**ENGO-625 LAB 1 REPORT**

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| *University of Calgary* | *Date: 16/11/2021* |

*NOTE: For the X-axis representing GPS-Time in the graphs in this report, the value of the first epoch is ‘239460’ seconds, but the X-axis has been offset or deducted by ‘239460’ seconds and set to ‘0’ for better readability.*

**TASK 1:**

1. **For the first 300 epoch, plot the coordinates of the satellites in 3D.**  
     
   Chart, scatter chart

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Figure : Satellite Coordinates 3D plot for first 300 epochs

1. **Discuss: How are the satellites distributed. Is 300 epochs enough time to see the satellite paths? If not, plot some more (possibly all) epochs and discuss.**  
   Satellites are distributed evenly in the space No, 300 epochs is not enough time to observe satellite paths. Fig 2. shows the trajectory for all 3600 epochs.  
     
   Chart, line chart

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Figure : 3D Satellite trajectory for all epochs

1. **For the first 300 epochs, plot the pseudorange, Doppler, and L1 carrier phase observed for each of the satellites.**

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Figure : Doppler vs Time

Chart

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Figure : Pseudorange vs Time

Graphical user interface, table

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Figure : Carrier Phase vs Time

1. **Discuss: How much does the pseudorange vary from epoch to epoch. Does this match the change in the carrier phase? Does the carrier phase change match what you think it should based on the Doppler and the pseudorange?**

Change in Pseudorange and Carrier Phase is directly proportional to the Doppler Offset.

The Range Rate multiplied by delta Time will produce delta range, . The change in Pseudorange from epoch to epoch equals the delta range + Errors. It can also be written as . It is the same for the Carrier phase given cycle slip, or loss of lock does not occur. In that case, the delta phase value between the epoch at which the slip occurred and the next epoch can be any arbitrary value.   
  
For the first 300 epochs, according to my analysis, I haven’t found any considerable deviation of delta L1 phase with delta Pseudorange(not more than 2 meter or 10 cycles), that doesn’t mean loss of lock haven’t occurred. Further investigation will be required to check for cycle slips. In general, variation in Pseudorange is consistent with Carrier Phase L1. Whereas for Doppler, there are a couple of instances during the trial of 300 epochs where the values of doppler derived delta range varies considerably with Pseudorange and Carrier Phase, barring that doppler derived delta range matches with the delta Pseudorange and delta Phase under 2m envelope or threshold.

**TASK 2:** **Use parametric least-squares (PLSQ) and pseudorange measurements to estimate the 3D single point position and clock offset.**

* Position solution has been estimated using Least Squares (LS) and Weighted Least Squares Method (WLS).
* Sigma UERE( value is taken as 1 meter.
* Weight Matrix used in WLS is a diagonal matrix with each diagonal entry having value = , where is the Elevation angle of the Satellite corresponding to the entry.

**a), b), c) - Results and Graphs:**

*NOTE: Following are the legend description for the graphs –*

|  |  |  |  |
| --- | --- | --- | --- |
| **LS** : Least Squares | **WLS** : Weighted Least Squares | **+SD** : Positive Standard deviation value( | **-SD** : Negative Standard deviation value( |

Chart, line chart

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Figure : East Error (in m) for Least Squares

Chart, line chart

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Figure : North Error (in m) for Least Squares

Chart, line chart

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Figure : Up Error (in m) for Least Squares

Chart, scatter chart

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Figure : Horizontal/2D Error (in m) for Least Squares

Chart, line chart

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Figure : 3D Error (in m) for Least Squares

Chart, scatter chart

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Figure : 2D error comparison b/w LS and WLS

Chart

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Figure : 3D RMSE comparison b/w LS and WLS

Table 1: RMSE(in m) for LS and WLS based SPP

|  |  |  |
| --- | --- | --- |
| RMSE(m)/Method | Least Squares | Weighted Least Squares |
| Easting | 4.3220 | 4.2613 |
| Northing | 2.7006 | 2.1999 |
| Up | 6.1129 | 5.6785 |
| 3d Error | 10.851 | 10.324 |
| 2d Error | 5.0964 | 4.7956 |
| Receiver Clock Offset \* Speed of Light | 7.3760 | 7.1653 |

Table 2: 95th percentile error(in m) for LS and WLS based SPP

|  |  |  |
| --- | --- | --- |
| 95th Percentile(m)/Method | Least Squares | Weighted Least Squares |
| Easting | 5.6044 | 5.0091 |
| Northing | 4.0758 | 2.6671 |
| Up | 9.5569 | 7.0008 |
| 3d Error | 15.048 | 12.012 |
| 2d Error | 6.8943 | 5.6239 |
| Receiver Clock Offset \* Speed of Light | 9.5515 | 8.2107 |

**d.) Discuss the differences between the true and estimated accuracy.**

The true accuracy/error is defined as the offset between estimated position solution and the true position solution. The following formula describes the math behind estimated accuracy –

A picture containing table

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Solving for Where H is the design matrix, we obtain matrix, whose first three diagonal elements represent the variance of ‘ (estimation error in ENU direction), whose root square value has been used to plot the estimated accuracy ‘envelope’. Note mean of ‘ is zero, . The value of is directly proportional to and the variance or envelope will contract or expand depending on the a-priori value set for .

Observing figures 6, 7, and 8 representing position errors in ENU directions, we can infer the difference between the true and estimated accuracy. All the figures mentioned above contain true accuracy/error represented by the ‘RED’ line/marker, the ‘BLUE’ and ‘GREEN’ lines represent estimated accuracy/error “envelope”. If a-priori is determined correctly, the estimated accuracy envelope will enfold the true accuracy. I have assumed the value of 1 meter for , which on observing Figures 6, 7 and 8 turns out to be under-determined.

**Results comparison of LS and WLS method**

* The accuracy performance WLS is superior to LS for the given data. Theoretically, WLS is a superior estimation method to LS.
* The 95th Percentile accuracy metric exhibits the efficiency and accuracy of WLS over LS more clearly than RMSE.
* Observing figure 11, we can infer that WLS method has fewer outliers compared to the LS method.

**TASK 3: Compute the HDOP and VDOP**

1. **Plot each DOP as a time series.**

Chart

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Figure : DOP vs Time

1. **Plot a time series of the number of satellites used in the solution.**

Chart, waterfall chart

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Figure : Satellite Count vs Time

1. **Compare the plots of 3a/3b with the accuracy plots from 2b/2c.**In fig 14., from epoch 2800 to 3600 seconds, the satellite count has been consistently around 10 and later drops down to 9 from 3500 seconds onwards. Simultaneously DOP also degrades and jumps to higher values; refer fig 13. The combined effect of lower satellite count and higher DOP can be seen in Figures 6,7,8, and 10 where position solution degrades from 2800 seconds onwards, especially from 3500 seconds corresponding with big DOP jump.
2. **Discuss the role of DOP in analyzing solutions.**

Referring from Task 2d answer, the DOP(Dilution of Precision) values relies upon diagonal entries of matrix, which in turn is directly proportional to the variance of errors in ENU direction.

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The value of PDOP, HDOP, and VDOP relates to error in 3D, 2D, or Horizontal and Vertical position solutions. As DOP relies on design matrix ‘H’ and the ‘H’ matrix depends on relative satellite-receiver geometry, DOP is entirely dependent upon relative geometry of any given satellite to a receiver. The more spread-out the satellites are w.r.t receiver, the lesser the DOP, the better the position solution and vice versa. The ideal DOP is which represents the highest achievable confidence level.

**TASK 4: Compute the residuals**

1. **Plot each satellite’s residuals as a time series.**

**Chart

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Figure : Satellite Residual vs Time

1. **Plot all the residuals as a function of satellite elevation angle.**

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Figure : Satellite Residual vs Elevation Angle

1. **Discuss the trends seen in plots 4a/4b and discuss what you see**.  
   Following is the equation to compute residual :  
     
   for a few satellites, the residual values are more significant than the other( PRN 18 and PRN 22 have larger residuals). This signifies that considerable errors are present in measurement values for those satellites.

Figure 16(4b) represents Satellite Residual over increasing Elevation angle. It can be inferred from the graphs that larger satellite residuals or equivalently larger measurement errors are present at lower elevation angles compared to higher angles. WLS or elevation angle mask ( satellite data for elevation angle lower than a threshold are rejected) can be used de-weigh or eliminate lower elevation satellite data.

**TASK 5: Repeat Tasks 1-4 using measurements differenced between the rover and base station.**

*NOTE: Same satellites are used during receiver-single-differencing mode as it was for single-point, therefore there will be no change regarding satellite data as well as design matrix used for DOP computation, and hence Task-1 and Task-3 answers will be the same for both modes.*

1. **Comment on the differences between the single-point and receiver-single-differencing mode**Following are the equations for Single Point and Receiver-Single-Differencing mode

* Single Point:
* Receiver-Single-Differencing:

|  |  |
| --- | --- |
| Single Point | Receiver-Single-Differencing |
| 1. Measurements from only Remote station are used | 1. Measurements of Base station are also incorporated along with Remote Station |
| 1. Errors are present | 1. Errors are removed due to differencing |
| 1. No prior info regarding any station coordinates | 1. Base Station coordinates are known |
| 1. Lower quality residuals | 1. More accurate residuals or misclosure vector |
| 1. Less accurate position solution | 1. More accurate position solution |

* **TASK 2:  
    
  Results and Graphs:**

*Following are the legend description for the graphs –*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **LS** : Least Squares | **WLS** : Weighted Least Squares | **+SD** : Positive Standard deviation value | **-SD** : Negative Standard deviation value | **BRSD:** Between Receiver Single Differenced |

A graph with red lines

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Figure : Receiver-Single-Differencing Mode East Error(in m)

**Chart, line chart

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Figure : Receiver-Single-Differencing Mode North Error(in m)

**Chart, line chart

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Figure : Receiver-Single-Differencing Mode Up Error(in m)

**Chart, scatter chart

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Figure : Receiver-Single-Differencing Mode 2D Error(in m)

Chart, line chart

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Figure : Receiver-Single-Differencing Mode 3D RMSE(in m)

**Chart, scatter chart

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Figure : Comparison b/w 2D error for Receiver-Single-Differencing Mode vs Single Point LS and WLS

Graphical user interface, chart

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Figure : Comparison b/w 3D RMSE for Receiver-Single-Differencing Mode vs Single Point LS and WLS

Table : RMSE(in m) for LS and WLS based SPP, and Between Single Receiver Differenced Mode

|  |  |  |  |
| --- | --- | --- | --- |
| RMSE(m)/Method | Single Point Least Squares | Single Point Weighted Least Squares | Between Single Receiver Differenced based Least Squares |
| Easting | 4.3220 | 4.2613 | 0.6043 |
| Northing | 2.7006 | 2.1999 | 0.5318 |
| Up | 6.1129 | 5.6785 | 0.9996 |
| 3d Error | 10.851 | 10.324 | 1.6565 |
| 2d Error | 5.0964 | 4.7956 | 0.8050 |
| Receiver Clock Offset \* Speed of Light | 7.3760 | 7.1653 | 1.0472 |

Table : 95th Percentile error (in m) for LS and WLS based SPP, and Between Single Receiver Differenced Mode

|  |  |  |  |
| --- | --- | --- | --- |
| 95th Percentile(m)/Method | Single Point Least Squares | Single Point Weighted Least Squares | Between Single Receiver Differenced based Least Squares |
| Easting | 5.6044 | 5.0091 | 1.2068 |
| Northing | 4.0758 | 2.6671 | 1.0790 |
| Up | 9.5569 | 7.0008 | 2.0513 |
| 3d Error | 15.048 | 12.012 | 3.0654 |
| 2d Error | 6.8943 | 5.6239 | 1.5329 |
| Receiver Clock Offset \* Speed of Light | 9.5515 | 8.2107 | 1.1748 |

**d.) Discuss the differences between the true and estimated accuracy.**Since design matrix ‘H’ is the same for Between Single Receiver Differenced Mode as it was for Single Point Mode, the definition of true and estimated accuracy also remains the same. But the assumed value of 1 meter for produces the estimated accuracy envelope, which enfolds the true accuracy for almost the whole dataset, refer to figure 14, 15, and 16. Therefore it can be inferred that the measurement errors in Between Single Receiver Differenced Mode are considerably less than Single Point Mode.

**Results comparison of LS and WLS method**

* The accuracy performance of Between Single Receiver Differenced Mode is superior to Single Point Mode.
* Measurement errors e.g. Atmospheric Errors(Ionospheric and Tropospheric), Satellite Ephemerides, Satellite Clock Offset etc. are removed due to differencing.
* A-priori value for = 1, applies more accurately to Between Single Receiver Differenced Mode than to Single Point Mode for the given dataset.
* **TASK 4: Compute the residuals**

1. **Plot each satellite’s residuals as a time series.**

Chart

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Figure : Satellite Residual vs Time

1. **Plot all the residuals as a function of satellite elevation angle.**

Chart

Description automatically generated

Figure : Satellite Residual vs Elevation Angle

1. **Discuss the trends seen in plots 4a/4b and discuss what you see**.  
   Following is the equation to compute residual :  
     
   The trend will be the same for Single Receiver Differenced mode as it is for Single point mode. The difference between the two modes lies in the residual magnitude and the formula used to compute the residual. Residual magnitude in Figure 24 and 25 is considerably less than residuals in figure 15 and 16, which signifies the misclosure vector is less erroneous and contain fewer outliers, and will result in a more accurate position solution.

**TASK 6:** **From the tasks above, draw an intuitive general conclusion on the quality of the position estimates obtained by a user from GPS based on the number of the satellites in view, their spatial distribution in the sky as well as the quality of the pseudorange.**

Position estimates are more accurate if satellite count is more, this trend is discussed in ‘Task 3c’ answer. Similarly, the relation between the spatial distribution of satellites in the sky and positional accuracy is again described in ‘Task 3c and 3d’ answers. It has been concluded in Task 3, that the more the satellites are spread out in the space relative to the receiver better are the position estimates and vice versa. This receiver-satellite relative geometry is quantified using DOP parameters. The quality of pseudoranges is quantified using and it observes the following relation -

A more detailed explanation is provided in the answer of ‘task 2d’  
  
***NOTE:*** *The source code for this Java based project can also be found on the github – (link:* [*naman4u13/ENGO625 (github.com)*](https://github.com/naman4u13/ENGO625)*). User only needs to modify the output file path, before compiling the code to output the solution. Graphs and Output file concerning estimation accuracy and errors can be found in ‘ENGO625 -> result’ folder in the home directory. Code is present inside ‘ENGO625 -> engo625\_lab’ folder. Python script used to plot satellite trajectory is present in the root folder with filename as SatPlot.py*