Class 36

18 March 2023 14:07

CLASS- 36

Modular Arithmetic, gcd, lcm, Euclid, Mod

factors / Divisors ?

How to find them through code

↳ $\boxed{O(n)}$

1 - 36

36 →

1	9
2	12
3	18
4	36
6	

$2 \times 9 = 18$

35

⇒ $\boxed{36}$

1	*	36
2	*	18
3	*	12
4	*	9
6	*	6

9.

$\frac{36}{4}$

$\frac{36}{2}$

$\frac{36}{2} = 18$

6

$\frac{36}{3} = 12$

$36 = \boxed{\text{factor 1}} * \boxed{\text{factor 2}}$

6

1 - 6

$36 \% 2 == 0$

$\sqrt{36} = 6$

$\frac{36}{2}$

$\frac{36}{3}$

O(n)

i

$\frac{36}{i}$

factor$_1$ = i

factor 2 = $\frac{36(n)}{i}$

$\boxed{\text{Arrays PrimeNo}}$

O(n)

$\boxed{i = 1 \text{ --- } i = n-1}$

$\boxed{\text{We don't need to check all the Nos}}$

24 → 1 * 24

n / i

n / i

$\boxed{\text{How much!}}$

How much!

24 → 1 * 24, 2 * 12, 3 * 8, 4 * 6

$\sqrt{24} = 4$

n ⇒ $\sqrt{n}$

$1 - \sqrt{n}$

for (i=1; i<=4; i++)
{ if (n % i == 0)
{ cout << i << n/i;

$n = fact_1 * fact_2$

$n = \sqrt{n} * \sqrt{n}$

$36 = \sqrt{36} * \sqrt{36}$

$\sqrt{36} = 6$

$\frac{n}{\sqrt{n}}$

36 → 1 * 36, 2 * 18, 3 * 12, 4 * 9, 6 * 6

$n = n_1 * n_2$

$n_1 \le \sqrt{n}, \quad n_2 \ge \sqrt{n}$

$n = n_1 * n_2$ [n_1, n_2 are integers]

$\frac{n}{n_2} = n_1$

$n = fact_1 * fact_2$

$\frac{n}{fact1} = fact2$

$n = fact_1 * fact2$

for (i - - -)

n/i

n % i == 0 ?

$50 = 5 * 10$

$\frac{50}{5} = 10$

50 % 5 == 0

$\frac{50}{5} = 10 * 5$ or $5 * 10$

n → All factors

$1 - \sqrt{n}$

n-1 All factors $(1 - \sqrt{n})$

```
for ( i = 1; i <= √n; i++ )
{   if ( n % i == 0 )
    {  cout << i << n/i << endl
    }
}
```

$36 \Rightarrow 1 * 36$

$2 * 18$

$3 * 12$

$4 * 9$

$\boxed{6} * 6$

$i = 1 - \sqrt{36}$

$36 \% 1 == 0$

$36 / 1 = 36$

$36 \% 9 == 0$

$\frac{36}{2} = 18$

$\boxed{36 \% 6 == 0}$

$\frac{36}{6} = 6$

$O(\sqrt{n})$

$Q = 10^7$

$\boxed{Q} \Rightarrow$

- $n_1 \rightarrow O(\sqrt{n})$
- n_2
- n_3

Total T.C = $(Q * \sqrt{n})$

count or sum

if ((n / i) != i)
{ }

divisor

Count and sum

Mathematical way

$n = p_1^{n_1} * p_2^{n_2} * p_3^{n_3} \ldots\ldots p_n^{n_n}$

$36 \Rightarrow 2^2 * 3^2$

$$\begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

Composite No = Prime No Product

1, N

48 → 2, 3

99 → 3

$36 = 2^2 * 3^2 \Rightarrow 2 * 2 * 3 * 3$

Subset

Divisor

1 * 36 ⇒ Primefactor

2 * 18

3 * 12

4 * 9

6 * 6

Subset

Pnc

①

⇒ 3 * 3 * 2 * 2

12

36 = 3 * 12 ⇒ 3 * 3 * 2 * 2

3 * 12

$1 * 36 \Rightarrow \underbrace{2 * 2 * 3 * 3}_{36} * 1$

$2 * 18 = \underbrace{2}_{2} * \underbrace{2 * 3 * 3}_{18}$

$3 * 12 \Rightarrow \underbrace{2 * 2 * 3}_{12} * 3$

4 * 9 =

36 = 2 * 2 * 3 * 3

divisors

(2, 3) = 6

Formula — Total No of divisors = ?

using Prime factorization

Detail

$2 * 2 * 3 * 3 \Rightarrow 2^2 * 3^2$

0, 1, 2

0, 1, 2

3 * 3

Combi

{3} {3}

9

Subset

Subset = 0, 1, 2 3 * 3 Combi

Total subset $3*3 = 9$ Total option = $x * y$

$n = p_1^a * p_2^b * p_3^c$

Total divisor = $(a+1)(b+1)(c+1)$

$2^2 * 3^2$

Maths

PnC

6 2 18

{2, 3} {2} {2, 3, 3}

12 {2, 2, 3} {2, 2} {2, 3, 3, 2} 36

{3} { } {3, 3}

3 1 9

$n = p_1^a * p_2^b * p_3^c \; - - - \; p_n^z$ (logn)

Total divisor = $(a+1)(b+1) - - - (z+1)$

Count divisors ⇒ $O(\sqrt{n})$ = O(logn)

O(n)

$10^{9/2} = 10^4$

$\sqrt{10^9}$

$\log_2 10^9 = 30$ logn

$\sqrt{n} \to 10{,}000$ 30

Sum of divisors

$n = p_1^a * p_2^b * p_3^c \; - - - \; p_n^z$

$(2 * (2 * (3 * 3)))$

$n = p_1^{n_1} * p_2^{n_2} - - - p_n^{n_n}$

Sum of Divisor = $(1 + \boxed{p_1} + \boxed{p_1^2} + \dots \boxed{p_1^{n_1}}) * \boxed{(1 + p_2 + p_2^2 + - - - p_2^{n_2})}$

$2 * 2 * 2 * 3 * 3$

$2 * 3$ | $2 * 3 * 3$ | 2 + $2 * 2$ | $2 * 2 * 3$ | $2 * 2 * 3 * 3$ + $2 * 2 * 2$ | $2 * 2 * 2 * 3$ | $2 * 2 * 2 * 3 * 3$ | 3 | $3 * 3$

{1} {2} {2 * 2} {2 * 2 * 2}

{1} {3} {3 3}

{3} {3 3}

Divisor

$n = p_1^{a_1} * p_2^{a_2} * p_3^{a_3} - - - p_n^{a_n}$

$(1 + p_1 + p_1^2 + \dots p_1^{n_1}) * (1 + p_2 + p_2^2 + \dots p_2^{n_2}) * - - - (1 + p_n^1 + p_n^2 + - - p_n^{n})$

G.P

Sum of divisor =

$$\left[\frac{p_1^{n+1} - 1}{p_1 - 1}\right] * \left[\frac{p_2^{n_2+1} - 1}{p_2 - 1}\right] * - - - \left(\frac{p_n^{n_n+1} - 1}{p_n - 1}\right)$$

$$\left(\frac{\ }{p_1 - 1}\right)\left(\frac{\ }{p_2 - 1}\right) \quad \left(\frac{\ }{p_n - 1}\right)$$

$n = p_1^{n_1} * p_2^{n_2} * p_3^{n_3} * \ldots$

$36 \Rightarrow \boxed{2^2 * 3^2}$ $\quad \boxed{\left(\frac{2^3 - 1}{1}\right) * \left(\frac{3^3 - 1}{2}\right)}$

$(2+1)(2+1)$ $\quad 7 * \cancel{26}\ 13 = 91$

$= 9$

Divisors → $O(n)$ $\quad O(\sqrt{n})$

1) count and sum of divisors → formula

Count: $n = p_1^{n_1} * p_2^{n_2} * p_3^{n_3} \ldots p_n^{n_n}$

$$\boxed{\begin{aligned} &\text{Count} = (n_1+1)(n_2+1) \ldots (n_n+1) \\ &\text{Sum} = \left(\frac{p_1^{n_1+1} - 1}{p_1 - 1}\right)\left(\frac{p_2^{n_2+1} - 1}{p_2 - 1}\right) \ldots \left(\frac{p_n^{n_n+1} - 1}{p_n - 1}\right)\end{aligned}}$$

Prime Nos

Prime Nos → 2 Factor $\quad$ 1, n

2, 3, 5, 7, 11, 13, 17, 19, 23 ...

Ques-1 $\quad$ How to check if a no is **Prime**

Que-1 How to check if a

No of divisors

for (i=1 ; i<=n ; i++)
{

}

For (i=1 ; $\boxed{i<=\sqrt{n}}$; i++)
{

}

1, n

Prime x10 x

(i=2 ; i<√n ; i++)

n

$i < \sqrt{n}$

$\boxed{i^2 < n}$

Prime factorization

GCD, LCM, Count of Divisor, Sum of Divisors

36 ⇒ 1, 2, 3, 4, 6, 9, 12, 18, 36

$36 = \boxed{2^2 * 3^2}$

1

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13,
14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26
27, 28, 29, 30

$6 = \boxed{2 * 3}$

Composite No = Prime no

$\boxed{36}$

$\frac{36}{2} = \frac{18}{2} \Rightarrow 9 = 3$

$n = p_1^{n1} * p_2^{n2} * p_3^{n3} * p_4^{n4}$

$\frac{36}{3}$

$\frac{36}{2} = \frac{18}{2} \Rightarrow 9 = \frac{3}{3}$

$36 = 2^2 * 3^2$

$4 = 2*2$

$36 = \boxed{3^2}$

int cnt=0
while(n%2==0)
{ n/=2; cnt++;
n=1 }

$2*2$ $8 = 2*2*2$

9 = 3*3   10 = 2*5   11

while(n%3==0)
{ n/=3
}

[Prime Nos can only divide n]

$n = 2 * 3 * 5 * 7 * 11$

composite

2 3 5 7 11 4

2

4, 6, 8, 10 = 2*5

$2^3 * 5^1$

$\frac{20}{10} = \frac{2*2*5}{2*5}$

10x

2, 5

40

2 → 3
5 → 1

$\boxed{40} \Rightarrow 2*2*2*5$

$40 = \cancel{2*2*2}*5$

1 - 39

2 3 4 5

Composite No → Prime Factors

Composite No → Prime factors

36 * 3 2 1

while(36 % 2 == 0)

n - $\sqrt{n}$

1 - n

2, 3

{ 36 /= 2;

36 / 4

4, 6 Composite No

⇒ 36 = 4 * 9 3 * 3

4

4

2 2

for(2 - $\sqrt{n}$)
{

}

Prime factor

n

Composite = $P_1 * P_2 * P_3$

n = - - - -

n → Prime factors

n = $P_1 * P_2 * P_3$ - - - P_n

1 - n-1