Analyzing S&P 500 Bear Markets

[1]:
```
import matplotlib.pyplot as plt import
numpy as np
import pandas as pd
import pandas_datareader as pdr import
seaborn as sns
```

[2]:
```
sns.set_theme()
```

1 Data

We calculate daily stock returns in project_2.ipynb and save them to sp500.csv. This file also includes the level of the S&P 500 index as column ^GSPC. To avoid imprecise correlation and Capital Asset Pricing Model (CAPM) beta estimates, we can remove months with fewer than 10 returns. We could also filter these months with few returns when we calculate correlations and CAPM betas.

[3]:
```
sp500 = (
    pd.read_csv(
        filepath_or_buffer='sp500.csv',
        parse_dates=['Date'], index_col=['Date']
    )
    .rename_axis(columns='Ticker')
)

gspc = sp500[['^GSPC']]
del sp500['^GSPC']

# keep only months with at least 10 returns for each stock
sp500 = (
    sp500
    .stack()
    .loc[
        (sp500.resample('M').transform('count') >= 10).stack()
    ]
    .unstack()
)
```

[4]:

```
ff = (
    pdr.DataReader(
        name='F-F_Research_Data_Factors_daily',
        data_source='famafrench',
        start='1900'
    ) [0]
    .div(100)
    .rename_axis(columns='Factor')
)
```

2: FutureWarning: The argument 'date_parser' is deprecated and will be removed in a future version.

<ipython-input-6-f6fa8e73ace0>: Please use 'date_format' instead, or read your data in as 'object' dtype and then call 'to_datetime'.

pdr.DataReader(

We calculate excess returns here to simplify our CAPM beta calculations later.

```
sp500_rf = ( sp500
    .sub(ff['RF'], axis=0)
    .join(ff[['Mkt-RF']])
    .loc['1980':'2023']
)
```

2 Functions

The following function calculates bear markets based on declines and recoveries given with the down and uparguments.

[6]:

```
def calc_bear(df, col, down=0.2, up=0.2): df_copy =
    df.copy()
    down = 1 - down up =
    1 + up

    bear = df_copy['Bear'] = False
    low = high = df_copy[col].iloc[0]

    for i, x in df_copy[col].items():
        if not bear and (x < down*high):
            bear = df_copy.at[i, 'Bear'] = True
            low = high = x
        elif bear and (x > up*low):
            bear = df_copy.at[i, 'Bear'] = False
            low = high = x
        else:
            df_copy.at[i, 'Bear'] = bear
```

```
            low = min(x, low) high =
            max(x, high)

    return df_copy
```

We resample the bear market data to combine it with the monthly and quarterly correlations. We consider

[7]:

```
def resample_bear(df, freq, n):
    return (
        df
        .resample(freq)
        ['Bear']
        .sum()
        .gt(n - 1)
        .astype(bool)
        .to_frame('Bear')
    )
```

The following function calculates the unique pairwise correlations each month or quarter.

[8]:

```
def calc_corr(df, freq):
    return (
        df
        .groupby(pd.Grouper(freq=freq))
        .corr()
        .stack()
        .rename_axis(index=['Date', 'T1', 'T2'])
        .to_frame('corr')
        .reset_index()
        .query('T1 < T2')
        .set_index('Date')
    )
```

The following function calculates the CAPM beta for each stock each month or quarter.

[9]:

```
def calc_beta(df, freq):
    grouped = df.groupby(pd.Grouper(freq=freq)) top =
    grouped.cov()['Mkt-RF']
    bot = grouped['Mkt-RF'].var()
    return (
        top
        .div(bot)
        .rename_axis(index=['Date',  'T1'])
        .to_frame('beta')
        .reset_index()
```

[1]After we discussed this solution in class, I tweaked calc_beta()so it drops the “Mkt-RF” beta inside the function instead of outside the function to match the style of calc_corr().

```
            .query('T1 != "Mkt-RF"')
      )
```

3 Calculations

First, we calculate all the bear markets and resample them to align with the monthly and quarterly data.

[10]:
```
bear_d = gspc.pipe(calc_bear, '^GSPC')
bear_m = bear_d.pipe(resample_bear, freq='M', n=10) bear_q =
bear_d.pipe(resample_bear, freq='Q', n=30)
```

Here are the bear markets.

[11]:
```
bear_d_p1 = bear_d['Bear'].astype(int).diff().loc['1980':'2023'].pipe(lambda x:␣
  ↪x[x == +1])
bear_d_m1 = bear_d['Bear'].astype(int).diff().loc['1980':'2023'].pipe(lambda x:␣
  ↪x[x == -1])
n = 47
print('=' * n)
print('List of Declines and Recoveries (Daily)') print('-' * n)
for p1, m1 in zip(bear_d_p1.index, bear_d_m1.index): print(f'Start on {p1:%b %d,
      %Y}, and Stop on {m1:%b %d, %Y}')
print('=' * n)
```

```
===============================================
List of Declines and Recoveries (Daily)
-----------------------------------------------
Start on Feb 22, 1982, and Stop on Sep 14, 1982
Start on Oct 19, 1987, and Stop on Mar 08, 1988
Start on Mar 12, 2001, and Stop on Dec 05, 2001
Start on Jul 10, 2002, and Stop on Aug 22, 2002
Start on Jul 09, 2008, and Stop on Dec 08, 2008
Start on Feb 23, 2009, and Stop on Mar 23, 2009
Start on Mar 12, 2020, and Stop on Apr 08, 2020
Start on Jun 13, 2022, and Stop on Jun 08, 2023
===============================================
```

Another intrepration is to calculate crises using only end-of-month prices.

[12]:
```
bear_m_alt = gspc.resample('M').last().pipe(calc_bear, '^GSPC')

bear_m_alt_p1 = bear_m_alt['Bear'].astype(int).diff().loc['1980':'2023'].
  ↪pipe(lambda x: x[x == +1])
```

```
bear_m_alt_m1 = bear_m_alt['Bear'].astype(int).diff().loc['1980':'2023'].

    pipe(lambda x: x[x == -1])
n = 47
print('=' * n)
print('List of Declines and Recoveries (Monthly)') print('-' * n)
for p1, m1 in zip(bear_m_alt_p1.index, bear_m_alt_m1.index): print(f'Start on
        {p1:%b %d, %Y}, and Stop on {m1:%b %d, %Y}')
print('=' * n)
```

```
===============================================
List of Declines and Recoveries (Monthly)
-----------------------------------------------
Start on Mar 31, 1982, and Stop on Oct 31, 1982
Start on Oct 31, 1987, and Stop on Oct 31, 1988
Start on Mar 31, 2001, and Stop on Jul 31, 2003
Start on Sep 30, 2008, and Stop on May 31, 2009
Start on Mar 31, 2020, and Stop on Jul 31, 2020
Start on Jun 30, 2022, and Stop on Jun 30, 2023
===============================================
```

...or end-of-quarter prices

[13]:

```
bear_q_alt = gspc.resample('Q').last().pipe(calc_bear, '^GSPC')

bear_q_alt_p1 = bear_q_alt['Bear'].astype(int).diff().loc['1980':'2023'].

    pipe(lambda x: x[x == +1])
bear_q_alt_m1 = bear_q_alt['Bear'].astype(int).diff().loc['1980':'2023'].

    pipe(lambda x: x[x == -1])
n = 47
print('=' * n)
print('List of Declines and Recoveries (Quarterly)') print('-' * n)
for p1, m1 in zip(bear_q_alt_p1.index, bear_q_alt_m1.index): print(f'Start on
        {p1:%b %d, %Y}, and Stop on {m1:%b %d, %Y}')
print('=' * n)
```

```
-----------------------------------------------
Start on Dec 31, 1987, and Stop on Jun 30, 1989
Start on Mar 31, 2001, and Stop on Sep 30, 2003
Start on Sep 30, 2008, and Stop on Sep 30, 2009
Start on Mar 31, 2020, and Stop on Sep 30, 2020
Start on Jun 30, 2022, and Stop on Jun 30, 2023
===============================================
```

We can also calculate S&P 500 drawdowns to provide another definition of crises. Here, we use a three-year window to approximate our bear market definition, but the details will not affect our conclusions much.

[14]:
```
dd_d = gspc.div(gspc.rolling('1095d').max())['^GSPC'].rename('Drawdown') dd_m =
dd_d.resample('M').last().loc['1980':'2023']
```

Second, we calculate the means of all the unique pairwise correlations. Here, we only calculate the means for simplicity, but our results are similar if we calculate medians instead. We can also calculate minimums and maximums to help us troubleshoot.

[15]:
```
corr_m = sp500.pipe(calc_corr, freq='M').groupby('Date')['corr'].mean().
    rename('Monthly')
corr_q = sp500.pipe(calc_corr, freq='Q').groupby('Date')['corr'].mean().
    rename('Quarterly')
```

Finally, we calculate the means of all the CAPM betas.

[16]:
```
sp500_rf.loc['1980-01'].pipe(calc_beta, freq='M')
```

[16]:

	Date	T1	beta
0	1980-01-31	A	NaN
1	1980-01-31	AAL	NaN
2	1980-01-31	AAPL	NaN
3	1980-01-31	ABBV	NaN
4	1980-01-31	ABNB	NaN
..	...	...	...
498	1980-01-31	YUM	NaN
499	1980-01-31	ZBH	NaN
500	1980-01-31	ZBRA	NaN
501	1980-01-31	ZION	NaN
502	1980-01-31	ZTS	NaN

[503 rows x 3 columns]

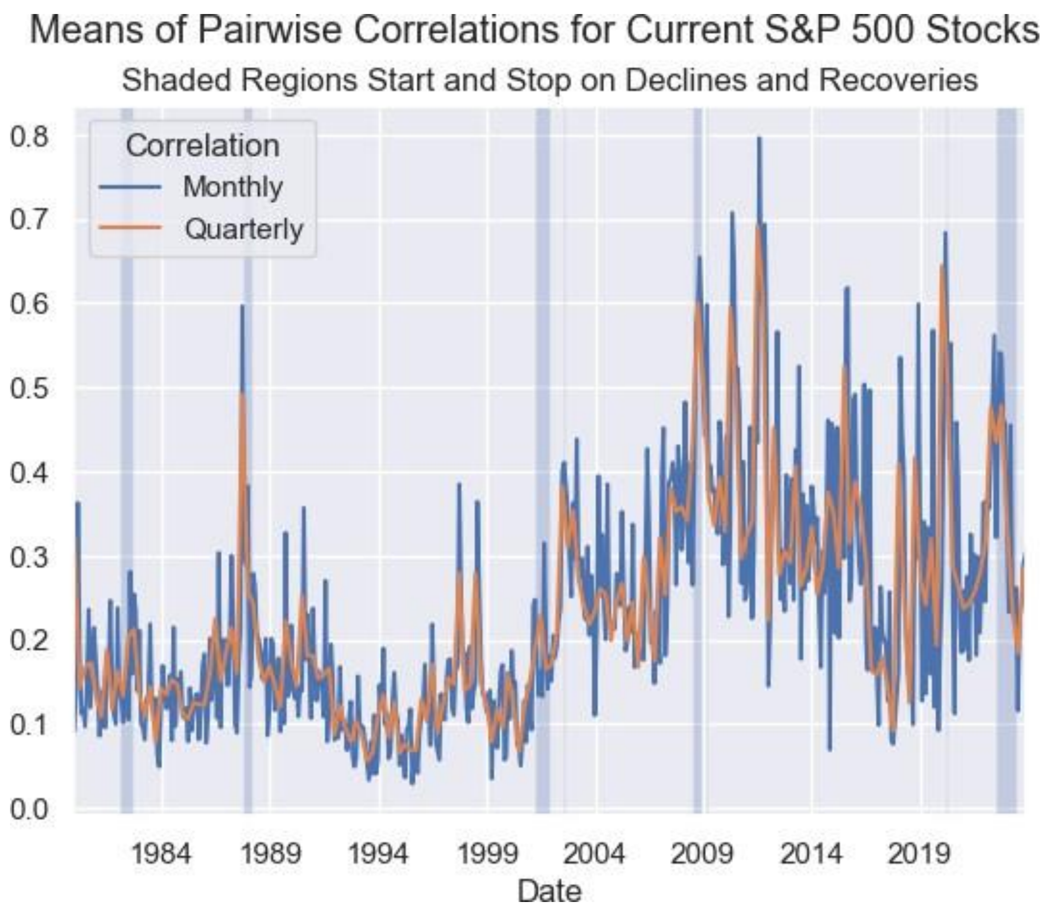
[17]:
```
beta_m = sp500_rf.pipe(calc_beta, freq='M').groupby('Date')['beta'].mean().
    rename('Monthly')
beta_q = sp500_rf.pipe(calc_beta, freq='Q').groupby('Date')['beta'].mean().
    rename('Quarterly')
```

4 Analysis

We see that correlations spike during bear markets for monthly and quarterly correlations. The spikes are not to one, so the cliche that "correlations go to one" during crises is dramatic but is directionally true.

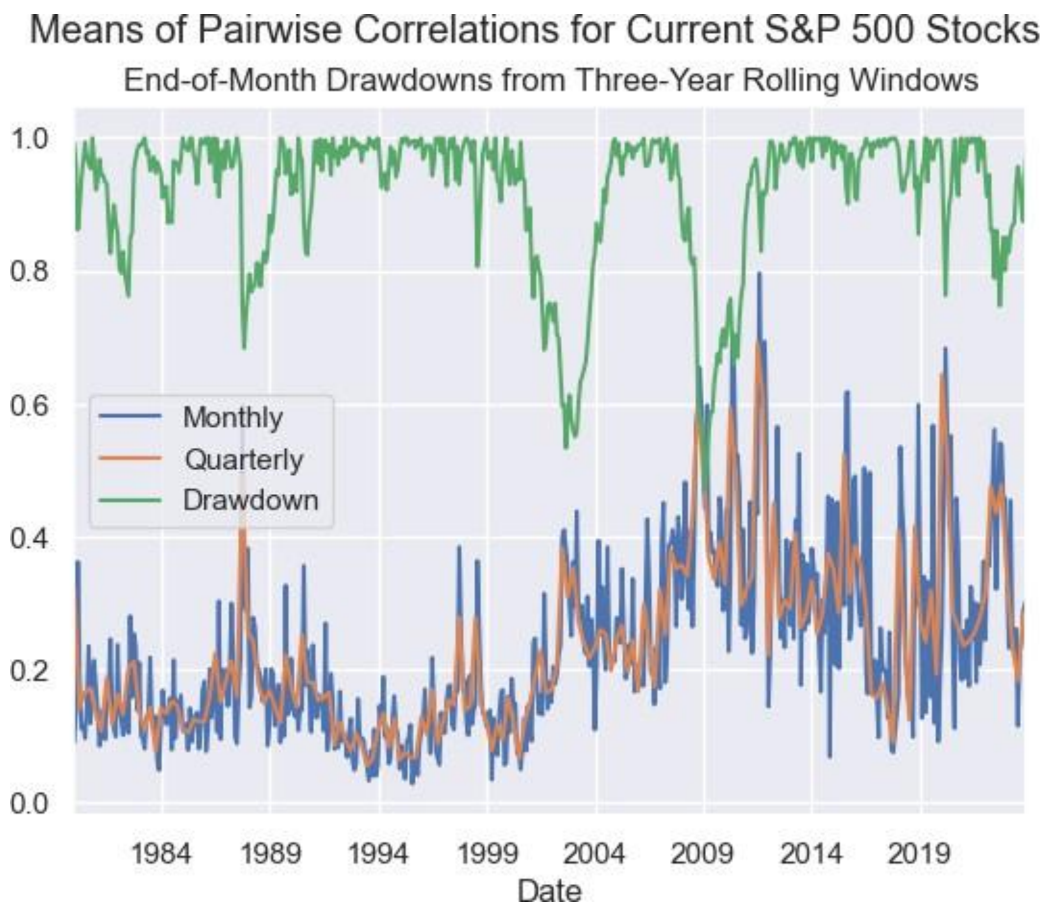
[18]:

```
fig, ax = plt.subplots() corr_m.plot(ax=ax)
corr_q.plot(ax=ax)
plt.legend(title='Correlation')
for p1, m1 in zip(bear_d_p1.index, bear_d_m1.index):
    ax.axvspan(xmin=p1, xmax=m1, alpha=0.25)
plt.suptitle('Means of Pairwise Correlations for Current S&P 500 Stocks') plt.title('Shaded
Regions Start and Stop on Declines and Recoveries') plt.show()
```



[19]:

```
fig, ax = plt.subplots()
corr_m.plot(ax=ax)
corr_q.plot(ax=ax)
dd_m.plot(ax=ax) plt.legend()
plt.suptitle('Means of Pairwise Correlations for Current S&P 500 Stocks') plt.title('End-of-
Month Drawdowns from Three-Year Rolling Windows') plt.show()
```



[20]:

```
pd.concat([corr_m, corr_q, dd_m], axis=1).corr()
```

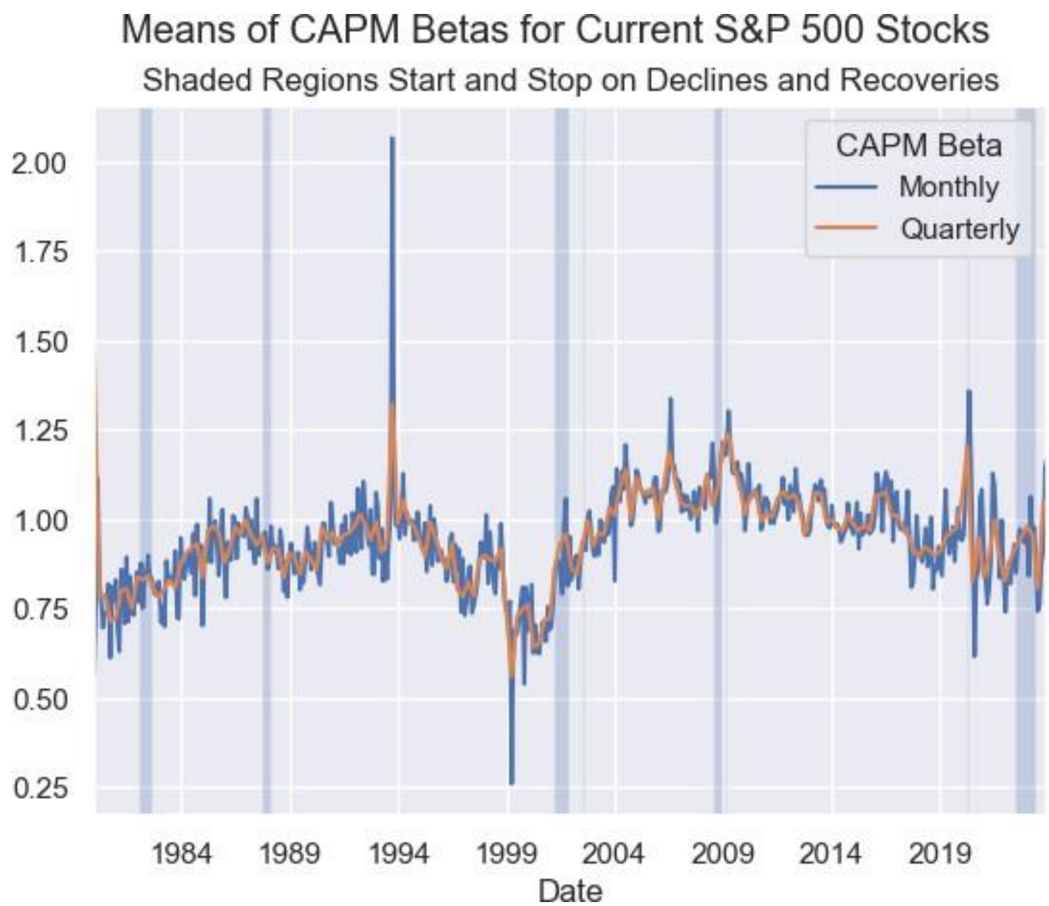
[20] :

	Monthly	Quarterly	Drawdown
Monthly	1.000000	0.900692	-0.422175
Quarterly	0.900692	1.000000	-0.468241
Drawdown	-0.422175	-0.468241	1.000000

We *do not* see this pattern for CAPM betas!

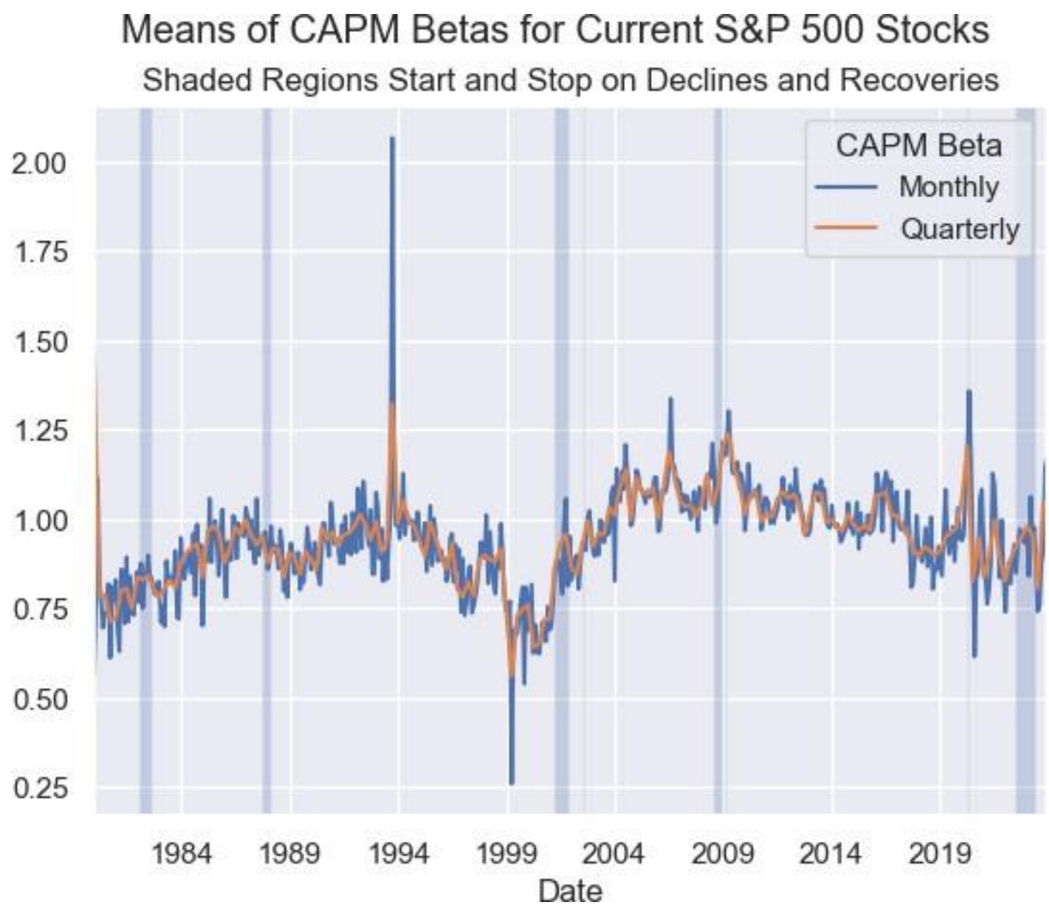
[21] :

```
fig, ax = plt.subplots() beta_m.plot(ax=ax)
beta_q.plot(ax=ax) plt.legend(title='CAPM
Beta')
for p1, m1 in zip(bear_d_p1.index, bear_d_m1.index):
    ax.axvspan(xmin=p1, xmax=m1, alpha=0.25)
plt.suptitle('Means of CAPM Betas for Current S&P 500 Stocks') plt.title('Shaded Regions Start
and Stop on Declines and Recoveries') plt.show()
```



[22] :

```
fig, ax = plt.subplots() beta_m.plot(ax=ax)
beta_q.plot(ax=ax) plt.legend(title='CAPM
Beta')
for p1, m1 in zip(bear_d_p1.index, bear_d_m1.index):
    ax.axvspan(xmin=p1, xmax=m1, alpha=0.25)
plt.suptitle('Means of CAPM Betas for Current S&P 500 Stocks') plt.title('Shaded Regions Start
and Stop on Declines and Recoveries') plt.show()
```



Finally, bar plots provide another perspective on these relations. The vertical black bars indicate 95% confidence intervals.

[23] :
```
df_m = bear_m.copy() df_m['Correlation']
=corr_m df_m['CAPM Beta'] = beta_m
```

[24] :
```
df_q = bear_q.copy() df_q['Correlation'] =
corr_q df_q['CAPM Beta'] = beta_q
```

[25] :
```
df = (
    pd.concat(
        objs=[df_m.dropna(), df_q.dropna()],
        keys=['Monthly', 'Quarterly'],
        names=['Frequency']
    )
    .reset_index()
```

```
    .melt(
        id_vars=['Frequency','Date','Bear'],
        var_name='Statistic', value_name='Value'
    )
)
```

[26] :

```
sns.catplot(
    data=df,
    x='Frequency',
    y='Value',
    col='Statistic',
    hue='Bear',
    kind='bar',
    sharey=False
)
plt.suptitle(
    t='Means of Correlations and CAPM Betas for Current S&P 500 Stocks' + '\n' +
    'Bear Markets Start and Stop on Declines and Recoveries', y=1.05
)
plt.show()
```