

Number Theory (Matrix)

- O A
- C P
- DSA

Basic Maths
using
Coding

Number Theory

- Modular Arithmetic
- Modular exponentiation
- Prime NOS, factors / divisors, Sieve
- GCD, LCM,
- Modular Multiplicative Inverse
- Fermat's Little theorem
- Euclid's Algorithm
- Conferences (H - S)
- Pigeonhole Principle
- PNL (factorial, combinatorial, Permutations)

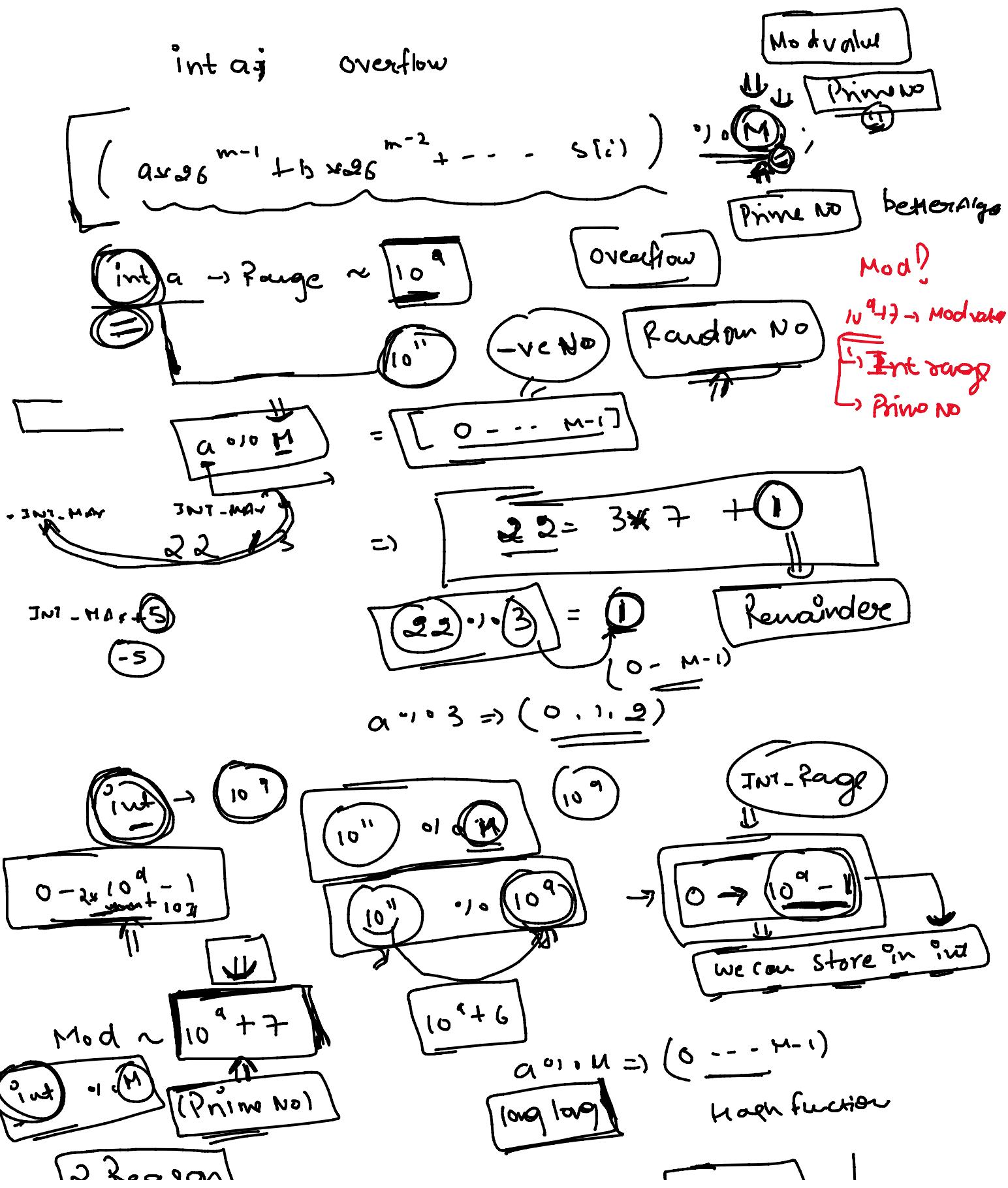
Mechanical

Easy/Medium

Division

Modular Arithmetic

Modular Arithmetic



2 Reason

int value

Result int Range

①

int range
 10^9 to -10^9

Mod: $10^9 + 7$

$$\begin{array}{r} 500 \cdot 10^9 = \\ - \\ 250 \cdot 10^9 = \\ - \\ 10^9 = \end{array}$$

② Fabinacci Main Function prime No
main value mod

Generic Mod value

Prime value

Overflow \rightarrow negative

divide Mod

$$22 = 3 \cdot 7 + 1$$

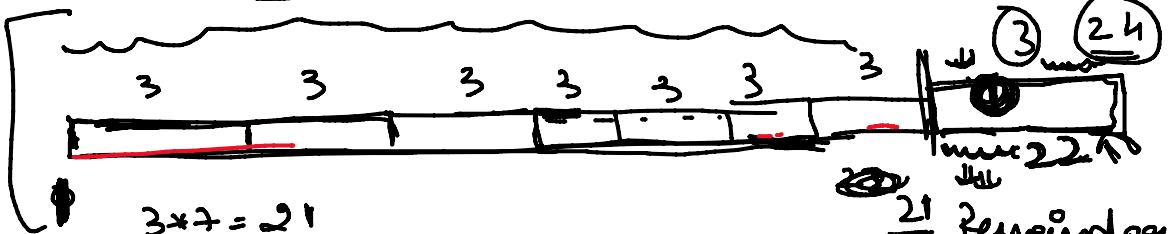
$$\frac{22}{3} = 7$$

1, 0, 0

22×3

$$22 \cdot 10^3 = 1$$

1, 0, 0



$$3 \cdot 7 = 21$$

$$3 - 22 \cdot 10^3$$

$$\Rightarrow 3 - 1 = 2$$



1 unit \rightarrow add 5
1 unit \rightarrow 1 (Remove)

2 unit \rightarrow add

1 unit \rightarrow Remove

Modular Arithmetic

① Addition

$(a+b) \text{ } \% M$

a, b

$$\begin{aligned} (a+b) \text{ } \% M &\Rightarrow \cancel{(a \% M + b \% M)} \text{ } \% M \\ (a-b) \text{ } \% M &\Rightarrow \cancel{(a \% M - b \% M + M)} \text{ } \% M \end{aligned}$$

$$\underline{(17 + 18) \% 6} \Rightarrow \underline{\underline{5}}$$

$$(17 \% 6 + 18 \% 6) \% 6 =$$

\downarrow
5 + 0

$$(5 + 0) \% 6 \Rightarrow \underline{\underline{5}}$$

$$\underline{(18 - 13) \% 4} = \underline{\underline{1}}$$

$$(18 \% 4 - 13 \% 4 \% 4) \% 4$$

$$\begin{aligned} (2 - 1 + \cancel{4}) \% 4 &= \underline{\underline{1}} \\ (\underline{1 + \cancel{4}}) \% 4 &\Rightarrow (\cancel{1 \% 4} + \cancel{4 \% 4}) \\ &= \underline{\underline{1}} \end{aligned}$$

$$\underline{\underline{13 - 15}} \% 4$$

$$\underline{\underline{(-2 \% 4) \% 4}}$$

-2

$$\left(-\frac{2+4}{4} \right) \% 4$$

$\frac{2}{4} \geq 0$

$$2 \% 4 = 2$$

Hash value

$\left[\begin{array}{l} \text{if } (t < 0) \\ \end{array} \right]$

Hash value

$$\begin{cases} \text{if } (t < 0) \\ \quad t += M \\ \text{else} \end{cases}$$

$$(a+b) \times m \Rightarrow a \times m + b \times m$$

(3) Multiplication

addition
 subtraction
 multiplication
 total

$$(a+b) \times m \Rightarrow (a \times m + b \times m) \times m$$

Total range $> \text{int}$

Overflow

Int range

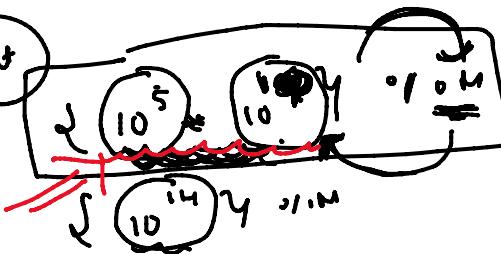
(n)

$$n = 10^9 + 1$$

$0 - n - 1$

10^{10}

int



$$\begin{aligned}
 &= (10^5 \times 10^4 \times m) + 10^{14} \times m \\
 &= (10^5 \times 10^4) \times m + 10^{14} \times m \\
 &\quad \text{overflow}
 \end{aligned}$$

overflow \Rightarrow Galat wrong

changed to

To prevent overflow in

Modular arithmetic

1 intermediate steps

first call return

int

Tight constraint (Memory)

long long

intermediate steps

call $> 10^9$

MLE, TLE

Modular Arithmetic

1

Addition
Subtraction

Saragh

$$\begin{aligned}
 &45 \times 10^4 = 1 \\
 &45 \rightarrow 45 \quad 1 \rightarrow 1
 \end{aligned}$$

Addition

Subtraction

Multiplication

$$\begin{array}{c} \text{if } 0 \leq a < M \\ \text{then } a + b \equiv a + b \pmod{M} \end{array}$$

$$(5+9) \pmod{4} = 1$$

$$(5+10+1 \pmod{4}) \pmod{4}$$

$$(11 \pmod{4}) \pmod{4} = 1$$

Modular arithmetic on Division

$$\left(\frac{a}{b}\right) \pmod{M} = \left(\frac{a \cdot b^{-1}}{b \pmod{M}}\right) \pmod{M}$$

if $b \pmod{M} = 0$

↳ Possible?

$$b^{-1} \neq \frac{1}{b}$$

$$\begin{aligned} \left(\frac{a}{b}\right) \pmod{M} &\Rightarrow \left[\frac{a \cdot (b^{-1})}{b \pmod{M}}\right] \pmod{M} \\ &\Rightarrow \left[\frac{a \pmod{M}}{b \pmod{M}} \cdot (b^{-1}) \pmod{M}\right] \pmod{M} \end{aligned}$$

Congruency rule

$$a \equiv b \pmod{M}$$

$$a \equiv b \pmod{M}$$

$$a \pmod{M} = b \pmod{M}$$

$$a \pmod{M} = b \pmod{M}$$

GCD and LCM

GCD (Greatest common Divisor) or HCF

$$4 \rightarrow 2 \times 2 \Rightarrow 4$$

$$12 \rightarrow 2 \times 2 \times 3$$

$$\rightarrow 2 \times 2 \times 3$$

$$2 \times 3 \Rightarrow 1$$

$$\begin{array}{c} \text{L} \\ \begin{array}{|c|c|c|} \hline & 2 & 1 \\ \hline 2 & \times & 3 \\ \hline 2 & 1 & 1 \\ \hline \end{array} \end{array}$$

$$\begin{array}{l} \text{LCM} \\ \hline 4 \rightarrow 2 \times 2 \quad \text{or} \quad \frac{2^2}{2^2} \times \frac{3^0}{3^1} \\ 12 \rightarrow 2 \times 2 \times 3 \quad \frac{2^3}{2^3} \times \frac{3^1}{3^1} \end{array} \quad 2^2 \times 3^1 = \boxed{12}$$

$$\begin{array}{c}
 \text{GCD} = 2^1 \times 3^1 = 6 \\
 \text{LCM} = 2^2 \times 3^2 = 36
 \end{array}$$

$$\begin{array}{r} \cancel{2 \times 3} \\ = a \times b = \end{array} \quad \begin{array}{c} 2^2 \times 3^1 + x^1 \times 3^2 \\ \hline 2^1 \times 3^1 \end{array}$$

$$\text{GCD} \times \text{LCM} = a \times b$$

Long Division

$$\frac{1^2}{2} \sqrt{18} L^1$$

$$\text{GCD} \leftarrow \frac{6}{12} = ?$$

$$\begin{array}{c}
 \text{a} \cdot \text{b} = \text{f} \\
 \hline
 \boxed{5 \cdot 9} \\
 \downarrow \\
 \boxed{(4) \underline{5})} \quad 5 \cdot 1 = \underline{1} \\
 \downarrow \qquad \qquad \qquad \downarrow \\
 (\underline{1} \cdot 4) \quad (4 \cdot 1 = \underline{0})
 \end{array}$$

$$\begin{array}{r}
 (114) \\
 (0, 11) \\
 \hline
 2 \overline{) 9} \quad 4 \\
 \underline{-8} \quad \underline{1} \\
 \hline
 2 \quad 2 \\
 \underline{-0} \quad \underline{2} \\
 \hline
 0
 \end{array}
 \quad
 \text{GCD} = 1 \quad | \quad \log n$$

GCD → Prime Factors → long Division (Euclid Algorithm)

$$\begin{array}{r}
 5 \quad 9 \\
 \hline
 1 \quad 5 \quad 9 \\
 \hline
 1 \quad 5 \quad 9
 \end{array}
 \quad
 \text{gcd}(5, 9) \rightarrow \text{gcd}(4, 5) \rightarrow \text{gcd}(114)$$

int gcd = 1;

long division gcd(4, 5);

Euclid Algorithm

```

int gcd ( int a, int b ) {
    if ( a == 0 ) return b;
    return gcd ( b % a, a );
}
    
```

$$lcm = \frac{a * b}{\text{gcd}}$$

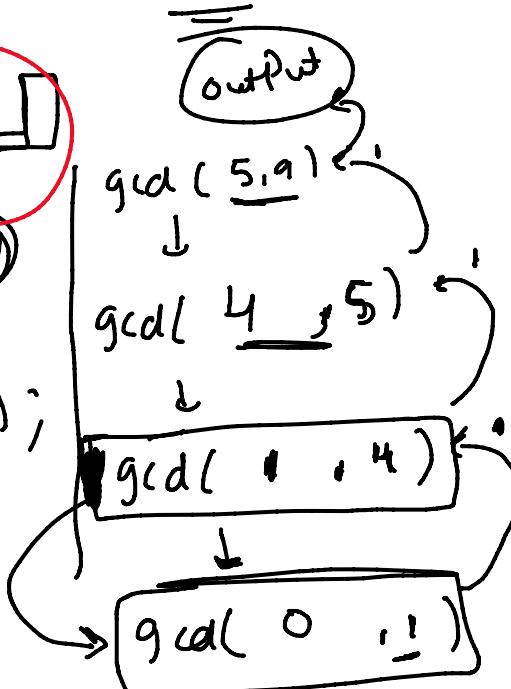
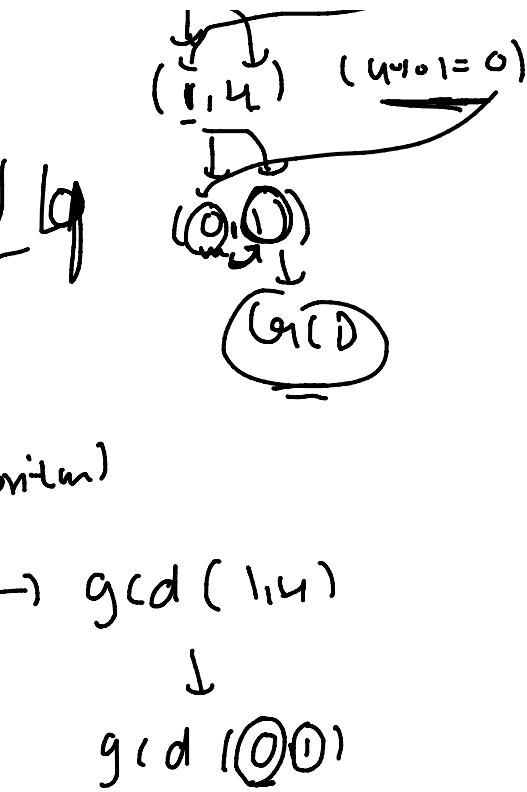
$$\text{gcd}(\underline{18}, \underline{12})$$

$$\text{gcd}(\underline{12}, \underline{18})$$

Binary Search → $\log_2 n$

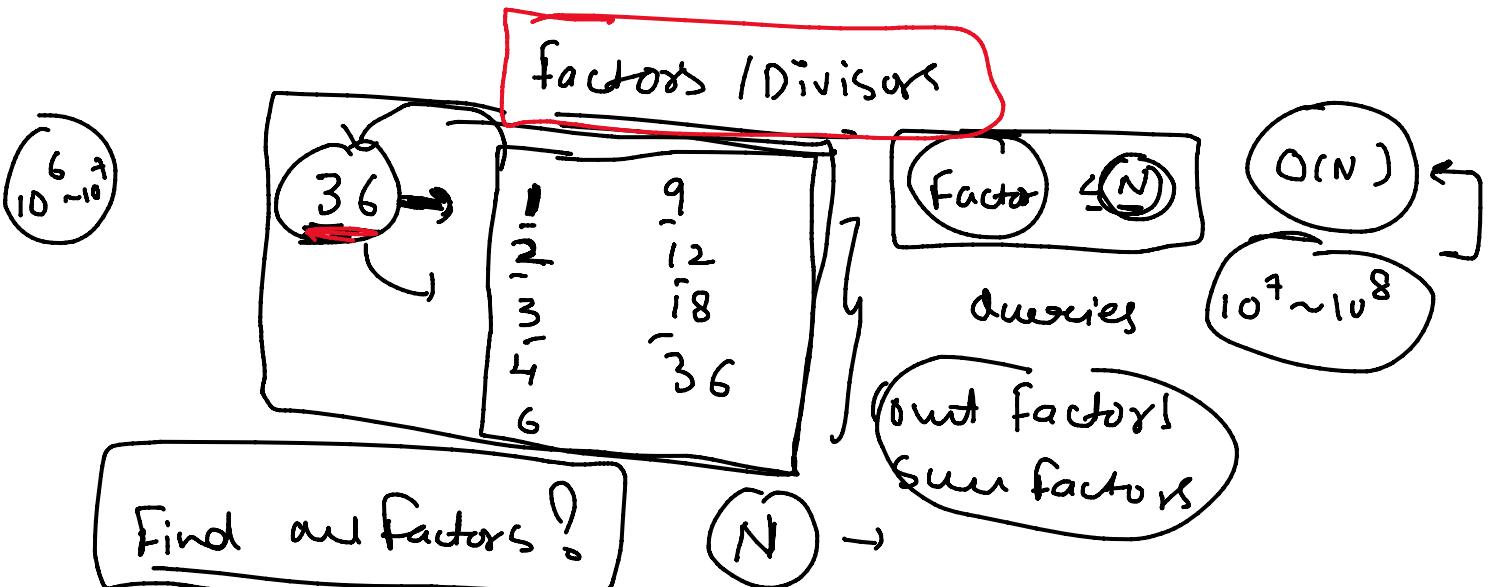
--gcd(a, b)

--gcd(a, b)



GCD, LCM, Modular Arithmetic, Extended GCD

GCD, LCM, Modular Arithmetic, Euclid Algorithm



`int count = 0; int sum = 0`

`for (i=1; i <= n; i++)`

`if (n % i == 0)`

`{ count <= i < " "; count++; sum+=i; }`

`}`

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