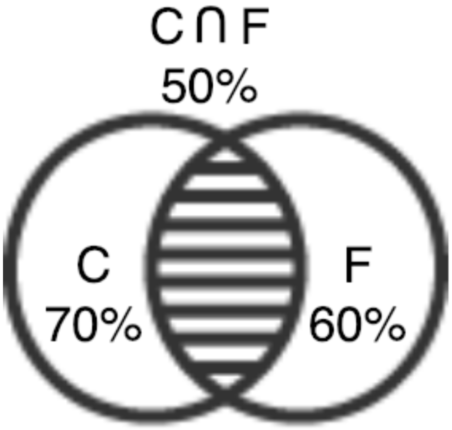
Question 2.4

Among employees of a certain firm, 70% know C/C++, 60% know Fortran, and 50% know both languages. What portion of programmers
(a) does not know Fortran?
(b) does not know Fortran and does not know C/C++?
(c) knows C/C++ but not Fortran?
(d) knows Fortran but not C/C++?
(e) If someone knows Fortran, what is the probability that he/she knows C/C++ too?
(f) If someone knows C/C++, what is the probability that he/she knows Fortran too?

Solution

Let 'C' be people knowing C/C++
and 'F' be people knowing Fortran,
With 'Ω' as the sample space.



Given:

$p(C) = 0.7$
$p(F) = 0.6$
$p(C, F) = 0.5$

a. $p(F)^1 = 1 - p(F)$
$= 1 - 0.6$
$= 0.4$

b. $p(F \cup C)^1 = 1 - p(F \cup C)$
$= 1 - (p(F) + p(C) - P(F \cap C))$
$= 1 - (0.6 + 0.7 - 0.5)$
$= 0.2$

c. $p(C - F) = p(C) - p(C \cap F)$
$= 0.7 - 0.5$
$= 0.2$

d. $p(F - C) = p(F) - p(C \cap F)$
$= 0.6 - 0.5$
$= 0.1$

e. p(C / F) = p(C ∩ F) / p(F)
= 0.5/0.6
= 0.8333

f. p(F / C) = p(C ∩ F) / p(C)
= 0.5/0.7
= 0.7143

Question 2.9

Successful implementation of a new system is based on three independent modules. Module 1 works properly with probability 0.96. For modules 2 and 3, these probabilities equal 0.95 and 0.90. Compute the probability that at least one of these three modules fails to work properly.

Solution

p(Failure) = 1 – p(Success)

We know that, p(Success) is the product of reliabilities of each independent module.
Therefore,

P(Failure) = 1 – p(Success)
= 1 – (p(A)*p(B)*p(C))
= 1 – (0.96)*(0.95)*(0.90)
= 1 – 0.8208
= 0.1792

Thus the probability that any one of the module might fail would be 17.92%

Question 2.16

A computer maker receives parts from three suppliers, S1, S2, and S3. Fifty percent come from S1, twenty percent from S2, and thirty percent from S3. Among all the parts supplied by S1, 5% are defective. For S2 and S3, the portion of defective parts is 3% and 6%, respectively.
(a) What portion of all the parts is defective?
(b) A customer complains that a certain part in her recently purchased computer is defective. What is the probability that it was supplied by S1?

Solution

Given:

p(D / S1) = 0.05
p(D / S2) = 0.03
p(D / S3) = 0.06
p(S1) = 0.5
p(S2) = 0.2
p(S3) = 0.3

a. p(D) = $\sum_{i=1}^{N} p\left(\frac{D}{X_i}\right) * p(X_i)$
= (0.05*0.5) + (0.03*0.2) + (0.06*0.3)
= 0.025 + 0.006 + 0.018
= 0.049

Therefore, 4.9% of all the parts is defective.

b. $p(S1 / D) = \frac{p\left(\frac{D}{S1}\right) * p(S1)}{p(D)}$ [Bayer's Rule]

= (0.05*0.5)/0.049
= (0.025)/0.049
= 0.5102

Therefore, probability that the defective piece was given by S1 is 51.02%

Question 2.19

At a plant, 20% of all the produced parts are subject to a special electronic inspection. It is known that any produced part which was inspected electronically has no defects with probability 0.95. For a part that was not inspected electronically this probability is only 0.7. A customer receives a part and find defects in it. What is the probability that this part went through an electronic inspection?

Solution

Let 'D' be Defective and 'E' be Electronic Inspection
Given:

$p(E) = 0.2$ $\Rightarrow p(E^1) = 0.8$
$p(D^1 / E) = 0.95$
$p(D^1 / E^1) = 0.70$

$p(E / D) = p(D / E) * p(E) / p(D)$

$p(D / E) = 1 - p(D^1 / E)$
$= 1 – 0.95$
$= 0.05$

$p(D) = 1 – p(D^1)$
$= 1 – (p(D^1 / E)*p(E) + p(D^1 / E^1)*p(E^1))$
$= 1 – [(0.95*0.2) + (0.70*0.8)]$
$= 1 – (0.19 + 0.56)$
$= 1 – 0.75$
$= 0.25$

$p(E / D) = p(D / E) * p(E) / p(D)$
$= (0.05 * 0.2) / (0.25)$
$= 0.01 / 0.25$
$= 0.04$

Question 2.21

In the system in Figure 2.7, each component fails with probability 0.3 independently of other components. Compute the system's reliability.

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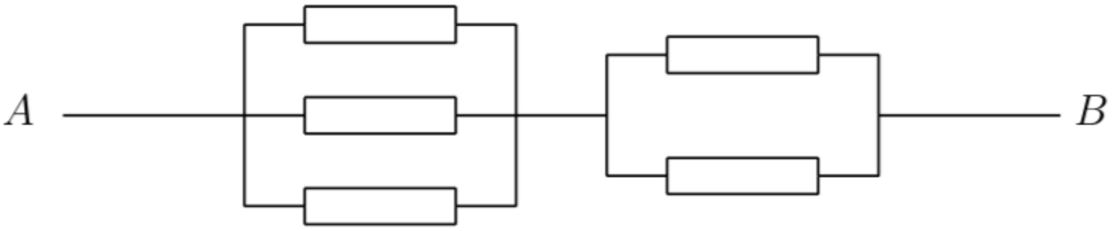
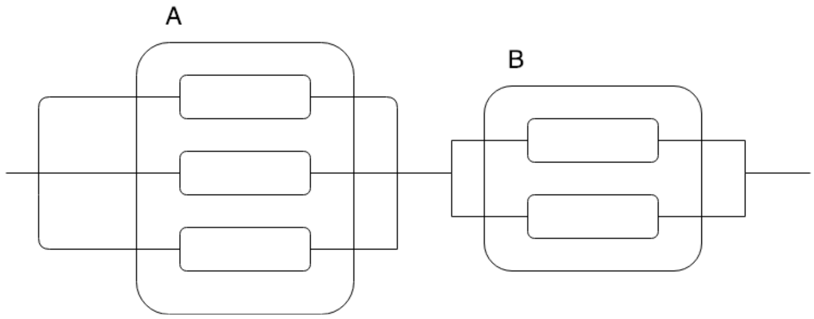


FIGURE 2.7: *Calculate reliability of this system (Exercise 2.21).*

Solution

Let A – Be the first system success
And B – be the second system success
And S – be the success for complete system



$p(A) = 1 - p(A^1)$
$= 1 - (0.3*0.3*0.3)$
$= 1 - 0.027$
$= 0.973$

$p(B) = 1 - p(B^1)$
$= 1 - (0.3*0.3)$
$= 1 - 0.09$
$= 0.91$

$p(S) = p(A) * p(B)$
$= 0.973 * 0.91$
$= 0.8854$

Therefore, Reliability of the system is 88.54%

Question 2.26

Two out of six computers in a lab have problems with hard drives. If three computers are selected at random for inspection, what is the probability that none of them has hard drive problems?

Solution

p(No hard disk problems in 3 computers)
= p(1^{st}, hard disk problem)*p(2^{nd}, no hard disk problem)*p(3^{rd}, no hard disk problem)

$= \frac{4}{6} * \frac{3}{5} * \frac{2}{4} = \frac{24}{120}$
= 0.2

Therefore, there is 20% chance of getting all the computers with no hard disk problems.

Question 3.7

The number of home runs scored by a certain team in one baseball game is a random variable with the distribution:

x	0	1	2
$P(x)$	0.4	0.4	0.2

The team plays 2 games. The number of home runs scored in one game is independent of the number of home runs in the other game. Let Y be the total number of home runs. Find E(Y) and Var(Y).

Solution

$P(Y = 0) = P(X_1 = 0) * P(X_2 = 0)$
$= 0.4 * 0.4 = 0.16$

$P(Y = 1) = [P(X_1 = 0) * P(X_2 = 1)] + [P(X_1 = 1) * P(X_2 = 0)]$
$= (0.4 * 0.4) + (0.4 * 0.4) = 0.16 + 0.16 = 0.32$

$P(Y = 2) = [P(X_1 = 0) * P(X_2 = 2)] + [P(X_1 = 1) * P(X_2 = 1)]$
$+ [P(X_1 = 2) * P(X_2 = 0)]$
$= (0.4 * 0.2) + (0.4 * 0.4) + (0.2 * 0.4)$
$= 0.08 + 0.16 + 0.08$
$= 0.32$

$P(Y = 3) = [P(X_1 = 1) * P(X_2 = 2)] + [P(X_1 = 2) * P(X_2 = 1)]$
$= (0.4 * 0.2) + (0.2 * 0.4)$
$= 0.16$

$P(Y = 4) = P(X_1 = 2) * P(X_2 = 2)$
$= 0.2 * 0.2$
$= 0.04$

$E(Y) = \sum_y y * P(y)$
$E(Y) = (0 * 0.16) + (1 * 0.32) + (2 * 0.32) + (3 * 0.16) + (4 * 0.04)$
$= 0 + 0.32 + 0.64 + 0.48 + 0.16$
$= 1.60$

$Var(Y) = \sum_y (y - E(y))^2 * P(y)$
$= [(0 - 1.6)^2 * 0.16] + [(1 - 1.6)^2 * 0.32] + [(2 - 1.6)^2 * 0.32]$
$+ [(3 - 1.6)^2 * 0.16] + [(4 - 1.6)^2 * 0.04]$
$= (2.56 * 0.16) + (0.36 * 0.32) + (0.16 * 0.32) + (1.96 * 0.16) + (5.76 * 0.04)$
$= 0.4096 + 0.1152 + 0.0512 + 0.3136 + 0.2304$
$= 1.12$

Question 3.18

Shares of company A cost \$10 per share and give a profit of X%. Independently of A, shares of company B cost \$50 per share and give a profit of Y% Deciding how to invest \$1000, Mr.X chooses between 3 portfolios.

a. *100 shares of A*
b. *50 shares of A and 10 shares of B*
c. *20 shares of B*

The distribution of X is given by probabilities:

$$P\{X = -3\} = 0.3,\ P\{X = 0\} = 0.2,\ P\{X = 3\} = 0.5.$$

The distribution of Y is given by probabilities:

$$P\{Y = -3\} = 0.4, P\{Y = 3\} = 0.6.$$

Compute expectations and variances of the total dollar profit generated by portfolios (a), (b), and (c). What is the least risky portfolio? What is the most risky portfolio?

Solution

Convert Distribution table from percent to dollar

$$P\left(X = -\frac{3}{100} * 10 = -0.3\right) = 0.3$$
$$P\left(X = \frac{0}{100} * 10 = 0\right) = 0.2$$
$$P\left(X = \frac{3}{100} * 10 = 0.3\right) = 0.5$$

$$P\left(Y = \frac{-3}{100} * 50 = -1.5\right) = 0.4$$
$$P\left(Y = \frac{3}{100} * 50 = 1.5\right) = 0.6$$

Let us compute E(X), Var(X), E(Y) and Var(Y) per share.

E(X) = $\sum_x x * P(x)$
= (-0.3 * 0.3) + (0 * 0.2) + (0.3 * 0.5)
= (-0.09 + 0.0 + 0.15)
= 0.06

Var(X) = $\sum_x (x - E(x))^2 * P(x)$
= $[(-0.3 - 0.06)^2 * 0.3] + [(0 - 0.06)^2 * 0.2] + [(0.3 - 0.06)^2 * 0.5]$
= $[(-0.36)^2 * 0.3] + [(-0.06)^2 * 0.2] + [(0.24)^2 * 0.5]$
= (0.1296 * 0.3) + (0.0036 * 0.3) + (0.0576 * 0.5)
= 0.03888 + 0.00108 + 0.0288
= 0.0684

E(Y) = $\sum_y y * P(y)$
= (-1.5 * 0.4) + (1.5 * 0.6)
= (-0.6 + 0.9)
= 0.3

$\text{Var(Y)} = \sum_y (y - E(y))^2 * P(y)$
$= [(-1.5 - 0.3)^2 * 0.4] + [(1.5 - 0.3)^2 * 0.6]$
$= [(-1.8)^2 * 0.4] + [(1.2)^2 * 0.6]$
$= (3.24 * 0.4) + (1.44 * 0.6)$
$= 1.296 + 0.864$
$= 2.160$

Now lets compute for the portfolios:

a. 100 shares of A

E(A) = 100 * E(X)
= 100 * 0.06
= 6

Var(A) = 100^2 * Var(X)
= 10000 * 0.0684
= 684

b. 50 shares of A and 10 shares of B

E(B) = 50 * E(X) + 10 * E(Y)
= (50 * 0.06) + (10 * 0.3)
= 3 + 3
= 6

Var(B) = 50^2 * Var(X) + 10^2 * Var(Y)
= (2500 * 0.0684) + (100 * 2.16)
= (171 + 216)
= 387

c. 20 shares of B

E(C) = 20 * E(Y)
= (20 * 0.3)
= 6

Var(C) = 20^2 * Var(Y)
= (400 * 2.16)
= 864

We know that the level of risk is directly proportional to the variance.
Therefore, the least risky is the portfolio 'B', while most risky being portfolio 'C'.

Question 3.22

Five percent of computer parts produced by a certain supplier are defective. What is the probability that a sample of 16 parts contains more than 3 defective ones?

Solution

Given:

p = 0.05 [Probability that a system is defective]
n = 16

Therefore, we can apply Binomial(n, p)

P(X > 3) = 1 – P(X <= 3) [Probability that there are more than 3 defective]
= 1 – 0.993
= 0.007

Question 3.26

After a computer virus entered the system, a computer manager checks the condition of all important files. She knows that each file has probability 0.2 to be damaged by the virus, independently of other files.

(a) Compute the probability that at least 5 of the first 20 files are damaged.

(b) Compute the probability that the manager has to check at least 6 files in order to find 3 undamaged files.

Solution

Given:

p = 0.2 [Probability of file damage]

a. n = 20 [Number of files]

Therefore, we can apply Binomial(n, p) Distribution

P(X >= 5) = 1 – P(X <= 4)
= 1 – 0.63
= 0.37

b. k = 3 [Number of damaged files]

Therefore, we can apply Negative Binomial (k, p) Distribution

$$P(X = x) = \binom{x-1}{k-1} * p^k * (1-p)^{x-k}$$

$$P(X = 6) = \binom{5}{2} * 0.2^3 * (1-0.2)^{6-3}$$

= 10 * 0.008 * 0.512
= 0.04096

Question 3.27

Messages arrive at an electronic message center at random times, with an average of 9 messages per hour.

(a) What is the probability of receiving at least five messages during the next hour?

(b) What is the probability of receiving exactly five messages during the next hour?

Solution

Given:

$\lambda = 9$ [Average messages per hour]

We can apply Poisson Distribution to get required probabilities.

a.

P(X >= 5) = 1 – P(X <= 4)
= 1 – 0.055
= 0.945

b.

P(X = 5) = P(X <= 5) – P(X <= 4)
= 0.116 – 0.055
= 0.061

Question 4.1

The lifetime, in years, of some electronic component is a continuous random variable with the density

$$f(x) = \begin{cases} \frac{k}{x^4} & for\ x \geq 1 \\ 0 & for\ x < 1 \end{cases}$$

Find k, the cumulative distribution function, and the probability of lifetime to exceed 2 years.

Solution

Finding value of k

We know that $\lim_{x\to\infty}(F(x)) \quad = 1$

That is $\int_{-\infty}^{\infty} f(x)dx = 1$

⇨ $1 = \int_{-\infty}^{1} 0\,dx + \int_{1}^{\infty} \frac{k}{x^4} dx$

⇨ $1 = \left.\frac{k}{-3x^3}\right]_1^\infty$

⇨ $1 = (0 - \frac{k}{-3})$

⇨ $1 = \frac{k}{3}$

⇨ k = 3

F(x) = $\int_{-\infty}^{x} f(y)dy$

⇨ F(x) = $\int_{-\infty}^{1} 0\,dy + \int_{1}^{x} \frac{3}{y^4} dy$

⇨ F(x) = $\left.\frac{3}{-3y^3}\right]_1^x$

⇨ F(x) = $-\frac{1}{x^3} + 1$

Therefore F(x) = $\begin{cases} 1 - \frac{1}{x^3} & for\ x \geq 1 \\ 0 & for\ x < 1 \end{cases}$

P(X > 2) = 1 – F(2)

$= 1 - \left(1 - \frac{1}{2^3}\right)$

= 1/8

= 0.125

Question 4.4

Lifetime of a certain hardware is a continuous random variable with density

$$f(x) = \begin{cases} K - \frac{x}{50} & for\ 0 < x < 10\ years \\ 0 & for\ all\ other\ x \end{cases}$$

(a) Find K.
(b) What is the probability of a failure within the first 5 years?
(c) What is the expectation of the lifetime?

Solution

a.

We know that $\lim_{x\to\infty}\left(F(x)\right) = 1$

That is $\int_{-\infty}^{\infty} f(x)dx = 1$

- $\Rightarrow 1 = \int_{-\infty}^{0} 0\,dx + \int_{0}^{10} K - \frac{x}{50}\,dx + \int_{10}^{\infty} 0\,dx$
- $\Rightarrow 1 = 0 + \left(Kx - \frac{x^2}{100}\Big]_0^{10}\right)$
- $\Rightarrow 1 = \left(10K - \frac{100}{100}\right) - (0 - 0)$
- $\Rightarrow 1 = 10K - 1$
- $\Rightarrow 2 = 10K$
- $\Rightarrow$ k = 0.2

Therefore,

$$f(x) = \begin{cases} 0.2 - \dfrac{x}{50} & for\ 0 < x < 10\ years \\ 0 & for\ all\ other\ x \end{cases}$$

b.

P(X <= 5) = $\int_{-\infty}^{5} f(x)dx$

- $\Rightarrow P(X \le 5) = \int_{-\infty}^{0} 0\,dx + \int_{0}^{5} 0.2 - \frac{x}{50}\,dx$
- $\Rightarrow P(X \le 5) = \left(0.2x - \frac{x^2}{100}\Big]_0^{5}\right)$
- $\Rightarrow P(X \le 5) = \left(0.2 * 5 - \frac{25}{100}\right) - 0$
- $\Rightarrow P(X \le 5) = (1 - 0.25)\ 0$
- $\Rightarrow P(X \le 5) = 0.75$

c.

$$E(X) = \int_{-\infty}^{\infty} x * f(x)\,dx$$

$$E(X) = \int_{-\infty}^{0} x * 0\,dx + \int_{0}^{10} x * \left(0.2 - \frac{x}{50}\right)\,dx + \int_{10}^{\infty} x * 0\,dx$$

$$E(X) = 0 + \int_{0}^{10} \left(0.2x - \frac{x^2}{50}\right)\,dx + 0$$

$$E(X) = \frac{0.2x^2}{2} - \frac{x^3}{3 * 50}\Bigg]_0^{10}$$

$$E(X) = \left(0.1x^2 - \frac{x^3}{150}\right)\Bigg]_0^{10}$$

$$E(X) = \left(0.1(100) - \frac{1000}{150}\right) - 0$$

$$E(X) = (10 - 6.6667)$$

$$E(X) = 3.3333$$

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Question 4.7

The time it takes a printer to print a job is an Exponential random variable with the expectation of 12 seconds. You send a job to the printer at 10:00 am, and it appears to be third in line. What is the probability that your job will be ready before 10:01?

Solution

Given:

$\lambda = 12$

$\alpha = 3$

t = 60

Therefore,

$E(X) = \left(\frac{1}{\lambda}\right) = \frac{1}{12}$

Using Gamma – Poisson Formula (λ, α)

$$\lambda t = \frac{1}{12} * 60 = 5$$

$$P(T > t) = \gamma(x < \alpha)$$
$$P(T < 60) = \gamma(x > 3) = P(x \geq 3)$$
$$= 1 - F(2)$$
$$= 1 - 0.125$$
$$= 0.875$$

Question 4.30

An internet service provider has two connection lines for its customers. Eighty percent of customers are connected through Line I, and twenty percent are connected through Line II. Line I has a Gamma connection time with parameters α = 3 and λ = 2 per min. Line II has a Uniform(a, b) connection time with parameters a = 20 sec and b = 50 sec. Compute the probability that it takes a randomly selected customer more than 30 seconds to connect to the internet.

Solution

(For Line 1)

Given:

$\lambda = \frac{2}{60} = \frac{1}{30}$

$\alpha = 3$

t = 30

Therefore, for Gamma Poisson Distribution (α, λ):

$$\lambda t = \frac{1}{30} * 30 = 1$$

$$P(T > t) = \gamma(x < \alpha)$$
$$P(T > 30) = \gamma(x < 3) = P(x < 3)$$
$$= F(2)$$
$$= 0.92$$

(For Line 2)

Given:

a = 20 sec

b = 50 sec

Therefore,

$$f(x) = \frac{1}{b-a} = \frac{1}{50-20} = \frac{1}{30}$$

$$P(30 < x < 50) = \int_{30}^{50} f(x)dx$$
$$= \int_{30}^{50} \frac{1}{30} dx$$
$$= \left(\frac{x}{30}\right)\Big]_{30}^{50}$$
$$= \frac{50}{30} - \frac{30}{30} = \frac{5}{3} - 1$$
$$= \frac{2}{3}$$

Therefore, required probability = $\left(\frac{80}{100} * 0.92\right) + \left(\frac{20}{100} * \frac{2}{3}\right)$ = 0.8693

References:

- Probability and Statistics for Computer Scientists, Second Edition, Michael Baron.