_F = (\varnothing_{SG}, \varnothing, \varnothing, \varnothing, \varnothing_{GM}) \\
\forall F_1, F_2 : Fr_0 \bullet F_1 \cup_F F_2 = \\
\quad (fSG\, F_1 \cup_{SG} fSG\, F_2, EsR\, F_1 \cup EsR\, F_2, srcR\, F_1 \cup srcR\, F_2, tgtR\, F_1 \cup tgtR\, F_2, fet\, F_1 \cup_{GM} fet\, F_2) \\
\bigcup_F \{\} = \varnothing_F \\
\forall F : Fr_0;\ Fs : \mathbb{P}\, Fr_0 \bullet \bigcup_F (\{F\} \cup Fs) = F \cup_F (\bigcup_F Fs)
\end{array}
$$

$$
\begin{array}{|l}
\rightsquigarrow : Fr_0 \nrightarrow V \nrightarrow V \\
\odot^{SG} : Fr_0 \nrightarrow SGr \\
rEsR : Fr_0 \nrightarrow \mathbb{P}\, E \\
\odot : Fr_0 \nrightarrow Fr_0 \\
\hline
\forall F : Fr_0 \bullet \rightsquigarrow F = (\leftrightsquigarrow F) \rhd (fLNs\, F) \\
\forall F : Fr_0 \bullet \odot^{SG} F = (fSG\, F) \odot^{SG} (\rightsquigarrow F) \\
\forall F : Fr_0 \bullet rEsR\, F = \mathrm{dom}((srcR\, F) \unrhd \mathrm{dom}(\rightsquigarrow F)) \\
\forall F : Fr_0 \bullet \odot F = (\odot^{SG} F, rEsR\, F, (rEsR\, F) \lhd (srcR\, F), (rEsR\, F) \lhd (tgtR\, F), fet\, F)
\end{array}
$$

$$Fr_a == \{F : Fr_0 \mid \otimes(\overset{G}{\leftrightsquigarrow} F)\}$$
$$Fr == \{F : Fr_a \mid \odot^{SG} F \in SGr\}$$

relation(refsLocal_)

$$
\begin{array}{|l}
\mathrm{refsLocal}\_ : \mathbb{P}\, Fr_0 \\
\hline
\forall F : Fr_0 \bullet \mathrm{refsLocal}\, F \Leftrightarrow fRNs\, F \subseteq fLNs\, F
\end{array}
$$

$$TFr == \{F : Fr_a \mid \mathrm{refsLocal}\, F \wedge \odot^{SG} F \in TSGr\}$$

relation($\boxminus$ _)
relation($\boxplus$ _)

$$
\begin{array}{|l}
\boxminus \_ : Fr \leftrightarrow Fr \\
\hline
\forall F_1, F_2 : Fr \bullet \boxminus(F_1, F_2) \Leftrightarrow fLNs\, F_1 \cap fLNs\, F_2 = \varnothing \wedge fEs\, F_1 \cap fEs\, F_2 = \varnothing
\end{array}
$$

$$[I]$$
$$\boxplus \_ : \mathbb{P}(I \rightarrowtail\!\!\!\!\!\!\;\mid\; Fr)$$
$$\forall\, Fs : I \mathrel{\ooalign{$\rightarrow$\cr\kern.3em$|$}} Fr \bullet \boxplus\ Fs \Leftrightarrow \forall\, i, j : \mathrm{dom}\, Fs \mid i \neq j \bullet \boxminus(Fs\ i, Fs\ j)$$

relation$(\_ \subseteq^{rs} \_)$
relation$(\_ \mapsto\!\!\Rightarrow \_)$

$$\_ \subseteq^{rs} \_ : Fr \leftrightarrow Fr$$
$$\_ \mapsto\!\!\Rightarrow \_ : Fr \leftrightarrow Fr$$
$$\forall\, F_1, F_2 : Fr \bullet F_1 \subseteq^{rs} F_2 \Leftrightarrow \mathrm{ran}(tgtR\ F_1) \cap fLNs\ F_2 \neq \varnothing$$
$$\forall\, F_1, F_2 : Fr \bullet F_1 \mapsto\!\!\Rightarrow F_2 \Leftrightarrow F_1 \subseteq^{rs} F_2 \wedge \neg\, (F_2 \subseteq^{rs} F_1)$$

function 10 **leftassoc** $(\_ \rightrightarrows_{\bullet} \_)$

$$\_ \rightrightarrows_{\bullet} \_ : GrM \times (Fr \times Fr) \rightarrow GrM$$
$$\forall\, m : GrM;\ F_s, F_t : Fr_0 \bullet$$
$$m \rightrightarrows_{\bullet} (F_s, F_t) = (((\rightsquigarrow F_s)^{\oplus} \boxdot (fLNs\ F_s))^{\sim} \mathbin{;} (fV\ m) \mathbin{;} ((\rightsquigarrow F_t)^{\oplus} \boxdot (fLNs\ F_t)), fE\ m)$$

function 1 **leftassoc** $(\_ \rightarrow_F \_)$

$$\_ \rightarrow_F \_ : Fr \times Fr \rightarrow \mathbb{P}\ GrM$$
$$\forall\, F_s, F_t : Fr \bullet F_s \rightarrow_F F_t = \{fv : fLNs\ F_s \rightarrow fLNs\ F_t;\ fe : fEsA\ F_s \rightarrow fEsA\ F_t \mid$$
$$(\circledcirc^{SG} F_s, (fv, fe) \rightrightarrows_{\bullet} (F_s, F_t)) \Rrightarrow^{SG} \circledcirc^{SG} F_t\}$$

relation$(\_ \Rrightarrow^{F} \_)$

$$\_ \Rrightarrow^{F} \_ : (Fr \times GrM) \leftrightarrow Fr$$
$$\forall\, m : GrM;\ F_s, F_t : Fr_0 \bullet (F_s, m) \Rrightarrow^{F} F_t \Leftrightarrow m \in F_s \rightarrow_F F_t$$

relation$(\_ \sqsupseteq^F \_)$

$$\begin{array}{|l}
_ \sqsupseteq^F _ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupseteq^F F_a \Leftrightarrow (F_c, m) \Rrightarrow^F F_a \\
\quad \wedge\ (\odot^{SG} F_c, m \rightrightarrows_\bullet (F_c, F_a)) \sqsupseteq^{SG_0} (\odot^{SG} F_a)
\end{array}$$

relation$(\_ \sqsupset^F \_)$

$$\begin{array}{|l}
_ \sqsupset^F _ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupset^F F_a \Leftrightarrow (F_c, m) \Rrightarrow^F F_a \wedge \{F_a, F_c\} \subseteq Fr \\
\quad \wedge\ (\odot^{SG} F_c, m \rightrightarrows_\bullet (F_c, F_a)) \sqsupset^{SG_0} (\odot^{SG} F_a)
\end{array}$$

relation$(\_ \Supset^F \_)$

$$\begin{array}{|l}
_ \Supset^F _ : GrwT \leftrightarrow Fr \\
\hline
\forall GwT : GrwT;\ F : Fr \bullet GwT \Supset^F F \Leftrightarrow GwT \Supset^{SG} \odot^{SG} F
\end{array}$$

relation$(\_ \dashv\vdash^F \_)$

$$\begin{array}{|l}
_ \dashv\vdash^F _ : Fr \leftrightarrow Fr \\
\hline
\forall m : GrM;\ F_s, F_t : Fr_0 \bullet F_s \dashv\vdash^F F_t \Leftrightarrow fet\, F_s \neq \varnothing_{GM} \wedge (\odot^{SG} F_s, fet\, F_s) \dashv\vdash^{SG} (\odot^{SG} F_t)
\end{array}$$

relation$(\_ \Vvdash^F \_)$

$$\begin{array}{|l}
_ \Vvdash^F _ : \mathbb{P}((GrwET \times Fr_0) \times (GrwT \times Fr_0)) \\
\hline
\forall GwET : GrwET;\ F_s, F_t : Fr_0;\ GwT : GrwT \bullet (GwET, F_s) \Vvdash^F (GwT, F_t) \Leftrightarrow GwET^{Gw} \Supset^F F_s \\
\quad \wedge\ GwT \Supset^F F_t \wedge \mathrm{domg}\,(GwET^{ET}) =_p \mathrm{domg}\,((fet\, F_s) \circ_G (GwET^{T})) \\
\quad \wedge\ GwET^{ET} \in (GwET^{G}) \twoheadrightarrow_G (GwT^{G}) \wedge F_s \dashv\vdash^F F_t
\end{array}$$

9 Global Fragment Graphs

section *Fragmenta_GFGs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$$GFGr == \{ G : Gr \mid \otimes(G \bowtie_{Es} (Es\ G \setminus EsId\ G)) \}$$

function$(\_^{\dashrightarrow})$

$$\begin{array}{|l}
_^{\dashrightarrow} : GFGr \rightarrow V \leftrightarrow V \\
\hline
\forall GFG : GFGr \bullet GFG^{\dashrightarrow} = (GFG^{\leftrightarrow})^{+}
\end{array}$$

10 Models

section *Fragmenta_Mdls* **parents** *standard_toolkit*, *Fragmenta_Frs*, *Fragmenta_GFGs*

$$Mdl_0 == \{ GFG : GFGr;\ fd : V \nrightarrow Fr \mid fd \in Ns\ GFG \rightarrow Fr \land \boxplus\ fd \}$$

$$\begin{array}{|l}
mGFG : Mdl_0 \rightarrow GFGr \\
mFD : Mdl_0 \rightarrow V \nrightarrow Fr \\
\hline
\forall GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mGFG(GFG, fd) = GFG \\
\forall GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mFD(GFG, fd) = fd
\end{array}$$

$$\begin{array}{|l}
mUFs : Mdl_0 \rightarrow Fr \\
\hline
mUFs = \bigcup_F \circ \operatorname{ran} \circ mFD
\end{array}$$

$$\begin{array}{|l}
\mathrm{from} : Mdl_0 \rightarrow V \nrightarrow V \\
\hline
\forall M : Mdl_0;\ v : V \bullet \mathrm{from}\ M\ v = (\mu\ vf : (Ns \circ mGFG) M \mid v \in fLNs(mFD\ M\ vf))
\end{array}$$

relation$(\Uparrow \_)$

$$
\begin{array}{|l}
\Uparrow \_ : \mathbb{P}\, Mdl_0 \\
\hline
\forall\, M : Mdl_0 \bullet \Uparrow M \Leftrightarrow \exists\, UF : Fr_0 \mid UF = mUFs\, M \bullet \\
\quad \forall\, p : (NsP \circ fSG)\, UF \bullet (from\, M\, p, from\, M\, (\leftrightsquigarrow\ UF\, p)) \in ((\_^{\dashrightarrow}) \circ mGFG) M
\end{array}
$$

$$
Mdl == \{M : Mdl_0 \mid (mUFs\, M) \in TFr \wedge \Uparrow M\}
$$

$$
\begin{array}{|l}
\odot^{M} : Mdl \to Fr \\
\hline
\forall\, M : Mdl_0 \bullet \odot^{M} = \odot \circ mUFs
\end{array}
$$

function 1 **leftassoc** $(\_ \to_M \_)$
relation$(\_ \Rrightarrow^{M} \_)$

$$
\begin{array}{|l}
\_ \to_M \_ : Mdl \times Mdl \to \mathbb{P}\, GrM \\
\_ \Rrightarrow^{M} \_ : \mathbb{P}((Mdl \times \mathbb{P}\, GrM) \times Mdl) \\
\hline
\forall\, M_s, M_t : Mdl \bullet M_s \to_M M_t = \{m : GrM \mid \\
\quad \exists\, UF_s, UF_t : Fr_0 \mid UF_s = mUFs\, M_s \wedge UF_t = mUFs\, M_t \bullet m \in UF_s \to_F UF_t\} \\
\forall\, M_s, M_t : Mdl;\ ms : \mathbb{P}\, GrM \bullet (M_s, ms) \Rrightarrow^{M} M_t \Leftrightarrow \bigcup_{GM} ms \in M_s \to_M M_t
\end{array}
$$

relation$(\_ \sqsupset^{M} \_)$

$$
\begin{array}{|l}
\_ \sqsupset^{M} \_ : (Mdl \times \mathbb{P}\, GrM) \leftrightarrow Mdl \\
\hline
\forall\, M_c, M_a : Mdl_0;\ ms : \mathbb{P}\, GrM \bullet (M_c, ms) \sqsupset^{M} M_a \\
\quad \Leftrightarrow \exists\, UF_c, UF_a : Fr_0 \mid UF_c = mUFs\, M_c \wedge UF_a = mUFs\, M_a \bullet (UF_c, \bigcup_{GM} ms) \sqsupset^{F} UF_a
\end{array}
$$

relation$(\_ \Supset^{M} \_)$

$$
\begin{array}{|l}
\_ \Supset^{M} \_ : GrwT \leftrightarrow Mdl \\
\hline
\forall\, GwT : GrwT;\ M : Mdl \bullet GwT \Supset^{M} M \Leftrightarrow GwT \Supset^{F} mUFs\, M
\end{array}
$$

relation$(\_ \dashv\vdash^{F} \_)$

$$
\begin{array}{|l}
\_ \dashv\vdash^F \_ : Mdl \leftrightarrow Mdl \\
\hline
\forall\, m : GrM;\ M_s, M_t : Mdl_0 \bullet M_s \dashv\vdash^F M_t \Leftrightarrow mUFs\, M_s \dashv\vdash^F mUFs\, M_t
\end{array}
$$

relation($\_ \Vdash^M \_$)

$$
\begin{array}{|l}
\_ \Vdash^M \_ : \mathbb{P}((GrwET \times Mdl_0) \times (GrwT \times Mdl_0)) \\
\hline
\forall\, GwET : GrwET;\ M_s, M_t : Mdl_0;\ GwT : GrwT \bullet \\
\quad (GwET, M_s) \Vdash^M (GwT, M_t) \Leftrightarrow (GwET, mUFs\, M_s) \Vdash^F (GwT, mUFs\, M_t)
\end{array}
$$