ENPM673: Perception for Autonomous Robots

Homework 1

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1. Question 1:

1.1 To Determine FOV

Given that the camera has a resolution of 5MP with a square shaped sensor of side 14mm. In addition, the focal length is given to be $f = 15mm$
We can determine the Field of View (FOV) using the following equation.

$$FOV = 2tan^{-1}(\frac{d}{2f}) \tag{1}$$

Here, d is the dimension of the sensor and f is the focal length.In this case, the horizontal and the vertical FOV will be equal, since the camera sensor is square shaped. Therefore, FOV can be obtained by:

$$\begin{aligned} FOV &= 2tan^{-1}(\frac{14}{2(15)}) \\ &\implies 2tan^{-1}(\frac{7}{(15)}) \\ &\implies 50.033 \text{ degrees} \end{aligned} \tag{2}$$

The Angular field of view in both the horizontal and vertical direction is 50.033 degrees.

1.2 To determine minimum number of pixels for the given condition

Given that a square shaped object of width 5cm is placed at a distance of 20m away from the camera.

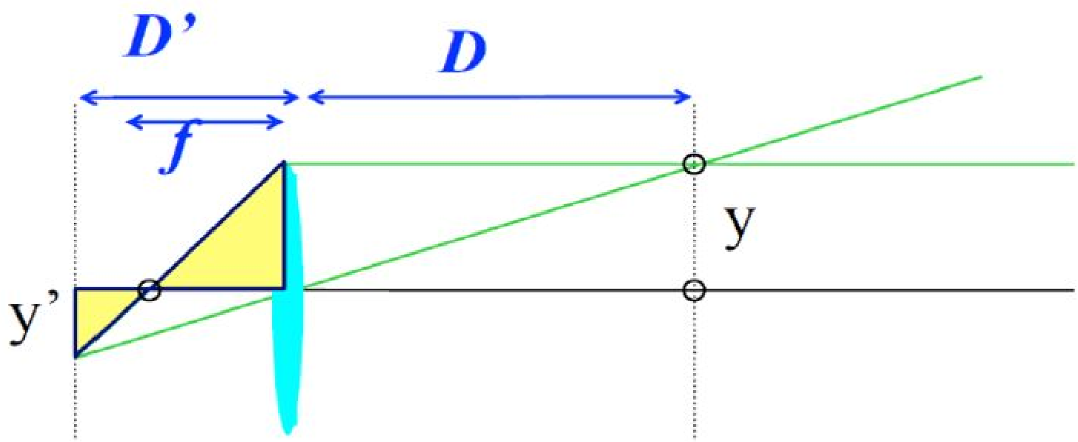

Figure 1: Thin lens formula

From the above figure we can write:

$$\frac{\tilde{y}}{y} = \frac{\tilde{D}}{D} \tag{3}$$

Here, $\tilde{y}$ and y are the heights of the image and the object respectively. $\tilde{D}$ and D are the distances from the lens of the image and the object respectively.

From equation (3): y=50mm and D=20000

$$
\begin{aligned}
\frac{\tilde{y}}{y} &= \frac{15}{20000} \\
\implies \frac{\tilde{y}}{50} &= \frac{15}{20000} \\
\implies \tilde{y} &= \frac{15}{400}
\end{aligned}
\tag{4}
$$

The area of the image can be given as:

$$\text{area of image} = (\frac{15}{400})^2 \tag{5}$$

Now, the 1 sq. unit of pixel can be determined:

$$\text{1 sq. unit pixel area} = \frac{5 \times 10^6}{14 \times 14} \tag{6}$$

From equations (5) and (6), the minimum number of pixels for this image can be determined:

$$
\begin{aligned}
\text{min. pixels} &= (\frac{15}{400})^2 \times \frac{5 \times 10^6}{(14)^2} \\
\implies \frac{1125 \times 10^6}{3136 \times 10^4} &= 0.3587 \times 10^2 \\
&\implies \mathbf{35.87} \text{ pixels}
\end{aligned}
\tag{7}
$$

Therefore the FOV is **50.033 degrees** and the minimum number of pixels required for image under given conditions is **35.87 pixels**

2. Question 2:

2.1 Part 1: Dataset 1

Below shown in figure 1 is the Least Square method to fit a curve for the given set of data from Dataset 1.csv. It is a type of regression analysis method providing a relationship between a known independent variable and an unknown dependent variable.

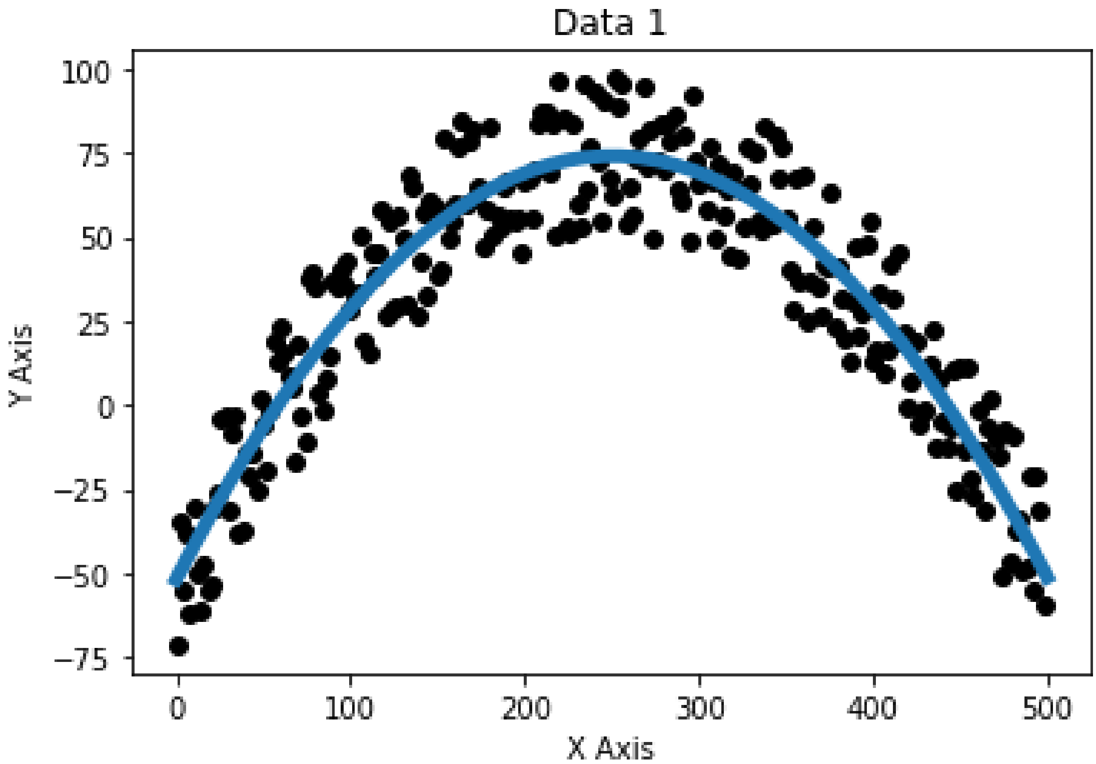

Figure 2: Curve Fitting using Least Square

2.2 Part 1: Dataset 2

For the dataset-2, we used RANSAC method to fit the curve because of the outliers. Below is the plot of the curve fitting using RANSAC method. Please refer to the code for better understanding.

2.3 Part 2

1. For dataset 1, we have applied Least Square method as there are no visible outliers present in data 1. Also, Least Squares is used because it produces smallest sum of squares of errors. This is the difference between the observed value and the anticipated value of the squared residuals. Since, our x values are in sequence without any noise while y values has noise in it, therefore, we used Least Square Method.

2. For dataset 2, we have outliers. An outlier is data that does not belong to the line. The reason we did not use Least Square method is because when calculating an error from an outlier, the squared error heavily penalizes the outliers. The least square method is not robust to the noise, the least square method with regularization amplifies the noise but RANSAC is robust to outliers. It removes the outliers from the math,

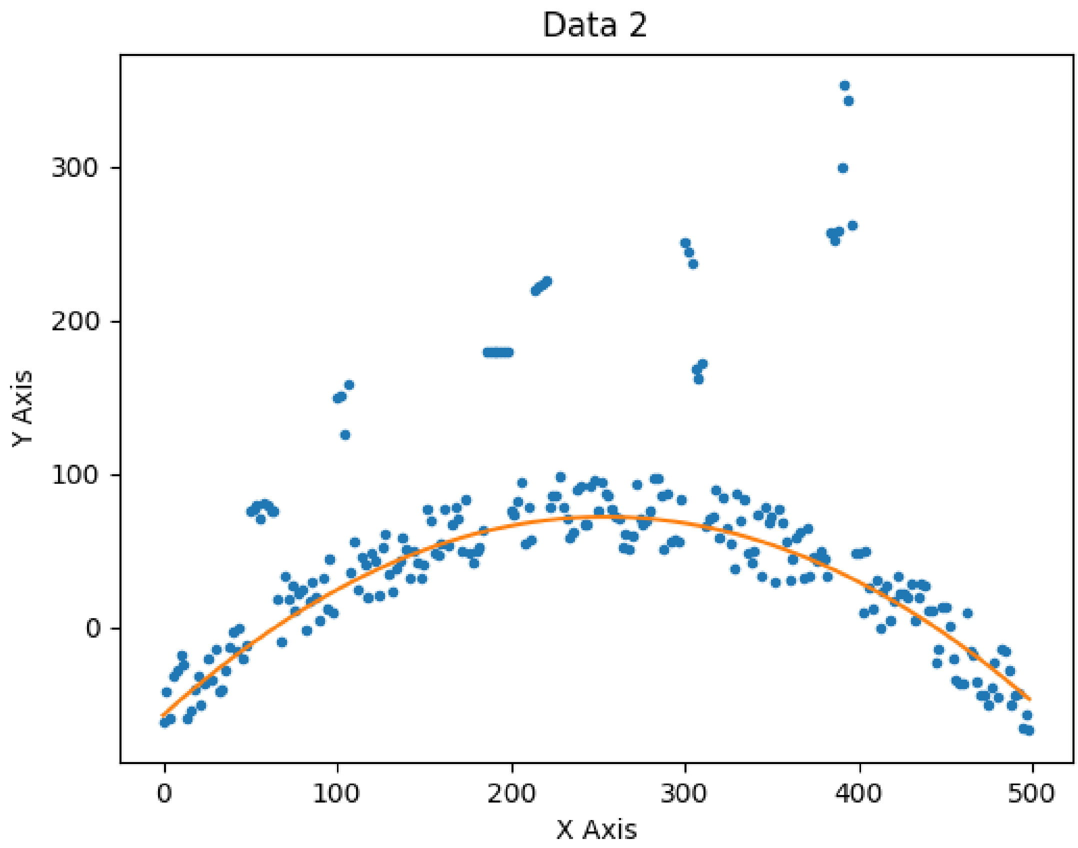

Figure 3: Curve Fitting using RANSAC

it takes 3 random points (for parabola), fits a model, find all the close points and rejects the rest as the outliers, and finds the best model. In this method, we faced problems for plotting the graph for a certain model that has maximum inliers. After few iterations, we decided to use probability technique, which gave us the probability of roughly around 0.86 for the threshold value of 30 taking 217 inliers in it.

3. Question 3:

3.1 Single Value Decomposition

$$A = \begin{bmatrix} -x1 & -y1 & -1 & 0 & 0 & 0 & x1*xp1 & y1*xp1 & xp1 \\ 0 & 0 & 0 & -x1 & -y1 & -1 & x1*yp1 & y1*yp1 & yp1 \\ -x2 & -y2 & -1 & 0 & 0 & 0 & x2*xp2 & y2*xp2 & xp2 \\ 0 & 0 & 0-x2 & & -y2 & -1 & x2*yp2 & y2*yp2 & yp2 \\ -x3 & -y3 & -1 & 0 & 0 & 0 & x3*xp3 & y3*xp3 & xp3 \\ 0 & 0 & 0 & -x3 & -y3 & -1 & x3*yp3 & y3*yp3 & yp3 \\ -x4 & -y4 & -1 & 0 & 0 & 0 & x4*xp4 & y4*xp4 & xp4 \\ 0 & 0 & 0 & -x4 & -y4 & -1 & x4*yp4 & y4*yp4 & yp4 \end{bmatrix}, x = \begin{bmatrix} H_{11} \\ H_{12} \\ H_{13} \\ H_{21} \\ H_{22} \\ H_{23} \\ H_{31} \\ H_{32} \\ H_{33} \end{bmatrix}$$

	x	y	xp	yp
1	5	5	100	100
2	150	5	200	80
3	150	150	220	80
4	5	150	100	200

Figure 4: Given Data for matrix A

The given system of equations is of the form $Ax = 0$.This cannot be solved using least squares method,because they form homogeneous linear equations. Therefore, the method of Single Value Decomposition(SVD) is used.

From the data given in figure (4), we can obtain the A matrix:

$$\begin{bmatrix} -5 & -5 & -1 & 0 & 0 & 0 & 500 & 500 & 100 \\ 0 & 0 & 0 & -5 & -5 & -1 & 500 & 500 & 100 \\ -150 & -5 & -1 & 0 & 0 & 0 & 30000 & 1000 & 200 \\ 0 & 0 & 0 & -150 & -5 & -1 & 12000 & 400 & 80 \\ -150 & -150 & -1 & 0 & 0 & 0 & 33000 & 33000 & 220 \\ 0 & 0 & 0 & -150 & -150 & -1 & 12000 & 12000 & 80 \\ -5 & -150 & -1 & 0 & 0 & 0 & 500 & 15000 & 100 \\ 0 & 0 & 0 & -5 & -150 & -1 & 1000 & 30000 & 200 \end{bmatrix} \tag{8}$$

SVD is expressed as:

$$A = U\Sigma V^T \tag{9}$$

U is $m \times m$ orthogonal matrix, where each column forms the Eigen Vectors of the matrix AA^T
V^T is a $n \times n$ orthogonal matrix, where each column forms the transpose of Eigen vectors of the matrix A^TA.
And, Σ is a $m \times n$ diagonal matrix with non-negative entries of the eigen values of A^TA. In addition, in this particular case, the absolute values of the Eigen values are considered.

The Eigen values which are of the form 1×9 are mapped to form a diagonal matrix of the order $m \times n$. The matrices U, Σ, V^T are multiplied to obtain the SVD value of matrix A which will of the order $m \times n$

Homography Matrix: In this case,Homography matrix corresponds to the last column of the V matrix in the SVD, output has been reshaped to form a 3×3 symmetric matrix.

Please refer the code for the implementation.

```
The Homography matrix H is:
[[ 3.50321685e-03  3.37762918e-03 -8.13642090e-01]
 [-2.55301166e-03 -2.92733046e-03  5.81332763e-01]
 [-3.66202493e-07 -2.00680613e-06  9.16829699e-06]]
```

Figure 5: Homography matrix: H