



**ELECTRONIC CITY CAMPUS**

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Hosur Road, Near Electronic City, Bangalore-100

## **MAT LAB**

**Subject: Linear Algebra and its Applications**

**Subject Code: UE20MA251**

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**SRN : PES2UG20CS209**

**Section: D**

**Branch: B.Tech - CSE**

**Marks awarded:**

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# Linear Algebra and its Applications

UE20MA251

Name: Naman Choudhary	SRN: PES2UG20CS209	Section: D
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## Gauss Elimination

Program Number	1
Program Qn	Solve the matrix by using gaussian elimination: $x+2y+z=3$ , $2x+y-2z=3$ , $-3x+y+z=-6$
Source Code	
<pre> C=[1 2 1;2 1 -2;-3 1 1] b=[3 3 -6] A=[C b]; n=size(A,1); x=zeros(n,1); for i=1:n-1     for j=i+1:n         m=A(j,i)/A(i,i)         A(j,:)=A(j,:)-m*A(i,:)     end end x(n)=A(n,n+1)/A(n,n) for i=n-1:-1:1     summ=0     for j=i+1:n         summ=summ + A(i,j)*x(j,:)     end     x(i,:)=(A(i,n+1)-summ)/A(i,i) end </pre>	
Output Screenshot	

C = 3×3

1  
2  
-3

b = 3×1

m = 2

A = 3×4

1  
0  
-3

m = -3

A = 3×4

1  
0  
0

m = -2.3333

A = 3×4

1.0000  
0 -  
0

x = 3×1

summ = 0

summ = -3.0000

x = 3×1

summ = 0

summ = 2.9606e-16

x = 3×1

summ = 0.7500

x = 3×1

<b>Program Number</b>	1 a
<b>Source Code</b>	
<pre>C=[1 5 1;1 1 -2;7 1 4] b=[-8 2 -6] ' A=[C b]; n=size(A,1); x=zeros(n,1); for i=1:n-1     for j=i+1:n         m=A(j,i)/A(i,i)         A(j,:)=A(j,:)-m*A(i,:)     end end x(n)=A(n,n+1)/A(n,n) for i=n-1:-1:1     summ=0     for j=i+1:n         summ=summ + A(i,j)*x(j,:)     end     x(i,:)=(A(i,n+1)-summ)/A(i,i) end</pre>	
<b>Output Screenshot</b>	

Command Window

&gt;&gt; P1a

C =

1	5	1
1	1	-2
7	1	4

b =

-8
2
-6

m =

1

A =

1	5	1	-8
0	-4	-3	10
7	1	4	-6

m =

7

A =

1	5	1	-8
0	-4	-3	10

fx

Command Window

0	-34	-3	50
---	-----	----	----

m =

8.5000

A =

1.0000	5.0000	1.0000	-8.0000
0	-4.0000	-3.0000	10.0000
0	0	22.5000	-35.0000

x =

0
0
-1.5556

summ =

0

summ =

4.6667

x =

0
-1.3333
-1.5556

fx

```

summ =
    0

summ =
   -6.6667

x =
   -1.3333
   -1.3333
   -1.5556

summ =
   -8.2222

x =
    0.2222
   -1.3333
   -1.5556
fx >> |

```

<b>Program Number</b>	1 b
<b>Source Code</b>	
<pre> C=[ 4  5  8;1  5 -2;-3  5  6] b=[ 2  1 -2] ' A=[C b]; n=size(A,1); x=zeros(n,1); for i=1:n-1     for j=i+1:n         m=A(j,i)/A(i,i)         A(j,:)=A(j,:)-m*A(i,:)     end end x(n)=A(n,n+1)/A(n,n) for i=n-1:-1:1     summ=0     for j=i+1:n         summ=summ + A(i,j)*x(j,:)     end     x(i,:)=(A(i,n+1)-summ)/A(i,i) end end </pre>	

## Output Screenshot

C = 3x3

4	5	8
1	5	-2
-3	5	6

b = 3x1

2
1
-2

m = 0.2500

A = 3x4

4.0000	5.0000	8.0000	2.0000
0	3.7500	-4.0000	0.5000
-3.0000	5.0000	6.0000	-2.0000

m = -0.7500

A = 3x4

4.0000	5.0000	8.0000	2.0000
0	3.7500	-4.0000	0.5000
0	8.7500	12.0000	-0.5000

m = 2.3333

A = 3x4

4.0000	5.0000	8.0000	2.0000
0	3.7500	-4.0000	0.5000
0	0	21.3333	-1.6667

x = 3x1

0
0
-0.0781

summ = 0

summ = 0.3125

```

x = 3x1
      0
    0.0500
   -0.0781

summ = 0
summ = 0.2500
x = 3x1
      0.4375
      0.0500
     -0.0781

summ = -0.3750
x = 3x1
      0.5938
      0.0500
     -0.0781

```

## Gauss Jordan

<b>Program Number</b>	2
<b>Program Qn</b>	Find the inverse of A using Gauss-Jordon Method where  A = 1 1 1 4 3 -1 3 5 3
<b>Source Code</b>	



```
A=[1,1,1;4,3,-1;3,5,3]
n=length(A(1,:));
Aug=[A,eye(n,n)]
for j=1:n-1
    for i=j+1:n
        Aug(i,j:2*n)=Aug(i,j:2*n)-Aug(i,j)/
Aug(j,j)*Aug(j,j:2*n)
    end
end
for j=n:-1:2
    Aug(1:j-1,:)=Aug(1:j-1,:)-Aug(1:j-1,j)/
Aug(j,j)*Aug(j,:)
end
for j=1:n
    Aug(j,:)=Aug(j,+)/Aug(j,j)
end
B=Aug(:,n+1:2*n)
```

**Output Screenshot**

A = 3×3

1	1	1
4	3	-1
3	5	3

Aug = 3×6

1	1	1	1	0	0
4	3	-1	0	1	0
3	5	3	0	0	1

Aug = 3×6

1	1	1	1	0	0
0	-1	-5	-4	1	0
3	5	3	0	0	1

Aug = 3×6

1	1	1	1	0	0
0	-1	-5	-4	1	0
0	2	0	-3	0	1

Aug = 3×6

1	1	1	1	0	0
0	-1	-5	-4	1	0
0	0	-10	-11	2	1

Aug = 3×6

1.0000	1.0000	0	-0.1000	0.2000	0.1000
0	-1.0000	0	1.5000	0	-0.5000
0	0	-10.0000	-11.0000	2.0000	1.0000

Aug = 3×6

1.0000	0	0	1.4000	0.2000	-0.4000
0	-1.0000	0	1.5000	0	-0.5000
0	0	-10.0000	-11.0000	2.0000	1.0000

Aug = 3×6

1.0000	0	0	1.4000	0.2000	-0.4000
0	-1.0000	0	1.5000	0	-0.5000
0	0	-10.0000	-11.0000	2.0000	1.0000

Aug = 3×6

1.0000	0	0	1.4000	0.2000	-0.4000
0	1.0000	0	-1.5000	0	0.5000
0	0	-10.0000	-11.0000	2.0000	1.0000

Aug = 3×6

1.0000	0	0	1.4000	0.2000	-0.4000
0	1.0000	0	-1.5000	0	0.5000
0	0	1.0000	1.1000	-0.2000	-0.1000

B = 3×3

1.4000	0.2000	-0.4000
-1.5000	0	0.5000
1.1000	-0.2000	-0.1000

<b>Program Number</b>	2 a
-----------------------	-----

### Source Code

```
A=[2,4,5;5,1,1;7,8,9]
n=length(A(1,:));
Aug=[A,eye(n,n)]
for j=1:n-1
    for i=j+1:n
        Aug(i,j:2*n)=Aug(i,j:2*n)-Aug(i,j)/
Aug(j,j)*Aug(j,j:2*n)
    end
end
for j=n:-1:2
    Aug(1:j-1,:)=Aug(1:j-1,:)-Aug(1:j-1,j)/
Aug(j,j)*Aug(j,:)
end
for j=1:n
    Aug(j,:)=Aug(j,:)/Aug(j,j)
end
B=Aug(:,n+1:2*n)
```

### Output Screenshot

A = 3x3

2	4	5
5	1	1
7	8	9

Aug = 3x6

2	4	5	1	0	0
5	1	1	0	1	0
7	8	9	0	0	1

Aug = 3x6

2.0000	4.0000	5.0000	1.0000	0	0
0	-9.0000	-11.5000	-2.5000	1.0000	0
7.0000	8.0000	9.0000	0	0	1.0000

Aug = 3x6

2.0000	4.0000	5.0000	1.0000	0	0
0	-9.0000	-11.5000	-2.5000	1.0000	0
0	-6.0000	-8.5000	-3.5000	0	1.0000

Aug = 3x6

2.0000	4.0000	5.0000	1.0000	0	0
0	-9.0000	-11.5000	-2.5000	1.0000	0
0	0	-0.8333	-1.8333	-0.6667	1.0000

Aug = 3x6

2.0000	4.0000	0	-10.0000	-4.0000	6.0000
0	-9.0000	0	22.8000	10.2000	-13.8000
0	0	-0.8333	-1.8333	-0.6667	1.0000

Aug = 3x6

2.0000	0	0	0.1333	0.5333	-0.1333
0	-9.0000	0	22.8000	10.2000	-13.8000
0	0	-0.8333	-1.8333	-0.6667	1.0000

Aug = 3x6

1.0000	0	0	0.0667	0.2667	-0.0667
0	-9.0000	0	22.8000	10.2000	-13.8000
0	0	-0.8333	-1.8333	-0.6667	1.0000

Aug = 3x6

1.0000	0	0	0.0667	0.2667	-0.0667
0	1.0000	0	-2.5333	-1.1333	1.5333
0	0	-0.8333	-1.8333	-0.6667	1.0000

Aug = 3×6

1.0000	0	0	0.0667	0.2667	-0.0667
0	1.0000	0	-2.5333	-1.1333	1.5333
0	0	1.0000	2.2000	0.8000	-1.2000

B = 3×3

0.0667	0.2667	-0.0667
-2.5333	-1.1333	1.5333
2.2000	0.8000	-1.2000

<b>Program Number</b>	2 b
-----------------------	-----

### Source Code

```
A=[ 2,5,7;-2,8,2;3,5,6]
n=length(A(1,:));
Aug=[A,eye(n,n)]
for j=1:n-1
    for i=j+1:n
        Aug(i,j:2*n)=Aug(i,j:2*n)-Aug(i,j)/
Aug(j,j)*Aug(j,j:2*n)
    end
end
for j=n:-1:2
    Aug(1:j-1,:)=Aug(1:j-1,:)-Aug(1:j-1,j)/
Aug(j,j)*Aug(j,:)
end
for j=1:n
    Aug(j,:)=Aug(j,:)/Aug(j,j)
end
B=Aug(:,n+1:2*n)
```

### Output Screenshot

A =  $3 \times 3$

2	5	7
-2	8	2
3	5	6

Aug =  $3 \times 6$

2	5	7	1	0	0
-2	8	2	0	1	0
3	5	6	0	0	1

Aug =  $3 \times 6$

2	5	7	1	0	0
0	13	9	1	1	0
3	5	6	0	0	1

Aug =  $3 \times 6$

2.0000	5.0000	7.0000	1.0000	0	0
0	13.0000	9.0000	1.0000	1.0000	0
0	-2.5000	-4.5000	-1.5000	0	1.0000

Aug =  $3 \times 6$

2.0000	5.0000	7.0000	1.0000	0	0
0	13.0000	9.0000	1.0000	1.0000	0
0	0	-2.7692	-1.3077	0.1923	1.0000

Aug =  $3 \times 6$

2.0000	5.0000	0.0000	-2.3056	0.4861	2.5278
0	13.0000	0	-3.2500	1.6250	3.2500
0	0	-2.7692	-1.3077	0.1923	1.0000

Aug =  $3 \times 6$

2.0000	0	0.0000	-1.0556	-0.1389	1.2778
0	13.0000	0	-3.2500	1.6250	3.2500
0	0	-2.7692	-1.3077	0.1923	1.0000

Aug =  $3 \times 6$

1.0000	0	0.0000	-0.5278	-0.0694	0.6389
0	13.0000	0	-3.2500	1.6250	3.2500
0	0	-2.7692	-1.3077	0.1923	1.0000

Aug =  $3 \times 6$

1.0000	0	0.0000	-0.5278	-0.0694	0.6389
0	1.0000	0	-0.2500	0.1250	0.2500
0	0	-2.7692	-1.3077	0.1923	1.0000

Aug = 3×6

1.0000	0	0.0000	-0.5278	-0.0694	0.6389
0	1.0000	0	-0.2500	0.1250	0.2500
0	0	1.0000	0.4722	-0.0694	-0.3611

B = 3×3

-0.5278	-0.0694	0.6389
-0.2500	0.1250	0.2500
0.4722	-0.0694	-0.3611

# LU Decomposition

<b>Program Number</b>	3
<b>Program Qn</b>	LU decomposition of A were  A = 1 1 -1 3 5 6 7 8 9
<b>Source Code</b>	
<pre>%LU Decomposition Ab = [1 1 -1;3 5 6;7 8 9]; %% Forward Elimination n= length(A);  L = eye(n); % With A(1,1) as pivot Element for i =2:3 alpha = Ab(i,1)/Ab(1,1); L(i,1) = alpha; Ab(i,:) = Ab(i,:) - alpha*Ab(1,:); end % With A(2,2) as pivot Element i=3; alpha = Ab(i,2)/Ab(2,2); L(i,2) = alpha Ab(i,:) = Ab(i,:) - alpha*Ab(2,:); U = Ab(1:n,1:n)</pre>	
<b>Output Screenshot</b>	

```
L = 3x3
    1.0000    0    0
    3.0000    1.0000    0
    7.0000    0.5000    1.0000
```

```
U = 3x3
    1.0000    1.0000   -1.0000
         0    2.0000    9.0000
         0         0   11.5000
```

<b>Program Number</b>	3 a
-----------------------	-----

### Source Code

```
%LU Decomposition
Ab = [3 5 -1;3 8 6;9 8 1];
%% Forward Elimination n= length(A);


L = eye(n);
% With A(1,1) as pivot Element
for i =2:3
alpha = Ab(i,1)/Ab(1,1); L(i,1) = alpha;
Ab(i,:) = Ab(i,:) - alpha*Ab(1,:);
end
% With A(2,2) as pivot Element i=3;
alpha = Ab(i,2)/Ab(2,2); L(i,2) = alpha
Ab(i,:) = Ab(i,:) - alpha*Ab(2,:); U = Ab(1:n,1:n)
```

### Output Screenshot

```
L = 3x3
    1.0000    0    0
    1.0000    1.0000    0
    3.0000   -2.3333    1.0000
```

```
U = 3x3
    3.0000    5.0000   -1.0000
         0    3.0000    7.0000
         0         0   20.3333
```



<b>Program Number</b>	3 b
<b>Source Code</b>	
<pre> %LU Decomposition Ab = [3 1 -1;6 4 6;-9 3 7]; %% Forward Elimination n= length(A);  L = eye(n); % With A(1,1) as pivot Element for i =2:3 alpha = Ab(i,1)/Ab(1,1); L(i,1) = alpha; Ab(i,:) = Ab(i,:) - alpha*Ab(1,:); end % With A(2,2) as pivot Element i=3; alpha = Ab(i,2)/Ab(2,2); L(i,2) = alpha Ab(i,:) = Ab(i,:) - alpha*Ab(2,:); U = Ab(1:n,1:n) </pre>	
<b>Output Screenshot</b>	
 <pre> L = 3x3     1     0     0     2     1     0    -3     3     1  U = 3x3     3     1    -1     0     2     8     0     0   -20 </pre>	

## Gram Schmidt Orthogonalization

<b>Program Number</b>	4
<b>Program Qn</b>	Apply the Gram-Schmidt process to the vectors (1,1,2), (0,0,1) and (1,0,0) to produce a set of Orthonormal vectors.
<b>Source Code</b>	

```
A=[1,1,2;0,0,1;1,0,0]
Q=zeros(3)
R=zeros(3)
for j=1:3
    v=A(:,j)
    for i=1:j-1
        R(i,j)=Q(:,i)'*A(:,j)
        v=v-R(i,j)*Q(:,i)
    end
    R(j,j)=norm(v)
    Q(:,j)=v/R(j,j)
end
```

**Output Screenshot**

A = 3×3

1	1	2
0	0	1
1	0	0

Q = 3×3

0	0	0
0	0	0
0	0	0

R = 3×3

0	0	0
0	0	0
0	0	0

v = 3×1

1
0
1

R = 3×3

1.4142	0	0
0	0	0
0	0	0

Q = 3×3

0.7071	0	0
0	0	0
0.7071	0	0

v = 3×1

1
0
0

R = 3×3

1.4142	0.7071	0
0	0	0
0	0	0

v = 3×1

0.5000
0
-0.5000

R = 3x3

1.4142	0.7071	0
0	0.7071	0
0	0	0

Q = 3x3

0.7071	0.7071	0
0	0	0
0.7071	-0.7071	0

v = 3x1

2  
1  
0

R = 3x3

1.4142	0.7071	1.4142
0	0.7071	0
0	0	0

v = 3x1

1.0000  
1.0000  
-1.0000

R = 3x3

1.4142	0.7071	1.4142
0	0.7071	1.4142
0	0	0

v = 3x1

-0.0000  
1.0000  
0.0000

R = 3x3

1.4142	0.7071	1.4142
0	0.7071	1.4142
0	0	1.0000

Q = 3x3

0.7071	0.7071	-0.0000
0	0	1.0000
0.7071	-0.7071	0.0000

Program Number	4 a
----------------	-----

Source Code

```
A=[1,0,1;0,0,2;3,1,0]
Q=zeros(3)
R=zeros(3)
for j=1:3
    v=A(:,j)
    for i=1:j-1
        R(i,j)=Q(:,i)'*A(:,j)
        v=v-R(i,j)*Q(:,i)
    end
    R(j,j)=norm(v)
    Q(:,j)=v/R(j,j)
end
```

**Output Screenshot**

$Q = 3 \times 3$

0.3162	0	0
0	0	0
0.9487	0	0

$v = 3 \times 1$

0  
0  
1

$R = 3 \times 3$

3.1623	0.9487	0
0	0	0
0	0	0

$v = 3 \times 1$

-0.3000  
0  
0.1000

$R = 3 \times 3$

3.1623	0.9487	0
0	0.3162	0
0	0	0

$Q = 3 \times 3$

0.3162	-0.9487	0
0	0	0
0.9487	0.3162	0

$v = 3 \times 1$

1  
2  
0

$R = 3 \times 3$

3.1623	0.9487	0.3162
0	0.3162	0
0	0	0

$v = 3 \times 1$

0.9000  
2.0000  
-0.3000

$R = 3 \times 3$

3.1623	0.9487	0.3162
--------	--------	--------

R = 3×3

3.1623	0.9487	0.3162
0	0.3162	-0.9487
0	0	0

v = 3×1

0.0000
2.0000
0.0000

R = 3×3

3.1623	0.9487	0.3162
0	0.3162	-0.9487
0	0	2.0000

Q = 3×3

0.3162	-0.9487	0.0000
0	0	1.0000
0.9487	0.3162	0.0000

<b>Program Number</b>	4 b
-----------------------	-----

### Source Code

```
A=[3,0,2;0,0,2;0,4,0]
Q=zeros(3)
R=zeros(3)
for j=1:3
    v=A(:,j)
    for i=1:j-1
        R(i,j)=Q(:,i)'*A(:,j)
        v=v-R(i,j)*Q(:,i)
    end
    R(j,j)=norm(v)
    Q(:,j)=v/R(j,j)
end
```

### Output Screenshot

A = 3×3

1	0	1
0	0	2
3	1	0

Q = 3×3

0	0	0
0	0	0
0	0	0

R = 3×3

0	0	0
0	0	0
0	0	0

v = 3×1

1
0
3

R = 3×3

3.1623	0	0
0	0	0
0	0	0

Q = 3×3

0.3162	0	0
0	0	0
0.9487	0	0

v = 3×1

0
0
1

R = 3×3

3.1623	0.9487	0
0	0	0
0	0	0

v = 3×1



Q = 3x3

0.3162	-0.9487	0
0	0	0
0.9487	0.3162	0

v = 3x1

1  
2  
0

R = 3x3

3.1623	0.9487	0.3162
0	0.3162	0
0	0	0

v = 3x1

0.9000  
2.0000  
-0.3000

R = 3x3

3.1623	0.9487	0.3162
0	0.3162	-0.9487
0	0	0

v = 3x1

0.0000  
2.0000  
0.0000

R = 3x3

3.1623	0.9487	0.3162
0	0.3162	-0.9487
0	0	2.0000

Q = 3x3

0.3162	-0.9487	0.0000
0	0	1.0000
0.9487	0.3162	0.0000

# In Built

## Eigen value Eigen vector

<b>Program Number</b>	5 a
<b>Program Qn</b>	Find the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$
<b>Source Code</b>	
<pre>A=[1,1,3;1,5,1;3,1,1] e=eig(A) det(A) prod(eig(A)) sum(eig(A)) trace(A) [V,D]=eig(A)</pre>	
<b>Output Screenshot</b>	

A = 3×3

1	1	3
1	5	1
3	1	1

e = 3×1

-2.0000
3.0000
6.0000

ans = -36

ans = -36.0000

ans = 7.0000

ans = 7

V = 3×3

-0.7071	0.5774	0.4082
0	-0.5774	0.8165
0.7071	0.5774	0.4082

D = 3×3

-2.0000	0	0
0	3.0000	0
0	0	6.0000

# QR Factorisation

Program Number	5 b
Program Qn	Find the QR factorisation of the matrix A where A= 1 1 0 1 0 1 0 1 1
Source Code	
A=[ 1,1,0;1,0,1;0,1,1 ] [Q,R]=qr(A)	
Output Screenshot	
<div>A = 3x3 1 1 0 1 0 1 0 1 1</div> <div>Q = 3x3 -0.7071 0.4082 -0.5774 -0.7071 -0.4082 0.5774 0 0.8165 0.5774</div> <div>R = 3x3 -1.4142 -0.7071 -0.7071 0 1.2247 0.4082 0 0 1.1547</div>	

# Projection of least squares

<b>Program Number</b>	5 c
<b>Program Qn</b>	Projection of least squares
<b>Source Code</b>	
<pre>A=[1,0;0,1;1,1] b=[1;3;4] x = lsqr(A,b)</pre>	
<b>Output Screenshot</b>	
<pre>A = 3x2     1    0     0    1     1    1  b = 3x1     1     3     4  Iteration 2 to a solution with relative residual 4.3e-17.  x = 2x1     1     3</pre>	

# Four fundamental subspaces

Program Number	5 d
Program Qn	Four fundamental subspaces
Source Code	
<pre>A=[1,2,3;2,-1,1]; % Row Reduced Echelon Form [R, pivot] = rref(A) % Rank rank = length(pivot) % basis of the column space of A columnsp = A(:,pivot) % basis of the nullspace of A nullsp = null(A,'r') % basis of the row space of A rowsp = R(1:rank,:)' % basis of the left nullspace of A leftnullsp = null(A','r')</pre>	
Output Screenshot	

R = 2×3

1	0	1
0	1	1

pivot = 1×2

1	2
---	---

rank = 2

columnsp = 2×2

1	2
2	-1

nullsp = 3×1

-1
-1
1

rowsp = 3×2

1	0
0	1
1	1

leftnullsp =

2×0 empty **double** matrix