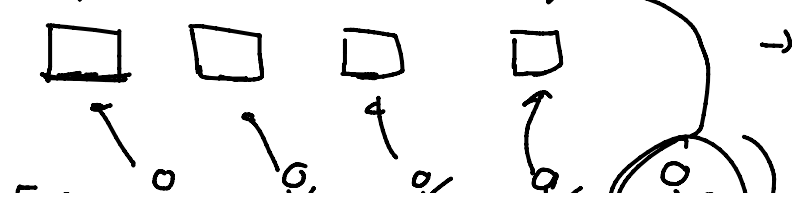
CLASS-40

Number Theory !! –

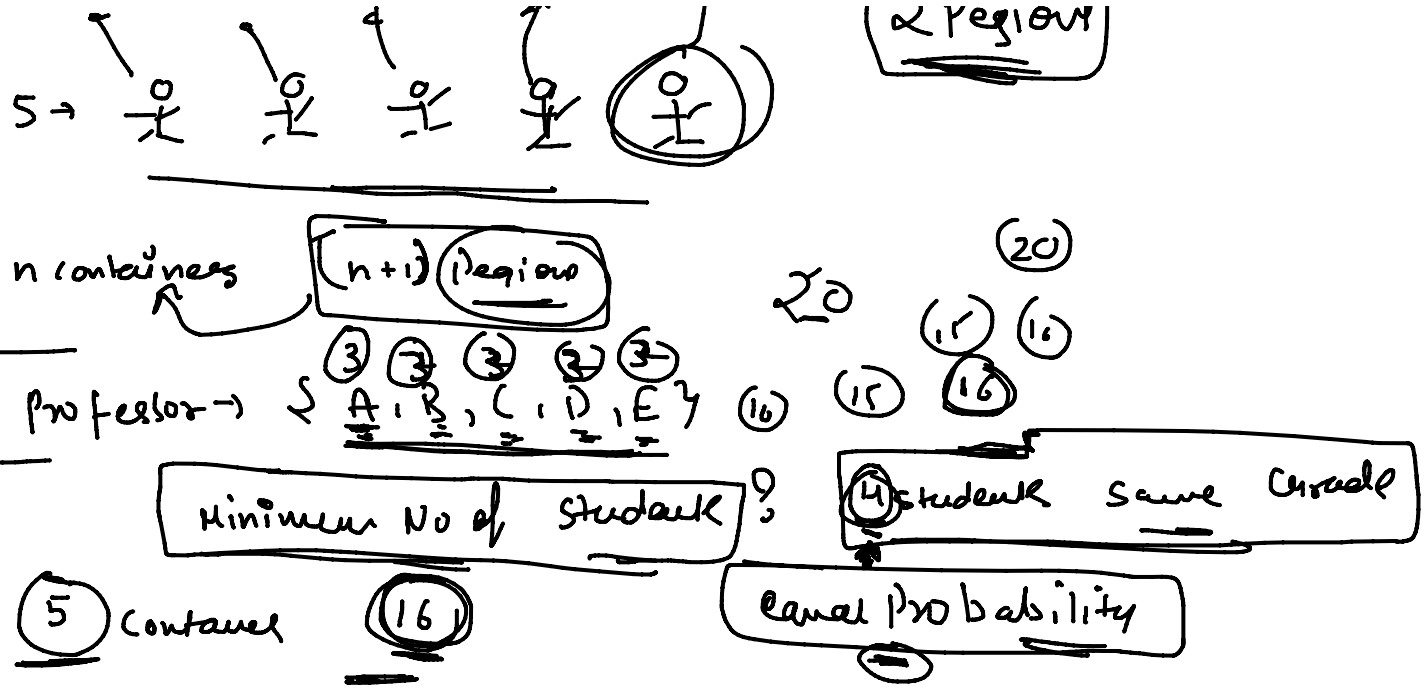
- Modular Arithmatic
- GCD / LCM (Euclid)
- Divisors / factors } 2-3 approaches
  - count
  - sum
- Prime No
- Prime factorization
- Seive → [0 - N]
  - Highest / lowest Prime factor
  - Prime factorization
  - Divisor for every No b/w [1 ton]
- Conjectural
- Binary Exponentiation
- Fermat's little
- Extended Euclidian
  - linear Diophantine equation
- Modular Multiplicative Inverse
- Euler Totient function
- Pegionhole

Pegionhole Principle

4 →



→ Atleast 1 container with 2 Pegion



Questions

12000 ⇒ 3

Trailing zeroes

n → n! Trailing zeroes!

3 → 3! ⇒ 6 → 0

5! = 120 → Trailing zeroes = 1

$0 \le n \le 10^4$

Brute Force → n! ** n!

n! = n * (n-1) * (n-2) * ... * 1

```
Vector<int> fact(n);
fact[0] = 1
fact[1] = 1;
for(i=2; i<=n; i++)
    fact[i] = (i * fact[i-1]) % mod
```

20! 30 → 40!

10 = 2 * 5

120 → 12 * 10

1260 → 12 * 10 * 10

12000 → 12 * 10 * 10 * 10

20! =

12000 → 12 * 10 * 10 * 10

20! =

30! =

5! = 5 * 4 * 3 * 2 * 1 → How many 10?

(2,5)

⇒ 5 * 2 * 2 * 3 * 2 * 1

= 10

Trailing zero = 1 ⇒ 10 * 2 * 2 * 2 * 1

1st step → No of 10

10 → 2 * 5

(2,5) Pairs = 10

No of (2,5) Pairs in n! ⇒ W/o Calculation n! Value

n! = n * n-1 * n-2 ---- 1

6! = 6 * 5 * 4 * 3 * 2 * 1 → (2,5)

3 * 2

2 * 2

Trailing zeroes = 1

No of 2 > No of 5

n!

No of (2,5) Pairs

= No of 5 available!

n! ⇒

50! = 1 * -- 5 * --- 10 * -- 15 * --- 20 * -- (25) * -- 30 ---

35 --- 40 --- 45 --- 50 = 5 * 2 * [

7[

⊥

5 * 5 * 3

9 source of 5

10 + 1 = 11 source of 5

$5 \times 5 \times 3$

Multiples of 5 → 1, 5 ... (10) } (12)

" " 25 → 1 (2)

" " 125 → 1 (0)

125! = 1 - - - - 100 - - - 125

25 → 125

$5 \times 5 \times 5$

$$n! \rightarrow \left\lfloor \frac{n}{5} \right\rfloor + \left\lfloor \frac{n}{5^2} \right\rfloor + \left\lfloor \frac{n}{5^3} \right\rfloor + \left\lfloor \frac{n}{5^4} \right\rfloor + \ldots$$

$\left\lfloor \frac{n}{5^n} \right\rfloor \neq 0$

$\frac{20}{2} = 10 \quad \frac{20}{4} = 5 \quad \frac{20}{8} = 2$

```
For ( i=5; (n/i) >= 1 ; i*=5 )
{   count += n/i;
}
```

$i = 5^2$

$\frac{n}{5} + \frac{n}{5^2} + \frac{n}{5^3}$

⇓

```
i=5
while ( (n/i) >= 1 )
{ ans += (n/i);
  i*=5;
}
```

$\left(\frac{50}{125} \right) = 0$

n=5

1, 2, 3, 4, 5 → 2, 3, 4, 1, 5

3, 4, 2, 1, 5

Prime Nos → (Prime No)! * (Non Prime no)!

Non Prime

→ 2, 3, 5

PnC

1, 2, 5, 4, 3

2 3 5

Total Prime No =

Non Prime = n - total Prime

1, 2, 3

3 2

2	3	1
3	2	1
1	2	3

→ orders? 3! = 6

B G G B G

4 5

3	1	4	2
2	1		3
1	3		2

5

2!

3!

a b ⇒ a*b

Total ways / orders

⇒ 2! * 3!

1, 2, 3, 4

B G G B

1	3	4	2
1	4	3	2
2	3	4	1
2	4	3	1

2! * 2!

orders ⇒ 4

G!

1 → 5

2*5 = 10

3*5

→ G!

B! * G!

→ n → Prime Nos

Seive nloglogn

→ n → Prime Nos | Seive | Ringtogel

{2, 4, 3, 7, 10, 6}

10 → 2*5

10080 → {$2^5 * 3^2 * 5 * 7$} unique Prime factor (4)

2 → 2*1
4 → 2*2
3 → 3*1
7 → 7*1
10 → 2*5
6 → 2*3

2 * 2 * 2 * 3 * 7 * 2 * 5 * 2 * 3

No of unique Prime factors

ma → 2 → 5
3 → 2
7 → 1
5 → 1

⇒ 4

Map → unique Elements

ma.size

Prime factorization = ?

Prime Nos

seive

np, dp

n log log(n)

```
vector<int> Prime = seive();
int ans = 0;
for (auto i : Prime)
{
    for (auto j : nums)
```

1-n

1-n

log n

log n

find ans

2, 4, 3, 7, 10, 6

for (auto j : nums)
{ if (j % i == 0)
{ ans++; break;
}
}

nlogn + nloglogn

2, 4, 3, 7, 10, 6

{2, 3, 5, 7}

ans = 4

4, 7, 8, 15, 3, 5

4, 7, 8 | 15, 3, 5

28*8   15*15

coprime

gcd(Part1, Part2) = 1

1

4 7 8 | 15

4*7*8

2*2
7*1
2*2*2

2 | 7   3 | 5

15 -> 5*3
2 -> 2
5 -> 5

=) 2*2*2*2*2*7 * 5*3

3*5   GCD=1

(intersection) => 1

gcd = 1

1

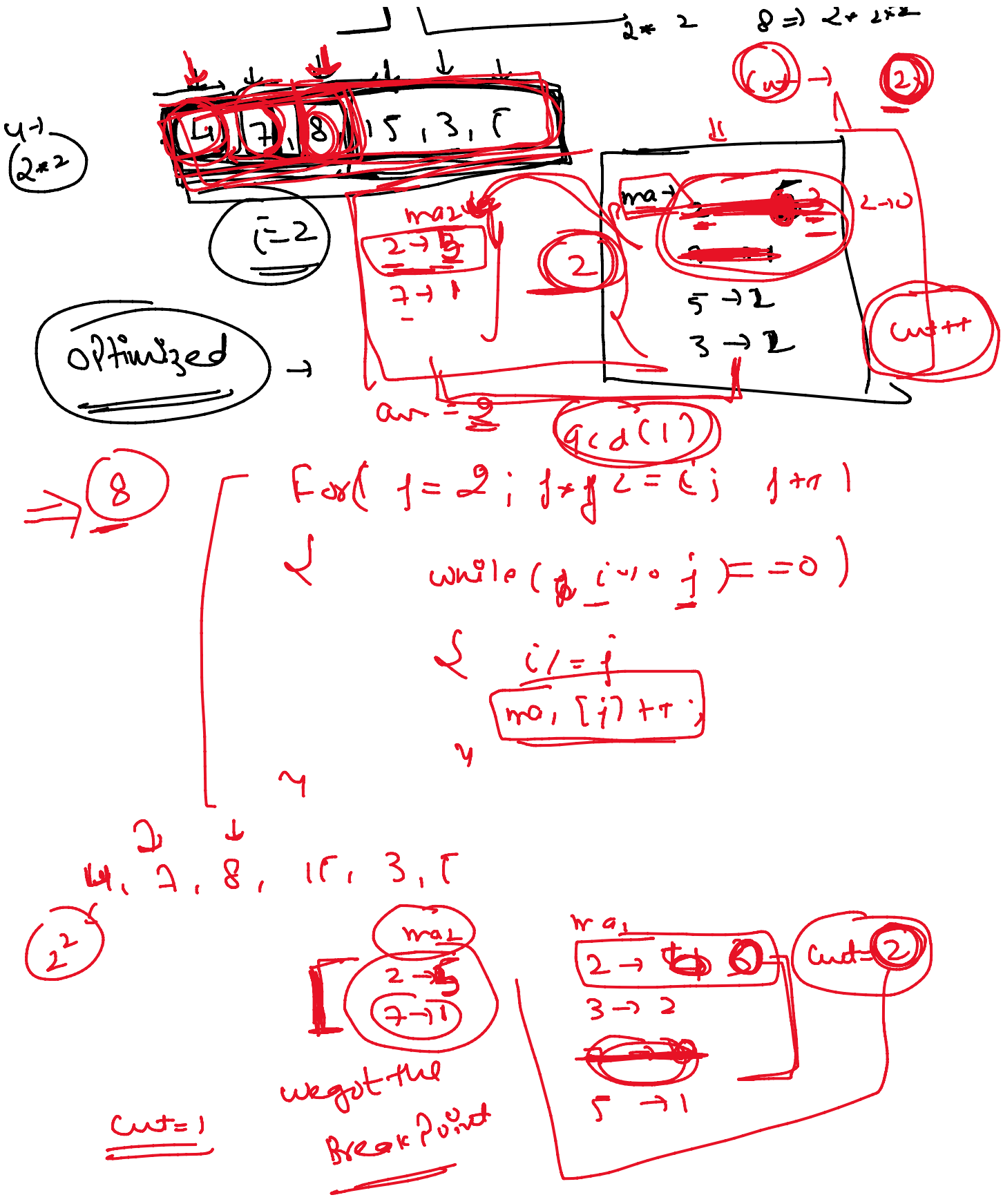
=) 2*2*2*2*2*7
5*3*3*5

GCD=1   GCD=15

Brute Force

=) i=0

2*2   8 =) 2*2*2


8 ⇒ 2 * 2 * 2
2 * 2
4 7 8 15 3 5
i = 2
ma2
2 → 5
7 → 1
ma1
2 → 0
3 → 2
5 → 1
cnt = 2
cnt++
ans = 2
gcd(1)
optimized
8
For(i = 2 ; i * i <= n ; i++)
while(n % i == 0)
n = n / i
ma1[i]++
4 7 8 15 3 5
2²
ma2
2 → 5
7 → 1
we got the Break Point
cnt = 1
ma1
2 → 0
3 → 2
5 → 1
cnt = 2