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Problem 1 - Exercise 1.1

a. *What are the least and greatest number of leaf nodes in a binary tree with n nodes?*
b. *What is the relationship between the number of nodes in a full binary tree and the number of leaf nodes?*

Solution

a. The least number of leaf nodes in a binary tree is 1. This is because when a binary tree is skewed, we only have the last node which is the leaf node.

   The max number of leaf nodes can be seen in a complete binary tree with nodes n. This is given by the floor value of $\frac{n+1}{2}$

   Therefore, the number of leaf nodes in a binary tree with n nodes is given by 1 to $floor\left(\frac{n+1}{2}\right)$, where n is positive number.

b. Let

   Number of nodes = n
   Number of leaf nodes = l
   Height of the tree = h

   Then we know that in a full binary tree:

   $$n = 2^{h+1} - 1 \; and \; l = \; 2^h$$

   Solving, the first equation:

   $$2^{h+1} = n + 1$$
   $$2^h * 2 = n + 1$$
   $$2^h = \frac{n+1}{2}$$

   We know that $l = \; 2^h$. Therefore,

$$l = \frac{n+1}{2} \; \longmapsto n = 2l - 1$$

Problem 2

a. *Insert the following 15 randomly generated objects into a binary search tree in the order they are listed. (20 p)*

   34, 15, 65, 62, 69, 42, 40, 80, 50, 59, 23, 46, 57, 3, 29

b. *Give two integers that could be inserted into this tree that would increase the height of this tree.*
c. *Remove the root node four times by copying up the smallest element of the right sub-tree, show the final tree*

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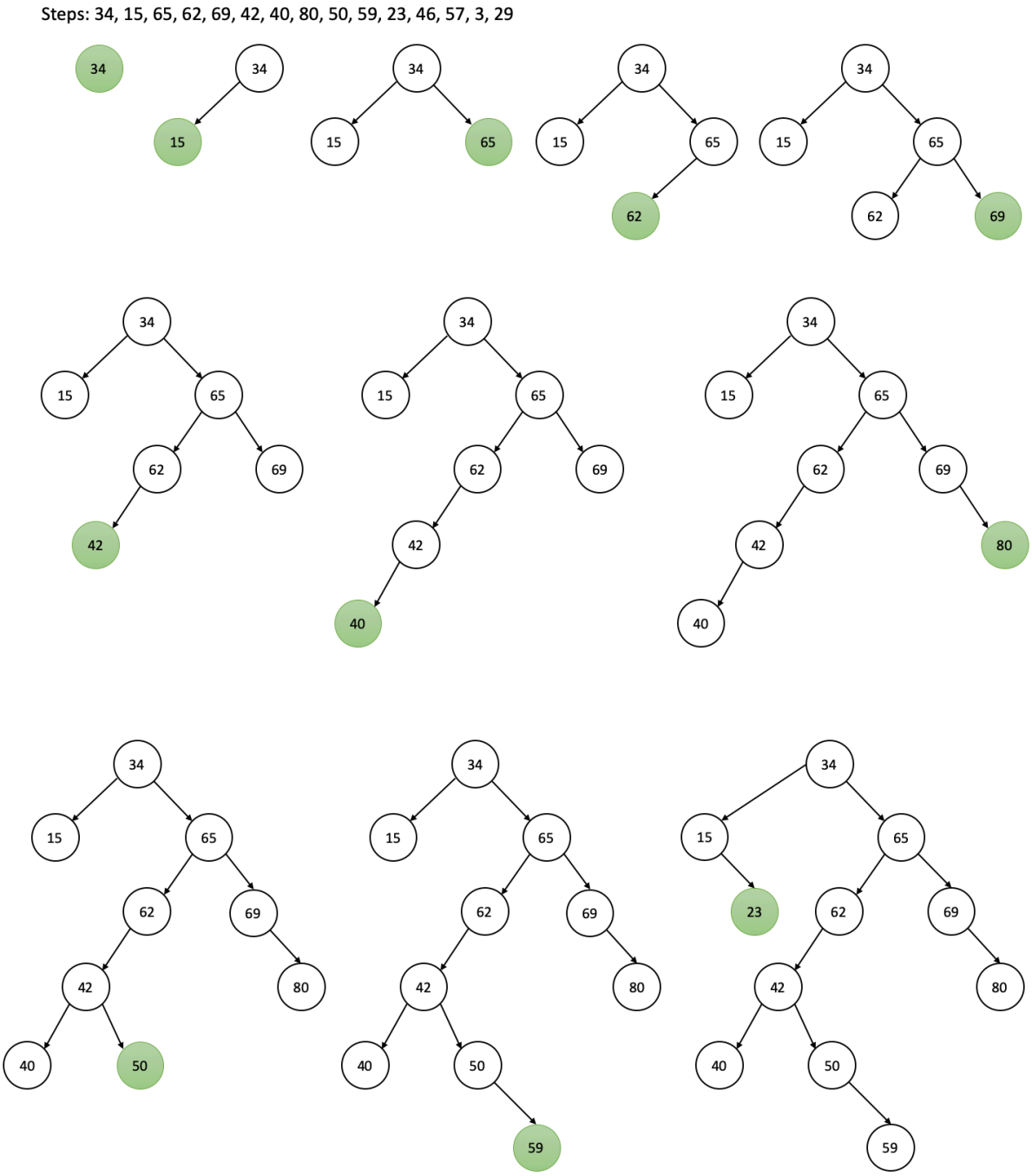
Solution

a.



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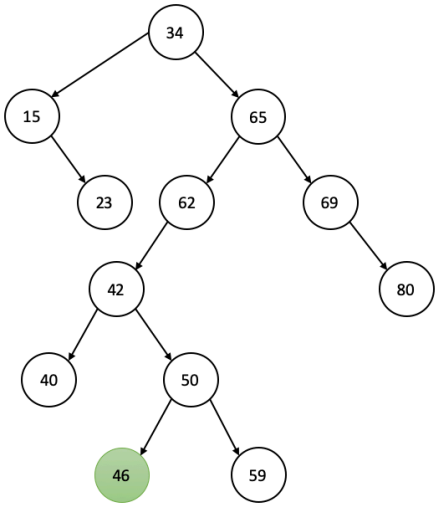

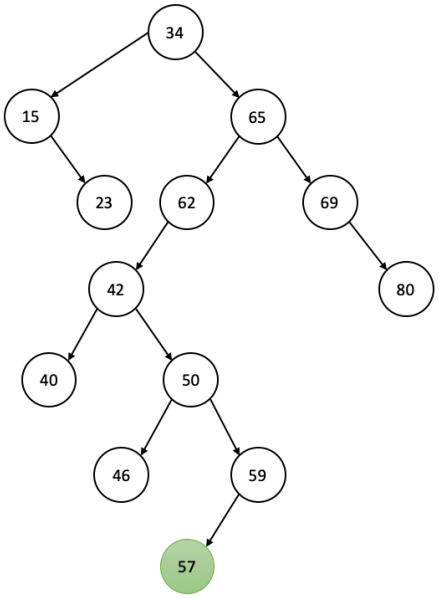

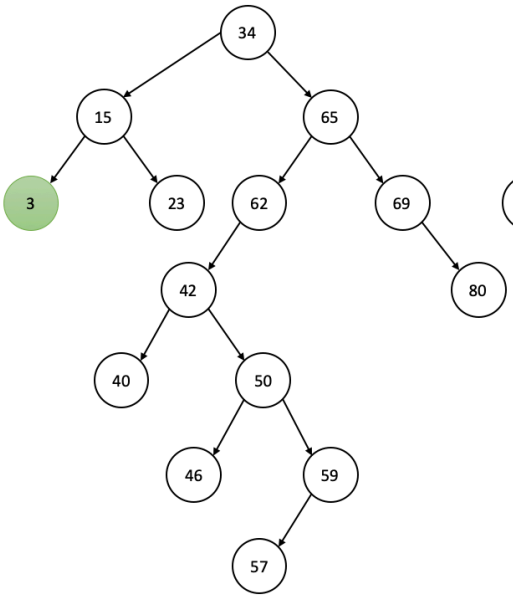

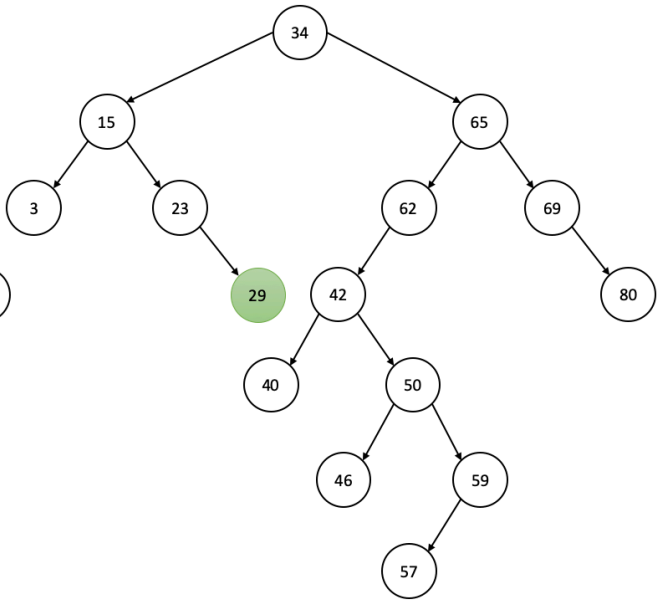

b. The height of the tree can be increased by adding any number from 51 to 58. This will generate a child node for 57 and increase the height by 1. The current height of the tree is 6 and then should increase by 7.
Numbers that can be added: 51, 52, 53, 54, 55, 56, 58

Note:
- Adding 2 numbers from the above set, say 51 and 52 will increase the height by 2.
- Adding numbers like 60 and 61 will increase the height by 1.

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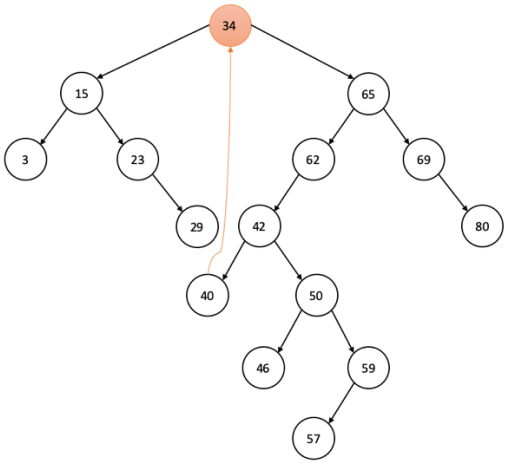
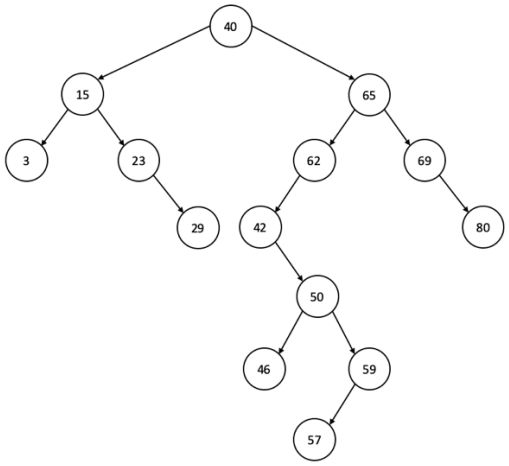
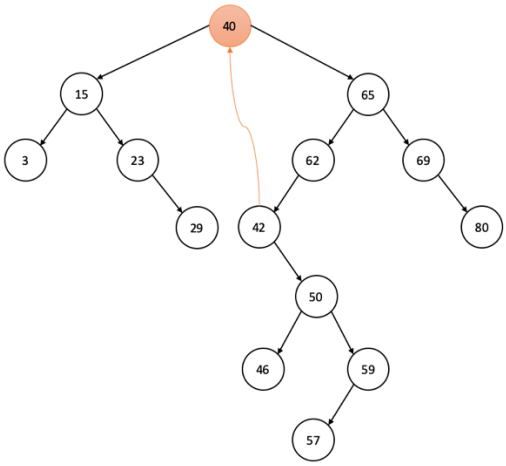
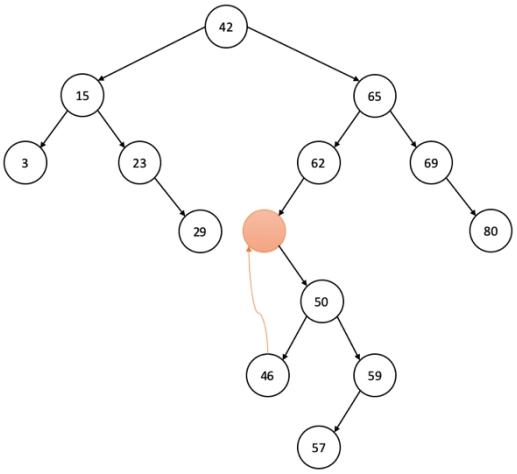
c. Deletion of root node 1st time:

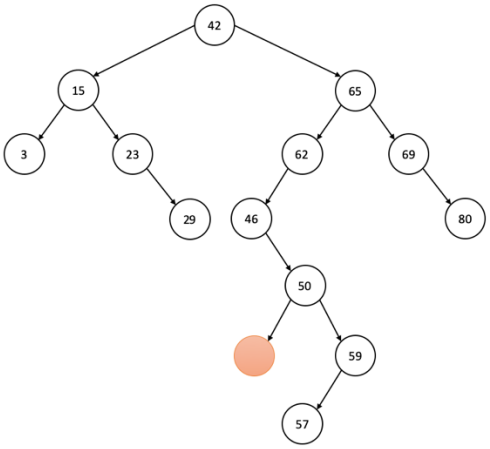


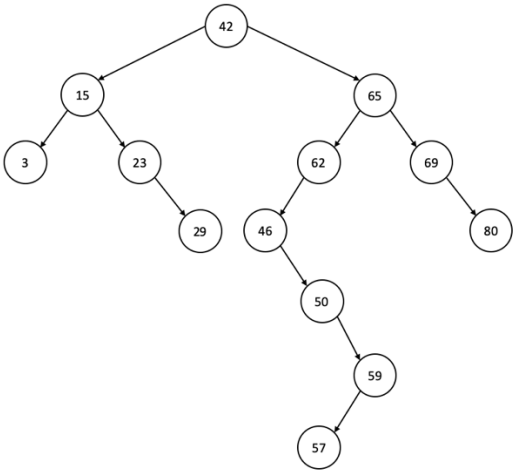


Deletion of root node 2nd time:









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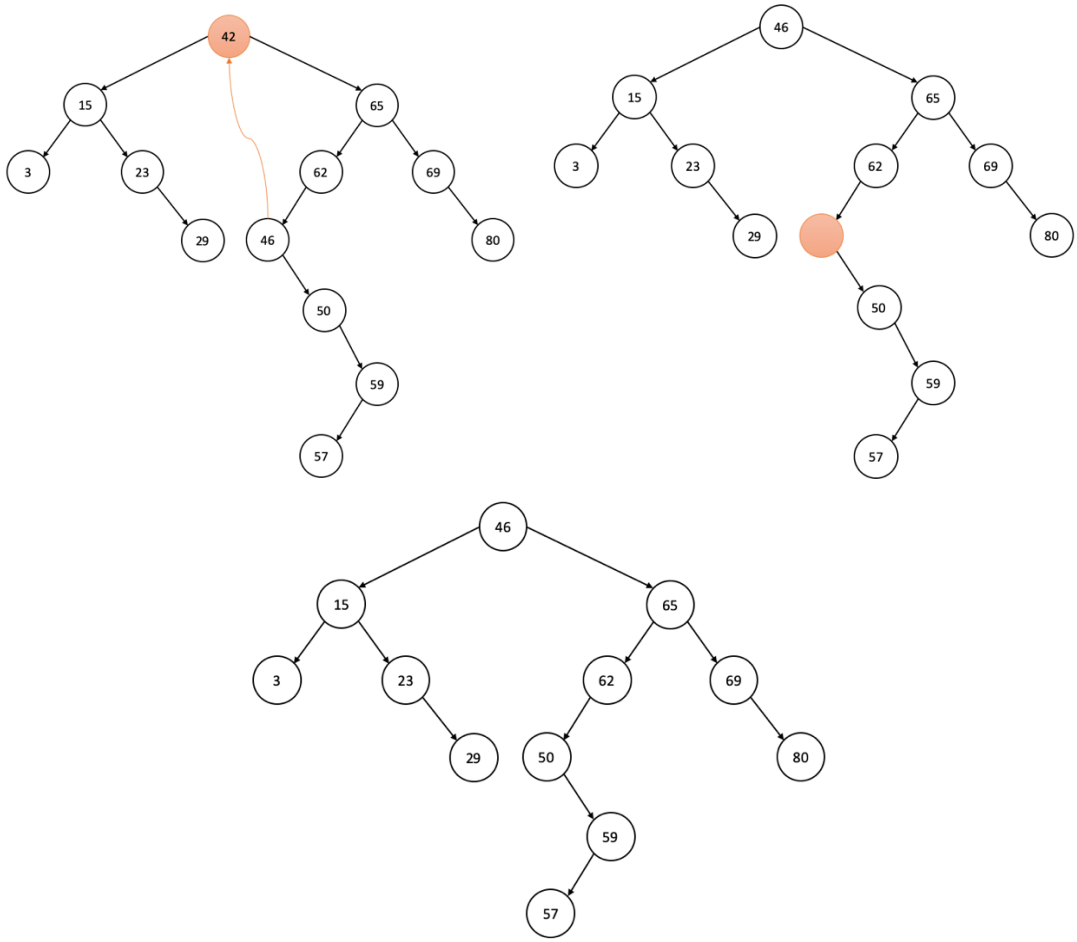
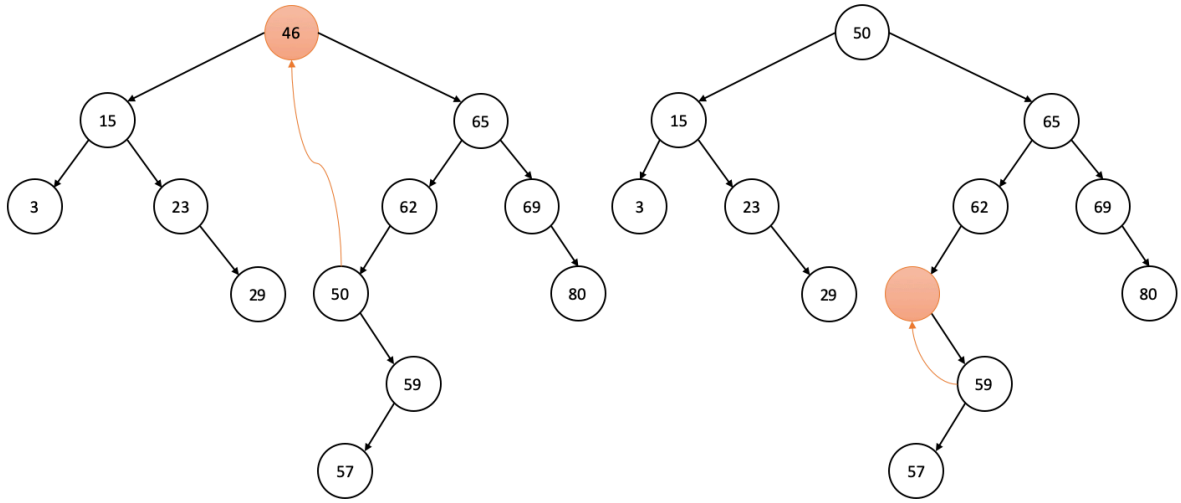
Deletion of root node 3rd time:



Deletion of root node 4th time:



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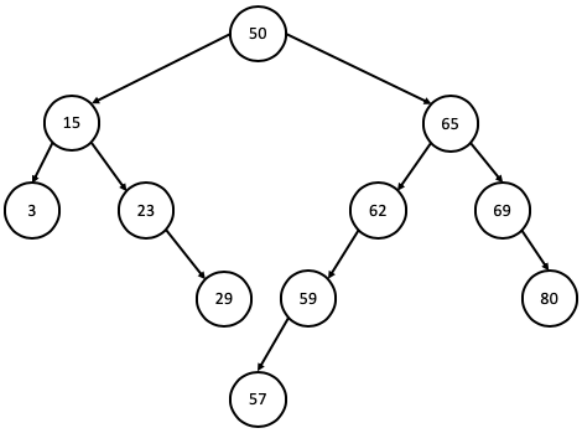
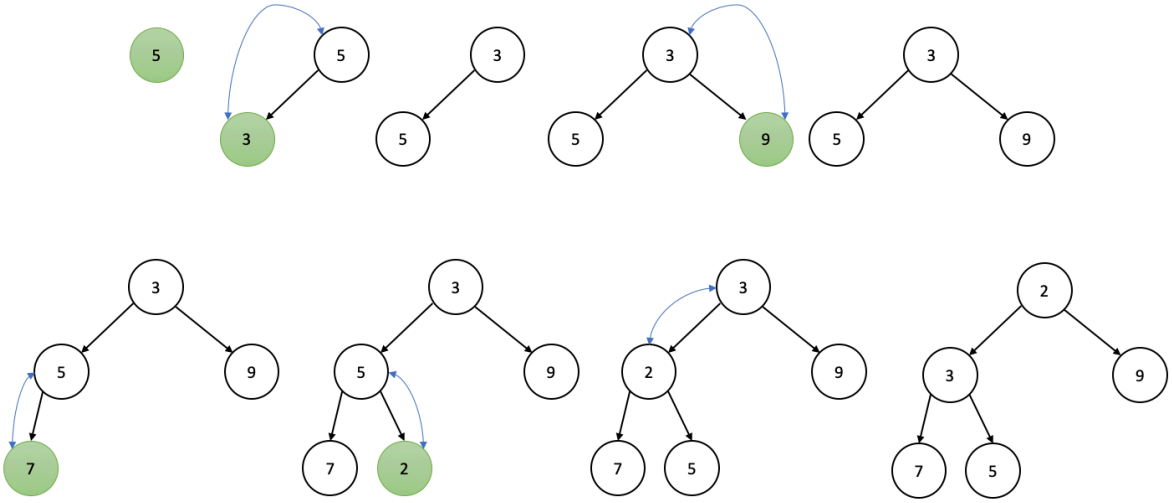


Problem 3

Insert the following n objects, in the order given, into a binary min-heap and place your answer into the following table.

5, 3, 9, 7, 2, 4, 6, 1, 8

Solution



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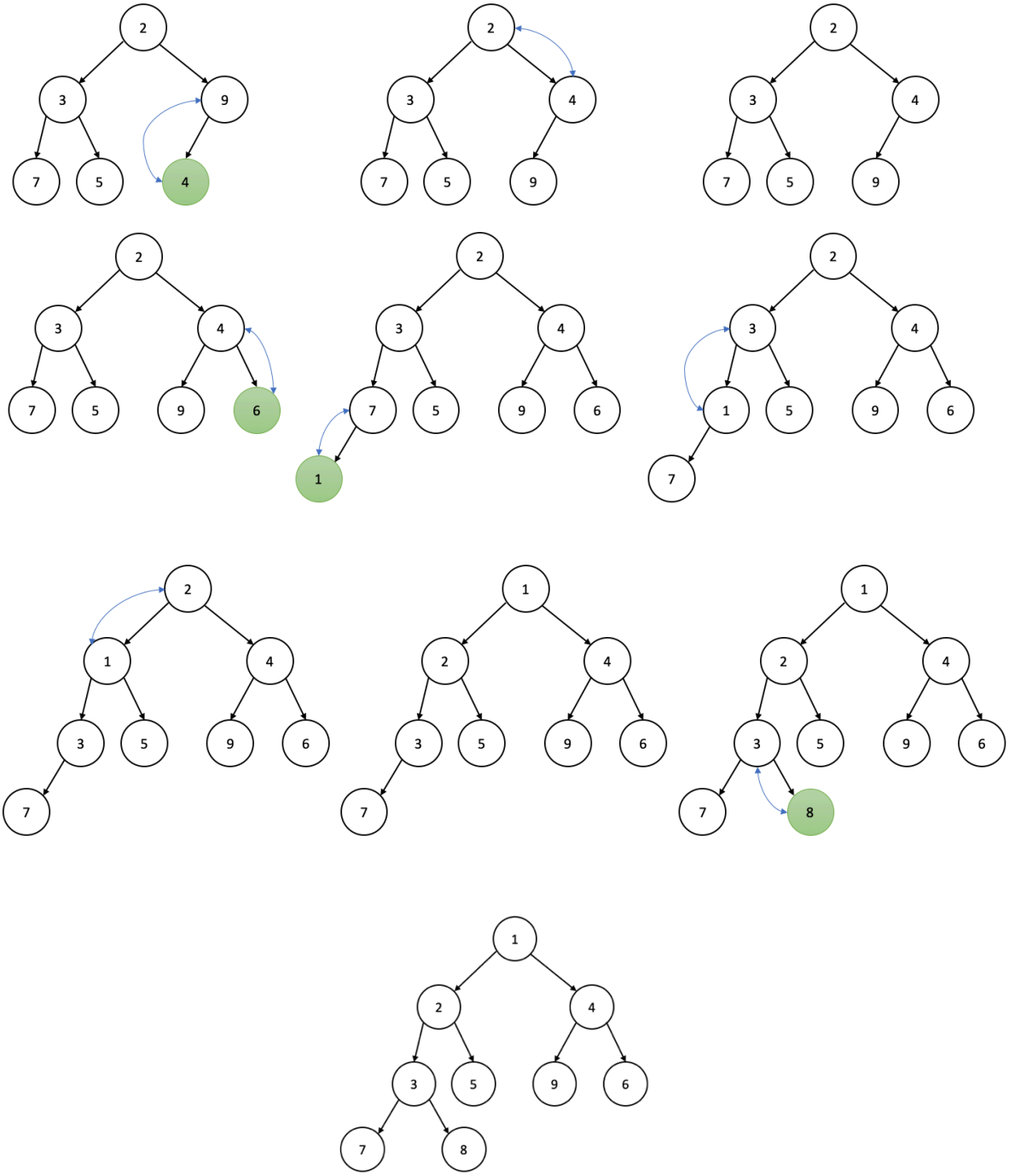

Required Array:

Index	0	1	2	3	4	5	6	7	7	8
Data		1	2	4	3	5	9	6	7	8