

CLASS - 36

Modular Arithmetic, gcd, lcm, Euclid, Mod

factors / Divisors ?

How to find them through code

$O(n)$

$i = 1 \dots n$

$36 \rightarrow$

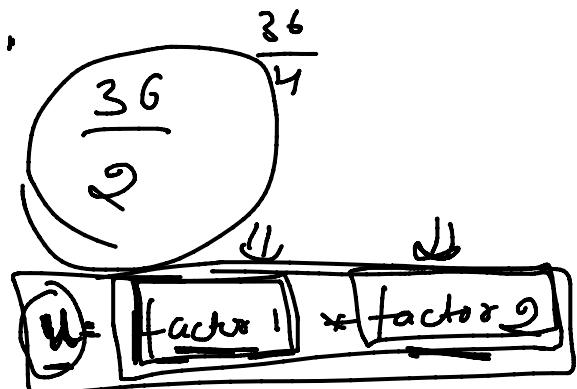
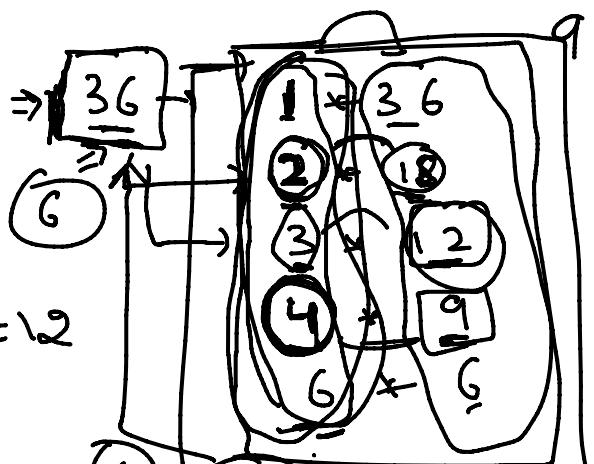
$$\begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ 6 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ 12 \\ 18 \\ 36 \\ \hline \end{array}$$

$$2 \times 9 = 18$$

$3 \times 12 = 36$

$$\begin{array}{l} 36 \\ \times 2 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 36 \\ \times 3 \\ \hline 12 \end{array}$$



$$36 \div 10 = 0$$

$O(n)$

i

$$\begin{array}{r} 36 \\ \hline i \end{array}$$

$O(n)$

arrays Primos

$$\text{factor}_1 = i$$

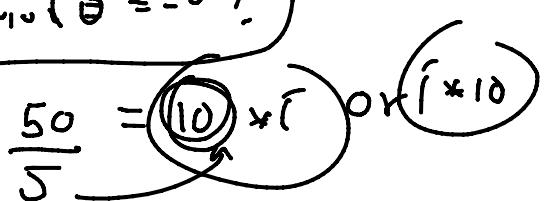
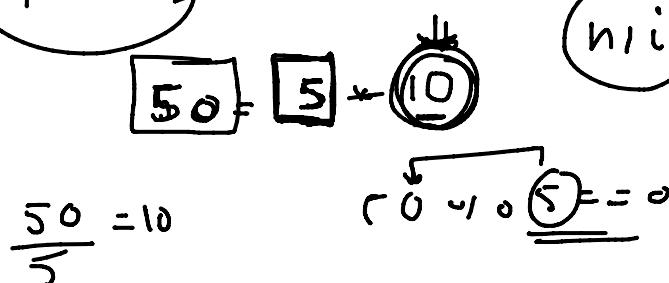
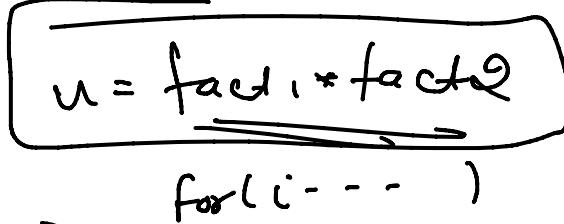
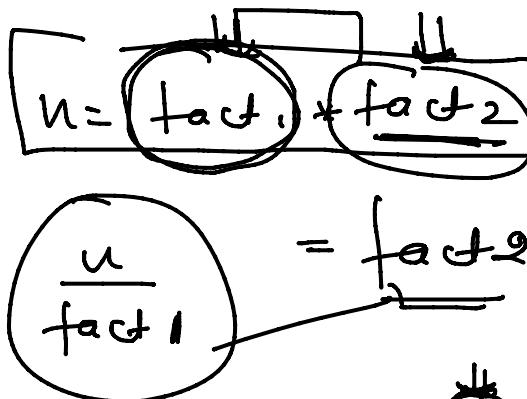
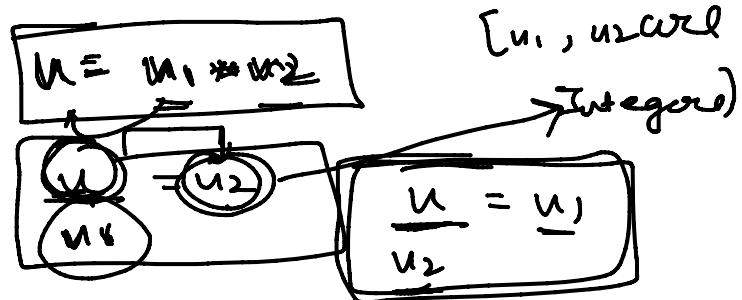
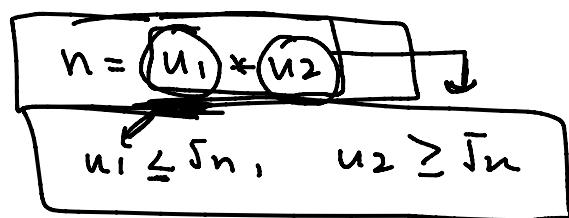
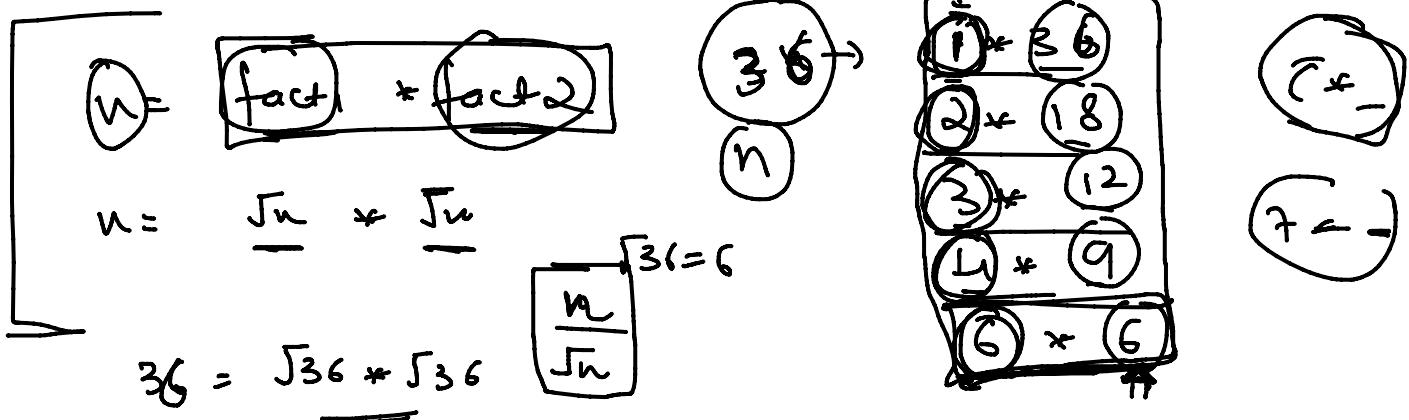
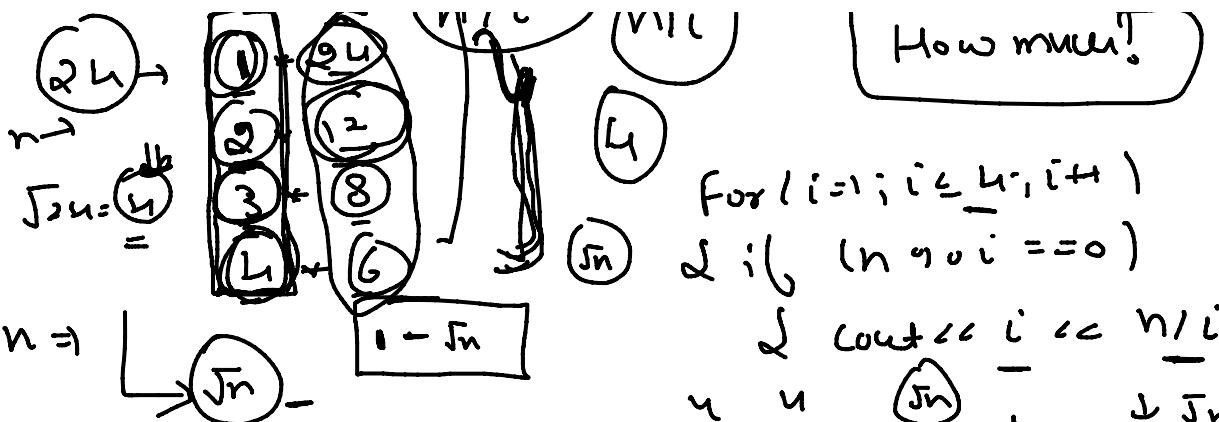
$$\text{factor}_2 = \frac{36}{i}$$

$i=1 \dots i=n-1$

We don't need to check all the nos

How much?

$24 \rightarrow$ $\frac{n}{i}$ $\frac{m}{i}$



$n - 1$ All factors

$1 - \sqrt{n}$

n -> All factors $(1 - \sqrt{n})$

for (i = 1 ; $i <= \sqrt{n}$; $i++$)

& if ($n \% i == 0$)

& cout << i << n / i else endl

4 4

$$36 \Rightarrow \frac{1}{1} \times 36$$

$$i = 1 - \sqrt{36}$$

$$\underline{2} \neq 18$$

$$\underline{36 \% 1} = 0$$

$$\underline{3} \neq 12$$

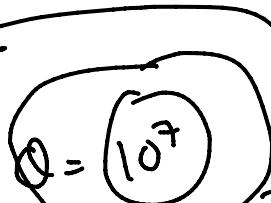
$$36 / 1 = 36$$

$$\underline{4} \neq 9$$

$$\boxed{6} \neq 6$$

$$\underline{36 \% 9} = 0$$

$$\frac{36}{2} = 18$$



Count or sum

if ($(n / i) != i$)

γ

divisor

Count and sum

$$\text{Total T.C.} = (\text{Q} + \sqrt{n})$$

Mathematical way

$$n = p_1^{n_1} * p_2^{n_2} * p_3^{n_3} * \dots * p_m^{n_m}$$

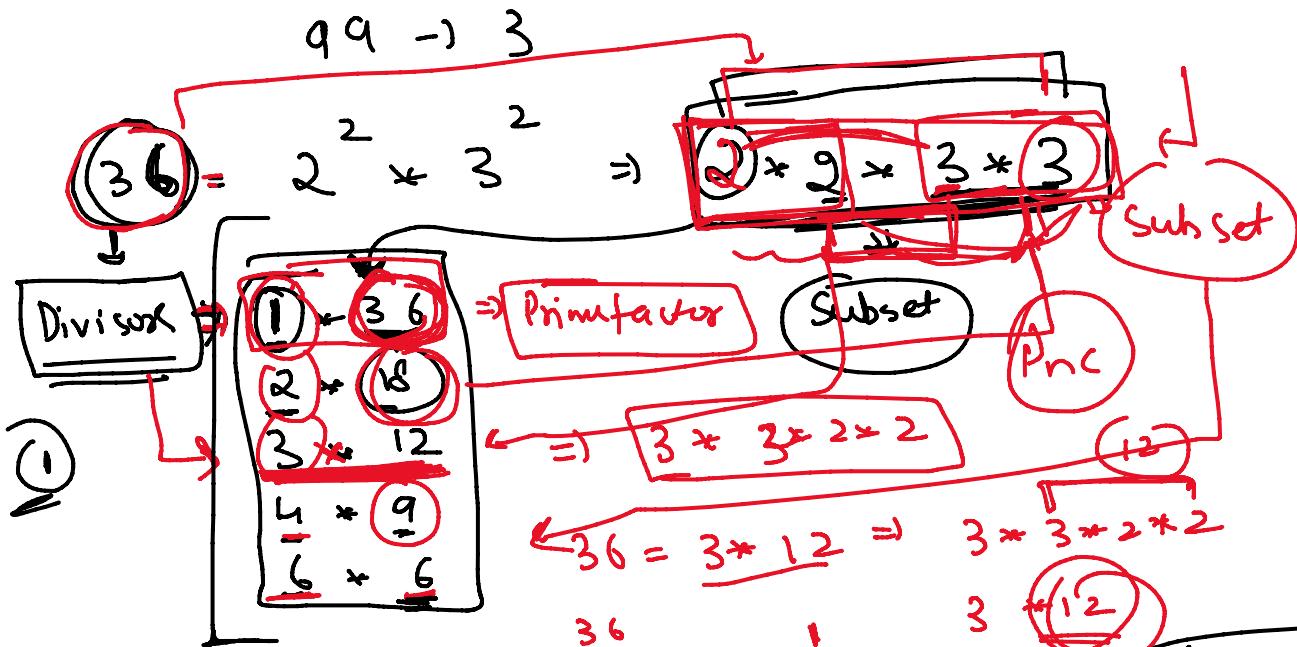
$$36 = 2^2 * 3^2$$

$$\begin{array}{r} 36 \\ \hline 2 | 18 \\ 2 | 9 \\ 3 | 9 \\ 3 | 3 \\ \hline 1 \end{array}$$

..... n .. n .. 1 .. 1

Composite No = Prime No Product

1, N 48 → 2, 3



$$1 \times 36 \Rightarrow 2 \times 2 \times 3 \times 3 \times 1$$

$$2 \times 18 = \underbrace{2}_{1} \times \underbrace{2 \times 3 \times 3}_{18}$$

$$3 \times 12 = \underbrace{2 \times 2 \times 3}_{12} \times 3$$

$$4 \times 9 =$$

Total No of divisor = ?

using prime factorization

Doubt!

$$2 \times 2 \times 3 \times 3 \Rightarrow$$

$$2^2 \times 3^2$$

$$2^3 \times 3^2$$

Subset

$$0, 1, 2$$

$$0, 1, 2$$

$$3 \times 3$$

Combi

Subset
≡

$$\boxed{0, 1, 2}$$

Total Subsets $\rightarrow 3 \times 3 = 9$

$n = P_1^a \times P_2^b \times P_3^c \dots P_n^z$

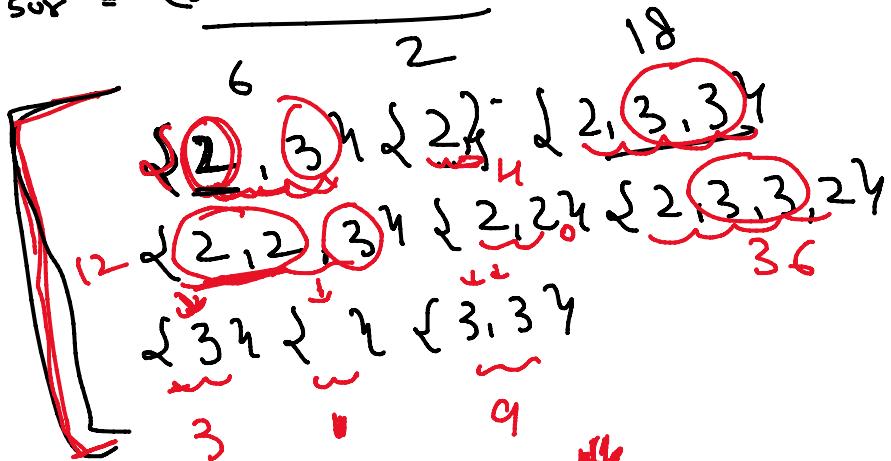
Total option = $a + b$

$$\text{Total divisors} = \frac{(a+1)(b+1)(c+1)}{2}$$

$$2^2 \times 3^2$$

Maths

FNC



$$n = P_1^a \times P_2^b \times P_3^c \dots P_n^z$$

$\log n$

$$\text{Total divisors} = (a+1)(b+1) \dots (z+1)$$

$$\text{Count divisors} = O(\log n) = O(\log n)$$

$$10^9 =$$

$$10^9$$

$$\sqrt{10^9}$$

$$\log_2 10^9 =$$

$$30$$

$$Jn \rightarrow 10,000,$$

$$30$$

Sum of divisors

$$n = P_1^a \times P_2^b \times P_3^c \dots P_n^z$$

$$(2 \times 2 \times 2^2 \times 3)$$

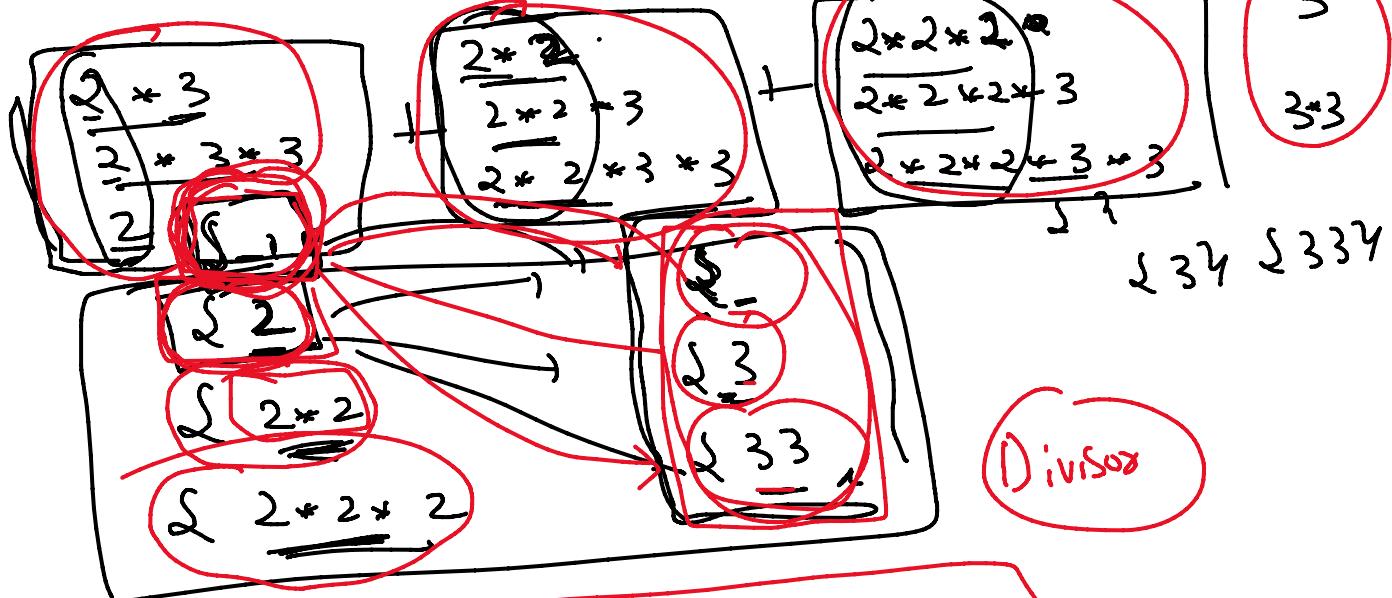
$$n = P_1^{m_1} \times P_2^{m_2}$$

$P_n^{m_n}$

Sum of Divisors =

$$1 + P_1 + P_1^2 + \dots + P_1^{m_1} \times (1 + P_2 + P_2^2 + \dots + P_2^{m_2})$$

$$2 \times 2 \times 2^2 \times 3 \times 3$$



$$n = P_1^{m_1} \times P_2^{m_2} \times P_3^{m_3} \dots P_n^{m_n}$$

$$(1 + P_1 + P_1^2 + \dots + P_1^{m_1}) \times (1 + P_2 + P_2^2 + \dots + P_2^{m_2}) \times \dots \times (1 + P_n + P_n^2 + \dots + P_n^{m_n})$$

Sum of divisors :-

$$\left[\frac{P_1^{m_1+1} - 1}{P_1 - 1} \right] \times \left[\frac{P_2^{m_2+1} - 1}{P_2 - 1} \right] \times \dots \times \left[\frac{P_n^{m_n+1} - 1}{P_n - 1} \right]$$

$$P_1 - 1 \quad P_2 - 1 \quad \dots \quad P_n - 1$$

$$n = P_1^{n_1} \times P_2^{n_2} \times P_3^{n_3} \times \dots \times P_n^{n_n}$$

$$36 \Rightarrow \left[\frac{2^2 \times 3^2}{-} \right] \times \left[\frac{2^3 - 1}{2 - 1} \right] \times \left[\frac{3^3 - 1}{2 - 1} \right]$$

$$(2+1)(2-1) \quad \text{Ans}$$

$$7 \times 26 \times 13 = \text{Ans}$$

Divisors $\rightarrow O(n) \quad O(\sqrt{n})$

b) count and sum of divisors \rightarrow Formula

$$\text{Count} = n = P_1^{n_1} \times P_2^{n_2} \times P_3^{n_3} \times \dots \times P_n^{n_n}$$

\rightarrow

$$\text{Count} = (n_1 + 1)(n_2 + 1) \dots (n_n + 1)$$

$$\text{Sum} = \left[\frac{P_1^{n_1+1} - 1}{P_1 - 1} \right] \left[\frac{P_2^{n_2+1} - 1}{P_2 - 1} \right] \dots \left[\frac{P_n^{n_n+1} - 1}{P_n - 1} \right]$$

Prime Nos

Prime Nos

\rightarrow 2 Factors

1, n

2, 3, 5, 7, 11, 13, 17, 19, 23, ...

Ques-1

How to check if a no is

Prime

ques-1

How to find no. of divisors

==

No. of divisors

for ($i=1$; $i \leq n$; $i++$)

{

n

For ($i=1$; $i \leq \sqrt{n}$; $i++$)

{

n

($i=2$; $i \leq \sqrt{n}$; $i++$)

n

$1, n$

Prime $\neq 10 \times$

$i < \sqrt{n}$

$i^2 < n$

Prime factorization

→ GCD, LCM, Count of Divisors,
Sum of Divisors

$36 \Rightarrow 1, \cancel{2}, \cancel{3}, 4, 6, 9, 12, 18, 36$

$36 = \boxed{2 \times 3}$

(1)

$\rightarrow \cancel{1}, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}, \cancel{7}, \cancel{8}, \cancel{9}, \cancel{10}, \cancel{11}, \cancel{12}, \cancel{13},$
 $14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26$
 $27, 28, 29, 30$

$\boxed{2 \times 3}$

Composite No. \neq Prime No.

$\boxed{36}$

$\boxed{36} = \frac{18}{2} \Rightarrow \boxed{18} = 3^2$

$$n = p_1^{n_1} + p_2^{n_2} + p_3^{n_3} + p_4^{n_4}$$

$$\begin{array}{c} (50) \\ | \\ 2 \end{array} \quad \begin{array}{c} (56) \\ | \\ 2 \end{array} = \frac{48}{2} = \begin{array}{c} (9) \\ | \\ 3 \end{array} = \frac{3}{3} = 1$$

$$36 = \boxed{3^2} \quad \boxed{\cancel{4}} \quad \boxed{\cancel{5}} \quad \boxed{\cancel{6}} \quad \boxed{\cancel{7}} \quad \boxed{\cancel{8}} \quad \boxed{\cancel{9}}$$

int cut = 0
while ($n \% 2 == 0$)

{ $n /= 2$; cut++ }

$\begin{array}{c} 2 \times 2 \\ 8 \\ 9 = 3 \times 3 \\ 10 = 2 \times 5 \\ 11 \end{array}$
while ($n \% 3 == 0$)
{ $n /= 3$ }

(Primes can only divide n)

$$u = \boxed{2} \times \boxed{3} \times \boxed{5} \times \boxed{7} \times \boxed{11} \quad \text{(Composite)}$$

$$4, 6, 8, 10 = \boxed{2} \times \boxed{r}$$

$$10 = \frac{2 \times 2 \times r}{\boxed{2 \times 5}}$$

$$10x$$

$$2 \times 5$$

$$\begin{array}{c} 2 \rightarrow 3 \\ 5 \rightarrow 0 \end{array}$$

$$40 = \frac{2 \times 2 \times 2 \times 5}{\boxed{2 \times 5}}$$

$$40 \Rightarrow 2 \times 2 \times 2 \times 5$$

$$1 - 39$$

$$(3, 4, 5)$$

(Composite No)

Prime Factors

Composite No → Prime factors

36 → ~~3~~ 21 ←

$$\text{while } (36 \cdot 4 \cdot 2 = 0)$$

$$2 \cdot 36 = 2;$$

2. 3

4. 6 Composite Nos

$$\Rightarrow 36 = \frac{4 \cdot 9}{4} = 3 \cdot 3$$

4

2. 2

For S

Prime factor

$$\text{Composite} = P_1 \times P_2 \times P_3$$

n = - - - - -

n → Prime factors

$$n = P_1 \times P_2 \times P_3 \cdots P_n$$

1 - n-1