

* Binary search using Recursion

$s - e \rightarrow$ element findout ?
 $(0 - n - 1) \quad (u)$

$arr = [1, 2, 3, 4, 5, 6, 7]$
 $u = 2$

Problem Statement \Rightarrow

1) Find index of u in the whole array



2) Subproblem ?
 $i - j$

$0, n-1$

use Binary search \uparrow
 \Rightarrow Mid

$\left(\text{int left}, \text{int right} \right)$

$\left\{ \text{if } \underline{\underline{\text{left}}} <= \underline{\underline{\text{right}}} \right\}$

$\left\{ \text{int mid} = (\text{left} + \text{right}) / 2;$

$\text{if } (a[\text{mid}]) == u)$

$\left\{ \text{return mid;}$

γ

$\text{else if } (a[\text{mid}] < u)$

$\left\{ \text{int ans} = \underline{\underline{\text{binary search}}}$

$(\text{mid} + 1, \text{right});$

$\text{int s} = 0;$
 $\text{int e} = n - 1;$

$\text{while } (s <= e)$

$\left\{ \text{int mid} = (s + e) / 2;$

$\text{if } (a[\text{mid}] == u)$

$\left\{ \text{return mid;}$

$\text{else if } (a[\text{mid}] < u)$

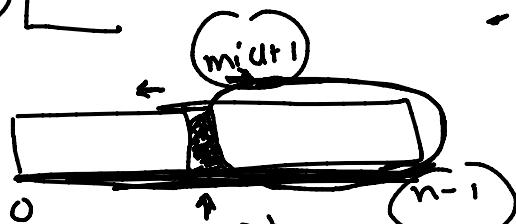
$\left\{ \text{mid}++;$

γ

else

$\left\{ \text{mid}--;$

γ



find u in range
 $(\text{mid} + 1) \rightarrow (n - 1)$

Recursion ($\text{int } n$)

$\left\{ \text{if some work you should do}$

$\left\{ \text{if some work from recursion}$

\leftarrow int ans = binarySearch(a, mid + 1, right);
 if (ans == -1)
 return ans;
 γ

If some work from recursion
 mid + right d.R

Recursion \rightarrow Redundant work

else if (a[mid] > u)

\leftarrow int ans = binarySearch(left, mid - 1);
 if (ans != -1)
 return ans;
 γ

return -1;

γ

l, r

int bs (vector<int> a, int l, int r, int u)

\leftarrow if (l > r)
 {
 return -1;
 }

// Base Case

\leftarrow else {
 int mid = (l + r) / 2;
 if (a[mid] == u)

0 + a = 1

\leftarrow if (a[mid] == u) x 8 == 7 x
 (red circle around this line)
 (red circle around this line)
 else if (a[mid] < u) x 11 x
 int ans = bs(a, mid + 1, right, u);

\leftarrow if (ans == -1)
 return ans;
 γ

\leftarrow if (ans == -1)
 return ans;

(-1)

\leftarrow else
 int ans = bs(a, l, mid - 1, u);
 if (ans == -1)

ans = 1

(mid = 3)

left

mid = 3

$\text{int ans} = \text{DSL}[1:n]$

If $\text{ans} != -1$ $\Rightarrow \text{ans} = 1$

Return ans; return +

return -1;

$(1, 2, 3, 4, 5, 6, 7)$
 $(0, n-1)$
 $\text{mid} = \frac{2+7}{2} = 4$

$\Rightarrow 2$
 $\text{bs}(a, 0, 5, 2)$

$\text{bs}(a, 0, 2, 2)$

$(1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$
 n

$\text{bs}(a, 0, 9, 7) \Rightarrow \underline{\alpha 11}$

$\text{mid} = 5$
 $\text{ans}, \text{mid} = 5 \Rightarrow 5$
 $T.C = O(\log n)$
 $\Rightarrow \text{No. of stacks} * T \text{ Time each}$

$S.C = O(\log_2 n)$

$\text{bs}(a, 5, 9, 7) \rightarrow h_{12}$

$\text{bs}(a, 5, 6, 7) \rightarrow h_{14}$

$\text{bs}(a, 6, 6, 7)$

$n \rightarrow h_{12} \rightarrow h_{14} \rightarrow \dots$

Print all Subsequences Using Recursion

Subsequence

$(1, 2, 3, 4)$

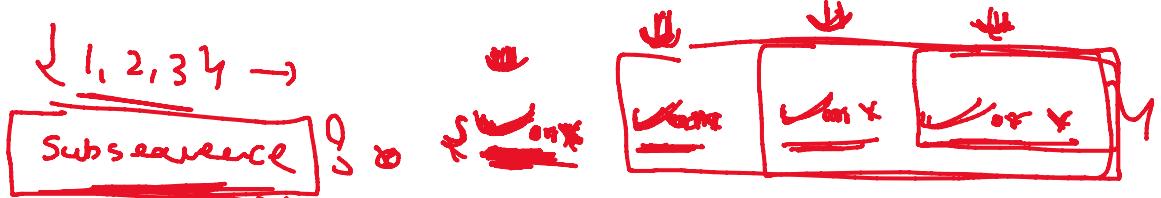
$(2, 4)$

Bit Masking

Steps

$\frac{n}{2^k} = 1 \Rightarrow 1 \leq \log_2 n$

$\downarrow 1, 2, 3, 4 \Rightarrow \{2, 4\} + \{ - \dots \leq 4$



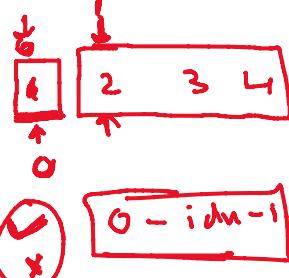
Main P.S. ? [Print all subsequence starting from index 0]

Subproblem ?
→ Generate all subsequence start from index 0
" " " "
" " " "
" " "

Void Subsequence (vector<int> a, int idu, vector<int> output)

```

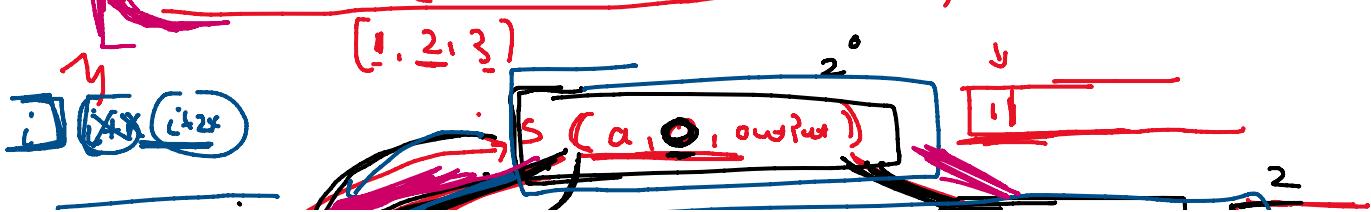
    int n = a.size();
    if (idu == n)
        // Base Case
        for (auto i : output)
            cout << i << " ";
        return;
    }
  
```

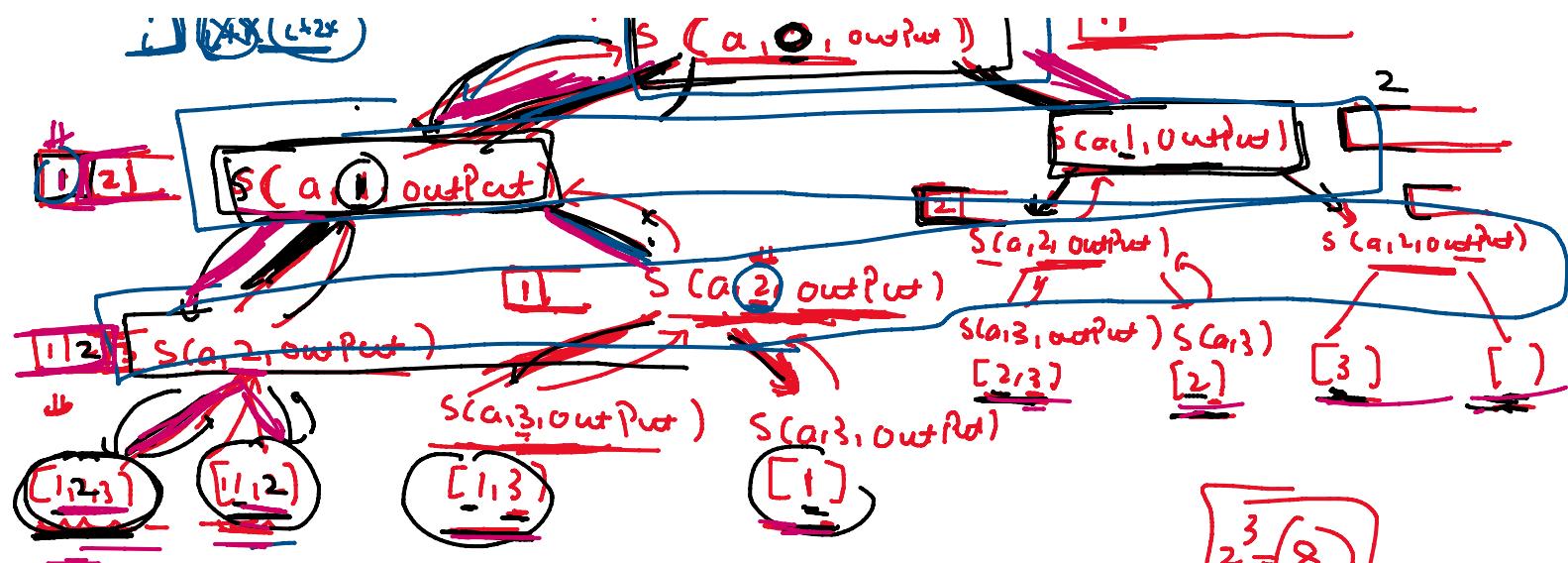


1/2 cases |

1/2 Case-1 include element idu
⇒ Output.push-back (a[idu]);
Subsequence (a, idu+1, output); (a, 2, output)

1/2 Case 2 → doesn't include element idu
⇒ Output.pop-back();
Subsequence (a, idu+1, output);





T.C =

$$2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^n$$

Total states

$$= 2^n + O(1) = O(2^n)$$

$$2^3 = 8$$