Z Specification of Fragmenta and PCs

Nuno Amálio

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1 Generics

section *Fragmenta_Generics* **parents** *standard_toolkit*

$$\mathrm{acyclic}[X] == \{ r : X \leftrightarrow X \mid r^{+} \cap \mathrm{id}\, X = \varnothing \}$$
$$\mathit{connected}[X] == \{ r : X \leftrightarrow X \mid \forall x : \mathrm{dom}\, r;\ y : \mathrm{ran}\, r \bullet x \mapsto y \in r^{+} \}$$
$$\mathit{tree}[X] == \{ r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \land r \in X \nrightarrow X \}$$
$$\mathit{forest}[X] == \{ r : X \leftrightarrow X \mid r \in \mathrm{acyclic} \land (\forall s : X \leftrightarrow X \mid s \subseteq r \land s \in \mathit{connected} \bullet s \in \mathit{tree}) \}$$
$$\mathrm{injrel}[X, Y] == \{ r : X \leftrightarrow Y \mid r^{\sim} \in Y \nrightarrow X \}$$
$$\mathrm{antireflexive}[X] == \{ r : X \leftrightarrow X \mid r \cap \mathit{id}(\mathrm{dom}\ r) = \varnothing \}$$

$$[X, Y, Z]$$
$$\mathrm{flip} : (X \to Y \nrightarrow Z) \to (Y \to X \nrightarrow Z)$$
$$\forall f : X \to Y \nrightarrow Z \bullet \mathrm{flip}\, f = (\lambda\, y : Y \bullet \lambda\, x : X \bullet f\ x\ y)$$

$$[X, Y, Z, W]$$
$$\mathrm{apply} : (X \to Z) \to (Y \to W) \to (X \times Y) \to (Z \times W)$$
$$\forall f : X \to Z;\ g : Y \to W;\ x : X;\ y : Y \bullet \mathrm{apply}\, f\ g\ (x, y) = (f\ x, g\ y)$$

$$[X, Y]$$
$$\mathrm{map} : (X \to Y) \to \mathbb{P}\, X \to \mathbb{P}\, Y$$
$$\mathrm{mapS} : (X \to Y) \to \mathrm{seq}\, X \to \mathrm{seq}\ Y$$
$$\forall f : X \to Y \bullet \mathrm{map}\, f \{\} = \{\}$$
$$\forall f : X \to Y;\ x : X;\ \mathit{xs} : \mathbb{P}\, X \bullet \mathrm{map}\, f (\{x\} \cup \mathit{xs}) = \{f\ x\} \cup (\mathrm{map}\, f\ \mathit{xs})$$
$$\forall f : X \to Y \bullet \mathrm{mapS}\, f \langle\rangle = \langle\rangle$$
$$\forall f : X \to Y;\ x : X;\ \mathit{xs} : \mathrm{seq}\, X \bullet \mathrm{mapS}\, f (\langle x \rangle \frown \mathit{xs}) = \langle f\ x \rangle \frown (\mathrm{mapS}\, f\ \mathit{xs})$$

function 10 **leftassoc** $(\_ \boxdot \_)$

$$[X]$$
$$\_ \boxdot \_ : ((X \to X) \times \mathbb{P}\, X) \to (X \nrightarrow X)$$
$$\forall f : X \to X;\ s : \mathbb{P}\, X \bullet f \boxdot s = (\mathrm{id}\ s) \oplus f$$

function$(\_^{\oplus})$

$$
\begin{array}{l}
[X] \\
\hline
\_^{\oplus} : (X \leftrightarrow X) \rightarrow (X \leftrightarrow X) \\
\hline
\forall r : X \leftrightarrow X \bullet r^{\oplus} = \textbf{if } r \oplus r \mathbin{;} r = r \textbf{ then } r \textbf{ else } (r \oplus r \mathbin{;} r)^{\oplus}
\end{array}
$$

$$\mathrm{opt}[X] == \{s : \mathbb{P}\, X \mid \#\, s \leq 1\}$$

$$
\begin{array}{l}
[X] \\
\hline
\mathrm{the} : \mathrm{opt}[X] \rightarrow\!\!\!\!\!\!+ \ X \\
\hline
\forall x : X \bullet \mathrm{the}\,\{x\} = x
\end{array}
$$

$$
\begin{array}{l}
[X, Y] \\
\hline
\mathit{flatten} : (X \rightarrow\!\!\!\!\!\!+ \ \mathbb{P}\, Y) \rightarrow (X \leftrightarrow Y) \\
\hline
\forall f : X \rightarrow\!\!\!\!\!\!+ \ \mathbb{P}\, Y \bullet \mathit{flatten}\, f = \{x : \mathrm{dom}\, f;\ y : Y \mid y \in f\, x\}
\end{array}
$$

2 Graphs with sequences

section *Fragmenta_GrSs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$$\mathit{GrSs} == \{vs : \mathbb{P}\, V;\ es : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!\!+ \ V;\ t : E \rightarrow\!\!\!\!\!\!+ \ \mathrm{seq}\, V \mid s \in es \rightarrow vs \wedge t \in es \rightarrow \mathrm{seq}\, vs\}$$

$$
\begin{array}{l}
ns : \mathit{GrSs} \rightarrow \mathbb{P}\, V \\
es : \mathit{GrSs} \rightarrow \mathbb{P}\, E \\
sou : \mathit{GrSs} \rightarrow E \rightarrow\!\!\!\!\!\!+ \ V \\
tar : \mathit{GrSs} \rightarrow E \rightarrow\!\!\!\!\!\!+ \ \mathrm{seq}\, V \\
tarr : \mathit{GrSs} \rightarrow E \leftrightarrow V \\
\hline
\forall vs : \mathbb{P}\, V;\ as : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!\!+ \ V;\ t : E \rightarrow\!\!\!\!\!\!+ \ \mathrm{seq}\, V \bullet ns(vs, as, s, t) = vs \\
\forall vs : \mathbb{P}\, V;\ as : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!\!+ \ V;\ t : E \rightarrow\!\!\!\!\!\!+ \ \mathrm{seq}\, V \bullet es(vs, as, s, t) = as \\
\forall vs : \mathbb{P}\, V;\ as : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!\!+ \ V;\ t : E \rightarrow\!\!\!\!\!\!+ \ \mathrm{seq}\, V \bullet sou(vs, as, s, t) = s \\
\forall vs : \mathbb{P}\, V;\ as : \mathbb{P}\, E;\ s : E \rightarrow\!\!\!\!\!\!+ \ V;\ t : E \rightarrow\!\!\!\!\!\!+ \ \mathrm{seq}\, V \bullet tar(vs, as, s, t) = t \\
\forall G : \mathit{GrSs} \bullet tarr\ G = \mathit{flatten}(\mathrm{ran} \circ (tar\ G))
\end{array}
$$

$$\begin{array}{|l} \varnothing_{GSs} : GrSs \\ \hline \varnothing_{GSs} = (\varnothing, \varnothing, \varnothing, \varnothing) \end{array}$$

$$\begin{array}{|l} \mathrm{EsIdGSs} : GrSs \to \mathbb{P}\, E \\ \hline \forall\, G : GrSs \bullet \mathrm{EsIdGSs}\; G = \{e : es\; G \mid \langle sou\; G\; e\rangle = tar\; G\; e\} \end{array}$$

function 10 **leftassoc** (_restrictEsGSs_)

$$\begin{array}{|l} \_\mathrm{restrictEsGSs}\_ : GrSs \times \mathbb{P}\, E \to GrSs \\ \hline \forall\, G : GrSs;\; as : \mathbb{P}\, E \bullet G\; \mathrm{restrictEsGSs}\; as = (ns\; G, es\; G \cap as, as \lhd sou\; G, as \lhd tar\; G) \end{array}$$

relation(adjacentGSs _)

$$\begin{array}{|l} \mathrm{adjacentGSs}\_ : \mathbb{P}(GrSs \times V \times V) \\ \hline \forall\, G : GrSs;\; v_1, v_2 : V \bullet \mathrm{adjacentGSs}(G, v_1, v_2) \Leftrightarrow \exists\, e : es\; G \bullet sou\; G\; e = v_1 \land v_2 \in \mathrm{ran}(tar\; G\; e) \end{array}$$

$$\begin{array}{|l} EsIncidentGSs : GrSs \to \mathbb{P}\, V \to \mathbb{P}\, E \\ \hline \forall\, G : GrSs;\; vs : \mathbb{P}\, V \bullet EsIncidentGSs\; G\; vs = (sou\; G)^{\sim} \limg vs \rimg \cup \{e : es\; G \mid \exists\, v : vs \bullet v \in \mathrm{ran}(tar\; G\; e)\} \end{array}$$

$$\begin{array}{|l} \mathrm{successorsGSs} : V \times GrSs \to \mathbb{P}\, V \\ \hline \forall\, v : V;\; G : GrSs \bullet \mathrm{successorsGSs}(v, G) = \{v_1 : ns\; G \mid \mathrm{adjacentGSs}(G, v, v_1)\} \end{array}$$

function($\_^{\rightleftarrows}$)

$$\begin{array}{|l} \_^{\rightleftarrows} : GrSs \nrightarrow GrSs \\ \hline \forall\, G : GrSs \bullet G^{\rightleftarrows} = (ns\; G, es\; G, (\lambda\, e : es\; G \bullet head(tar\; G\; e)), (\lambda\, e : es\; G \bullet \langle sou\; G\; e\rangle)) \end{array}$$

function($\_^{\leftrightarrow}$)

$$\_^{\leftrightarrow} : GrSs \rightarrow V \leftrightarrow V$$
$$\forall\, G : GrSs \bullet G^{\leftrightarrow} = \{v_1, v_2 : ns\ G \mid \mathrm{adjacentGSs}(G, v_1, v_2)\}$$

relation($\otimes$ _)

$$\otimes\_ : \mathbb{P}\ GrSs$$
$$\forall\, G : GrSs \bullet \otimes\ G \Leftrightarrow G^{\leftrightarrow} \in \mathrm{acyclic}$$

relation($\boxminus_{Es}$ _)
relation($\boxminus$ _)

$$\boxminus_{Es}\_, \boxminus\_ : \mathbb{P}(GrSs \times GrSs)$$
$$\forall\, G_1, G_2 : GrSs \bullet \boxminus_{Es}(G_1, G_2) \Leftrightarrow es\ G_1 \cap es\ G_2 = \varnothing$$
$$\forall\, G_1, G_2 : GrSs \bullet \boxminus(G_1, G_2) \Leftrightarrow ns\ G_1 \cap ns\ G_2 = \varnothing \wedge \boxminus_{Es}(G_1, G_2)$$

relation($\boxplus$ _)

$$[I]$$
$$\boxplus\_ : \mathbb{P}(I \nrightarrow GrSs)$$
$$\forall\, Gs : I \nrightarrow GrSs \bullet \boxplus\ Gs \Leftrightarrow \forall\, i, j : \mathrm{dom}\ Gs \mid i \neq j \bullet \boxminus(Gs\ i, Gs\ j)$$

function 10 **leftassoc** ($\_ \cup_{GSs} \_$)

$$\_ \cup_{GSs} \_ : GrSs \times GrSs \rightarrow GrSs$$
$$\forall\, G_1, G_2 : GrSs \bullet G_1 \cup_{GSs} G_2 = (ns\ G_1 \cup ns\ G_2, es\ G_1 \cup es\ G_2, sou\ G_1 \cup sou\ G_2, tar\ G_1 \cup tar\ G_2)$$

function 10 **leftassoc** ($\_ \odot \_$)

$$
\begin{array}{|l}
\_ \odot \_ : GrSs \times (V \leftrightarrow V) \nrightarrow GrSs \\
\hline
\forall G : GrSs;\ s : V \leftrightarrow V \mid s \in ns\ G \nrightarrow ns\ G \land s \in \text{antireflexive} \bullet \\
\quad G \odot\ s = (ns\ G \setminus \operatorname{dom} s, es\ G, (s \boxdot ns\ G) \circ (sou\ G), (\lambda\, e : es\ G \bullet mapS(s \boxdot ns\ G)(tar\ G\ e)))
\end{array}
$$

$$
GrMSs == (V \nrightarrow V) \times (E \times \mathbb{N} \nrightarrow E)
$$

$$
\begin{array}{|l}
\phi_V : GrMSs \to V \nrightarrow V \\
\phi_E : GrMSs \to E \times \mathbb{N} \nrightarrow E \\
sou' : GrSs \to E \times \mathbb{N} \nrightarrow V \\
tar' : GrSs \to E \times \mathbb{N} \nrightarrow V \\
\hline
\phi_V = \mathit{first} \\
\phi_E = \mathit{second} \\
\forall G : GrSs;\ e : E;\ n : \mathbb{N} \bullet sou'\ G\,(e, n) = sou\ G\ e \\
\forall G : GrSs;\ e : E;\ n : \mathbb{N} \bullet tar'\ G\,(e, n) = (tar\ G\ e)\ n
\end{array}
$$

$$
\begin{array}{|l}
\text{idgss} : GrSs \to GrMSs \\
\hline
\forall G : GrSs \bullet \text{idgss}\ G = (id\,(ns\ G), map(\lambda\, e : E \bullet (e, 1) \mapsto e)(es\ G))
\end{array}
$$

$$
\begin{array}{|l}
\varnothing_{GMSs} : GrMSs \\
\hline
\varnothing_{GMSs} = (\{\}, \{\})
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_{GMSs} \_)$

$$
\begin{array}{|l}
\_ \cup_{GMSs} \_ : GrMSs \times GrMSs \nrightarrow GrMSs \\
\bigcup_{GMSs} : \mathbb{P}\ GrMSs \to GrMSs \\
\hline
\forall f, g : GrMSs \bullet f \cup_{GMSs} g = (\phi_V\ f \cup \phi_V\ g, \phi_E\ f \cup \phi_E\ g) \\
\bigcup_{GMSs} \varnothing = \varnothing_{GMSs} \\
\forall f : GrMSs;\ fs : \mathbb{P}\ GrMSs \bullet \bigcup_{GMSs} (\{f\} \cup fs) = f \cup_{GMSs} (\bigcup_{GMSs} fs)
\end{array}
$$

function 10 **leftassoc** $(\_ \to_{GSs2G} \_)$

$$
\begin{array}{|l}
\_ \to_{GSs2G} \_ : GrSs \times Gr \to \mathbb{P}\ GrMSs \\
\hline
\forall G_1 : GrSs;\ G_2 : Gr \bullet G_1 \to_{GSs2G} G_2 = \{fv : ns\ G_1 \to Ns\ G_2;\ fe : es\ G_1 \times \mathbb{N} \to Es\ G_2 \mid \\
\quad src\ G_2 \circ fe = fv \circ sou'\ G_1 \land tgt\ G_2 \circ fe = fv \circ tar'\ G_1\}
\end{array}
$$

3 Graphs with typing

section *Fragmenta_GrswT* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$GrwT == \{G : Gr;\ t : GrM \mid \mathrm{domg}\, t = G\}$

$$
\begin{array}{|l}
gOf : GrwT \rightarrow Gr \\
ty : GrwT \rightarrow GrM \\
\hline
\forall G : Gr;\ sm : V \nrightarrow \mathrm{seq}\, V;\ t : GrM \bullet gOf(G, t) = G \\
\forall G : Gr;\ sm : V \nrightarrow \mathrm{seq}\, V;\ t : GrM \bullet ty(G, t) = t
\end{array}
$$

$$
\begin{array}{|l}
\varnothing_{GwT} : GrwT \\
\hline
\varnothing_{GwT} = (\varnothing_G, \varnothing_{GM})
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_{GwT} \_)$

$$
\begin{array}{|l}
\_ \cup_{GwT} \_ : GrwT \times GrwT \rightarrow GrwT \\
\hline
\forall G_1, G_2 : GrwT \bullet G_1 \cup_{GwT} G_2 = ((gOf\ G_1) \cup_G (gOf\ G_2), (ty\ G_1) \cup_{GM} (ty\ G_2))
\end{array}
$$

4 Graphs with typing and node sequences

section *Fragmenta_GrswTSs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_GrswT*

$GrwTSs == \{G : GrwT;\ sm : V \nrightarrow (\mathrm{seq}\ V) \mid sm \in (Ns \circ gOf)\ G \nrightarrow \mathrm{seq}(Ns \circ gOf)\ G$
$\quad \land (\forall n : \mathrm{dom}\, sm \bullet \mathrm{ran}(sm\ n) \cap \mathrm{dom}\ sm = \varnothing)\}$

$$
\begin{array}{|l}
gwt : GrwTSs \to GrwT \\
seqm : GrwTSs \to (V \nrightarrow \text{seq } V) \\
sNs : GrwTSs \to \mathbb{P}\, V \\
aNs : GrwTSs \to \mathbb{P}\, V \\
sEs : GrwTSs \to \mathbb{P}\, E \\
tgtseq : GrwTSs \to E \leftrightarrow V \\
gsrc : GrwTSs \to E \nrightarrow V \\
gtgt : GrwTSs \to E \leftrightarrow V \\
\hline
\forall G : GrwT;\ sm : V \nrightarrow \text{seq } V \bullet gwt(G, sm) = G \\
\forall G : GrwT;\ sm : V \nrightarrow \text{seq } V \bullet seqm(G, sm) = sm \\
\forall G : GrwTSs \bullet sNs\ G = (\text{dom} \circ seqm)\ G \\
\forall G : GrwTSs \bullet aNs\ G = (Ns \circ gOf \circ gwt)\ G \setminus sNs\ G \\
\forall G : GrwTSs \bullet tgtseq\ G = flatten(\text{ran} \circ (seqm\ G) \circ ((tgt \circ gOf \circ gwt)\ G)) \\
\forall G : GrwTSs \bullet sEs\ G = \text{dom}((tgt \circ gOf \circ gwt)\ G \rhd (sNs\ G)) \\
\forall G : GrwTSs \bullet gsrc\ G = (src \circ gOf \circ gwt)\ G \\
\forall G : GrwTSs \bullet gtgt\ G = ((tgt \circ gOf \circ gwt)\ G \unrhd (sNs\ G)) \cup tgtseq\ G
\end{array}
$$

$$
\begin{array}{|l}
\varnothing_{GwTSs} : GrwTSs \\
\hline
\varnothing_{GwTSs} = (\varnothing_{GwT}, \varnothing)
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_{GwTSs} \_)$

$$
\begin{array}{|l}
\_ \cup_{GwTSs} \_ : GrwTSs \times GrwTSs \to GrwTSs \\
\hline
\forall G_1, G_2 : GrwTSs \bullet G_1 \cup_{GwTSs} G_2 = ((gwt\ G_1) \cup_{GwT} (gwt\ G_2), (seqm\ G_1) \cup (seqm\ G_2))
\end{array}
$$

5 SG Element Types

section *Fragmenta_SGElemTys* **parents** *standard_toolkit*, *Fragmenta_Generics*

$SGNT ::= nnrml \mid nabst \mid nprxy \mid nenum \mid nval \mid nvirt \mid nopt$

$SGED ::= dbi \mid duni$

$SGET ::= einh \mid ecomp\langle\langle SGED\rangle\rangle \mid erel\langle\langle SGED\rangle\rangle \mid ewander$

relation $(\_ \prec_{NT} \_)$

$$
\begin{array}{|l}
\_ \prec_{NT} \_ : SGNT \leftrightarrow SGNT \\
\hline
\forall nt_1, nt_2 : SGNT \bullet nt_1 \prec_{NT} nt_2 \Leftrightarrow (nt_2 = nenum \Leftrightarrow nt_1 = nval) \\
\quad \land (nt_1 = nvirt \Rightarrow nt_2 = nvirt) \land (nt_1 = nabst \Rightarrow nt_2 \in \{nabst, nvirt, nprxy\}) \\
\quad \land nt_1 \notin \{nprxy, nenum\} \land nt_2 \notin \{nopt\}
\end{array}
$$

relation$(\_ \leq_{rNT} \_)$

$$
\begin{array}{|l}
\_ \leq_{rNT} \_ : SGNT \leftrightarrow SGNT \\
\hline
\forall nt_1, nt_2 : SGNT \bullet nt_1 \leq_{rNT} nt_2 \Leftrightarrow nt_1 = nt_2 \lor nt_1 = nprxy \\
\quad \lor nt_2 = nabst \land nt_1 \in \{nnrml, nvirt\} \lor nt_2 \in \{nnrml, nopt\}
\end{array}
$$

relation$(\_ =_{ET} \_)$

$$
\begin{array}{|l}
\_ =_{ET} \_ : SGET \leftrightarrow SGET \\
\hline
\forall et_1, et_2 : SGET \bullet et_1 =_{ET} et_2 \Leftrightarrow et_1 = et_2 \\
\quad \lor (\forall d_1, d_2 : SGED \bullet et_1 = erel\ d_1 \\
\quad \land et_2 = erel\ d_2 \lor et_1 = ecomp\ d_1 \land et_2 = ecomp\ d_2)
\end{array}
$$

relation$(\_ \leq_{ET} \_)$

$$
\begin{array}{|l}
\_ \leq_{ET} \_ : SGET \leftrightarrow SGET \\
\hline
\forall et_1, et_2 : SGET \bullet et_1 \leq_{ET} et_2 \Leftrightarrow et_2 = ewander \land et_1 \neq einh \\
\quad \lor et_1 =_{ET} et_2
\end{array}
$$

6 Multiplicities

section *Fragmenta_Mult* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_SGElemTys*

$MultVal ::= \mathbf{v}\langle\langle \mathbb{N} \rangle\rangle \mid *$

$MultC ::= mr\langle\langle \mathbb{N} \times MultVal \rangle\rangle \mid ms\langle\langle MultVal \rangle\rangle$

relation$(\_ \leq_{mv} \_)$

$$\_ \leq_{mv} \_ : MultVal \leftrightarrow MultVal$$

$$\forall m_1, m_2 : MultVal \bullet m_1 \leq_{mv} m_2 \Leftrightarrow m_2 = * \lor \exists j, k : \mathbb{N} \mid m_1 = \mathbf{v}\ j \land m_2 = \mathbf{v}\ k \bullet j \leq k$$

$$Mult == \{mc : MultC \mid \exists lb : \mathbb{N};\ ub : MultVal \bullet mc = mr(lb, ub) \land \mathbf{v}\ lb \leq_{mv} ub$$
$$\lor \exists mv : MultVal \bullet mc = ms\ mv\}$$

$$MultMany == \{ms\, *, mr(0, *)\}$$

$$MultRange == \{m : MultC \mid \exists k : \mathbb{N} \mid k > 1 \bullet m = ms\,(\mathbf{v}\ k)$$
$$\lor \exists lb : \mathbb{N};\ umv : MultVal \mid \mathbf{v}\ 2 \leq_{mv} umv \bullet m = mr(lb, umv)\}$$

relation$(\_ \between \_)$

$$\_ \between \_ : \mathbb{P}(\mathbb{N} \times (MultVal \times MultVal))$$

$$\forall k : \mathbb{N};\ lb, ub : MultVal \bullet k \between (lb, ub) \Leftrightarrow lb \leq_{mv} \mathbf{v}\ k \land \mathbf{v}\ k \leq_{mv} ub$$

$$mlb, mub : MultC \rightarrow\!\!\!\!\!\!\!+\; MultVal$$

$$mlb(ms\, *) = \mathbf{v}\ 0$$

$$\forall k : \mathbb{N} \bullet mlb(ms(\mathbf{v}\ k)) = \mathbf{v}\ k$$

$$\forall k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ k$$

$$\forall mv : MultVal \bullet mub(ms\ mv) = mv$$

$$\forall k, m : \mathbb{N} \bullet mlb(mr(k, \mathbf{v}\ m)) = \mathbf{v}\ m$$

relation$(\_ \leq_{\mathcal{M}} \_)$

$$\_ \leq_{\mathcal{M}} \_ : MultC \leftrightarrow MultC$$

$$\forall m_1, m_2 : MultC \bullet m_1 \leq_{\mathcal{M}} m_2 \Leftrightarrow mlb\ m_2 \leq_{mv} mlb\ m_1 \land mub\ m_1 \leq_{mv} mub\ m_2$$

relation$(\_ \propto \_)$

$$\_ \propto \_ : \mathbb{P}(SGET \times (MultC \times MultC))$$

$$\forall et : SGET;\ m_1, m_2 : MultC \bullet et \propto (m_1, m_2) \Leftrightarrow et = erel\ dbi$$
$$\lor et = ecomp\ duni \land m_1 = ms(\mathbf{v}\ 1) \lor et = erel\ duni \land m_1 \in MultMany$$
$$\lor et = ecomp\ dbi \land m_1 \in \{ms(\mathbf{v}\ 1), mr(0, \mathbf{v}\ 1)\}$$
$$\lor et = ewander \land (m_1, m_2) \in MultMany \times MultMany$$

relation(rbounded _)

$$
\begin{array}{|l}
\multicolumn{1}{l}{[X, Y]} \\
\hline
\mathrm{rbounded}\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\, X \times \mathit{MultC}) \\
\hline
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ m : \mathit{MultC} \bullet \\
\quad \mathrm{rbounded}(r, s, m) \Leftrightarrow \forall\, x : s \bullet \#(r \llparenthesis \{x\} \rrparenthesis) \between (\mathit{mlb}\, m, \mathit{mub}\, m)
\end{array}
$$

relation($r\mathcal{M}Ok\_$)

$$
\begin{array}{|l}
\multicolumn{1}{l}{[X, Y]} \\
\hline
r\mathcal{M}Ok\_ : \mathbb{P}((X \leftrightarrow Y) \times \mathbb{P}\, X \times \mathbb{P}\, Y \times \mathit{MultC} \times \mathit{MultC}) \\
\hline
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, ms(\mathbf{v}\ 1), ms(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail\!\!\!\rightarrow t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr(0, \mathbf{v}\ 1), mr(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail\!\!\!\!\!+ t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr(0, \mathbf{v}\ 1), ms(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrowtail t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y \bullet \\
\quad r\mathcal{M}Ok(r, s, t, ms(\mathbf{v}\ 1), mr(0, \mathbf{v}\ 1)) \Leftrightarrow r^{\sim} \in t \rightarrowtail s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mm : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mm, ms(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrow t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mm : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, ms(\mathbf{v}\ 1), mm) \Leftrightarrow r^{\sim} \in t \rightarrow s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mm_1, mm_2 : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mm_1, mm_2) \Leftrightarrow r \in s \leftrightarrow t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mm : \mathit{MultMany};\ mr : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mm, mr) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r, s, mr) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mm : \mathit{MultMany};\ mr : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr, mm) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r^{\sim}, t, mr) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mm : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mm, mr(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \nrightarrow t \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mm : \mathit{MultMany} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr(0, \mathbf{v}\ 1), mm) \Leftrightarrow r^{\sim} \in t \nrightarrow s \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mr_1, mr_2 : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr_1, mr_2) \Leftrightarrow r \in s \leftrightarrow t \land \mathrm{rbounded}(r, s, mr_2) \land \mathrm{rbounded}(r^{\sim}, t, mr_1) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mr : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr, ms(\mathbf{v}\ 1)) \Leftrightarrow r \in s \rightarrow t \land \mathrm{rbounded}(r^{\sim}, t, mr) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ mr : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, ms(\mathbf{v}\ 1), mr) \Leftrightarrow r^{\sim} \in t \rightarrow s \land \mathrm{rbounded}(r, s, mr) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ m : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, m, mr(0, \mathbf{v}\ 1)) \Leftrightarrow r \in s \nrightarrow t \land \mathrm{rbounded}(r^{\sim}, t, m) \\
\forall\, r : X \leftrightarrow Y;\ s : \mathbb{P}\, X;\ t : \mathbb{P}\, Y;\ m : \mathit{MultRange} \bullet \\
\quad r\mathcal{M}Ok(r, s, t, mr(0, \mathbf{v}\ 1), m) \Leftrightarrow r^{\sim} \in t \nrightarrow s \land \mathrm{rbounded}(r, s, m)
\end{array}
$$

7 Structural Graphs

section *Fragmenta_SGs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*, *Fragmenta_SGElemTys*, *Fragmenta_Mult*, *Fragmenta_GrswT*

$$SGr_0 == \{ G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult \mid nt \in Ns\ G \rightarrow SGNT \land et \in Es\ G \rightarrow SGET \}$$

$$\begin{array}{l}
gr : SGr_0 \rightarrow Gr \\
sg_Ns : SGr_0 \rightarrow \mathbb{P}\ V \\
sg_Es : SGr_0 \rightarrow \mathbb{P}\ E \\
sg_src, sg_tgt : SGr_0 \rightarrow E \pfun V \\
nty : SGr_0 \rightarrow V \pfun SGNT \\
ety : SGr_0 \rightarrow E \pfun SGET \\
srcm, tgtm : SGr_0 \rightarrow E \pfun Mult \\
\hline
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult \bullet gr(G, nt, et, sm, tm) = G \\
sg_Ns = Ns \circ gr \\
sg_Es = Es \circ gr \\
sg_src = src \circ gr \\
sg_tgt = tgt \circ gr \\
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult \bullet \\
\quad nty(G, nt, et, sm, tm) = nt \\
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult \bullet \\
\quad ety(G, nt, et, sm, tm) = et \\
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult \bullet \\
\quad srcm(G, nt, et, sm, tm) = sm \\
\forall G : Gr;\ nt : V \pfun SGNT;\ et : E \pfun SGET;\ sm, tm : E \pfun Mult \bullet \\
\quad tgtm(G, nt, et, sm, tm) = tm
\end{array}$$

$$\begin{array}{l}
\emptyset_{SG} : SGr_0 \\
\hline
\emptyset_{SG} = (\emptyset_G, \emptyset, \emptyset, \emptyset, \emptyset)
\end{array}$$

$$\begin{array}{l}
NsTy : SGr_0 \rightarrow \mathbb{P}\ SGNT \rightarrow \mathbb{P}\ V \\
EsTy : SGr_0 \rightarrow \mathbb{P}\ SGET \rightarrow \mathbb{P}\ E \\
\hline
\forall SG : SGr_0;\ nts : \mathbb{P}\ SGNT \bullet NsTy\ SG\ nts = (nty\ SG)^{\sim} (\!| nts |\!) \\
\forall SG : SGr_0;\ ets : \mathbb{P}\ SGET \bullet EsTy\ SG\ ets = (ety\ SG)^{\sim} (\!| ets |\!)
\end{array}$$

$$
\begin{array}{|l}
EsA, EsW, EsI, EsC : SGr_0 \rightarrow \mathbb{P}\, E \\
\hline
\forall\, SG : SGr_0 \bullet \\
\quad EsA\; SG = EsTy\; SG\; (erel \llparenthesis SGED \rrparenthesis \cup ecomp \llparenthesis SGED \rrparenthesis) \\
\forall\, SG : SGr_0 \bullet EsW = (\text{flip}\; EsTy)\; \{ewander\} \\
\forall\, SG : SGr_0 \bullet EsI = (\text{flip}\; EsTy)\; \{einh\} \\
\forall\, SG : SGr_0 \bullet EsC\; SG = EsA\; SG \cup EsW\; SG
\end{array}
$$

$$
\begin{array}{|l}
NsP, NsEther, NsO, NsSeq : SGr_0 \rightarrow \mathbb{P}\, V \\
\hline
NsP = (\text{flip}\; NsTy)\; \{nprxy\} \\
NsEther = (\text{flip}\; NsTy)\; \{nabst, nvirt, nenum\} \\
NsO = (\text{flip}\; NsTy)\; \{nopt\}
\end{array}
$$

$$
\begin{array}{|l}
\pitchfork : SGr_0 \rightarrow Gr \\
\prec : SGr_0 \rightarrow V \leftrightarrow V \\
\hline
\forall\, SG : SGr_0 \bullet \pitchfork SG = gr\; SG \bowtie EsI\; SG \\
\prec = (\_^{\leftrightarrow}) \circ \pitchfork
\end{array}
$$

$$
\begin{array}{|l}
srcma : SGr_0 \rightarrow (E \nrightarrow Mult) \\
\hline
\forall\, SG : SGr_0 \bullet srcma\; SG = \\
\quad (srcm\; SG) \oplus (EsTy\; SG\; \{ecomp\; duni\} \times \{ms(\mathbf{v}\; 1)\}) \oplus (EsTy\; SG\; \{erel\; duni\} \times \{ms\; *\})
\end{array}
$$

relation($\mathcal{M}etysOk\,\_$)

$$
\begin{array}{|l}
\mathcal{M}etysOk\,\_ : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \mathcal{M}etysOk\;\; SG \Leftrightarrow \forall\, e : EsC\; SG \bullet (ety\; SG\; e) \propto (srcma\; SG\; e, tgtm\; SG\; e)
\end{array}
$$

relation(optsVoluntary_)

$$
\begin{array}{|l}
\text{optsVoluntary}\_ : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \\
\quad \text{optsVoluntary}\; SG \Leftrightarrow (ety\; SG) \llparenthesis EsIncident(gr\; SG)(NsO\; SG) \rrparenthesis \subseteq \{ewander\}
\end{array}
$$

relation(inhNtysOk_)

$$\begin{array}{|l}
\text{inhNtysOk_} : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \\
\quad \text{inhNtysOk}\, SG \Leftrightarrow \forall\, v, v' : sg_Ns\; SG \mid (v, v') \in (\prec\, SG) \bullet nty\; SG\; v \prec_{NT} nty\; SG\; v'
\end{array}$$

relation(seqsOk_)

$$\begin{array}{l}
SGr == \{SG : SGr_0 \mid \{srcma\; SG, tgtm\; SG\} \subseteq EsC\; SG \rightarrow Mult \\
\quad \wedge\, \mathcal{M}etysOk\;\; SG \wedge \text{optsVoluntary}\; SG \wedge \text{inhNtysOk}\, SG \wedge \circleddash(\pitchfork\; SG)\}
\end{array}$$

relation(etherealAreInherited _)

$$\begin{array}{|l}
\text{etherealAreInherited_} : \mathbb{P}\, SGr_0 \\
\hline
\forall\, SG : SGr_0 \bullet \text{etherealAreInherited}\, SG \Leftrightarrow NsEther\; SG \subseteq \text{ran}(\prec\, SG)
\end{array}$$

$$TSGr == \{SG : SGr \mid \text{etherealAreInherited}\, SG\}$$

$$\begin{array}{|l}
\preceq\, : SGr \rightarrow V \leftrightarrow V \\
\hline
\forall\, SG : SGr \bullet \preceq\, SG = (\prec\, SG)^{\,*}
\end{array}$$

$$\begin{array}{|l}
srcr, tgtr : SGr \rightarrow E \leftrightarrow V \\
src_0^*, src^*, tgt_0^*, tgt^* : SGr \rightarrow E \leftrightarrow V \\
\hline
\forall\, SG : SGr \bullet srcr\; SG = sg_src\; SG \cup (EsW\; SG \lhd sg_tgt\; SG) \\
\forall\, SG : SGr \bullet tgtr\; SG = sg_tgt\; SG \cup (EsW\; SG \lhd sg_src\; SG) \\
\forall\, SG : SGr \bullet src_0^*\;\; SG = EsC\; SG \lhd (srcr\; SG) \\
\forall\, SG : SGr \bullet src^*\;\; SG = (src_0^*\;\; SG) \mathbin{⨾} (\preceq\, SG)^{\sim} \\
\forall\, SG : SGr \bullet tgt_0^*\;\; SG = EsC\; SG \lhd (tgtr\; SG) \\
\forall\, SG : SGr \bullet tgt^*\;\; SG = (tgt_0^*\;\; SG) \mathbin{⨾} (\preceq\, SG)^{\sim}
\end{array}$$

relation$(\boxminus_{SGs}\_)$

$$\boxminus_{SGs}\_ : \mathbb{P}(SGr \times SGr)$$
$$\forall SG_1, SG_2 : SGr \bullet \boxminus_{SGs}(SG_1, SG_2) \Leftrightarrow \boxminus(gr\ SG_1, gr\ SG_2)$$

function 10 **leftassoc** $(\_ \cup_{SG} \_)$

$$\_ \cup_{SG} \_ : SGr \times SGr \nrightarrow SGr$$
$$\forall SG_1, SG_2 : SGr \bullet SG_1 \cup_{SG} SG_2 = (gr\ SG_1 \cup_G gr\ SG_2, nty\ SG_1 \cup nty\ SG_2, \\ ety\ SG_1 \cup ety\ SG_2, srcm\ SG_1 \cup srcm\ SG_2, tgtm\ SG_1 \cup tgtm\ SG_2)$$

function 10 **leftassoc** $(\_ \odot^{SG} \_)$

$$\_ \odot^{SG} \_ : SGr \times (V \nrightarrow V) \nrightarrow SGr$$
$$\forall SG : SGr;\ s : V \nrightarrow V \mid s \in NsP\ SG \nrightarrow sg\_Ns\ SG \land s \in antireflexive \bullet \\ SG \odot^{SG} s = (gr\ SG \odot s, (dom\ s) \lhd nty\ SG, ety\ SG, srcm\ SG, tgtm\ SG)$$

function 10 **leftassoc** $(\_ \rightarrow_{SG} \_)$

$$\_ \rightarrow_{SG} \_ : SGr \times SGr \rightarrow \mathbb{P}\ GrM$$
$$\forall SG_s, SG_t : SGr \bullet \\ SG_s \rightarrow_{SG} SG_t = \{fv : sg\_Ns\ SG_s \rightarrow sg\_Ns\ SG_t;\ fe : EsC\ SG_s \rightarrow EsC\ SG_t \mid \\ fv \circ src^*\ SG_s \subseteq src^*\ SG_t \circ fe \land fv \circ tgt^*\ SG_s \subseteq tgt^*\ SG_t \circ fe \\ \land fv \circ \preceq SG_s \subseteq \preceq SG_t \circ fv\}$$

relation$(\_ \Rrightarrow^{SG} \_)$

$$\_ \Rrightarrow^{SG} \_ : \mathbb{P}((SGr \times GrM) \times SGr)$$
$$\forall SG_s, SG_t : SGr;\ m : GrM \bullet (SG_s, m) \Rrightarrow^{SG} SG_t \Leftrightarrow m \in SG_s \rightarrow_{SG} SG_t$$

function 10 **leftassoc** $(\_ \rightarrow_{G2SG} \_)$

$$\_ \rightarrow_{G2SG} \_ : Gr \times SGr \rightarrow \mathbb{P}\, GrM$$

$$\forall\, G : Gr;\ SG : SGr \bullet G \rightarrow_{G2SG} SG = \{ fv : Ns\ G \rightarrow sg\_Ns\ SG;\ fe : Es\ G \rightarrow EsC\ SG \mid fv \circ src\ G \subseteq src^*\ SG \circ fe \wedge fv \circ tgt\ G \subseteq tgt^*\ SG \circ fe \}$$

relation$(\_ \Rrightarrow^{GwT} \_)$

$$\_ \Rrightarrow^{GwT} \_ : (GrwT \leftrightarrow SGr)$$

$$\forall\, GwT : GrwT;\ SG : SGr \bullet GwT \Rrightarrow^{GwT} SG \Leftrightarrow (ty\ GwT) \in (gOf\ GwT) \rightarrow_{G2SG} SG$$

$$insOf : GrM \times SGr \times \mathbb{P}\, V \rightarrow \mathbb{P}\, V$$
$$iesOf : GrM \times \mathbb{P}\, E \rightarrow \mathbb{P}\, E$$
$$igRMEs : GrwT \times \mathbb{P}\, E \rightarrow Gr$$

$$\forall\, m : GrM;\ SG : SGr;\ mns : \mathbb{P}\, V \bullet insOf(m, SG, mns) = (fV\ m)^{\sim} (\!| (\prec SG)^{\sim} (\!| mns |\!) |\!)$$

$$\forall\, m : GrM;\ mes : \mathbb{P}\, E \bullet iesOf(m, mes) = (fE\ m)^{\sim} (\!| mes |\!)$$

$$\forall\, GwT : GrwT;\ mes : \mathbb{P}\, E \bullet igRMEs(GwT, mes) = (gOf\ GwT) \bowtie iesOf((ty\ GwT), mes)$$

relation$(\mathrm{inverted_E}\_)$

$$\mathrm{inverted_E}\_ : \mathbb{P}(GrwT \times SGr \times E)$$
$$gOfwei, igRMEsW : GrwT \times SGr \times E \rightarrow Gr$$
$$gOfweis : GrwT \times SGr \times \mathbb{P}\, E \rightarrow Gr$$

$$\forall\, G : Gr;\ m : GrM;\ SG : SGr;\ e : E \bullet \mathrm{inverted_E}((G, m), SG, e) \Leftrightarrow ((sg\_tgt\ SG) \circ (fE\ m))e = ((fV\ m) \circ (src\ G))e$$

$$\forall\, GwT : GrwT;\ SG : SGr;\ e : E \bullet gOfwei(GwT, SG, e) = \textbf{if}\ \mathrm{inverted_E}(GwT, SG, e)\ \textbf{then}\ ((gOf\ GwT) \bowtie \{e\})^{\rightleftarrows}\ \textbf{else}(gOf\ GwT) \bowtie \{e\}$$

$$\forall\, GwT : GrwT;\ SG : SGr \bullet gOfweis(GwT, SG, \{\}) = \varnothing_G$$

$$\forall\, GwT : GrwT;\ SG : SGr;\ e : E;\ es : \mathbb{P}\, E \bullet gOfweis(GwT, SG, \{e\} \cup es) = gOfwei(GwT, SG, e) \cup_G gOfweis(GwT, SG, es)$$

$$\forall\, GwT : GrwT;\ SG : SGr;\ e : E \bullet igRMEsW(GwT, SG, e) = \textbf{if}\ e \notin EsW\ SG\ \textbf{then}\ igRMEs(GwT, \{e\})\ \textbf{else}\ gOfweis(GwT, SG, ((fE \circ ty)\ GwT)^{\sim} (\!| \{e\} |\!))$$

relation$(\_ \sqsupseteq^{SG} \_)$
relation$(\_ \sqsupseteq^{SG_0} \_)$
relation$(\_ \sqsupseteq_{NT} \_)$
relation$(\_ \sqsupseteq_{ET} \_)$
relation$(\_ \sqsupseteq_{\mathcal{M}} \_)$

$$
\begin{array}{l}
\_ \sqsupseteq_{NT} \_, \_ \sqsupseteq_{ET} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq_{NT}\ SG_a \Leftrightarrow \forall n : sg\_Ns\ SG_c \bullet (nty\ SG_c)\ n \leq_{rNT} ((nty\ SG_a) \circ (fV\ m))\ n \\
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq_{ET}\ SG_a \Leftrightarrow \forall e : EsC\ SG_c \bullet (ety\ SG_c)\ e \leq_{ET} ((ety\ SG_a) \circ (fE\ m))\ e
\end{array}
$$

$$
\begin{array}{l}
\_ \sqsupseteq_{\mathcal{M}} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq_{\mathcal{M}}\ SG_a \Leftrightarrow \forall e : EsC\ SG_c \bullet (srcma\ SG_c)\ e \leq_{\mathcal{M}} ((srcma\ SG_a) \circ (fE\ m))\ e \\
\qquad \wedge\ (tgtm\ SG_c)\ e \leq_{\mathcal{M}} ((tgtm\ SG_a) \circ (fE\ m))\ e
\end{array}
$$

$$
\begin{array}{l}
\_ \sqsupseteq^{SG} \_, \_ \sqsupseteq^{SG_0} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq^{SG_0}\ SG_a \Leftrightarrow (SG_c, m) \sqsupseteq_{NT}\ SG_a \wedge (SG_c, m) \sqsupseteq_{ET}\ SG_a \wedge (SG_c, m) \sqsupseteq_{\mathcal{M}}\ SG_a \\
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupseteq^{SG}\ SG_a \Leftrightarrow m \in SG_c \rightarrow_{SG} SG_a \wedge (SG_c, m) \sqsupseteq^{SG_0}\ SG_a
\end{array}
$$

relation$(\_ \sqsupset^{SG} \_)$
relation$(\_ \sqsupset^{SG_0} \_)$
relation$(\_ \sqsupset_{AEs} \_)$
relation(_OkRefinedIn_)
relation$(\_ \sqsupset_{ANNs} \_)$

$$
\begin{array}{l}
\_ \sqsupset_{ANNs} \_ : \mathbb{P}(GrM \times SGr) \\
\hline
\forall SG_a : SGr;\ m : GrM \bullet \\
\quad m \sqsupset_{ANNs} SG_a \Leftrightarrow \forall nn : NsTy\ SG_a\{nnrml\} \bullet (\preceq\ SG_a)\ (\!|\ \{nn\}\ |\!) \cap \operatorname{ran}(fV\ m) = \varnothing
\end{array}
$$

$$
\begin{array}{l}
\_\mathrm{OkRefinedIn}\_ : \mathbb{P}((SGr \times E) \times (SGr \times GrM)) \\
\_ \sqsupset_{AEs} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM;\ ae : E \bullet \\
\quad (SG_a, ae)\mathrm{OkRefinedIn}(SG_c, m) \Leftrightarrow \\
\qquad \exists r : V \leftrightarrow V;\ s, t : \mathbb{P}\ V \mid r = (\preceq\ SG_c) \mathbin{⨾} igRMEsW((gr\ SG_c, m), SG_a, ae)^{\leftrightarrow} \mathbin{⨾} (\preceq\ SG_c)^{\sim} \\
\qquad\quad \wedge\ s = insOf(m, SG_a, (sg\_src\ SG_a\ (\!|\ \{ae\}\ |\!))) \setminus ((NsEther\ SG_c) \setminus \operatorname{dom}\ r) \\
\qquad\quad \wedge\ t = insOf(m, SG_a, (sg\_tgt\ SG_a\ (\!|\ \{ae\}\ |\!))) \setminus ((NsEther\ SG_c) \setminus \operatorname{ran}\ r) \\
\qquad\qquad \bullet\ r \in s \leftrightarrow t \wedge r \neq \varnothing \\
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset_{AEs}\ SG_a \Leftrightarrow \forall e : (EsA\ SG_a) \bullet (SG_a, e)\mathrm{OkRefinedIn}(SG_c, m)
\end{array}
$$

$$
\begin{array}{|l}
\_ \sqsupset^{SG} \_, \_ \sqsupset^{SG_0} \_ : \mathbb{P}((SGr \times GrM) \times SGr) \\
\hline
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset^{SG_0}\ SG_a \Leftrightarrow (SG_c, m) \sqsupseteq^{SG_0}\ SG_a \land m \sqsupset_{ANNs} SG_a \land (SG_c, m) \sqsupset_{AEs}\ SG_a \\
\forall SG_c, SG_a : SGr;\ m : GrM \bullet \\
\quad (SG_c, m) \sqsupset^{SG}\ SG_a \Leftrightarrow m \in SG_c \to_{SG} SG_a \land (SG_c, m) \sqsupset^{SG_0}\ SG_a
\end{array}
$$

relation$(\_ \Supset^{SG} \_)$
relation$(\_ \Supset_{\mathcal{M}} \_)$
relation$(\_ \Supset_{FI} \_)$
relation$(\_ \Supset_{PNS} \_)$
relation$(\_ \ MEMOk\ \_)$

$$
\begin{array}{|l}
\_ \ ME\mathcal{M}Ok \ \_ : \mathbb{P}((SGr \times E) \times GrwT) \\
\hline
\forall GwT : GrwT;\ SG : SGr;\ me : E \bullet (SG, me)\ ME\mathcal{M}Ok\ GwT \Leftrightarrow \\
\quad \exists r : V \leftrightarrow V;\ s, t : \mathbb{P}\ V \mid r = igRMEsW(GwT, SG, me)^{\leftrightarrow} \\
\qquad \land s = insOf(ty\ GwT, SG, (src^*\ SG) \lparen \{me\} \rparen) \\
\qquad \land t = insOf(ty\ GwT, SG, (tgt^*\ SG) \lparen \{me\} \rparen) \\
\qquad\quad \bullet\ r\mathcal{M}Ok(r, s, t, srcma\ SG\ me, tgtm\ SG\ me)
\end{array}
$$

$$
\begin{array}{|l}
\_ \Supset_{\mathcal{M}} \_ : GrwT \leftrightarrow SGr \\
\_ \Supset_{FI} \_ : GrwT \leftrightarrow SGr \\
\_ \Supset_{PNS} \_ : GrwT \leftrightarrow SGr \\
\hline
\forall GwT : GrwT;\ SG : SGr \bullet GwT \Supset_{\mathcal{M}} SG \Leftrightarrow \forall me : EsC\ SG \bullet (SG, me)\ ME\mathcal{M}Ok\ GwT \\
\forall GwT : GrwT;\ SG : SGr \bullet GwT \Supset_{FI} SG \Leftrightarrow ((fV \circ ty)\ GwT)^{\sim} \lparen NsEther\ SG \rparen = \varnothing \\
\forall GwT : GrwT;\ SG : SGr \bullet \\
\quad GwT \Supset_{PNS} SG \Leftrightarrow igRMEs(GwT, EsTy\ SG\ \{ecomp\ dbi, ecomp\ duni\})^{\leftrightarrow} \in \mathrm{injrel}
\end{array}
$$

$$
\begin{array}{|l}
\_ \Supset^{SG} \_ : GrwT \leftrightarrow SGr \\
\hline
\forall GwT : GrwT;\ SG : SGr \bullet \\
\quad GwT \Supset^{SG} SG \Leftrightarrow GwT \Rrightarrow^{GwT} SG \land GwT \Supset_{\mathcal{M}} SG \land GwT \Supset_{FI} SG \land GwT \Supset_{PNS} SG
\end{array}
$$

8 Fragments

section *Fragmenta_Frs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_SGs*, *Fragmenta_GrswT*

$$
\begin{array}{l}
Fr_0 == \{ SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V \mid esr \cap (sg\_Es\ SG) = \varnothing \\
\quad \land\ sr \in esr \rightarrowtail\!\!\!\!\twoheadrightarrow (NsP\ SG) \land tr \in esr \rightarrow V \}
\end{array}
$$

$$
\begin{array}{|l}
fSG : Fr_0 \rightarrow SGr \\
EsR : Fr_0 \rightarrow \mathbb{P}\, E \\
srcR, tgtR : Fr_0 \rightarrow E \nrightarrow V \\
\hline
\forall\, SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V \bullet fSG(SG, esr, sr, tr) = SG \\
\forall\, SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V \bullet EsR(SG, esr, sr, tr) = esr \\
\forall\, SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V \bullet srcR(SG, esr, sr, tr) = sr \\
\forall\, SG : SGr;\ esr : \mathbb{P}\, E;\ sr, tr : E \nrightarrow V \bullet tgtR(SG, esr, sr, tr) = tr
\end{array}
$$

$$
\begin{array}{|l}
fLEs, fEs, fEsC : Fr_0 \rightarrow \mathbb{P}\, E \\
fLNs, fRNs, fNs : Fr_0 \rightarrow \mathbb{P}\, V \\
srcF, tgtF : Fr_0 \rightarrow E \nrightarrow V \\
\hline
fLEs = (sg\_Es \circ fSG) \\
\forall\, F : Fr_0 \bullet fEs\ F = fLEs\ F \cup EsR\ F \\
fEsC = EsC \circ fSG \\
fLNs = sg\_Ns \circ fSG \\
fRNs = \mathrm{ran} \circ tgtR \\
\forall\, F : Fr_0 \bullet fNs\ F = fLNs\ F \cup fRNs\ F \\
\forall\, F : Fr_0 \bullet srcF\ F = (sg\_src \circ fSG)\ F \cup srcR\ F \\
\forall\, F : Fr_0 \bullet tgtF\ F = (sg\_tgt \circ fSG)\ F \cup tgtR\ F
\end{array}
$$

$$
\begin{array}{|l}
\overset{G}{\leftrightsquigarrow} : Fr_0 \rightarrow Gr \\
\leftrightsquigarrow : Fr_0 \rightarrow V \leftrightarrow V \\
\hline
\forall\, F : Fr_0 \bullet \overset{G}{\leftrightsquigarrow}\ F = ((NsP \circ fSG) F \cup fRNs\ F, EsR\ F, srcR\ F, tgtR\ F) \\
\forall\, F : Fr_0 \bullet \leftrightsquigarrow\ F = (\overset{G}{\leftrightsquigarrow}\ F)^{\leftrightarrow}
\end{array}
$$

function 10 **leftassoc** $(\_ \cup_F \_)$

$$
\begin{array}{|l}
\varnothing_F : Fr_0 \\
\_ \cup_F \_ : Fr_0 \times Fr_0 \rightarrow Fr_0 \\
\bigcup_F : \mathbb{P}\, Fr_0 \rightarrow Fr_0 \\
\hline
\varnothing_F = (\varnothing_{SG}, \varnothing, \varnothing, \varnothing) \\
\forall\, F_1, F_2 : Fr_0 \bullet F_1 \cup_F F_2 = \\
\quad (fSG\, F_1 \cup_{SG} fSG\, F_2, EsR\, F_1 \cup EsR\, F_2, srcR\, F_1 \cup srcR\, F_2, tgtR\, F_1 \cup tgtR\, F_2) \\
\bigcup_F \{\} = \varnothing_F \\
\forall\, F : Fr_0;\; Fs : \mathbb{P}\, Fr_0 \bullet \bigcup_F (\{F\} \cup Fs) = F \cup_F (\bigcup_F Fs)
\end{array}
$$

$$
\begin{array}{|l}
\rightsquigarrow : Fr_0 \nrightarrow V \nrightarrow V \\
\circledbullet^{SG} : Fr_0 \nrightarrow SGr \\
rEsR : Fr_0 \nrightarrow \mathbb{P}\, E \\
\circledbullet : Fr_0 \nrightarrow Fr_0 \\
\hline
\forall\, F : Fr_0 \bullet \rightsquigarrow F = (\leftrightsquigarrow F) \rhd (fLNs\, F) \\
\forall\, F : Fr_0 \bullet \circledbullet^{SG} F = (fSG\, F) \odot^{SG} (\rightsquigarrow F) \\
\forall\, F : Fr_0 \bullet rEsR\, F = \operatorname{dom}((srcR\, F) \unrhd \operatorname{dom}(\rightsquigarrow F)) \\
\forall\, F : Fr_0 \bullet \circledbullet F = (\circledbullet^{SG} F, rEsR\, F, (rEsR\, F) \lhd (srcR\, F), (rEsR\, F) \lhd (tgtR\, F))
\end{array}
$$

$$Fr_a == \{F : Fr_0 \mid \otimes(\overset{G}{\leftrightsquigarrow} F)\}$$

$$Fr == \{F : Fr_a \mid \circledbullet^{SG} F \in SGr\}$$

relation(refsLocal_)

$$
\begin{array}{|l}
\text{refsLocal\_} : \mathbb{P}\, Fr_0 \\
\hline
\forall\, F : Fr_0 \bullet \text{refsLocal}\, F \Leftrightarrow fRNs\, F \subseteq fLNs\, F
\end{array}
$$

$$TFr == \{F : Fr_a \mid \text{refsLocal}\, F \wedge \circledbullet^{SG} F \in TSGr\}$$

relation($\boxminus$ _)
relation($\boxplus$ _)

$$
\begin{array}{|l}
\boxminus\_ : Fr \leftrightarrow Fr \\
\hline
\forall\, F_1, F_2 : Fr \bullet \boxminus(F_1, F_2) \Leftrightarrow \boxminus_{SGs}(fSG\, F_1, fSG\, F_2) \wedge EsR\, F_1 \cap EsR\, F_2 = \varnothing
\end{array}
$$

$$[I]$$

$$\boxplus\_ : \mathbb{P}(I \pfun Fr)$$

$$\forall Fs : I \pfun Fr \bullet \boxplus\ Fs \Leftrightarrow \forall i, j : \operatorname{dom} Fs \mid i \neq j \bullet \boxminus(Fs\, i, Fs\, j)$$

relation$(\_ \subseteq^{rs} \_)$
relation$(\_ \Mapsto \_)$

$$\_ \subseteq^{rs} \_ : Fr \leftrightarrow Fr$$
$$\_ \Mapsto \_ : Fr \leftrightarrow Fr$$

$$\forall F_1, F_2 : Fr \bullet F_1 \subseteq^{rs} F_2 \Leftrightarrow \operatorname{ran}(tgtR\ F_1) \subseteq fLNs\ F_2$$
$$\forall F_1, F_2 : Fr \bullet F_1 \Mapsto F_2 \Leftrightarrow F_1 \subseteq^{rs} F_2 \land \neg\ (F_2 \subseteq^{rs} F_1)$$

function 10 **leftassoc** $(\_ \rightrightarrows_{\bullet} \_)$

$$\_ \rightrightarrows_{\bullet} \_ : GrM \times (Fr \times Fr) \rightarrow GrM$$

$$\forall m : GrM;\ F_s, F_t : Fr_0 \bullet$$
$$m \rightrightarrows_{\bullet} (F_s, F_t) = (((\rightsquigarrow F_s)^{\oplus} \boxdot (fLNs\ F_s))^{\sim} \,⨾ (fV\ m) \,⨾ ((\rightsquigarrow F_t)^{\oplus} \boxdot (fLNs\ F_t)), fE\ m)$$

function 1 **leftassoc** $(\_ \rightarrow_F \_)$

$$\_ \rightarrow_F \_ : Fr \times Fr \rightarrow \mathbb{P}\ GrM$$

$$\forall F_s, F_t : Fr \bullet F_s \rightarrow_F F_t = \{fv : fLNs\ F_s \rightarrow fLNs\ F_t;\ fe : fEsC\ F_s \rightarrow fEsC\ F_t \mid$$
$$(\circledcirc^{SG} F_s, (fv, fe) \rightrightarrows_{\bullet} (F_s, F_t)) \Rrightarrow^{SG} \circledcirc^{SG} F_t\}$$

relation$(\_ \Rrightarrow^{F} \_)$

$$\_ \Rrightarrow^{F} \_ : (Fr \times GrM) \leftrightarrow Fr$$

$$\forall m : GrM;\ F_s, F_t : Fr_0 \bullet (F_s, m) \Rrightarrow^{F} F_t \Leftrightarrow m \in F_s \rightarrow_F F_t$$

relation$(\_ \sqsupseteq^F \_)$

$$
\begin{array}{|l}
\_ \sqsupseteq^F \_ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupseteq^F F_a \Leftrightarrow (F_c, m) \Rrightarrow^F F_a \\
\quad \land (\circledbullet^{SG} F_c, m \rightrightarrows_\bullet (F_c, F_a)) \sqsupseteq^{SG_0} (\circledbullet^{SG} F_a)
\end{array}
$$

relation$(\_ \sqsupset^F \_)$

$$
\begin{array}{|l}
\_ \sqsupset^F \_ : (Fr \times GrM) \leftrightarrow Fr \\
\hline
\forall F_c, F_a : Fr_0;\ m : GrM \bullet (F_c, m) \sqsupset^F F_a \Leftrightarrow (F_c, m) \Rrightarrow^F F_a \\
\quad \land (\circledbullet^{SG} F_c, m \rightrightarrows_\bullet (F_c, F_a)) \sqsupset^{SG_0} (\circledbullet^{SG} F_a)
\end{array}
$$

relation$(\_ \Supset^F \_)$

$$
\begin{array}{|l}
\_ \Supset^F \_ : GrwT \leftrightarrow Fr \\
\hline
\forall GwT : GrwT;\ F : Fr \bullet GwT \Supset^F F \Leftrightarrow GwT \Supset^{SG} \circledbullet^{SG} F
\end{array}
$$

9 Global Fragment Graphs

section *Fragmenta_GFGs* **parents** *standard_toolkit*, *Fragmenta_Generics*, *Fragmenta_Graphs*

$GFGr == \{G : Gr \mid \circleddash(G \bowtie (Es\ G \setminus EsId\ G))\}$

function$(\_^{\dashrightarrow})$

$$
\begin{array}{|l}
\_^{\dashrightarrow} : GFGr \rightarrow V \leftrightarrow V \\
\hline
\forall GFG : GFGr \bullet GFG^{\dashrightarrow} = (GFG^{\leftrightarrow})^+
\end{array}
$$

10 Models

section *Fragmenta_Mdls* **parents** *standard_toolkit*, *Fragmenta_Frs*, *Fragmenta_GFGs*

$Mdl_0 == \{GFG : GFGr;\ fd : V \nrightarrow Fr \mid fd \in Ns\ GFG \rightarrow Fr \wedge \boxplus\ fd\}$

$$
\begin{array}{|l}
mGFG : Mdl_0 \rightarrow GFGr \\
mFD : Mdl_0 \rightarrow V \nrightarrow Fr \\
\hline
\forall\ GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mGFG(GFG, fd) = GFG \\
\forall\ GFG : GFGr;\ fd : V \nrightarrow Fr \bullet mFD(GFG, fd) = fd
\end{array}
$$

$$
\begin{array}{|l}
mUFs : Mdl_0 \rightarrow Fr \\
\hline
mUFs = \bigcup_F \circ \operatorname{ran} \circ mFD
\end{array}
$$

$$
\begin{array}{|l}
\mathrm{from} : Mdl_0 \rightarrow V \nrightarrow V \\
\hline
\forall\ M : Mdl_0;\ v : V \bullet \mathrm{from}\ M\ v = (\mu\ vf : (Ns \circ mGFG)M \mid v \in fLNs(mFD\ M\ vf))
\end{array}
$$

relation$(\Uparrow \_)$

$$
\begin{array}{|l}
\Uparrow \_ : \mathbb{P}\ Mdl_0 \\
\hline
\forall\ M : Mdl_0 \bullet \Uparrow M \Leftrightarrow \exists\ UF : Fr_0 \mid UF = mUFs\ M \bullet \\
\quad \forall\ p : (NsP \circ fSG)\ UF \bullet (from\ M\ p, from\ M\ (\leftrightsquigarrow\ UF\ p)) \in ((\_ ^{\dashrightarrow}) \circ mGFG)M
\end{array}
$$

$Mdl == \{M : Mdl_0 \mid (mUFs\ M) \in TFr \wedge \Uparrow M\}$

$$
\begin{array}{|l}
\odot^M : Mdl \rightarrow Fr \\
\hline
\forall\ M : Mdl_0 \bullet \odot^M = \odot \circ mUFs
\end{array}
$$

function 1 **leftassoc** $(\_ \rightarrow_M \_)$
relation$(\_ \Rrightarrow^M \_)$

$$
\begin{array}{|l}
\_ \rightarrow_M \_ : Mdl \times Mdl \rightarrow \mathbb{P}\, GrM \\
\_ \Rrightarrow^M \_ : \mathbb{P}((Mdl \times \mathbb{P}\, GrM) \times Mdl) \\
\hline
\forall M_s, M_t : Mdl \bullet M_s \rightarrow_M M_t = \{ m : GrM \mid \\
\quad \exists\, UF_s, UF_t : Fr_0 \mid UF_s = mUFs\, M_s \land UF_t = mUFs\, M_t \bullet m \in UF_s \rightarrow_F UF_t \} \\
\forall M_s, M_t : Mdl;\ ms : \mathbb{P}\, GrM \bullet (M_s, ms) \Rrightarrow^M M_t \Leftrightarrow \bigcup_{GM} ms \in M_s \rightarrow_M M_t
\end{array}
$$

relation$(\_ \sqsupset^M \_)$

$$
\begin{array}{|l}
\_ \sqsupset^M \_ : (Mdl \times \mathbb{P}\, GrM) \leftrightarrow Mdl \\
\hline
\forall M_c, M_a : Mdl_0;\ ms : \mathbb{P}\, GrM \bullet (M_c, ms) \sqsupset^M M_a \\
\quad \Leftrightarrow \exists\, UF_c, UF_a : Fr_0 \mid UF_c = mUFs\, M_c \land UF_a = mUFs\, M_a \bullet (UF_c, \bigcup_{GM} ms) \sqsupset^F UF_a
\end{array}
$$

relation$(\_ \Supset^M \_)$

$$
\begin{array}{|l}
\_ \Supset^M \_ : GrwT \leftrightarrow Mdl \\
\hline
\forall\, GwT : GrwT;\ M : Mdl \bullet GwT \Supset^M M \Leftrightarrow GwT \Supset^F mUFs\, M
\end{array}
$$

11 PC Trees

section *PCTrees* **parents** *standard_toolkit*, *Fragmenta_Generics*

$$
\begin{array}{|l}
[X, Y, Z] \\
fst_t : X \times Y \times Z \rightarrow X \\
\hline
\forall\, x : X;\ y : Y;\ z : Z \bullet fst_t(x, y, z) = x
\end{array}
$$

$CMMT ::= Atom \mid StartCompound \mid ACompound \mid Compound \mid Operator \mid OpExtChoice \mid OpIntChoice$
$\quad \mid OpParallel \mid OpInterleave \mid OpThrow \mid OpIf \mid Stop \mid Skip$

relation$(\_ <_{CMM} \_)$
relation$(\_ \leq_{CMM} \_)$

$$
\begin{array}{|l}
\_ <_{CMM} \_ : CMMT \leftrightarrow CMMT \\
\_ \leq_{CMM} \_ : CMMT \leftrightarrow CMMT \\
\hline
(\_ <_{CMM} \_) = \{ACompound \mapsto Compound, StartCompound \mapsto ACompound\} \\
(\_ \leq_{CMM} \_) = (\_ <_{CMM} \_)^{*}
\end{array}
$$

$[Mdl, GrM, SGr]$

$MMI == Mdl \times Mdl \times GrM \times SGr$

$[Id, E, G, PC]$

$Ids == \mathbb{P}\ Id$

$IdS == \mathrm{seq}\ Id$

$Es == \mathrm{seq}\ E$

$Ren == Id \nrightarrow Id$

$Bool ::= True \mid False$

$[CT_0]$

$TOp ::= \Box \mid \sqcap \mid \rightarrow_{\shortmid}\langle\langle Bool\rangle\rangle \mid \|\langle\langle Es\rangle\rangle \mid \||| \mid \Theta\langle\langle Es\rangle\rangle \mid \iota\langle\langle G\rangle\rangle$

$$
\begin{array}{l}
PCT ::= \alpha\langle\langle Id \times \mathrm{opt}[G] \times \mathrm{opt}[Id \times E]\rangle\rangle \mid \kappa\langle\langle CT_0\rangle\rangle \mid \gamma\langle\langle TOp \times PCT \times PCT\rangle\rangle \\
\quad \mid \rho\langle\langle Id \times \mathrm{opt}[G] \times Es \times Ren\rangle\rangle \mid \Lambda \mid \theta \mid \chi
\end{array}
$$

$CT ::= \varkappa\langle\langle Id \times Es \times \mathrm{seq}\, CT \times PCT\rangle\rangle$

$PCTD == Id \times \mathrm{seq}\, CT$

$$
\begin{array}{|l}
mkAuxId : Id \rightarrowtail Id \\
toCT_0 : CT \rightarrowtail CT_0 \\
the\_ct : PCT \nrightarrow CT \\
rearrangeT : PCT \nrightarrow CT \\
\hline
\forall\, ct : CT \bullet the\_ct\,(\kappa\,(toCT_0\ ct)) = ct \\
\forall\, n : Id;\ ps : Es;\ cts : \mathrm{seq}\, CT;\ t_1, t_2 : PCT \bullet \\
\quad rearrangeT(\gamma(\rightarrow_{\shortmid}\ False, (\kappa \circ toCT_0 \circ \varkappa)(n, ps, cts, t_1), t_2)) \\
\qquad = \varkappa(n, ps, \langle\rangle, \gamma(\rightarrow_{\shortmid}\ False, (\kappa \circ toCT_0 \circ \varkappa)(mkAuxId\ n, \langle\rangle, cts, t_1), t_2)) \\
\forall\, t : PCT \bullet rearrangeT\ t = the\_ct\ t
\end{array}
$$

relation(isOperator_)

$$\begin{array}{|l} \mathrm{isOperator}\_ : \mathbb{P}\, PCT \\ \hline \forall\, t : PCT \bullet \mathrm{isOperator}\, t \Leftrightarrow \exists\, op : TOp;\ t_1, t_2 : PCT \bullet t = (\gamma(op, t_1, t_2)) \end{array}$$

$$\begin{array}{|l} cpctd : Id \rightarrow \mathrm{seq}\, CT \rightarrow PCTD \\ idpctd : PCTD \rightarrow Id \\ ctspctd : PCTD \rightarrow \mathrm{seq}\, CT \\ \hline \forall\, n : Id;\ cts : \mathrm{seq}\ CT \bullet cpctd\ n\ cts = (n, cts) \\ idpctd = first \\ ctspctd = second \end{array}$$

$PCTCxt == PCT \times Ids \times Ids$

$$\begin{array}{|l} toTOp : CMMT \times \mathrm{opt}[G] \times Es \rightarrow TOp \\ \hline \forall\, og : \mathrm{opt}[G];\ ps : Es \bullet toTOp(OpExtChoice, og, ps) = \Box \\ \forall\, og : \mathrm{opt}[G];\ ps : Es \bullet toTOp(OpIntChoice, og, ps) = \sqcap \\ \forall\, og : \mathrm{opt}[G];\ ps : Es \bullet toTOp(OpParallel, og, ps) = \parallel ps \\ \forall\, og : \mathrm{opt}[G];\ ps : Es \bullet toTOp(OpInterleave, og, ps) = ||| \\ \forall\, og : \mathrm{opt}[G];\ ps : Es \bullet toTOp(OpThrow, og, ps) = \Theta\ ps \\ \forall\, og : \mathrm{opt}[G];\ ps : Es \bullet toTOp(OpIf, og, ps) = \iota\,(\mathrm{the}\ og) \end{array}$$

function 10 **leftassoc** $(\_ :\rightarrow \_)$
function 10 **leftassoc** $(\_ :\rightarrow_O \_)$
function 10 **leftassoc** $(\_ : \twoheadrightarrow \_)$
function 10 **leftassoc** $(\_ \multimap \_)$

$$\begin{array}{|l} \_ :\rightarrow \_ : PCTCxt \times PCTCxt \rightarrow PCTCxt \\ \_ :\rightarrow_O \_ : PCTCxt \times PCTCxt \rightarrow PCTCxt \\ \_ : \twoheadrightarrow \_ : PCTCxt \times PCTCxt \rightarrow PCTCxt \\ \_ \multimap \_ : PCTCxt \times (TOp \times PCTCxt) \rightarrow PCTCxt \\ \hline \forall\, t_1, t_2 : PCT;\ rs, rs', cs, cs' : Ids \bullet \\ \quad (t_1, rs, cs)\ :\rightarrow\ (t_2, rs', cs') = (\gamma(\rightarrow\!\shortmid\ False, t_1, t_2), rs \cup rs', cs \cup cs') \\ \forall\, t_1, t_2 : PCT;\ rs, rs', cs, cs' : Ids \bullet \\ \quad (t_1, rs, cs)\ :\rightarrow_O\ (t_2, rs', cs') = (\gamma(\rightarrow\!\shortmid\ True, t_1, t_2), rs \cup rs', cs \cup cs') \\ \forall\, t_1, t_2 : PCT;\ rs, rs', cs, cs' : Ids \bullet (t_1, rs, cs)\ : \twoheadrightarrow\ (t_2, rs', cs') \\ \quad = \mathbf{if}\ t_2 = \Lambda\ \mathbf{then}\ (t_1, rs, cs)\ \mathbf{else}\ (t_1, rs, cs)\ :\rightarrow\ (t_2, rs', cs') \\ \forall\, t_1, t_2 : PCT;\ rs, rs', cs, cs' : Ids;\ op : TOp \bullet (t_1, rs, cs)\ \multimap\ (op, (t_2, rs', cs')) \\ \quad = \mathbf{if}\ t_2 = \Lambda\ \mathbf{then}\ (t_1, rs, cs)\ \mathbf{else}\ (\gamma\,(op, t_1, t_2), rs \cup rs', cs \cup cs') \end{array}$$

relation(isOfTy_)
relation(openAC_)

$$
\begin{array}{|l}
nmOf : PC \rightarrow Id \rightarrow Id \\
stCompound : MMI \times PC \rightarrow Id \\
\mathrm{isOfTy\_} : \mathbb{P}(MMI \times PC \times Id \times CMMT) \\
nxtAfterC : MMI \times PC \times Id \rightarrow \mathrm{opt}[Id] \\
nxtNAfter : MMI \times PC \times Id \rightarrow \mathrm{opt}[Id] \\
compoundStart : MMI \times PC \times Id \rightarrow Id \\
paramsOf : PC \times Id \rightarrow Es \\
renamingsOf : PC \times Id \rightarrow Ren \\
nmOfRefF : MMI \times PC \times Id \rightarrow Id \\
psOfRef : MMI \times PC \times Id \rightarrow Es \\
branchesOfOp : MMI \times PC \times Id \rightarrow IdS \\
opBGuard : MMI \times PC \times Id \rightarrow G \\
opValOfOp : MMI \times PC \times Id \rightarrow CMMT \\
nextNode : MMI \times PC \times Id \rightarrow \mathrm{opt}[Id] \\
guardOf : PC \times Id \rightarrow \mathrm{opt}[G] \\
nmOfPC : PC \rightarrow Id \\
anyExpOf : PC \times Id \rightarrow \mathrm{opt}[Id \times E] \\
\mathrm{openAC\_} : \mathbb{P}(PC \times Id) \\
inRefs : MMI \times PC \times Id \rightarrow Ids \\
commonInKs : MMI \times PC \times Id \rightarrow Ids
\end{array}
$$

$$
\begin{array}{|l}
atLeaf : PC \times Id \rightarrow PCTCxt \\
\hline
\forall pc : PC;\ n : Id \bullet atLeaf(pc, n) = (\alpha(nmOf\ pc\ n, guardOf(pc, n), anyExpOf(pc, n)), \{\}, \{\})
\end{array}
$$

relation(openACOf_)

$$
\begin{array}{|l}
optB : MMI \times PC \times \mathrm{opt}[Id] \times Ids \rightarrow PCTCxt \\
atomB : MMI \times PC \times Id \times Ids \rightarrow PCTCxt \\
\mathrm{openACOf\_} : \mathbb{P}(MMI \times PC \times Id) \\
\hline
\forall mmi : MMI;\ pc : PC;\ n : Id \bullet \mathrm{openACOf}(mmi, pc, n) \\
\quad \Leftrightarrow \exists\, oca : \mathrm{opt}[Id] \bullet oca = nxtAfterC(mmi, pc, n) \wedge oca \neq \varnothing \wedge \mathrm{openAC}(pc, \mathrm{the}\ oca) \\
\forall mmi : MMI;\ pc : PC;\ n : Id;\ gcs : Ids;\ tc : PCTCxt \bullet atomB(mmi, pc, n, gcs) = tc \\
\quad \Leftrightarrow \exists\, tc_1, tc_2 : PCTCxt \mid tc_1 = atLeaf(pc, n) \\
\qquad \wedge\ tc_2 = optB(mmi, pc, nxtNAfter(mmi, pc, n), gcs) \bullet \\
\qquad tc = \mathbf{if}\ \mathrm{openACOf}(mmi, pc, n)\ \mathbf{then}\ tc_1 \ :\rightarrow_O\ tc_2\ \mathbf{else}\ tc_1\ :\rightarrow\ tc_2
\end{array}
$$

$$refLeaf : MMI \times PC \times Id \times Ids \rightarrow PCTCxt$$

$$\forall mmi : MMI;\ pc : PC;\ n, rn : Id;\ cs : Ids \mid rn = nmOfRefF(mmi, pc, n) \bullet refLeaf(mmi, pc, n, cs)$$
$$= (\rho(rn, guardOf(pc, n), psOfRef(mmi, pc, n), renamingsOf(pc, n)),$$
$$\textbf{if } rn \in cs \lor \neg\ \mathrm{isOfTy}(mmi, pc, n, Compound) \textbf{ then } \{\} \textbf{ else } \{rn\}, \{\})$$

$$compoundAB : MMI \times PC \times Id \times Ids \rightarrow PCTCxt$$
$$seqCTs : MMI \times PC \times Ids \times Ids \rightarrow (\mathrm{seq}\ CT \times Ids \times Ids)$$

$$\forall mmi : MMI;\ pc : PC;\ gcs : Ids \bullet seqCTs(mmi, pc, \{\}, gcs) = (\langle\rangle, \{\}, gcs)$$

$$\forall mmi : MMI;\ pc : PC;\ n : Id;\ ns, rns', gcs, gcs' : Ids;\ cts : \mathrm{seq}\ CT \bullet$$
$$seqCTs(mmi, pc, \{n\} \cup ns, gcs) = (cts, rns', gcs')$$
$$\Leftrightarrow \exists t' : PCT;\ rns_1, rns_2, gcs_1, gcs_2 : Ids;\ cts' : \mathrm{seq}\ CT \mid$$
$$(t', rns_1, gcs_1) = compoundAB(mmi, pc, n, gcs)$$
$$\land (cts', rns_2, gcs_2) = seqCTs(mmi, pc, ns \cup rns_1 \setminus gcs_1, gcs \cup gcs_1) \bullet$$
$$cts = \langle rearrangeT\ t' \rangle \frown cts' \land rns' = ns \cup rns_1 \cup rns_2 \land gcs' = gcs \cup gcs_1 \cup gcs_2$$

$$consB : MMI \times PC \times Id \times Ids \rightarrow PCTCxt$$
$$compound : MMI \times PC \times Id \times Ids \rightarrow PCTCxt$$

$$\forall mmi : MMI;\ pc : PC;\ n : Id;\ cs : Ids;\ tc : PCTCxt \bullet compound(mmi, pc, n, cs) = tc$$
$$\Leftrightarrow \exists tcs : \mathrm{seq}\ CT;\ rs_1, rs_2, cs_1, cs_2 : Ids;\ t : PCT \mid$$
$$(tcs, rs_1, cs_1) = seqCTs(mmi, pc, inRefs(mmi, pc, n) \cup commonInKs(mmi, pc, n), \{n\} \cup cs)$$
$$\land (t, rs_2, cs_2) = consB(mmi, pc, compoundStart(mmi, pc, n), \{n\} \cup cs \cup cs_1) \bullet$$
$$tc = ((\kappa \circ toCT_0 \circ \varkappa)(n, paramsOf(pc, n), tcs, t), rs_1 \cup rs_2, \{n\} \cup cs_1 \cup cs_2)$$

$$\forall mmi : MMI;\ pc : PC;\ n : Id;\ cs : Ids \bullet compoundAB(mmi, pc, n, cs)$$
$$= compound(mmi, pc, n, cs) :\twoheadrightarrow optB(mmi, pc, nxtNAfter(mmi, pc, n), \{n\} \cup cs)$$

$$\forall mmi : MMI;\ pc : PC;\ on : \mathrm{opt}[Id];\ cs : Ids \bullet$$
$$optB(mmi, pc, on, cs) = \textbf{if } on = \varnothing \textbf{ then } (\Lambda, \{\}, \{\}) \textbf{ else } consB(mmi, pc, \mathrm{the}\ on, cs)$$

$$
\begin{array}{|l}
opTree : MMI \times PC \times Id \times Ids \rightarrow PCTCxt \\
opBranches : MMI \times PC \times TOp \times IdS \times Ids \rightarrow PCTCxt \\
\hline
\forall mmi : MMI;\ pc : PC;\ n : Id;\ cs : Ids;\ tc : PCTCxt \bullet opTree(mmi, pc, n, cs) = tc \\
\quad \Leftrightarrow \exists\, bs : IdS;\ ps : Es \mid \\
\quad\quad bs = branchesOfOp(mmi, pc, n) \land ps = paramsOf\,(pc, n) \bullet \\
\quad\quad tc = opBranches(mmi, pc, toTOp(opValOfOp(mmi, pc, n), guardOf(pc, n), ps), bs, cs) \\
\\
\forall mmi : MMI;\ pc : PC;\ op : TOp;\ cs : Ids \bullet \\
\quad opBranches(mmi, pc, op, \{\}, cs) = (\Lambda, \{\}, \{\}) \\
\\
\forall mmi : MMI;\ pc : PC;\ op : TOp;\ b : Id;\ bs : IdS;\ rs, cs, cs' : Ids;\ t : PCT \bullet \\
\quad opBranches(mmi, pc, op, \langle b \rangle \frown bs, cs) = (t, rs, cs') \\
\quad \Leftrightarrow \exists\, t' : PCT;\ rs', cs_0 : Ids \mid (t', rs', cs_0) = consB(mmi, pc, b, cs) \bullet \\
\quad\quad (t, rs, cs') = (t', rs', cs_0) \multimap (op, opBranches(mmi, pc, op, bs, cs \cup cs_0))
\end{array}
$$

$$
\begin{array}{|l}
refB : MMI \times PC \times Id \times Ids \rightarrow PCTCxt \\
refOrCompound : MMI \times PC \times Id \times Ids \rightarrow PCTCxt \\
\hline
\forall mmi : MMI;\ pc : PC;\ n : Id;\ gcs : Ids \bullet refB(mmi, pc, n, gcs) \\
\quad = refLeaf(mmi, pc, n, gcs) :\twoheadrightarrow optB(mmi, pc, nxtNAfter(mmi, pc, n), gcs) \\
\\
\forall mmi : MMI;\ pc : PC;\ n : Id;\ cs : Ids \bullet refOrCompound(mmi, pc, n, cs) \\
\quad = \mathbf{if}\ n \in cs\ \mathbf{then}\ (\rho\,(n, \{\}, \langle\rangle, \{\}), \{\,\}, \{\,\})\ \mathbf{else}\ compoundAB(mmi, pc, n, cs)
\end{array}
$$

$$
\begin{array}{|l}
\hline
\forall mmi : MMI;\ pc : PC;\ n : Id;\ gcs : Ids \mid isOfTy(mmi, pc, n, Atom) \bullet \\
\quad consB(mmi, pc, n, gcs) = atomB(mmi, pc, n, gcs) \\
\\
\forall mmi : MMI;\ pc : PC;\ n : Id;\ gcs : Ids \mid isOfTy(mmi, pc, n, Compound) \bullet \\
\quad consB(mmi, pc, n, gcs) = refOrCompound(mmi, pc, n, gcs) \\
\\
\forall mmi : MMI;\ pc : PC;\ n : Id;\ gcs : Ids \mid isOfTy(mmi, pc, n, Operator) \bullet \\
\quad consB(mmi, pc, n, gcs) = opTree(mmi, pc, n, gcs) \\
\\
\forall mmi : MMI;\ pc : PC;\ n : Id;\ gcs : Ids \mid isOfTy(mmi, pc, n, Stop) \bullet \\
\quad consB(mmi, pc, n, gcs) = (\theta, \{\}, \{\}) \\
\\
\forall mmi : MMI;\ pc : PC;\ n : Id;\ gcs : Ids \mid isOfTy(mmi, pc, n, Skip) \bullet \\
\quad consB(mmi, pc, n, gcs) = (\chi, \{\}, \{\})
\end{array}
$$

$$
\begin{array}{|l}
toPCTD : MMI \rightarrow PC \rightarrow PCTD \\
\hline
\forall mmi : MMI;\ pc : PC;\ pctd : PCTD \bullet \\
\quad toPCTD\ mmi\ pc = pctd \Leftrightarrow \\
\quad\quad \exists\, cts : \mathrm{seq}\, CT;\ rs, gcs : Ids \mid (cts, rs, gcs) = seqCTs(mmi, pc, \{stCompound\,(mmi, pc)\}, \{\}) \bullet \\
\quad\quad\quad pctd = cpctd\,(nmOfPC\ pc)\ cts
\end{array}
$$

$$
\begin{array}{|l}
atomsPCTD : PCTD \rightarrow \mathbb{P}\, Id \\
atomsCTs : \mathrm{seq}\, CT \rightarrow \mathbb{P}\, Id \\
atomsPCT : PCT \rightarrow \mathbb{P}\, Id \\
\hline
\forall\, pctd : PCTD \bullet atomsPCTD\ pctd = atomsCTs\,(ctspctd\ pctd) \\
atomsCTs\langle\rangle = \{\} \\
\forall\, ct : CT;\ cts : \mathrm{seq}\, CT \bullet atomsCTs(\langle ct\rangle \frown cts) = (atomsPCT\ ((\kappa \circ toCT_0)\ ct)) \cup (atomsCTs\ cts) \\
\forall\, n : Id;\ g : \mathrm{opt}[G];\ ae : \mathrm{opt}[Id \times E] \bullet atomsPCT(\alpha\,(n, g, ae)) = \{n\} \\
\forall\, n : Id;\ ps : Es;\ cts : \mathrm{seq}\, CT;\ t : PCT \bullet atomsPCT((\kappa \circ toCT_0 \circ \varkappa)(n, ps, cts, t)) = atomsPCT\ t \\
\forall\, op : TOp;\ pct_1, pct_2 : PCT \bullet atomsPCT(\gamma(op, pct_1, pct_2)) = atomsPCT\ pct_1 \cup atomsPCT\ pct_2 \\
\forall\, n : Id;\ og : \mathrm{opt}[G];\ ps : Es;\ r : Ren \bullet atomsPCT(\rho(n, og, ps, r)) = \{\} \\
atomsPCT\ \Lambda = \{\} \\
atomsPCT\ \theta = \{\}
\end{array}
$$

relation(isAtom_)

$$
\begin{array}{|l}
\mathrm{isAtom\_} : \mathbb{P}\, PCT \\
\hline
\forall\, t : PCT \bullet \mathrm{isAtom}\ t \Leftrightarrow \exists\, n : Id;\ g : \mathrm{opt}[G];\ ae : \mathrm{opt}[Id \times E] \bullet t = \alpha\,(n, g, ae)
\end{array}
$$

relation(isAtomAny_)

$$
\begin{array}{|l}
\mathrm{isAtomAny\_} : \mathbb{P}\, PCT \\
\hline
\forall\, t : PCT \bullet \mathrm{isAtomAny}\ t \Leftrightarrow \exists\, n : Id;\ g : \mathrm{opt}[G];\ ae : \mathrm{opt}[Id \times E] \bullet t = \alpha\,(n, g, ae) \wedge ae \neq \{\}
\end{array}
$$

relation(isSole_)

$$
\begin{array}{|l}
\mathrm{isSole\_} : \mathbb{P}\, PCT \\
\hline
\forall\, t : PCT \bullet \mathrm{isSole}\ t \Leftrightarrow t \in \mathrm{ran}\, \alpha \vee t \in \mathrm{ran}\, \rho \\
\quad \vee\ t \in \{\Lambda, \theta\} \vee \exists\, op : TOp;\ t_1, t_2 : PCT \bullet t = \gamma(op, t_1, t_2) \wedge \mathrm{isAtom}\ t_1
\end{array}
$$

12 PCs to CSP

section *PCsToCSP* **parents** *standard_toolkit*, *PCTrees*, *Fragmenta_Generics*

$[Exp_0]$

$Decl ::= channel\langle\!\langle Ids\rangle\!\rangle \mid include\langle\!\langle Ids\rangle\!\rangle \mid =_d\langle\!\langle Exp_0 \times Exp_0\rangle\!\rangle$

$Decls == \operatorname{seq} Decl$

$IdM == Id \nrightarrow Id$

$$
\begin{aligned}
Exp ::= {} & e_{id}\langle\!\langle Id\rangle\!\rangle \mid e_G\langle\!\langle G\rangle\!\rangle \mid e_\beta\langle\!\langle Id \times Es\rangle\!\rangle \mid e_{\between}\langle\!\langle Exp\rangle\!\rangle \mid e_\Gamma\langle\!\langle Exp \times Exp\rangle\!\rangle \mid e_\iota\langle\!\langle Exp \times Exp \times Exp\rangle\!\rangle \\
& \mid e_\rightarrow\langle\!\langle Exp \times Exp\rangle\!\rangle \mid e_\Box\langle\!\langle Exp \times Exp\rangle\!\rangle \mid e_{\Box}\langle\!\langle Id \times E \times Exp\rangle\!\rangle \mid e_\sqcap\langle\!\langle Exp \times Exp\rangle\!\rangle \\
& \mid e_{\fatsemi}\langle\!\langle Exp \times Exp\rangle\!\rangle \mid e_{\|}\langle\!\langle Es \times Exp \times Exp\rangle\!\rangle \mid e_\Theta\langle\!\langle Es \times Exp \times Exp\rangle\!\rangle \mid e_{|||}\langle\!\langle Exp \times Exp\rangle\!\rangle \\
& \mid e_\theta \mid e_\Lambda \mid e_\zeta\langle\!\langle Decls \times Exp\rangle\!\rangle \mid e_{[\![/]\!]}\langle\!\langle Exp \times IdM\rangle\!\rangle
\end{aligned}
$$

$CSP == Decls$

$toExp_0 : Exp \rightarrowtail Exp_0$

relation(isAtomic_)

$isAtomic\_ : \mathbb{P}\, Exp$

$\forall n : Id \bullet \mathrm{isAtomic}(e_{id}\ n)$

$\forall g : G \bullet \mathrm{isAtomic}(e_G\ g)$

$\forall n : Id;\ es : Es \bullet \mathrm{isAtomic}(e_\beta(n, es))$

$\forall e : Exp \bullet \mathrm{isAtomic}(e_{\between}\ e)$

$\forall e_1, e_2 : Exp \bullet isAtomic(e_\Gamma(e_1, e_2))$

$\forall e_1, e_2, e_3 : Exp \bullet isAtomic(e_\iota(e_1, e_2, e_3))$

$\forall e_1, e_2 : Exp \bullet isAtomic(e_\rightarrow(e_1, e_2))$

$isAtomic(e_\theta)$

$isAtomic(e_\Lambda)$

$\forall ds : Decls;\ e : Exp \bullet isAtomic(e_\zeta(ds, e))$

$\forall e : Exp;\ r : IdM \bullet isAtomic(e_{[\![/]\!]}(e, r))$

$\forall e : Exp \bullet \neg\ (isAtomic(e))$

relation(isExtChoice_)

$$\begin{array}{|l}
\text{isExtChoice}_ : \mathbb{P}\,\mathit{Exp} \\
\hline
\forall\, e : \mathit{Exp} \bullet \text{isExtChoice}\; e \Leftrightarrow \exists\, e_1, e_2 : \mathit{Exp} \bullet e = e_{\Box}(e_1, e_2)
\end{array}$$

$\mathit{compExps} == \{ e : \mathit{Exp} \mid \neg\, (\text{isAtomic}\; e) \}$

$\mathit{compNotExtChs} == \{ e : \mathit{Exp} \mid \neg\, (\text{isAtomic}\; e \lor \text{isExtChoice}\; e) \}$

$\mathit{ECtxt} == \mathit{Exp} \times \mathit{Decls}$

$$\begin{array}{|l}
\mathit{exp} : \mathit{ECtxt} \to \mathit{Exp} \\
\mathit{decls} : \mathit{ECtxt} \to \mathit{Decls} \\
\hline
\forall\, e : \mathit{Exp};\ \mathit{ds} : \mathit{Decls} \bullet \mathit{exp}(e, \mathit{ds}) = e \\
\forall\, e : \mathit{Exp};\ \mathit{ds} : \mathit{Decls} \bullet \mathit{decls}(e, \mathit{ds}) = \mathit{ds}
\end{array}$$

function 10 **leftassoc** $(\_ \dashv\vdash \_)$

$$\begin{array}{|l}
_ \dashv\vdash _ : \mathit{ECtxt} \times \mathit{ECtxt} \to (\mathit{Exp} \times \mathit{Exp} \times \mathit{Decls}) \\
\hline
\forall\, e_1, e_2 : \mathit{Exp};\ \mathit{ds}_1, \mathit{ds}_2 : \mathit{Decls} \bullet (e_1, \mathit{ds}_1) \dashv\vdash (e_2, \mathit{ds}_2) = (e_1, e_2, \mathit{ds}_1 \frown \mathit{ds}_2)
\end{array}$$

$$\begin{array}{|l}
\mathit{mkPMod} : \mathit{Id} \to \mathit{Id} \\
\mathit{mkBaseMod} : \mathit{Id} \to \mathit{Id}
\end{array}$$

$$\begin{array}{|l}
\mathit{importsOf} : \mathit{MMI} \times \mathit{PC} \to \mathbb{P}\,\mathit{Id} \\
\mathit{importedAtoms} : \mathit{MMI} \times \mathit{PC} \to \mathbb{P}\,\mathit{Id} \\
\mathit{getIdOfPC} : \mathit{PC} \to \mathit{Id}
\end{array}$$

$$\begin{array}{|l}
\mathit{cspChannels} : \mathit{MMI} \to \mathit{PC} \to \mathit{PCTD} \to \mathit{Decl} \\
\hline
\forall\, \mathit{mmi} : \mathit{MMI};\ \mathit{pc} : \mathit{PC};\ \mathit{ts} : \mathit{PCTD} \bullet \\
\quad \mathit{cspChannels}\; \mathit{mmi}\; \mathit{pc}\; \mathit{ts} = \mathit{channel}(\mathit{importedAtoms}(\mathit{mmi}, \mathit{pc}) \cup (\mathit{atomsPCTD}\; \mathit{ts}))
\end{array}$$

$$\begin{array}{|l}
\mathit{cspPImports} : \mathit{MMI} \to \mathit{PC} \to \mathit{Decl} \\
\mathit{cspMainImports} : \mathit{PC} \to \mathit{Decl} \\
\hline
\forall\, \mathit{mmi} : \mathit{MMI};\ \mathit{pc} : \mathit{PC} \bullet \mathit{cspPImports}\; \mathit{mmi}\; \mathit{pc} = \mathit{include}(\text{map}\; \mathit{mkPMod}\; (\mathit{importsOf}(\mathit{mmi}, \mathit{pc}))) \\
\forall\, \mathit{pc} : \mathit{PC} \bullet \mathit{cspMainImports}\; \mathit{pc} = \mathit{include}(\{(\mathit{mkPMod} \circ \mathit{getIdOfPC})\; \mathit{pc}, (\mathit{mkBaseMod} \circ \mathit{getIdOfPC})\; \mathit{pc}\})
\end{array}$$

$$
\begin{array}{l}
cspExp : PCT \rightarrow ECtxt \\
cspPDef : PCT \rightarrow Exp \\
\hline
\forall\, t : PCT;\; e : Exp \bullet cspPDef\ t = e \\
\quad \Leftrightarrow \exists\, e' : Exp;\; ds : Decls \mid (e', ds) = cspExp\ t \bullet e = \textbf{if}\ ds = \langle\rangle\ \textbf{then}\ e'\ \textbf{else}\ e_{\varsigma}(ds, e')
\end{array}
$$

$$
\begin{array}{l}
cspPRef : Id \rightarrow \mathrm{opt}[G] \rightarrow Es \rightarrow IdM \rightarrow Exp \\
\hline
\forall\, n : Id;\; g : \mathrm{opt}[G];\; ps : Es;\; rs : IdM;\; e : Exp \bullet cspPRef\ n\ g\ ps\ rs = e \\
\quad \Leftrightarrow \exists\, e_1, e_2 : Exp \mid e_1 = \textbf{if}\ ps = \langle\rangle\ \textbf{then}\ e_{id}\ \ n\ \textbf{else}\ \ e_{\beta}\,(n, ps) \\
\quad e_2 = \textbf{if}\ \ rs = \{\}\ \textbf{then}\ \ e_1\ \textbf{else}\ e_{[\![/]\!]}(e_1, rs) \bullet \\
\qquad e = \textbf{if}\ g = \{\}\ \textbf{then}\ e_2\ \textbf{else}\ e_{\Gamma}((e_G \circ \mathrm{the})\ g, e_2)
\end{array}
$$

$$
BFSN ::= Both\langle\!\langle \mathbb{P}\, Exp \times \mathbb{P}\, Exp \rangle\!\rangle \mid Fst\langle\!\langle \mathbb{P}\, Exp \rangle\!\rangle \mid Snd\langle\!\langle \mathbb{P}\, Exp \rangle\!\rangle \mid None
$$

$$
\begin{array}{l}
oparExp : Exp \rightarrow \mathbb{P}\, Exp \rightarrow Exp \\
calcExp1 : Exp \rightarrow BFSN \rightarrow Exp \\
calcExp2 : Exp \rightarrow BFSN \rightarrow Exp \\
cspExps : PCT \rightarrow PCT \rightarrow BFSN \rightarrow (Exp \times Exp \times Decls) \\
\hline
\forall\, e : Exp;\; es : \mathbb{P}\, Exp \bullet oparExp\ e\ es = \textbf{if}\ e \in es\ \textbf{then}\ e_{\between}\ \ e\ \textbf{else}\ e \\[1ex]
\forall\, e : Exp;\; es, es' : \mathbb{P}\, Exp \bullet calcExp1\ e\,(Both(es, es')) = oparExp\ e\ es \\[1ex]
\forall\, e : Exp;\; es : \mathbb{P}\, Exp \bullet calcExp1\ e\,(Fst\ es) = oparExp\ e\ es \\[1ex]
\forall\, e : Exp;\; es : \mathbb{P}\, Exp \bullet calcExp1\ e\,(Snd\ es) = e \\[1ex]
\forall\, e : Exp;\; es : \mathbb{P}\, Exp \bullet calcExp1\ e\ None = e \\[1ex]
\forall\, e : Exp;\; es, es' : \mathbb{P}\, Exp \bullet calcExp2\ e\,(Both(es, es')) = oparExp\ e\ es' \\[1ex]
\forall\, e : Exp;\; es : \mathbb{P}\, Exp \bullet calcExp2\ e\,(Fst\ es) = e \\[1ex]
\forall\, e : Exp;\; es : \mathbb{P}\, Exp \bullet calcExp2\ e\,(Snd\ es) = oparExp\ e\ es \\[1ex]
\forall\, e : Exp;\; es : \mathbb{P}\, Exp \bullet calcExp2\ e\ None = e \\[1ex]
\forall\, t_1, t_2 : PCT;\; ds : Decls;\; e_1, e_2 : Exp;\; bf : BFSN \bullet cspExps\ t_1\ t_2\ bf = (e_1, e_2, ds) \\
\quad \Leftrightarrow \exists\, e_1', e_2' : Exp \mid (e_1', e_2', ds) = (cspExp\ t_1) \dashv\vdash (cspExp\ t_2) \bullet \\
\quad e_1 = calcExp1\ e_1'\ bf \wedge e_2 = calcExp2\ e_2'\ bf
\end{array}
$$

$$
\begin{array}{l}
cspDecls : \mathrm{seq}\ CT \rightarrow CSP \\
\hline
cspDecls \langle\rangle = \langle\rangle \\[1ex]
\forall\, ct : CT;\; cts : \mathrm{seq}\ CT \bullet cspDecls\ (\langle ct \rangle \frown cts) = ((decls \circ cspExp)\ ((\kappa \circ toCT_0)\ ct)) \frown (cspDecls\ cts)
\end{array}
$$

$$
\begin{array}{|l}
cspExpPrfx : PCT \rightarrow PCT \rightarrow ECtxt \\
cspExpSeqC : PCT \rightarrow PCT \rightarrow ECtxt \\
\hline
\forall\, t_1, t_2 : PCT;\ e : Exp;\ ds : Decls \bullet cspExpPrfx\ t_1\ t_2 = (e, ds) \\
\quad \Leftrightarrow \exists\, e_1, e_2 : Exp \mid (e_1, e_2, ds) = cspExps\ t_1\ t_2\ (Snd\ compExps) \bullet e = e_{\rightarrow}(e_1, e_2) \\
\forall\, t_1, t_2 : PCT;\ e : Exp;\ ds : Decls \bullet cspExpPrfx\ t_1\ t_2 = (e, ds) \\
\quad \Leftrightarrow \exists\, e_1, e_2 : Exp \mid (e_1, e_2, ds) = cspExps\ t_1\ t_2\ (Snd\ compExps) \bullet e = e_{\fatsemi}(e_1, e_2)
\end{array}
$$

$$
\begin{array}{|l}
cspExpREC : PCT \rightarrow PCT \rightarrow ECtxt \\
\hline
\forall\, n : Id;\ g : \mathrm{opt}[G];\ t_2 : PCT;\ atv : Id;\ ats : E;\ e : Exp;\ ds : Decls \bullet \\
\quad cspExpREC(\alpha(n, g, \{(atv, ats)\}))t_2 = (e, ds) \\
\quad \Leftrightarrow \exists\, e_1, e_2 : Exp;\ ds_1 : Decls \mid (e_1, ds_1) = cspExp\ t_2 \\
\qquad \wedge\ e_2 = e_{\between}(e_{\Box}(atv, ats, e_{\rightarrow}(e_{id}\ \ atv, e_1))) \bullet \\
\qquad ds = ds_1 \wedge e = \textbf{if}\ \ g = \{\}\ \textbf{then}\ e_2\ \textbf{else}\ e_{\Gamma}(e_G\ (\mathrm{the}\, g), e_2)
\end{array}
$$

$$
\begin{array}{l}
\mathit{cspExp}\ \Lambda = (e_\Lambda, \langle\rangle) \\[1ex]
\mathit{cspExp}\ \chi = (e_\Lambda, \langle\rangle) \\[1ex]
\mathit{cspExp}\ \theta = (e_\theta, \langle\rangle) \\[1ex]
\forall\, n : \mathit{Id};\ g : \mathrm{opt}[G];\ e : \mathit{Exp} \bullet \mathit{cspExp}(\alpha\,(n, g, \{\})) = (e, \langle\rangle) \\
\quad \Leftrightarrow e = \mathbf{if}\ g = \varnothing\ \mathbf{then}\ e_{id}\ n\ \mathbf{else}\ e_\Gamma\,(e_G\,(\mathrm{the}\, g), e_{id}\ n) \\[1ex]
\forall\, n : \mathit{Id};\ ps : \mathit{Es};\ cts : \mathrm{seq}\, \mathit{CT};\ t : \mathit{PCT};\ e : \mathit{Exp};\ ds : \mathit{Decls} \bullet \\
\quad \mathit{cspExp}((\kappa \circ \mathit{toCT}_0 \circ \varkappa)(n, ps, cts, t)) = (e, ds) \\
\quad \Leftrightarrow \exists\, e_1, e_2, e_2' : \mathit{Exp} \mid e_1 = \mathit{cspPRef}\ n\ \{\}\ ps\ \{\} \\
\quad\quad \wedge\ e_2 = \mathit{cspPDef}\ t \wedge e_2' = \mathbf{if}\ cts \neq \langle\rangle\ \mathbf{then}\ e_\varsigma(\mathit{cspDecls}\ cts, e_2)\ \mathbf{else}\ e_2 \bullet \\
\quad\quad\quad e = e_1 \wedge ds = \langle =_d (\mathit{toExp}_0\, e_1, \mathit{toExp}_0\, e_2') \rangle \\[1ex]
\forall\, n : \mathit{Id};\ g : \mathrm{opt}[G];\ ps : \mathit{Es};\ r : \mathit{Ren} \bullet \mathit{cspExp}(\rho(n, g, ps, r)) = (\mathit{cspPRef}\ n\ g\ ps\ r, \langle\rangle) \\[1ex]
\forall\, g : G;\ t_1, t_2 : \mathit{PCT};\ e : \mathit{Exp};\ ds : \mathit{Decls} \bullet \mathit{cspExp}(\gamma(\iota\ g, t_1, t_2)) = (e, ds) \\
\quad \Leftrightarrow \exists\, e_1, e_2 : \mathit{Exp} \mid (e_1, e_2, ds) = \mathit{cspExps}\ t_1\ t_2\ \mathit{None} \bullet e = e_\iota(e_G\ g, e_1, e_2) \\[1ex]
\forall\, t_1, t_2 : \mathit{PCT};\ o : \mathit{Bool} \mid \mathrm{isAtomAny}\, t_1 \wedge (o = \mathit{True} \vee t_2 = \Lambda) \bullet \mathit{cspExp}(\gamma(\rightarrow_{\shortmid}\ o, t_1, t_2)) \\
\quad = \mathit{cspExpREC}\ t_1\ t_2 \\[1ex]
\forall\, t_1, t_2 : \mathit{PCT};\ e : \mathit{Exp};\ ds : \mathit{Decls} \mid \mathrm{isAtomAny}\, t_1 \bullet \mathit{cspExp}(\gamma(\rightarrow_{\shortmid}\ \mathit{False}, t_1, t_2)) = (e, ds) \\
\quad \Leftrightarrow \exists\, e_1, e_2 : \mathit{Exp};\ ds_1, ds_2 : \mathit{Decls} \mid (e_1, ds_1) = \mathit{cspExpREC}\ t_1\ \Lambda \wedge (e_2, ds_2) = \mathit{cspExp}\ t_2 \bullet \\
\quad\quad e = e_{⨾}(e_1, e_2) \wedge ds = ds_1 \frown ds_2 \\[1ex]
\forall\, t_1, t_2 : \mathit{PCT};\ o : \mathit{Bool} \bullet \mathit{cspExp}(\gamma(\rightarrow_{\shortmid}\ o, t_1, t_2)) \\
\quad = \mathbf{if}\ \mathit{isAtom}\ t_1\ \mathbf{then}\ \mathit{cspExpPrfx}\ t_1\ t_2\ \mathbf{else}\ \mathit{cspExpSeqC}\ t_1\ t_2 \\[1ex]
\forall\, t_1, t_2 : \mathit{PCT};\ e : \mathit{Exp};\ ds : \mathit{Decls} \bullet \mathit{cspExp}(\gamma(\Box, t_1, t_2)) = (e, ds) \\
\quad \Leftrightarrow \exists\, e_1, e_2 : \mathit{Exp} \mid \\
\quad\quad (e_1, e_2, ds) = \mathit{cspExps}\ t_1\ t_2\ (\mathit{Both}\,(\mathit{compExps}, \mathit{compNotExtChs})) \bullet \\
\quad\quad\quad e = e_\Box(e_1, e_2) \\[1ex]
\forall\, t_1, t_2 : \mathit{PCT};\ e : \mathit{Exp};\ ds : \mathit{Decls} \bullet \mathit{cspExp}(\gamma(\sqcap, t_1, t_2)) = (e, ds) \\
\quad \Leftrightarrow \exists\, e_1, e_2 : \mathit{Exp} \mid (e_1, e_2, ds) = \mathit{cspExps}\ t_1\ t_2\ (\mathit{Both}\,(\mathit{compExps}, \mathit{compExps})) \bullet e = e_\sqcap(e_1, e_2) \\[1ex]
\forall\, t_1, t_2 : \mathit{PCT};\ ps : \mathit{Es};\ e : \mathit{Exp};\ ds : \mathit{Decls} \bullet \mathit{cspExp}(\gamma(\parallel\ ps, t_1, t_2)) = (e, ds) \\
\quad \Leftrightarrow \exists\, e_1, e_2 : \mathit{Exp} \mid (e_1, e_2, ds) = \mathit{cspExps}\ t_1\ t_2\ (\mathit{Both}\,(\mathit{compExps}, \mathit{compExps})) \bullet e = e_\parallel(ps, e_1, e_2) \\[1ex]
\forall\, t_1, t_2 : \mathit{PCT};\ ps : \mathit{Es};\ e : \mathit{Exp};\ ds : \mathit{Decls} \bullet \mathit{cspExp}(\gamma(\Theta\ ps, t_1, t_2)) = (e, ds) \\
\quad \Leftrightarrow \exists\, e_1, e_2 : \mathit{Exp} \mid (e_1, e_2, ds) = \mathit{cspExps}\ t_1\ t_2\ (\mathit{Both}\,(\mathit{compExps}, \mathit{compExps})) \bullet e = e_\Theta(ps, e_1, e_2) \\[1ex]
\forall\, t_1, t_2 : \mathit{PCT};\ e : \mathit{Exp};\ ds : \mathit{Decls} \bullet \mathit{cspExp}(\gamma(|||, t_1, t_2)) = (e, ds) \\
\quad \Leftrightarrow \exists\, e_1, e_2 : \mathit{Exp} \mid (e_1, e_2, ds) = \mathit{cspExps}\ t_1\ t_2\ (\mathit{Both}\,(\mathit{compExps}, \mathit{compExps})) \bullet e = e_{|||}(e_1, e_2)
\end{array}
$$

$$
\begin{array}{l}
\mathit{toCSP} : \mathit{MMI} \rightarrow \mathit{PC} \rightarrow (\mathit{CSP} \times \mathit{CSP} \times \mathit{CSP}) \\
\hline
\forall\, mmi : \mathit{MMI};\ pc : \mathit{PC};\ csp_1, csp_2, csp_3 : \mathit{CSP} \bullet \mathit{toCSP}\ mmi\ pc = (csp_1, csp_2, csp_3) \\
\quad \Leftrightarrow \exists\, pctd : \mathit{PCTD} \mid pctd = \mathit{toPCTD}\ mmi\ pc \bullet csp_1 = \langle \mathit{cspChannels}\ mmi\ pc\ pctd \rangle \\
\quad \wedge\ csp_2 = \mathit{cspDecls}\,(\mathit{ctspctd}\ pctd) \wedge csp_3 = \langle \mathit{cspMainImports}\ pc \rangle \frown \langle \mathit{cspPImports}\ mmi\ pc \rangle
\end{array}
$$