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A cross-sectional application of the Nelson-Siegel-Svensson model to several negative yield cases

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Abstract: The appearance of negative bond yields presents significant challenges for the fixed income markets, which mainly concern related forecasting models. The Nelson-Siegel-Svensson model (NSS) is one of the models that is most frequently used by central banks to estimate the term structure of interest rates. The objective of this study is to evaluate the application of the NSS model to fit the yield curve of a set of 20 countries, the majority from the Eurozone, which registered negative sovereign bond yields. We conclude that the model adjusted well for all countries' yield curves, although no changes or constraints were introduced. In addition, a comparison was carried out between market instantaneous interest rate and the interest rate for the very distant future, which the model can predict, with good results for the instantaneous interest rate. An evaluation of the possible behaviour of shared debt securities (i.e. Eurobonds) was also analysed. In conclusion, the NSS model seems to remain a valuable, easy to use, and adaptable tool, to fit negative yield curves, for monetary policy institutions and market players alike.

Subjects: Public Finance; Credit & Credit Institutions; Investment & Securities

Keywords: yield curve; negative bond yields; Eurobonds; Nelson-Siegel-Svensson model

JEL classification: C02; C18; E43; E47; G12; G17

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PUBLIC INTEREST STATEMENT

Central banks and market participants need to estimate the term structure of interest rates, also known as yield curve. The term structure reflects expectations of market participants about future changes in interest rates and their assessment of monetary policy conditions. It is fundamental to measure pension liabilities and to pursue asset-liability management. The Nelson-Siegel-Svensson model (NSS) is one of the models that is most frequently used due to its parsimony and assumptions. The objective of this study is to evaluate the suitability of the NSS model to perform estimations under the appearance of negative bond yields. The study considers 295 different government bonds, from a group of 20 countries that registered negative yield to maturity at the data access date. The main conclusion is that the NSS model seems to remain a valuable tool, easy to use, to estimate the term structure of interest rates.

1. Introduction

The existence of negative bond yields presents significant challenges for the fixed income markets. Some of these challenges are related to modelling and forecasting methods, and others are due to the actual size of assets with negative yields (\$13,4 trillion, Financial Times, 2016). The final challenge is to detect the impact of negative bond yields on financial theory and the implications for bondholders and issuers.

In this study, the Nelson-Siegel-Svensson model (NSS) (Nelson & Siegel, 1987; Svensson, 1994) is used to evaluate the yield curves of a set of countries which registered negative sovereign bond yields which constitute an unusual situation. This model was considered due to its features, namely allowing for negative interest rates and non-normal interest rates distribution. This model is also usually adopted by central banks to estimate the term structure of interest rates (BIS, 2005).

Negative yields are a recent phenomena and to some degree can be an outcome of various important aspects. For example, the 2008 financial crisis led the Federal Reserve (Fed) to start quantitative easing programmes up until 29 October 2014, which were later followed by the European Central Bank (ECB) (ECB, 2017a) in the aftermath of the 2010/2011 European government debt crisis and the significant reduction in the directorate interest rate of ECB. Japan led the fixed income markets to search for “safe heavens”, as a result of its lost decades, characterised by the economic stagnation of Japan in the 1990s (Hayashi & Prescott, 2002), and low-interest rates, compounded by the reduction in GDP growth of China and world. These “safe heavens” issuers are those that have higher ratings and therefore they can provide a greater certainty that their debts will be serviced entirely. In a certain way, the high debt levels of European Union countries, and the highest debts in the world, such as that of Japan (234% of GDP in 2015—OECD, 2017), should demand greater yields for these issuers. However, ratings (that seems to be more favourable for developed countries (Cantor & Packer, 1996)) and the lack of the possibility for emerging countries to capture the fixed income markets with intensity, have led to the present situation, which is characterised by the issuers of higher debt in relation to GDP, with, in some cases, the lowest yields, and, awkwardly, cases of negative yields, which are not so predictable and common.

Given that the market players (e.g. insurance companies, pension funds, and banks) need to estimate and model the term structure of interest rates with these recent negative bond yields, this study analyses the applicability of the use of the NSS model in this context, by means of friendly, widely available, and simple tools. Indeed, the NSS model allows negative interest rates, does not restrict interest rates distribution, and is calibrated with market data. Accordingly, the objectives of this study are twofold. Firstly, to evaluate the adequacy of the NSS model through the fit of the yield curve, at a certain date, with at least one negative yield value, as well as the comparison between the interest rates values deducted from the model and market data, with an easy-to-use approach. Secondly, to evaluate the results of the model with partial market bond yields data (short, intermediate and long term).

The paper is comprised of the literature review, the methodology, the results and the conclusion sections. The literature review section presents and describes the NSS, its application and importance, and also the approaches carried out to fit negative yields market data. In the methodology section, the NSS model and parameters are described in detail, as well as the calibration method, the analysis procedure, and the data and software definitions to accomplish data analysis. The results prepare the way for further research. Given that the majority of countries under study are European and in the Eurozone, a comparison is conducted between their yield curves and some effects of a possible future shared Eurozone debt security (i.e. Eurobonds). The conclusion section presents the main findings.

2. Literature review

The term structure of interest rates or yield curve, is a key variable of economics and finance (Büttler, 2007). The direct relation between term structure of interest rates and yield curve should be clarified. Málek (2005), in Hladíková and Radová (2012), places the distinction to three equivalent descriptions of the term structure of interest rates:

- the discount function, which specifies zero-coupon bond prices as a function of maturity;
- the spot yield curve, which specifies zero-coupon bond yields (spot rates) as a function of maturity;
- the forward yield curve, which specifies zero-coupon bond forward yields (forward rates) as a function of maturity.

The discount function entails some undesirable conditions. Bond prices are insensitive to yields changes for shorter maturities. Sometimes, minimising price errors result in large yield errors for bonds for these shorter maturities (Svensson, 1994). Furthermore, monetary policymakers and economic discussions, generally focus on interest rates, rather than prices (Geyer & Mader, 1999). For these reasons, the discount function cannot be a suitable description of the term structure of interest rates.

To the purpose of an entire evaluation of the yield curve (maturities can be as high as 30, 50, and even 100 years), the forward market products are not adequate, as they have a short time limit, and therefore the forward yield curve can only be a proper description of the yield curve for shorter maturities.

In the case of the spot yield curve, the market has no zero-coupon bonds for all maturities, and only a few sets of countries issue these instruments, so therefore coupon government bonds should be considered. The use of coupon bonds, with different coupon rates instead of zero-coupon bonds, have a negligible impact, according to Kariya et al. (2013, in Inui, 2015). Svensson (1994) mentioned that obtaining implied forward interest rates from yield to maturity (YTM) on coupon bonds is more complicated than on zero coupon bonds. The YTM obtained from market data will give implied spot rates, instead of real spot rates, since one cannot compute the entire yield curve with all maturities (i.e. the spot yield curve) from zero-coupon bond yields, although Cox, Ingersoll, and Ross (1985) stated that “the expectations hypothesis postulates that bonds are priced so that the implied forward rates are equal to the expected spot rates”. In synthesis, the term structure of interest rates, or the yield curve, is computed through the YTM of government coupon bonds, and through the YTM that will obtain the implied rates.

One of the objectives and usefulness of fit in the yield curve is to provide the monetary policy institutions with indicators of rates evolution and expectations (e.g. inflation). The need for monetary policy institutions to have these indicators increased when flexible exchange rates replaced fixed exchange rates (Svensson, 1994). Another significant purpose is related to fixed income market participants (e.g. hedging strategies or assets allocation for pension funds).

There are several methods to fit the yield curve (Sundaresan, 2009). However, some do not allow for negative interest rate while others assume certain interest rates distribution (usually normal or log-normal). These include:

- the Vasicek model (Vasicek, 1977), which is a mean reversion process, allows for negative rates, but does not calibrate with market data, invalidating its use in this study; the Rendleman and Bartter model (Rendleman & Bartter, 1980) follows a simple multiplicative random walk. However, rates are assumed to be log-normally distributed, which invalidates its use in the case of negative yields;
- the Cox, Ingersoll and Ross (CIR) model (Cox et al., 1985) is a mean reversion model, but it does not permit negative interest rates, neither does it calibrate with market data, which invalidates its use in this study;

- the Ho and Lee model (Ho & Lee, 1986) is calibrated with market yields, interest rates can be negative, but assumes a normal distribution for interest rates, which constitutes a limitation in our study;
- the BDT model (Black, Derman, & Toy, 1990) can be calibrated through market equity options data, but it assumes that rates follow a lognormally distribution, which invalidates its use in the case of negative yields;
- the Black and Karasinski model (Black & Karasinski, 1991) is calibrated with market yields and volatilities, but also assumes that rates follow a lognormally distribution, which invalidates its use in this study;
- the Bootstrapping method generates a zero-coupon yield curve from existing market data such as bond prices, but lacks robustness (Martellini, Priaulet, & Priaulet, 2003);
- the NSS model (Nelson & Siegel, 1987; Svensson, 1994) uses an exponential function to approximate the unknown forward rate function; allows negative interest rates, does not restricts interest rates distribution, and is calibrated with market data.

In this context, the NSS model is the only model that is able to address negative interest rates and to allow a non-normal interest rate distribution. In fact, the NSS model has been widely used by market participants. It is parsimonious, although it is sensitive to the starting values of the parameters (Annaert, Claes, Ceuster, & Zhang, 2010). The NSS model respects the restrictions imposed by the economic and financial theory (interest rates take real numbers and not complex ones, and are higher for longer terms) and considers any yield curve form which is empirically observed in the market (Diebold & Rudebusch, 2013 in Ibáñez, 2015). Furthermore, if the NSS behaves satisfactorily in a negative yield market, then this would be of utmost importance for hedging strategies (mainly for market participants, to hedge against the flattening or steepening of the yield curve) and also for obtaining forecasts for interest rates levels (which is very useful for monetary policymakers).

In this study, we decided to evaluate the application of the NSS model to fit the yield curve of a set of countries which registered negative sovereign bond yields. In fact, several curve fitting spline methods have been criticised for having undesirable economic properties and for being “black box” models (Seber & Wild, 2003 in Annaert et al., 2010).

Accordingly, our purpose is to obtain a static value of instantaneous interest rate (IIR) and the interest rate of a very distant future (IRVDF), and also to check if the values given by the model are in accordance with the market ones. Additionally, another objective is to use a friendly, widely available tool for a not so in-depth user of maths tools or software.

3. Methodology

The yield curve that can be estimated from bond yields of a certain economic region is of utmost importance for monetary and economic authorities to support decision processes and to establish policies, as well as to market participants for their investments and actions (Martellini et al., 2003).

This study evaluates the NSS model, with a curve-fitting statistical model, under negative yields and all along the yield curve. This model provides values for instantaneous and distant future interest rates.

The approach adopted does not add more factors, parameters, or terms to the NSS model. It computes all yield curves for each of the selected countries and tries to obtain economic and financial data to evaluate the forecast adequacy of the model, even in cases of issuers with few negative yields. Therefore, it is not an objective to consider the NSS model parameters time series, neither to forecast its values to obtain a yield curve evolution. Hence, a cross-sectional fitting was adopted to check how the NSS model works with negative yields at some part of the yield curve (Saunders, Lewis, & Thornhill, 2016).

The NSS model, Equation (1), is a parametric curve-fitting method procedure, which is statistical in its approach.

$$\gamma(\theta) = \beta_1 + \beta_2 \left[\frac{1 - e^{-\frac{\theta}{\lambda_1}}}{\frac{\theta}{\lambda_1}} \right] + \beta_3 \left[\frac{1 - e^{-\frac{\theta}{\lambda_1}}}{\frac{\theta}{\lambda_1}} - e^{-\frac{\theta}{\lambda_1}} \right] + \beta_4 \left[\frac{1 - e^{-\frac{\theta}{\lambda_2}}}{\frac{\theta}{\lambda_2}} - e^{-\frac{\theta}{\lambda_2}} \right] \quad (1)$$

As clearly described by Guedes (2008), the NSS model parameters can have an economic interpretation, namely:

- $\gamma(\theta)$ is the yield to maturity value (spot rate) at the time of data access, for the maturity θ ;
- β_1 is the IRVDF;
- $\beta_1 + \beta_2$ is the yield curve initial value and can be interpreted as the IIR;
- $-\beta_2$ is the spread between IRVDF and IIR (i.e. the average slope of the curve);
- $\beta_{1,2}$ and β_3 determine how short and long interest rates interchange and are responsible for the hump that the yield curve shows;
- β_4 is the extension of the model proposed by Svensson (1994), which can be interpreted as an independent decay parameter, which will introduce a new hump to fit the model better;
- λ_1 and λ_2 are the parameters responsible for how inclination and curvature behave, which does not have an economic interpretation, although determining the interchange between IIR and IRVDF.

Until negative bond yields appear in some markets, the NSS model did not present much difficulty in its application and is thus widely used.

Guedes (2008) stated that $\beta_1 + \beta_2 > 0$ which for the paradigm of that time, and up until then, appeared to be a very reasonable economic and financial condition. The general perception that rates or at least nominal rates, would always be positive, empirically leads to the definition of limits under which the model should work. However, time and markets have shown that $\beta_1 + \beta_2$ (interpreted as the IIR) can be lower than zero. Therefore, this study tries to show that when $\beta_1 + \beta_2 < 0$ the IIR interpretation remains.

For a first approach, it is expected that the yield curve fitting with some negative bond yields would be more difficult, due to the calibration process, which usually calculates the minimum value of the sum of squared residuals (SSR). As stated by Svensson (1994), the parameters are obtained by minimising the sum of squared yield errors between estimated and observed yields. Our analysis follows the NSS model and the SSR. Gilli, Große, and Schumann (2010) stated that one possibility for the calibration is to use Equation (2) to calculate the SSR, where y is estimated yield using the NSS model, and y^M is the market yield value:

$$\min_{\beta, \lambda} \sum (y - y^M)^2 \quad (2)$$

In this study, the market values are the bond yields for each maturity, for each country. Using the Microsoft Excel Solver (Frontline Systems, 2017a) function, we obtain the residuals' minimum value, which allows one to obtain the values of the parameters $\beta_{1,2,3,4}$ and $\gamma_{1,2}$. The parametrisation of Solver for the data used in this paper is presented in detail in Section 3.2.

For forecasting purposes, only a few market bond yields maturities were tested, and the NSS model was used to adjust the curve for the missing maturities. Partial market data was considered following the classification of the beginning of the 1990s, that bond markets used for bond maturities, namely: short, intermediate, and long term (Martellini et al., 2003). The most usual time frame for each division are as follows: bonds with maturities until 5 years are called short-term bonds; from 5 to 10/12 years they are called intermediate bonds, and; higher than 10/12 years are called long bonds.

When the NSS model was used for forecasting short-term maturity bonds, the 5 years' time frame was not considered as a fixed period, because the model does not produce good-fitting results. The NSS model seems to need at least one negative yield market data to proceed with proper calibration. Taking this into consideration, the short-term time frame was different for every country, ranging from 2 to 5 years.

The inferior limit of the intermediate period is defined by the higher value found from the short-term forecast (STF). The upper limit was defined by the best-observed fitting, but whenever possible, this was no more than 10 years (Lithuania is a special case, as it has no bonds with maturities higher than 7 years), and the wider period that was considered with no market data to calibrate the model (Switzerland is a special case, where the limit is 25 years).

The adequacy of the NSS model to obtain accurate enough parameter values with partial market data was evaluated for three sectors of the yield curve: short, intermediate, and long term. For STF, the model was calibrated only with market yields for intermediate and long-term maturities, and thus obtained different values for the parameters to the ones obtained when all the market data was used to calibrate the model. The parameters values and the countries' yields curves with lower forecasts can be assessed in Appendix II. Similarly, the same action was carried out when calculating the intermediate and long-term maturities forecasts. For each of the forecast maturities, the model only had access to the other maturities, for which the values of the factors that best fitted the curve were computed. The Solver function was run as many times as possible, in order to get the best forecast fit values.

3.1. Data

The study considers 295 different government bonds, from a group of 20 countries (Austria, 16; Belgium, 14; Bulgaria, 9; the Czech Republic, 12; Denmark, 6; Finland, 12; France, 26; Germany, 38; Ireland, 12; Italy, 15; Japan, 18; Lithuania, 11; Luxembourg, 6; the Netherlands, 14; Portugal, 13; Slovakia, 12; Slovenia, 13; Spain, 15; Sweden, 16; and Switzerland, 17) with at least one negative yield to maturity government bond at the data access date. These dates were: 15 March 2017, for Austria, Denmark, Finland, France, the Netherlands, Sweden and Switzerland; 16 March 2017, for Germany and Japan; and 5 May 2017, for Belgium, Bulgaria, the Czech Republic, Ireland, Italy, Lithuania, Luxembourg, Portugal, Slovakia, Slovenia and Spain. The data source used to obtain bonds information used in the study was Bloomberg, through a Bloomberg Terminal. Inflation-indexed bonds were not considered.

The number of countries was chosen taking into consideration two main purposes: first, to try to get more issuers to evaluate model adequacy for a wider set of data; and second, as most are from Europe and subject to the ECB monetary policy, to try to obtain a wider, detailed sample, in order to obtain a conclusion that could apply to Europe and the Eurozone.

From the 19 of the 28 EU member countries (European Union, 2017), which use the Euro as their official currency and are subject to the ECB monetary policy, 14 are included in this study. The other five Eurozone countries (Cyprus, Estonia, Greece, Latvia, and Malta) were not included in the study, as they did not present any fixed income security with a negative yield, during the study dates of 15 and 16 March of 2017 and 5 May 2017.

At present, the European Union has 28 members (European Union, 2017), and therefore half of the members had negative bond yields at the time of the study dates. Croatia had negative yields for the period of the end of 2016 to the beginning of 2017, although, by 5 May 2017, yields for all maturities were positive. Iceland, Norway and the United Kingdom did not present negative bond yields at that date.

Tables 1 and 2 show the countries included in the study, their date of data access, the corresponding monetary policy institution, the currency, whether the country belongs to the

Table 1. Theoretical and observed values—Countries subject to the ECB monetary policy

Country	Date	Parameters	Theoretical value	Observed value	Δ	Notes	Monetary Policy Institution	Currency	European Union
Austria	03/15/2017	θ_1	2.0225%	-0.1100%	2.1325%	Euribor 12 M	ECB	Euro	Yes
		$\theta_1 + \theta_2$	0.0980%	-0.3540%	0.4520%	Eonia			
Belgium	05/05/2017	θ_1	2.2917%	-0.1240%	2.4157%	Euribor 12 M	ECB	Euro	Yes
		$\theta_1 + \theta_2$	-0.6863%	-0.3570%	-0.3293%	Eonia			
Finland	03/15/2017	θ_1	1.8388%	-0.1100%	1.9488%	Euribor 12 M	ECB	Euro	Yes
		$\theta_1 + \theta_2$	-0.1306%	-0.3540%	0.2234%	Eonia			
France	03/15/2017	θ_1	2.5685%	-0.1100%	2.6785%	Euribor 12 M	ECB	Euro	Yes
		$\theta_1 + \theta_2$	-0.6198%	-0.3540%	-0.2658%	Eonia			
Germany	03/16/2017	θ_1	1.7462%	-0.1110%	1.8572%	Euribor 12 M	ECB	Euro	Yes
		$\theta_1 + \theta_2$	-0.7518%	-0.3540%	-0.3978%	Eonia			
Ireland	05/05/2017	θ_1	2.5090%	-0.1240%	2.6330%	Euribor 12 M	ECB	Euro	Yes
		$\theta_1 + \theta_2$	-0.8571%	-0.3570%	-0.5001%	Eonia			
Italy	05/05/2017	θ_1	-0.4124%	-0.1240%	-0.2884%	Euribor 12 M	ECB	Euro	Yes
		$\theta_1 + \theta_2$	-1.2632%	-0.3570%	-0.9062%	Eonia			
Lithuania	05/05/2017	θ_1	2.9534%	-0.1240%	3.0774%	Euribor 12 M	ECB	Euro	Yes
		$\theta_1 + \theta_2$	0.2070%	-0.3570%	0.5640%	Eonia			
Luxembourg	05/05/2017	θ_1	1.8750%	-0.1240%	1.9990%	Euribor 12 M	ECB	Euro	Yes
		$\theta_1 + \theta_2$	-0.6429%	-0.3570%	-0.2859%	Eonia			
Netherlands	03/15/2017	θ_1	1.5679%	-0.1100%	1.6779%	Euribor 12 M	ECB	Euro	Yes
		$\theta_1 + \theta_2$	0.0971%	-0.3540%	0.4511%	Eonia			

(Continued)

Table 1. (Continued)

Country	Date	Parameters	Theoretical value	Observed value	Δ	Notes	Monetary Policy Institution	Currency	European Union
Portugal	05/05/2017	θ_1	4.7638%	-0.1240%	4.8878%	Euribor 12 M	ECB	Euro	Yes
		$\theta_1 + \theta_2$	-0.2846%	-0.3570%	0.0724%	Eonia			
Slovakia	05/05/2017	θ_1	2.8222%	-0.1240%	2.9462%	Euribor 12 M	ECB	Euro	Yes
		$\theta_1 + \theta_2$	-0.6126%	-0.3570%	-0.2556%	Eonia			
Slovenia	05/05/2017	θ_1	2.8705%	-0.1240%	2.9945%	Euribor 12 M	ECB	Euro	Yes
		$\theta_1 + \theta_2$	-0.6078%	-0.3570%	-0.2508%	Eonia			
Spain	05/05/2017	θ_1	3.4861%	-0.1240%	3.6101%	Euribor 12 M	ECB	Euro	Yes
		$\theta_1 + \theta_2$	-0.0656%	-0.3570%	0.2914%	Eonia			

Table 2. Theoretical and observed values—Countries not subject to the ECB monetary policy.

Country	Date	Parameters	Theoretical value	Observed value	Δ	Notes	Monetary Policy Institution	Currency	European Union
Bulgaria	05/05/2017	θ_1	-2.7195%	0.7822%	-3.5015%	SOFIBOR (Sofia Interbank Offered Rate)	Bulgarian National Bank	BGN	Yes
		$\theta_1 + \theta_2$	0.8329%	-0.4000%	1.2329%	LEONIA (LEV Overnight Index Average) Reference Rate			
Czech Republic	05/05/2017	θ_1	2.8872%	0.0500%	2.8372%	Deposit Facility	Czech National Bank	CZK	Yes
		$\theta_1 + \theta_2$	-1.8763%	0.0500%	-1.9263%	2W repo rate			
Denmark	03/15/2017	θ_1	1.7728%	0.0950%	1.6778%	CIBOR 12M	Denmark National Bank	DNK	Yes
		$\theta_1 + \theta_2$	-0.1107%	-0.4857%	0.3750%	Tomorrow/next (T/N) Rate			
Japan	03/16/2017	θ_1	1.3822%	0.3000%	1.0822%	Basic Discount Rates and Basic Loan Rates	Bank of Japan	Yen	No
		$\theta_1 + \theta_2$	0.0010%	-0.0430%	0.0440%	Average value of Uncollateralised Overnight Call Rate For Mar. 16			
Sweden	03/15/2017	θ_1	2.8118%	-0.3650%	3.1768%	STIBOR Fixing 6M	Sweden National Bank	SEK	Yes
		$\theta_1 + \theta_2$	-0.1883%	-0.5000%	0.3117%	Repo rate			
Switzerland	03/15/2017	θ_1	0.5743%	-0.7300%	1.3043%	3-month LIBOR CHF	Swiss National Bank	CHF	No
		$\theta_1 + \theta_2$	-0.7354%	-0.7300%	-0.0054%	SARON (formerly repo overnight index (SNB))			

European Union, the β_1 and $\beta_1 + \beta_2$ theoretical values (obtained from the fitting process), the observed values, and explanatory notes. Table 1 presents all countries subject to the ECB monetary policy, which use the Euro as their currency. Table 2 displays all the other countries, including Bulgaria, the Czech Republic, Denmark and Sweden, which determine their interest rates independently of the ECB, and are able to control their currency exchange rate (Bulgaria has a fixed exchange rate pegged to the Euro).

In this study, the IIR considered is the overnight rate (in practice, the instantaneous rate can be identified with an overnight forward rate (Svensson, 1994)) supervised by the countries' monetary policy institution. For countries subject to ECB rules, the rate considered is the unsecured overnight lending rate, Eonia® (Euro OverNight Index Average), Retrieved from <https://www.emmi-benchmarks.eu/euribor-eonia-org/eonia-rates.html> and accessed 6 August 2017. Eonia® is the observed value that compares the theoretical obtained from the NSS model.

The definition of a very distant future and its correspondent interest rate for that time horizon is, in a certain way, a not concrete date. Due to the present market situation of the ECB monetary-easing policy, that is intended to run until the end of December 2017 or beyond, if necessary (ECB, 2017b), the rate chosen as the observed value to compare with β_1 was the most time-distant rate at which Euro interbank term deposits are offered, Euribor® 12 months, Retrieved from <https://www.emmi-benchmarks.eu/euribor-org/about-euribor.html> and accessed 6 August 2017.

In Table 2, due to the uniqueness of each country's monetary policy institution, the rates considered to be the benchmark for β_1 (IRVDF) and $\beta_1 + \beta_2$ (IIR) are diversified. For $\beta_1 + \beta_2$, the corresponding overnight rate was chosen, or the repo rate, with the shorter time horizon (a repo rate is the rate at which banks can borrow from their Central bank). Hladíková and Radová (2012) also used the repo rate to compare with the starting value of the estimated forward rate. These two rates are very close to each other (Martellini et al., 2003). Similarly, for β_1 (IRVDF), the corresponding rate equivalent to the country's Euribor was chosen.

As the definition of very distant future is not concrete, two additional possibilities were considered for the theoretical value and observed value, respectively:

- theoretical value: the YTM of the lowest maturity bond (1 year).
- observed value: the YTM of the highest maturity bond.

Tables 3 and 4 show, for the two sets of countries, the fitting results when the yield to maturity of the lowest maturity bond (1 year) is considered as the theoretical value for the IRVDF.

Tables 5 and 6 show the fitting results when the yield to maturity of the highest maturity bond is assumed to be the observed value for the IRVDF.

A descriptive statistical analysis (with the calculation of: mean, median, standard deviation, kurtosis, asymmetry, minimum and maximum) was carried out for the differences of the theoretical and observed values. This exercise, together with a comparison between theoretical and observed values, can help obtain more substantiated conclusions. This analysis was applied to all the study countries, for both the IIR and the IRVDF.

3.2. Analysis

The application of the Solver function to all bonds took into consideration the following conditions: a Generalised Reduced Gradient (GRG) nonlinear algorithm for optimising non-linear problems as the resolution method; a restriction precision value of 10^{-8} (the standard value used by Solver is 10^{-6} , whereby a lower value provides a more precise value, although this increases the time Solver spends to arrive at a solution); the default selection for Solver to use automatic rounding was used;

Table 3. IRVDF (lowest maturity bond)—Countries subject to the ECB monetary policy

Country	Date	Theoretical value (considered as the YTM of the lowest maturity bond—1 year)	Observed value	Δ	Notes	Monetary Policy Institution	Currency	European Union
Austria	03/15/2017	-0.7037%	-0.1100%	-0.5937%	Euribor 12M	ECB	Euro	Yes
Belgium	05/05/2017	-0.6123%	-0.1240%	-0.4883%	Euribor 12M	ECB	Euro	Yes
Finland	03/15/2017	-0.8094%	-0.1100%	-0.6991%	Euribor 12M	ECB	Euro	Yes
France	03/15/2017	-0.5786%	-0.1100%	-0.4686%	Euribor 12M	ECB	Euro	Yes
Germany	03/16/2017	-0.8841%	-0.1110%	-0.7731%	Euribor 12M	ECB	Euro	Yes
Ireland	05/05/2017	-0.4194%	-0.1240%	-0.2954%	Euribor 12M	ECB	Euro	Yes
Italy	05/05/2017	-0.3088%	-0.1240%	-0.1848%	Euribor 12M	ECB	Euro	Yes
Lithuania	05/05/2017	-0.0152%	-0.1240%	0.1088%	Euribor 12M	ECB	Euro	Yes
Luxembourg	05/05/2017	-0.3402%	-0.1240%	-0.2162%	Euribor 12M	ECB	Euro	Yes
Netherlands	03/15/2017	-0.7479%	-0.1100%	-0.6379%	Euribor 12M	ECB	Euro	Yes
Portugal	05/05/2017	-0.1181%	-0.1240%	0.0059%	Euribor 12M	ECB	Euro	Yes
Slovakia	05/05/2017	-0.2671%	-0.1240%	-0.1431%	Euribor 12M	ECB	Euro	Yes
Slovenia	05/05/2017	-0.2533%	-0.1240%	-0.1233%	Euribor 12M	ECB	Euro	Yes
Spain	05/05/2017	-0.3368%	-0.1240%	-0.2128%	Euribor 12M	ECB	Euro	Yes

Table 4. IRVDF (lowest maturity bond)—Countries not subject to the ECB monetary policy

Country	Date	Theoretical value (considered as the YTM of the lowest maturity bond—1 year)	Observed value	Δ	Notes	Monetary Policy Institution	Currency	European Union
Bulgaria	05/05/2017	0.0960%	0.7820%	-0.6860%	SOFIBOR (Sofia Interbank Offered Rate)	Bulgarian National Bank	BGN	Yes
Czech Republic	05/05/2017	-0.4917%	0.0500%	-0.5417%	Deposit Facility	Czech National Bank	CZK	Yes
Denmark	03/15/2017	-0.6339%	0.0950%	-0.7289%	CBOR 12M	Denmark National Bank	DNK	Yes
Japan	03/16/2017	-0.2303%	0.3000%	-0.5303%	Basic Discount and Basic Loan Rates	Bank of Japan	Yen	No
Sweden	03/15/2017	-0.5647%	-0.3650%	-0.1997%	STIBOR Fixing 6M	Swedden National Bank	SEK	Yes
Switzerland	03/15/2017	-0.9485%	-0.7300%	-0.2185%	3-month LIBOR CHF	Swiss National Bank	CHF	No

Table 5. IRVDF (highest maturity bond)—Countries subject to the ECB monetary policy

Country	Date	Parameters	Theoretical value	Observed value (considered as the YTM of the highest maturity bond)	Δ	Monetary Policy Institution	Currency	European Union
Austria	03/15/2017	θ_1	2.0225%	1.8931%	0.1294%	ECB	Euro	Yes
Belgium	05/05/2017	θ_1	2.2917%	2.1217%	0.1700%	ECB	Euro	Yes
Finland	03/15/2017	θ_1	1.8388%	1.3619%	0.4769%	ECB	Euro	Yes
France	03/15/2017	θ_1	2.5685%	2.2526%	0.3158%	ECB	Euro	Yes
Germany	03/16/2017	θ_1	1.7462%	1.2170%	0.5291%	ECB	Euro	Yes
Ireland	05/05/2017	θ_1	2.5090%	1.9774%	0.5317%	ECB	Euro	Yes
Italy	05/05/2017	θ_1	-0.4124%	3.4239%	-3.8363%	ECB	Euro	Yes
Lithuania	05/05/2017	θ_1	2.9534%	0.6983%	2.2551%	ECB	Euro	Yes
Luxembourg	05/05/2017	θ_1	1.8750%	1.3796%	0.4953%	ECB	Euro	Yes
Netherlands	03/15/2017	θ_1	1.5679%	1.2591%	0.3088%	ECB	Euro	Yes
Portugal	05/05/2017	θ_1	4.7638%	4.1513%	0.6125%	ECB	Euro	Yes
Slovakia	05/05/2017	θ_1	2.8222%	1.8597%	0.9625%	ECB	Euro	Yes
Slovenia	05/05/2017	θ_1	2.8748%	2.3451%	0.5297%	ECB	Euro	Yes
Spain	05/05/2017	θ_1	3.4861%	3.1956%	0.2904%	ECB	Euro	Yes

Table 6. IRVDF (highest maturity bond)—Countries not subject to the ECB monetary policy

Country	Date	Parameters	Theoretical value	Observed value (considered as the YTM of the highest maturity bond)	Δ	Monetary Policy Institution	Currency	European Union
Bulgaria	05/05/2017	θ_1	-2.7195%	1.6040%	-4.3235%	Bulgarian National Bank	BGN	Yes
Czech Republic	05/05/2017	θ_1	2.8872%	2.3068%	0.5804%	Czech National Bank	CZK	Yes
Denmark	03/15/2017	θ_1	1.7728%	1.1336%	0.6391%	Denmark National Bank	DNK	Yes
Japan	03/16/2017	θ_1	1.3822%	0.9289%	0.4533%	Bank of Japan	Yen	No
Sweden	03/15/2017	θ_1	2.8118%	1.7023%	1.1095%	Sweden National Bank	SEK	Yes
Switzerland	03/15/2017	θ_1	0.5743%	0.4627%	0.1116%	Swiss National Bank	CHF	No

the value chosen for the Convergence (value between 0 and 1) was 10^{-8} , which defines the upper limit for the relative change in the destiny cell, for the last five iterations; a criteria for Solver to stop (i.e. if during the last five iterations the relative change in the value of the destination cell is less than $10^{-6}\%$, then Solver stops trying to converge even more) (Microsoft, 2017a).

The results obtained with direct differentiation (default on Solver) for all yield curves fitting computation were very good.

Solver uses a Generalised Reduced Gradient algorithm for optimising non-linear problems (Microsoft, 2017b), which provides a locally optimal solution for a reasonably well scaled, non-convex model (Frontline Systems, 2017b). Function f is convex, if the function f is below any line segment between two points on f (Tomioka, 2012).

The starting values for $\beta_{1,2,3,4}$ and $\gamma_{1,2}$ should be in or as near as possible, the order of magnitude of the expected values. Values near or below 0.01 for β_i and 1 to γ_j were used. After the first solution provided by Solver, the parameters values were submitted to small changes and the Solver function was ran again, in order to obtain an SSR as low as possible. Only when Solver provided the message that after five iterations the fitting curve had not changed, was that solution considered as the final one. No restrictions were applied to any of the values that $\beta_{1,2,3,4}$ and $\gamma_{1,2}$ assumed.

Theoretical and observed IRVDF and IIR can be compared in Figures 1 and 2. The other two possibilities are depicted in Figures 3 and 4.

When modelling the entire yield curve using the NSS model to access all the market yields to obtain SSR or when modelling the entire yield curve with part of the market data available (i.e. the cases of short term, intermediate and long-term bonds maturities), the parameters $\beta_{1,2,3,4}$ and $\gamma_{1,2}$ could take any value, as no restriction was applied to them. The parameters values obtained for each country are shown in the appendix in Table A1 (NSS model using all market yields available), Table A2 (short-term maturities forecast, or simply STF), Table A3 (intermediate-term maturities forecast or simply, intermediate-term forecast (ITF)), and Table A4 (long-term maturities forecast

Figure 1. Theoretical and observed IIR.

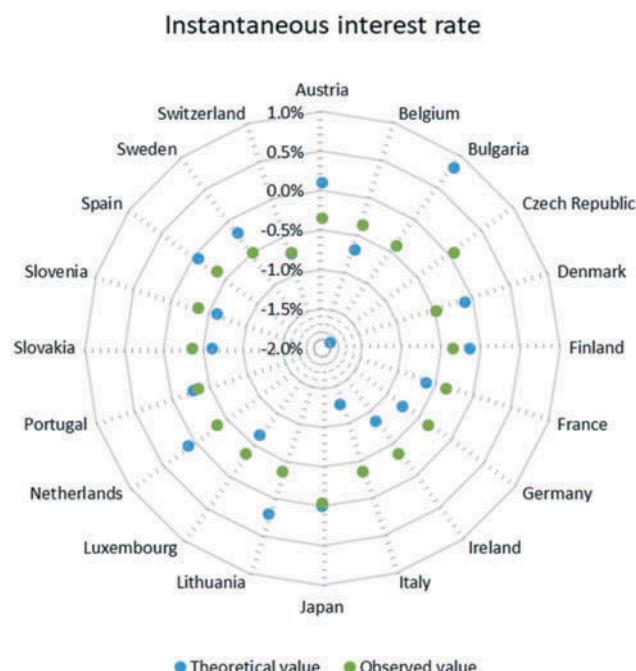


Figure 2. IRVDF (with observed value considered as Euribor 12 M).

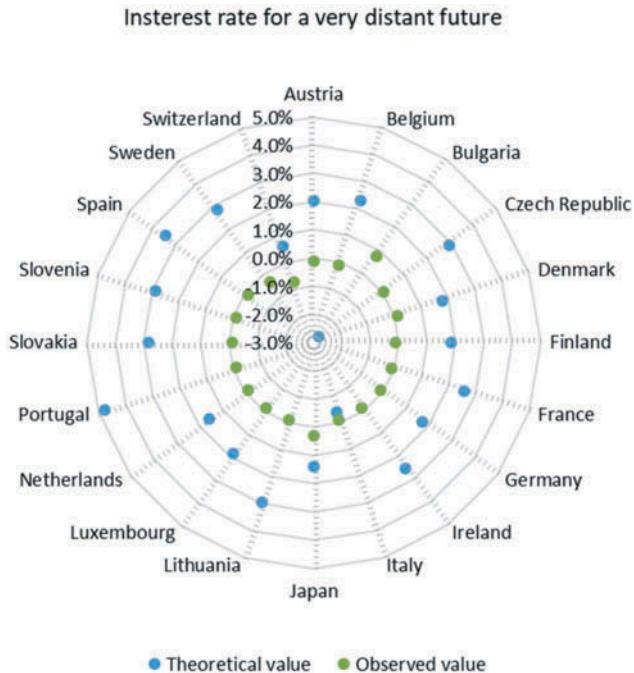
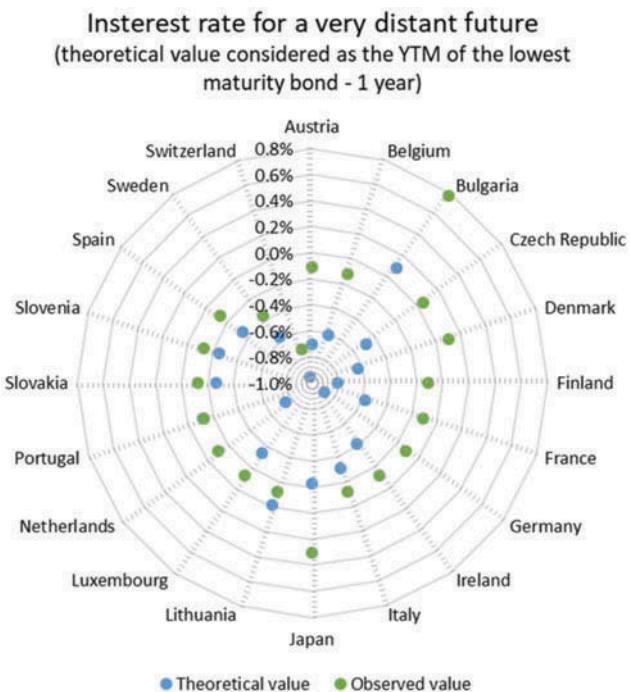


Figure 3. IRVDF (with theoretical value considered as the YTM of the lowest maturity bond (1 year)).



or simply, long-term forecast (LTF)). In addition, Figures A1 to A80, in the appendix, present each case for each of the 20 countries.

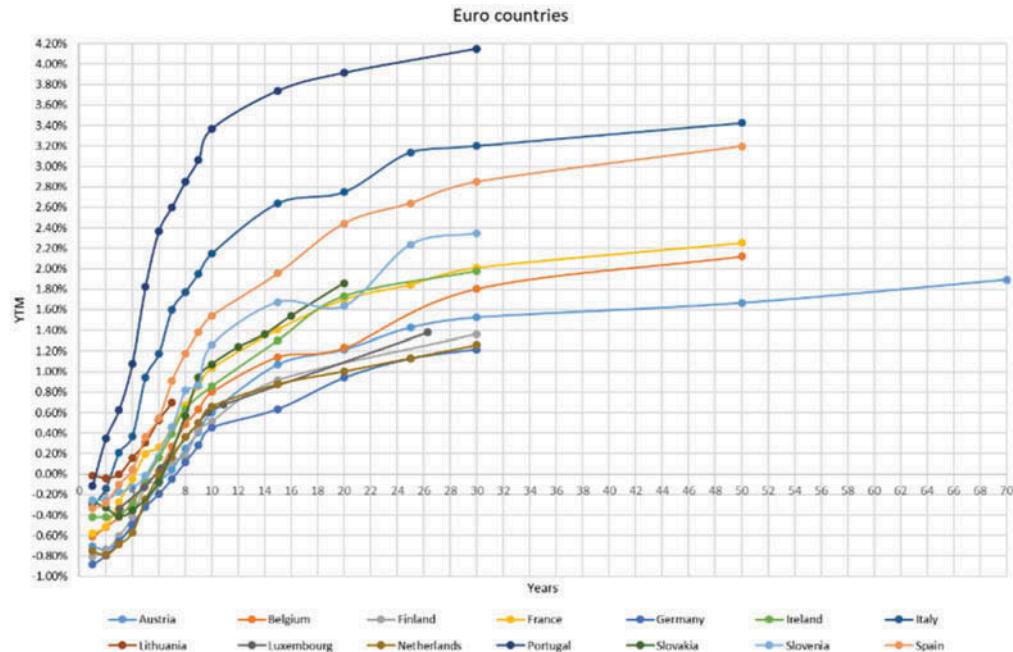
As the majority of countries in the study are from Europe, we compared all yield curves for these issuers (Figure 5). The spectrum of maturities that each country chooses or can have access to, in the market, is very different, as are the yields that each can have. The differences

Figure 4. IRVDF (with observed value considered as the YTM of the highest maturity bond).

Insterest rate for a very distant future
 (observed value considered as the YTM of the highest maturity bond)



Figure 5. Selected European countries yield curves.



for the yield curves are related to the premiums required by the market and they are dependent on ratings, political risk, GDP growth, debt levels, and economic development, among other variables.

The 10-year maturity bonds yield is one of the most used and widely compared one in financial markets. For the set of European countries, only Lithuania did not have maturities higher than 7 years, and thus it cannot be compared with its fellow European countries.

As a theoretical exercise, if the Eurozone countries eventually agreed on a shared debt security (i.e. Eurobonds), bonds with 10-year maturities could be issued at an initial phase, with higher maturities (>10 years) being just the choice of each country. Figure 6 shows this set of countries (without Lithuania) and their yield curves.

For the Eurozone countries, it was analysed whether the differences between the theoretical and observed rates values, for β_1 (IRVDF), could be explained by the rate difference that each country has in comparison to Germany (as Germany has the highest credit rating and its Sovereign Country Default Spreads (CDS), net of US, is 0.00%), using the Moody's credit ratings, for each country. This is Retrieved from http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ctryprem.html and accessed date: 10 June 2017.

Figure 7 shows the differences between the theoretical and observed interest rates values, for β_1 (IRVDF), for two interpretations of the very distant future. The first difference is the comparison between Sovereign CDS, net of US (or net of Germany, as both have the same value) (blue bar), and the observed value for β_1 , considered as the YTM of the highest maturity bond (green bar). For example, for Portugal, the difference is 2.9342%, which means that the YTM of its highest maturity bond is 2.9342% higher than the YTM of the highest maturity bond of Germany, with the relation with the Sovereign CDS, net of US.

Figure 6. Selected European countries yield curves (maturities until 10 years).

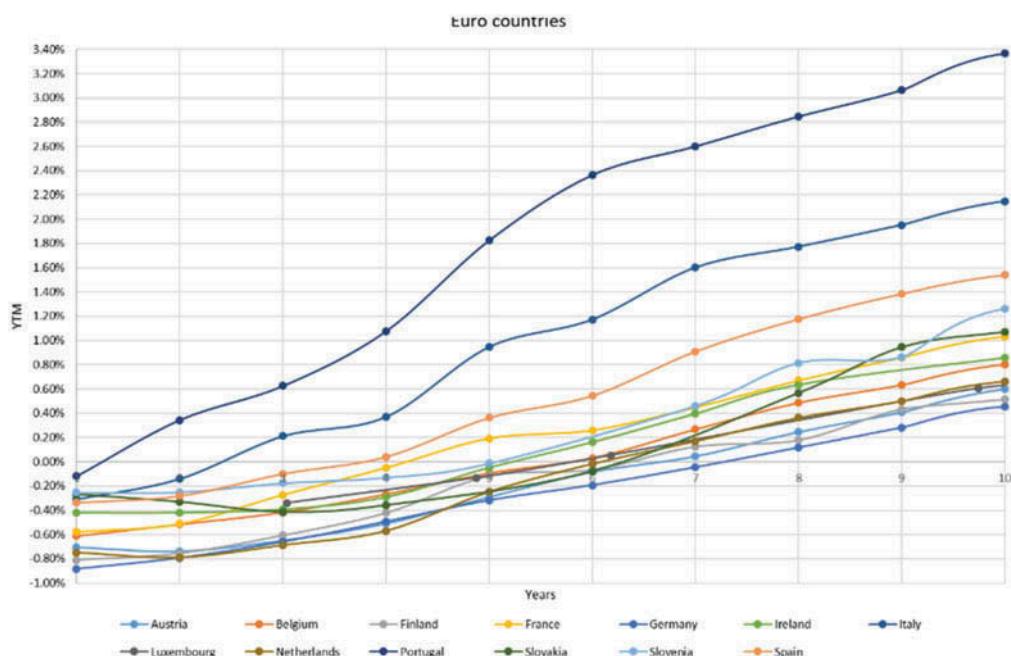
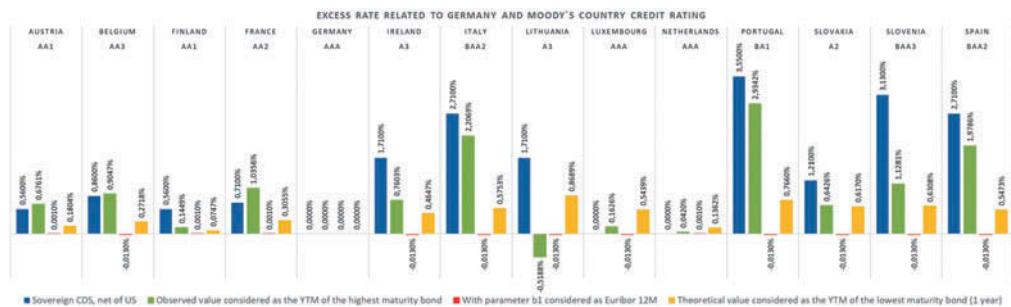


Figure 7. Interest rate difference—Countries in comparison to Germany.



The second difference is between the observed value for the β_1 parameter (considered as Euribor at 12 Months) and Germany's observed value (also, Euribor at 12 Months); and the difference between the theoretical value for β_1 (considered as YTM of the lowest maturity bonds, 1 year) of each country and the correspondent value of Germany.

4. Results and discussion

The NSS model fitting process, with no restrictions on the parameters values, adjusts the yield curve well for the wide variety of countries and range of maturities.

The values obtained for β_1 and $\beta_1 + \beta_2$ interpreted as IIR and IRVDF, respectively, show that theoretical and observed values are closer to each other for the IIR, than for the IRVDF, which presents a wider difference.

If the observed value for the IRVDF is considered as the highest maturity of the YTM, then the values are very similar to the theoretical ones. Specifically, the difference rate, in comparison to Germany, can be almost fully explained.

The difference between theoretical and observed IIR, for all countries, seems to have a normal distribution (kurtosis = 3.14), with a mean of -0.055%, a median of 0.019%, a standard deviation of 0.644%, a minimum of -1.926%, and a maximum of 1.233%. These results show a very wide range, which is probably influenced by different monetary policies. Indeed, when only the countries subject to the ECB monetary policy are considered, a platykurtic distribution is suitable (kurtosis = -0.67), with a mean of -0.081%, a median of -0.251%, a standard deviation of 0.429%, a minimum of -0.906%, and a maximum of 0.564%, which represents a shorter range, suggesting the same monetary policy.

On the other hand, the difference between theoretical and observed IRVDF, for all countries, suggests to have a leptokurtic distribution (kurtosis = 5.92), with a mean of 2.058%, a median of 2.274%, a standard deviation of 1.688%, a minimum of -3.501%, and a maximum of 4.888%, showing significant dispersion. Again, when only the countries subject to the ECB monetary policy are analysed, a platykurtic distribution is obtained (kurtosis = 2.69), with a mean of 2.470%, a median of 2.524%, a standard deviation of 1.154%, a minimum of -0.288%, and a maximum of 4.888%, which also shows a wide range.

The NSS model theoretical values for β_1 (IRVDF) are generally the value of the yield of the longest maturity in the yield curve (except for the extreme cases of Bulgaria, Italy, Lithuania and Sweden). To a certain degree, this is the most very distant future that is available for each country, and therefore, if the highest maturity for each country is the market interpretation of very distant future, then the model provides good values. Otherwise, if for very distant future one considers the one-year time frame, then the model is not so good.

The results for short, intermediate, and long-term forecasts, were also obtained. The short-term forecast shows that the model has difficulty in fitting the yield curve, given that the beginning of the yield curves is less smooth than the intermediate and long terms. Furthermore, negative yields appear in the shorter term. The intermediate and long-term forecasts show very acceptable fitting results, revealing that the NSS model can adjust for the entire curve in some cases and very few maturities.

The idea of issuing shared debt security (i.e. Eurobonds) is analysed. The findings indicate that the market would lower the risk premium and the yields for the most stressed countries (those that show higher yields). For the lower risk premium issuers, this initiative will increase yields. Since all countries share the risk, these risk premiums are thus reflected in yields, which could be a price to pay to obtain a more equal and less stressful financial system in the Eurozone. Indeed, the

evaluation of interest rate differences in comparison to Germany, reveals noticeable values, for the majority of countries considered.

5. Conclusion

The application of the NSS model to 20 countries with negative yields gives good estimates of the entire yield curves, fitting the data well. The methodology used is friendly and can be used as a simple and widely available tool.

The forecast of the IIR seems to be good, as the differences between theoretical and observed values appear to be small. If the IRVDF is considered to be the rate at the highest bond maturity, then the model presents good values.

The interpretation of the parameters of the NSS model seems to be adequate.

In the case of countries subject to the ECB monetary policy, the interest rate is defined by the ECB, however, in practice, European countries in the Eurozone are very different in essence (e.g. economic models, debt levels, financial history, weight, and importance on financial markets). Accordingly, all the countries are expected to have the same rates from the model, which seems not to be a realistic hypothesis. It can be concluded that rates should not all be the same, as the market requests a country risk premium for each rate, which is related to their ratings, debt level, GDP, national budgets and deficits, and political risk, among other factors. If the Eurozone countries had the same debt securities, such as Eurobonds, then rates would be the same, and the yield curve would be only one, and therefore the expected rate values obtained using the NSS model would be more precise and a good proxy for the market participants.

The difference rate in comparison to Germany, calculated from Moody's ratings and the corresponding Sovereign CDS, net of US, for countries subject to the ECB monetary policy, can be explained from the model parameters when considering the IRVDF to be the yield to maturity of the highest maturity for that country. The countries that presented a difference higher than 1%, are Ireland, Lithuania and Slovenia.

The forecast outputs show good fitting data for real values for both intermediate-term and long-term maturities. In the case of short-term maturity, forecast values are not as accurate as expected, which leads to the conclusion that, in this case, it is not a good model. The reasons for this can be the instability of monetary policy and the volatility of short-term interest rates.

In conclusion, the NSS model seems to remain a valuable tool to fit yield curves with negative yields, available for monetary policy institutions and market players alike. Further research could analyse the performance of the NSS model using longitudinal data.

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Appendix

Table A1. NSS model $\theta_{1,2,3,4}$ and $\gamma_{1,2}$ factors (fitting the entire yield curve)

	θ_1	θ_2	θ_3	θ_4	γ_1	γ_2
Austria	0.020225	-0.019245	-0.120936	-0.076041	0.089915	1.791626
Belgium	0.022917	-0.029780	-0.885637	-0.074036	0.017113	1.960027
Bulgaria	-0.027195	0.035524	-0.090480	0.184881	2.357249	6.002618
Czech Republic	0.028872	-0.047634	-0.000031	-0.080808	0.587137	2.992772
Denmark	0.017728	-0.018835	-0.103959	-0.069599	0.071836	1.643519
Finland	0.018388	-0.019695	-0.028984	-0.044361	0.762873	2.345685
France	0.025685	-0.031882	0.002701	-0.038365	2.496532	2.566768
Germany	0.017462	-0.024980	-0.026692	-0.018188	1.736145	3.919552
Ireland	0.025090	-0.033661	-0.046439	-0.082346	0.172151	1.845687
Italy	-0.004124	-0.008508	-0.075057	0.128453	0.040498	19.122517
Japan	0.013822	-0.013812	-0.024702	-4.345566	4.373515	0.000334
Lithuania	0.029534	-0.027464	0.046821	-0.066374	5.818697	3.420608
Luxembourg	0.018750	-0.025178	-0.310594	-0.067273	0.019172	1.841284
Netherlands	0.015679	-0.014708	-0.064738	-0.069723	0.087805	1.341456
Portugal	0.047638	-0.050484	-0.219748	-0.125183	0.062279	1.106536
Slovakia	0.028222	-0.034348	-0.198596	-0.095590	0.054860	1.893719
Slovenia	0.028748	-0.034780	-0.213193	-0.088685	0.059622	1.919800
Spain	0.034861	-0.035517	-0.240889	-0.100771	0.068538	1.810130
Sweden	0.028118	-0.030002	-1.285713	-0.071285	0.026309	2.652613
Switzerland	0.005743	-0.013097	-0.026070	-0.000303	1.632627	0.002583

Figure A1. Austria market and NSS yield curve (15 March 2017).

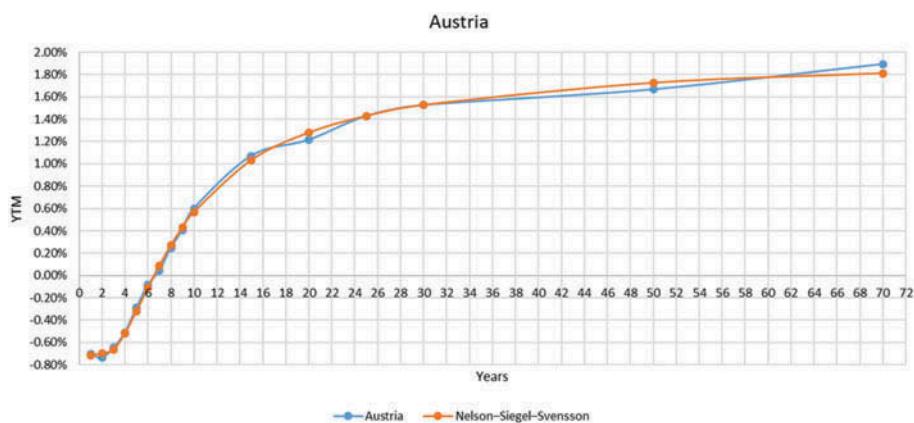


Figure A2. Belgium market and NSS yield curve (5 May 2017).

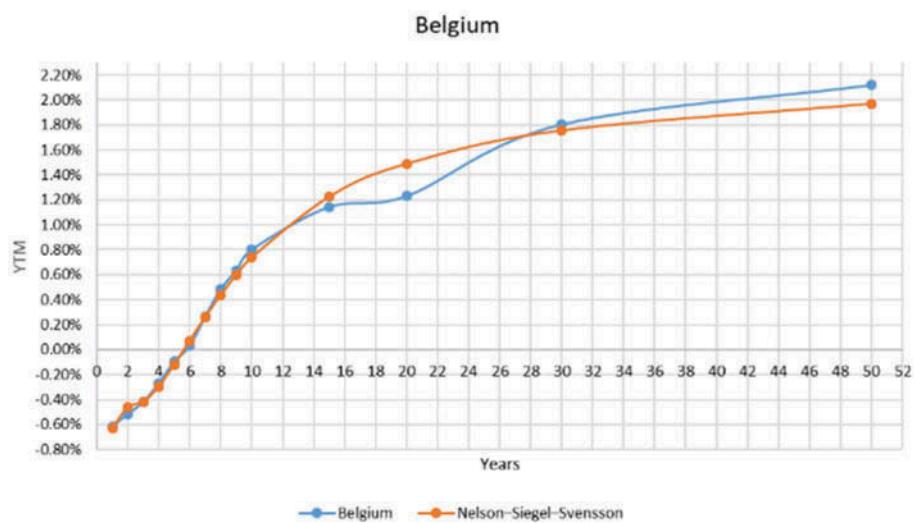


Figure A3. Bulgaria market and NSS yield curve (5 May 2017).

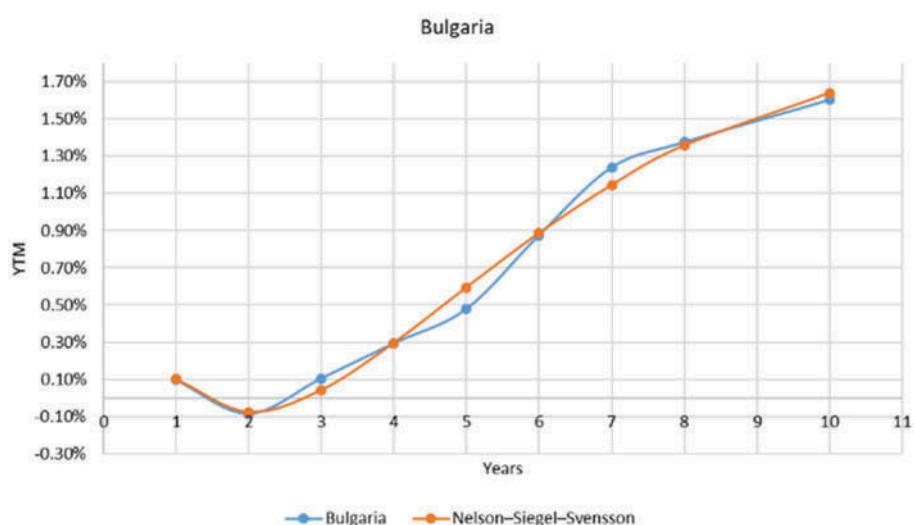


Figure A4. The Czech Republic market and NSS yield curve (5 May 2017).

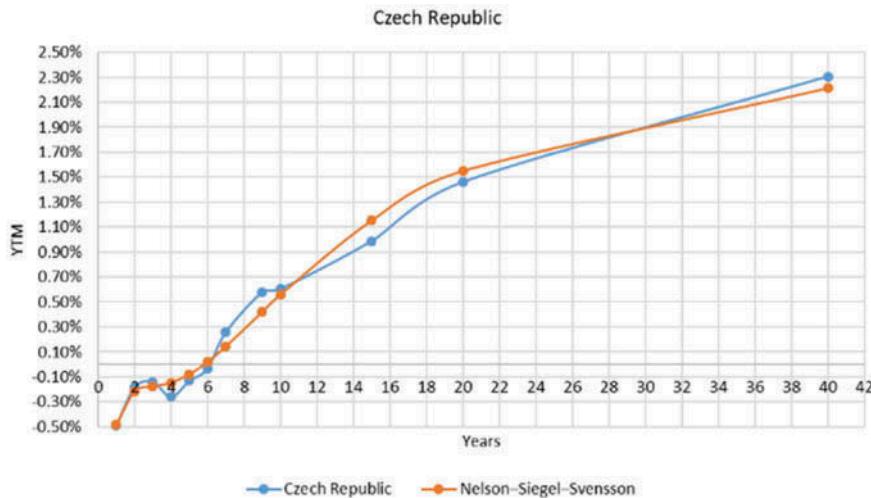


Figure A5. Denmark market and NSS yield curve (15 March 2017).

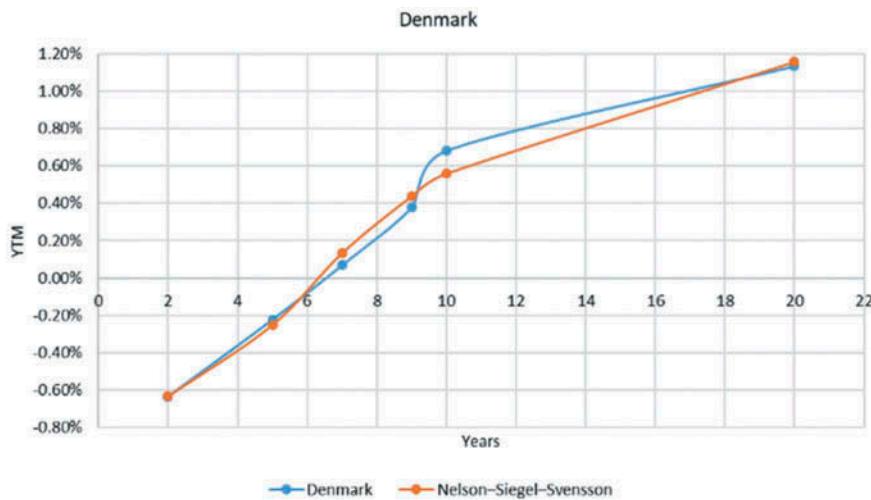


Figure A6. Finland market and NSS yield curve (15 March 2017).

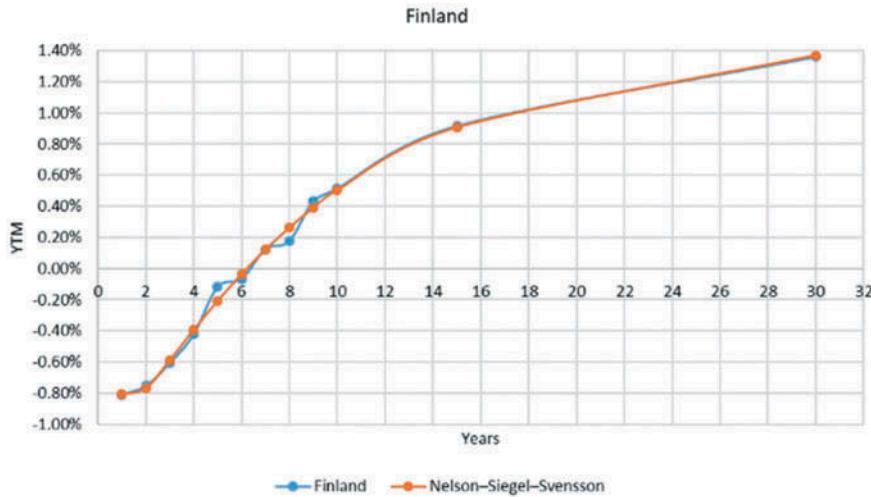


Figure A7. France market and NSS yield curve (15 March 2017).

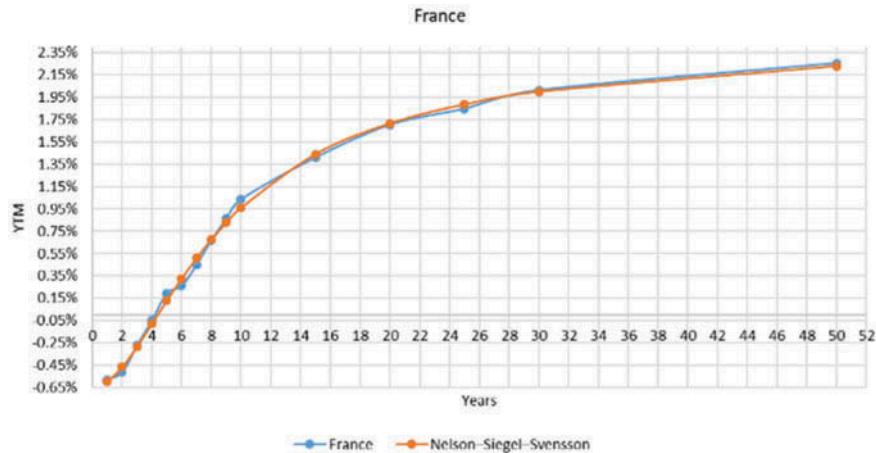


Figure A8. Germany market and NSS yield curve (16 March 2017).

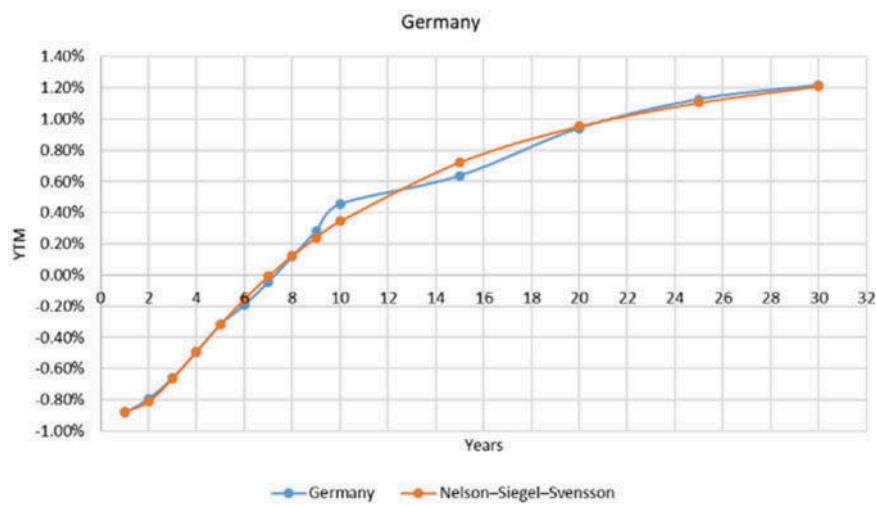


Figure A9. Ireland market and NSS yield curve (5 May 2017).

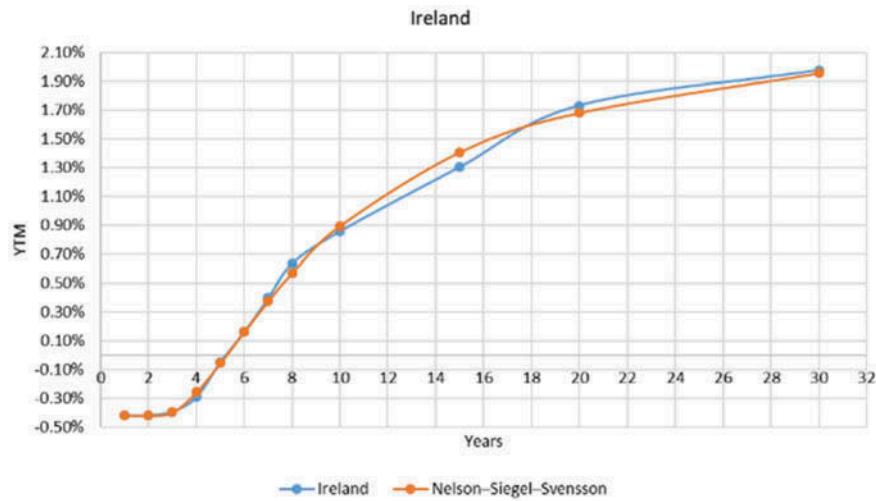


Figure A10. Italy market and NSS yield curve (5 May 2017).

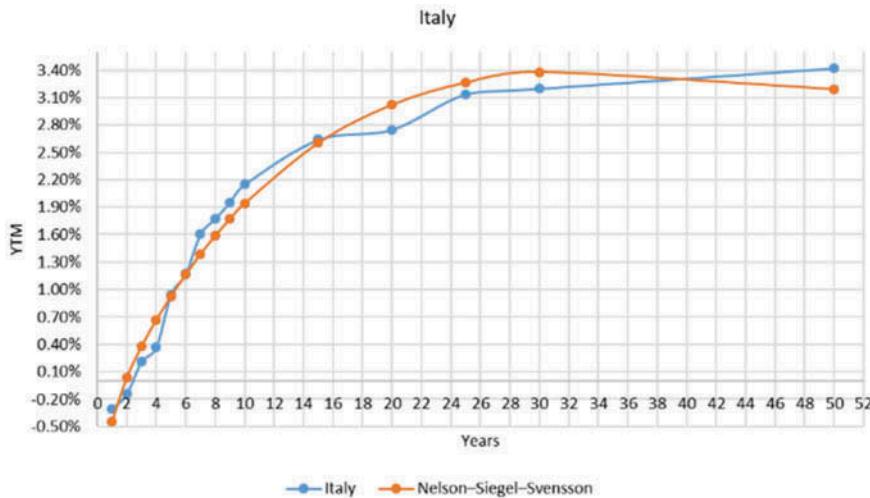


Figure A11. Japan market and NSS yield curve (16 March 2017).

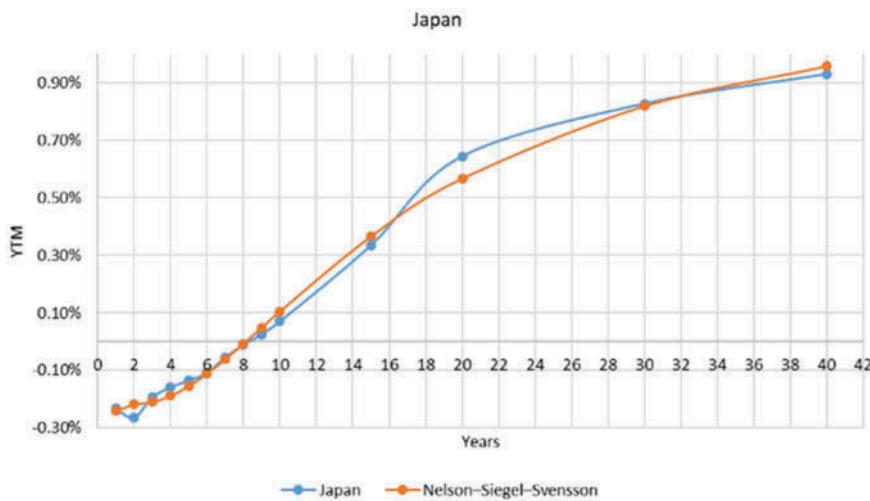


Figure A12. Lithuania market and NSS yield curve (5 May 2017).

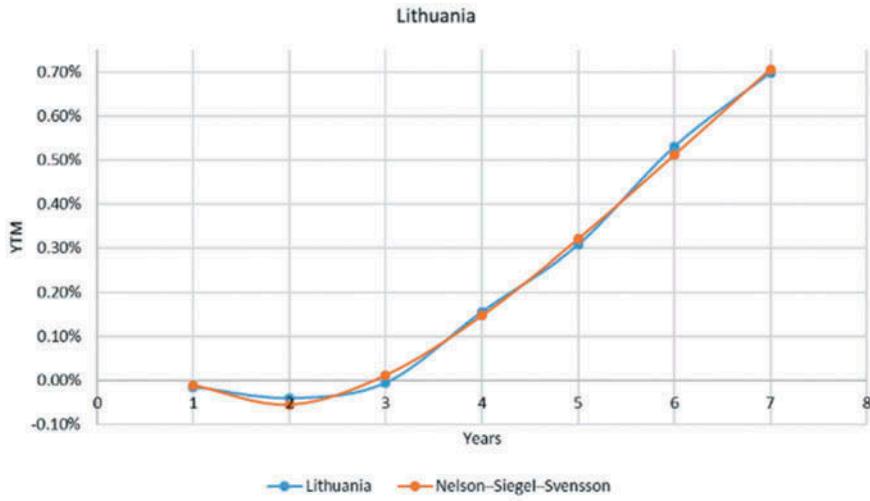


Figure A13. Luxembourg market and NSS yield curve (5 May 2017).

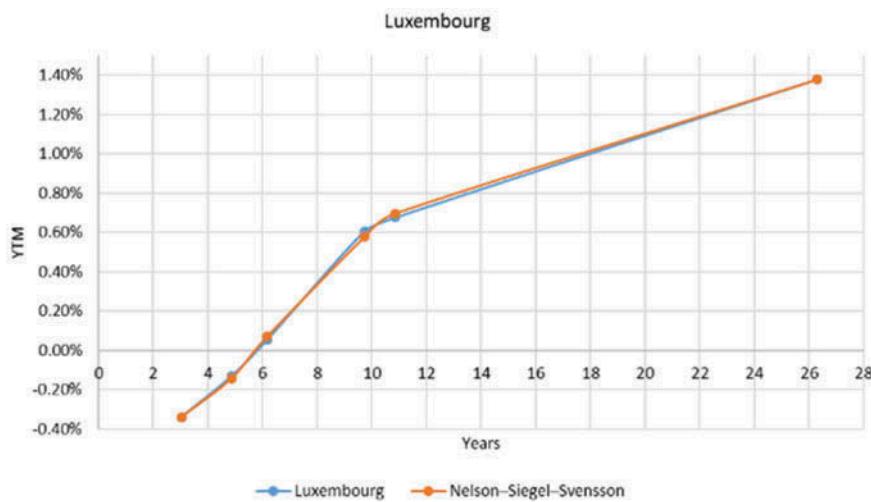


Figure A14. The Netherlands market and NSS yield curve (15 March 2017).

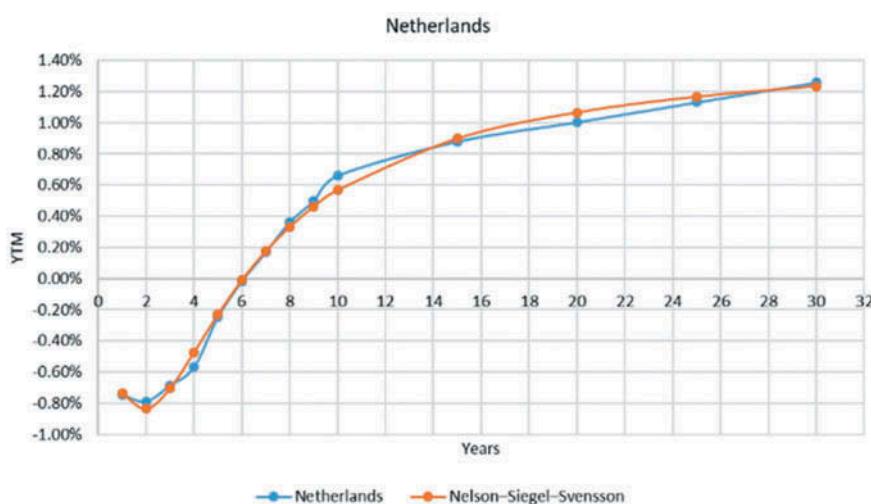


Figure A15. Portugal market and NSS yield curve (5 May 2017).

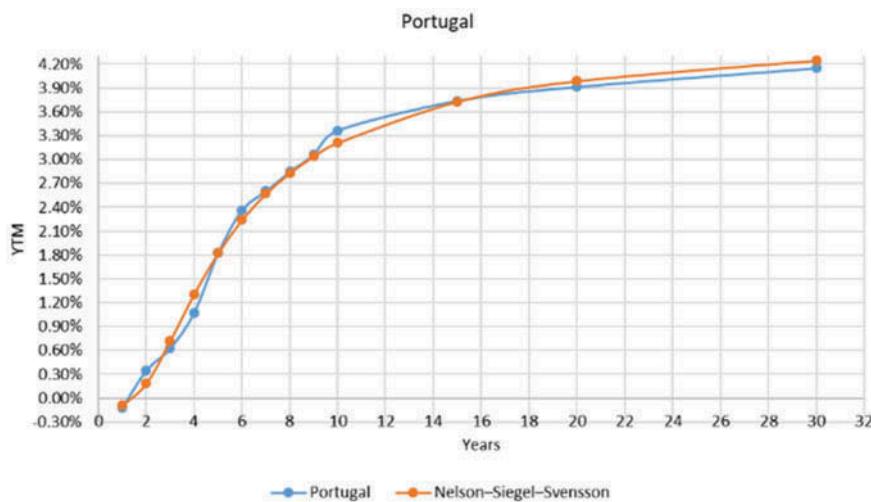


Figure A16. Slovakia market and NSS yield curve (5 May 2017).

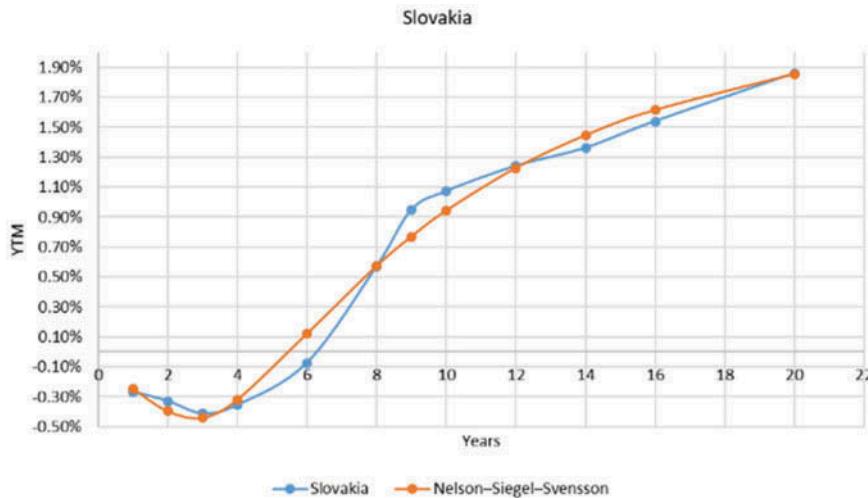


Figure A17. Slovenia market and NSS yield curve (5 May 2017).

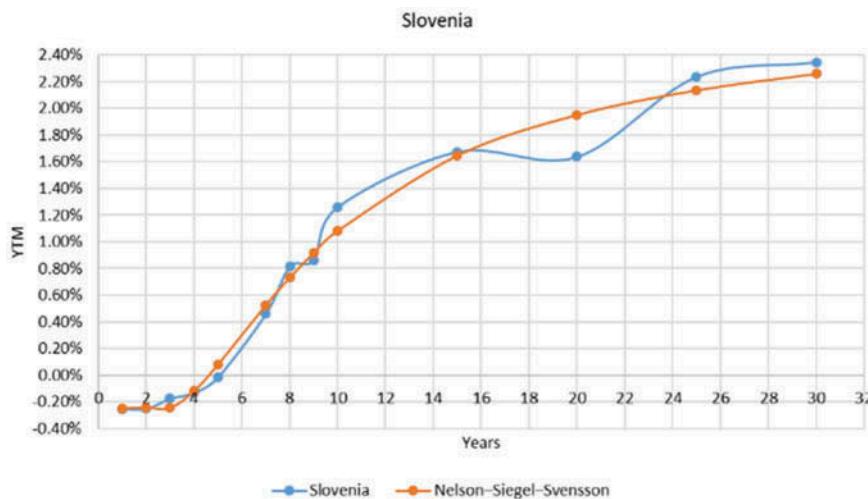


Figure A18. Spain market and NSS yield curve (5 May 2017).

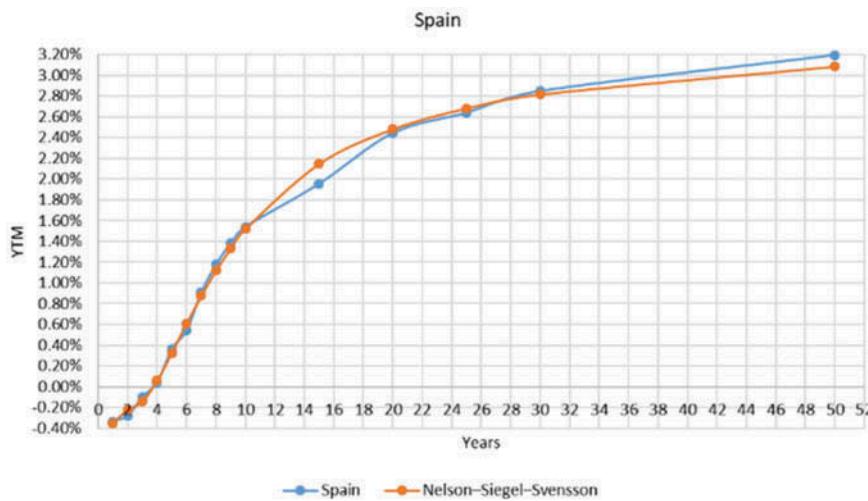


Figure A19. Sweden market and NSS yield curve (15 March 2017).

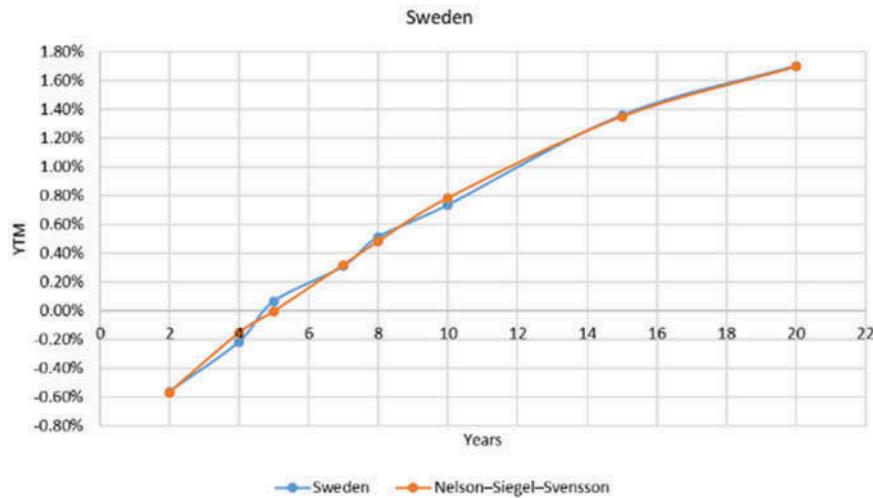


Figure A20. Switzerland market and NSS yield curve (15 March 2017).

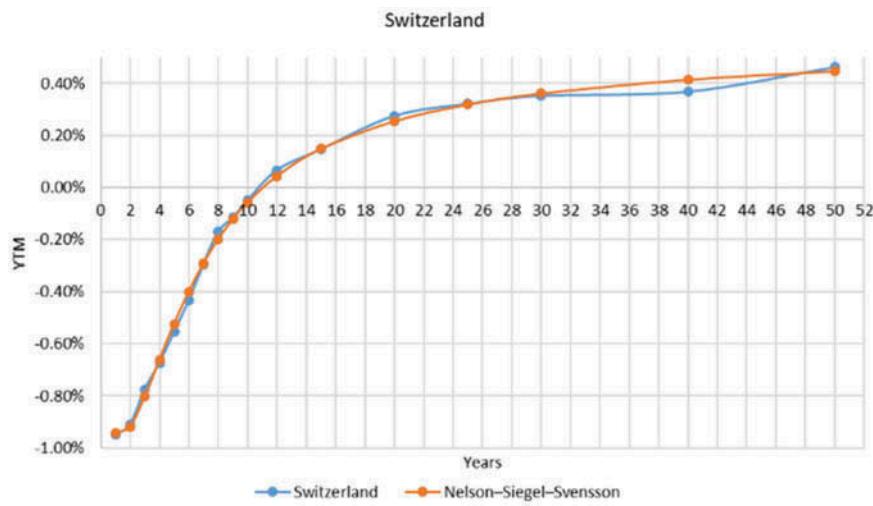


Table A2. NSS model $\theta_{1,2,3,4}$ and $\gamma_{1,2}$ factors (short term maturities forecast)

	θ_1	θ_2	θ_3	θ_4	γ_1	γ_2
Austria	0.020361	-0.008210	-0.066401	-0.029749	1.920081	0.304698
Belgium	0.025376	-0.037959	0.000077	0.000356	6.074789	0.000071
Bulgaria	0.030944	-0.013937	-0.000103	-0.082933	1.466186	1.491495
Czech Republic	0.011306	-0.017488	-0.026006	0.068027	8.156023	21.630683
Denmark	0.017778	-0.030000	-0.000051	-0.076807	0.009986	1.607286
Finland	0.020819	-0.033745	-0.017979	-2.010245	3.274978	19.997235701
France	0.025718	-0.030718	0.002344	-0.040094	2.463138	2.549057
Germany	0.016590	-0.026660	-0.000508	-0.027134	2.521240	2.616770
Ireland	0.025208	-0.032391	-0.065830	-0.078686	0.175415	1.908317
Italy	0.036929	-0.055425	-0.847133	0.806172	1.054440	0.987306
Japan	0.000566	-0.002726	0.291586	-0.270061	11.485179	10.347461
Lithuania	0.026793	-0.023638	0.036527	-0.072303	3.530776	2.682399
Luxembourg	0.018728	-0.019993	-0.308232	-0.070849	0.009939	1.783962
Netherlands	0.014816	-0.002200	0.220139	-0.289188	1.927579	1.744006
Portugal	0.046880	-0.386878	1.638792	-1.236006	0.489257	0.612912
Slovakia	0.026371	-0.060242	1.051857	-1.042339	0.917185	1.031749
Slovenia	0.027932	0.621663	-0.392937	-1.359934	1.232878	0.330309
Spain	0.035156	-0.035580	-0.251233	-0.094009	0.089443	1.937823
Sweden	0.029016	-0.013191	-1.083106	-0.077766	0.023536	2.750295
Switzerland	0.005696	-0.010164	-0.032022	0.117770	1.498992	0.003039

Figure A21. Austria market and NSS yield curve (15 March 2017)—STF.

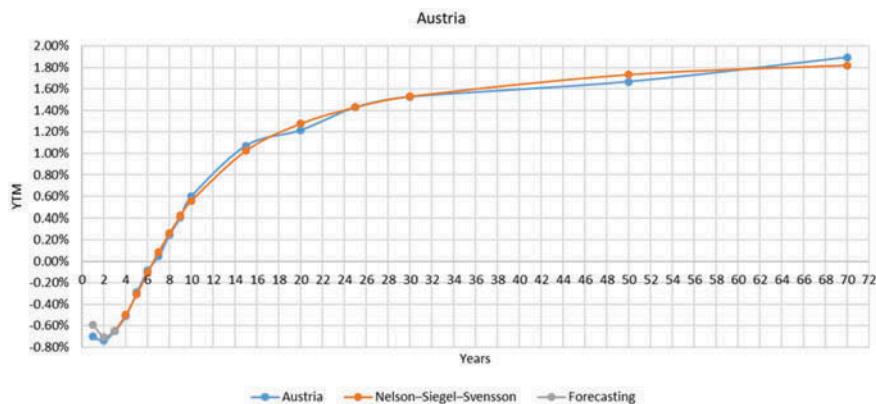


Figure A22. Belgium market and NSS yield curve (5 May 2017)—STF.

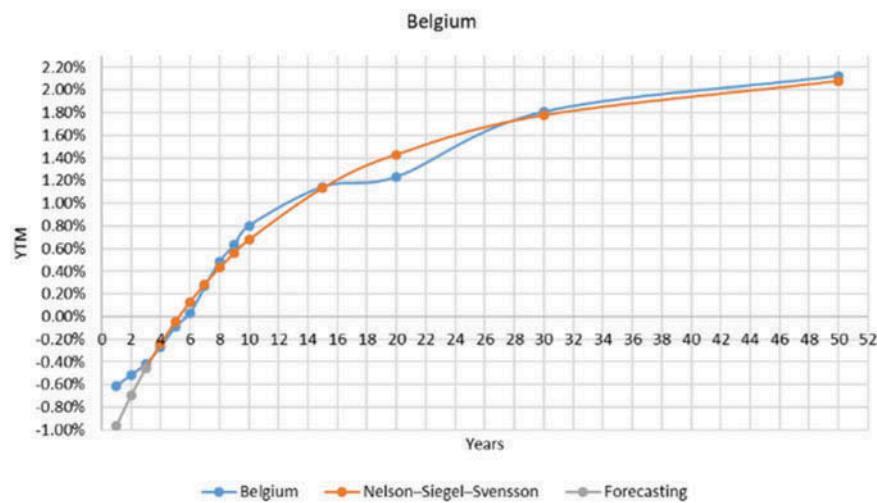


Figure A23. Bulgaria market and NSS yield curve (5 May 2017)—STF.

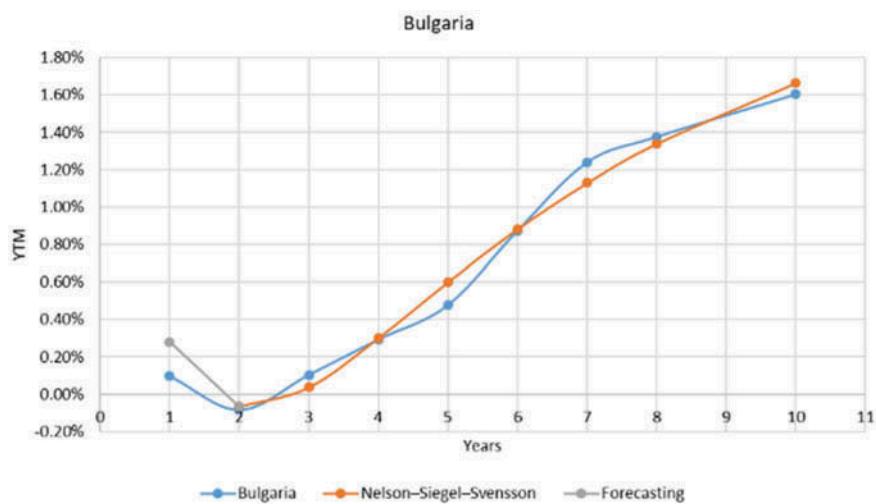


Figure A24. The Czech Republic market and NSS yield curve (5 May 2017)—STF.

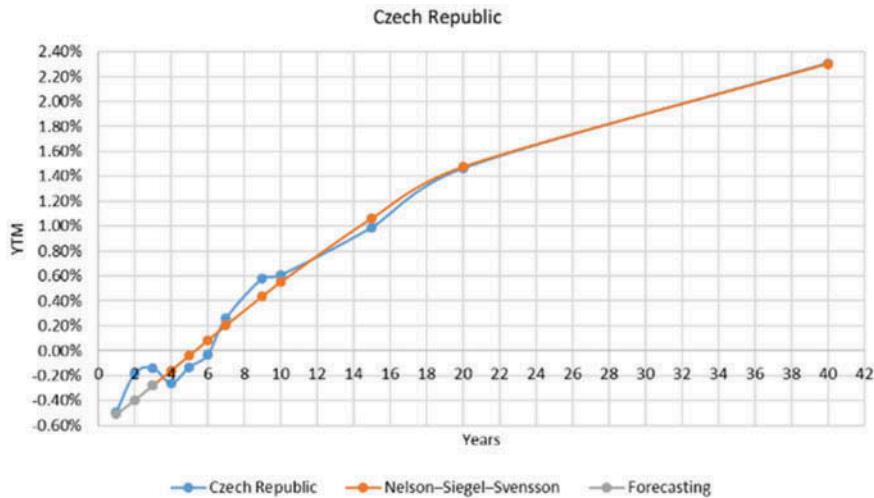


Figure A25. Denmark market and NSS yield curve (15 March 2017)—STF.

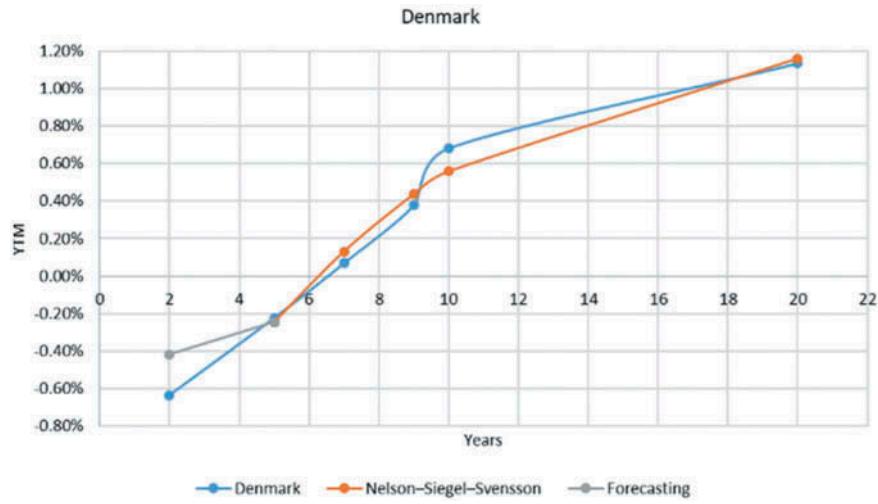


Figure A26. Finland market and NSS yield curve (15 March 2017)—STF.

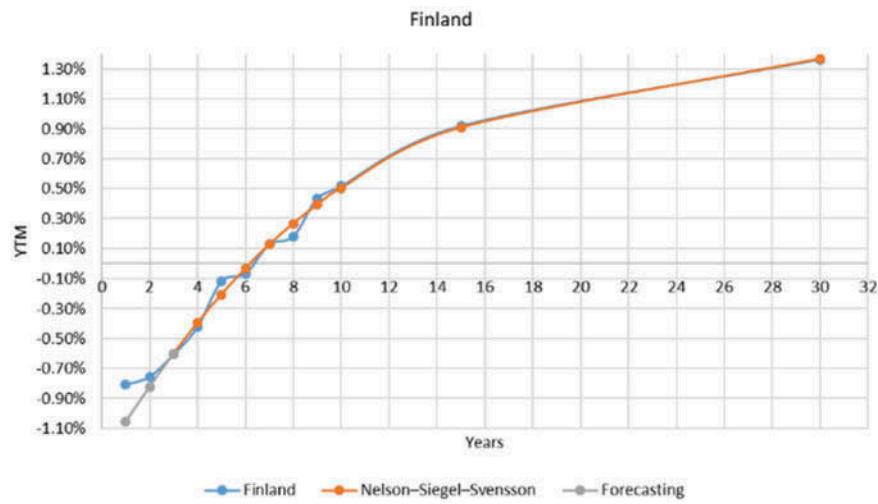


Figure A27. France market and NSS yield curve (15 March 2017)—STF.

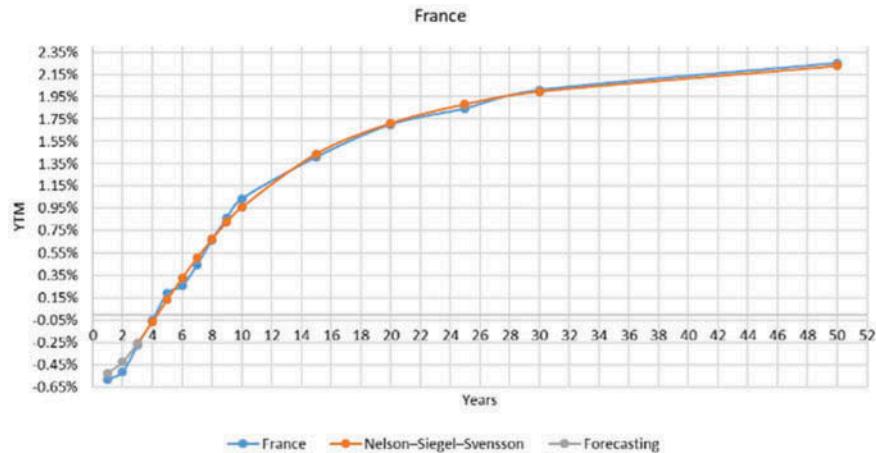


Figure A28. Germany market and NSS yield curve (16 March 2017)—STF.

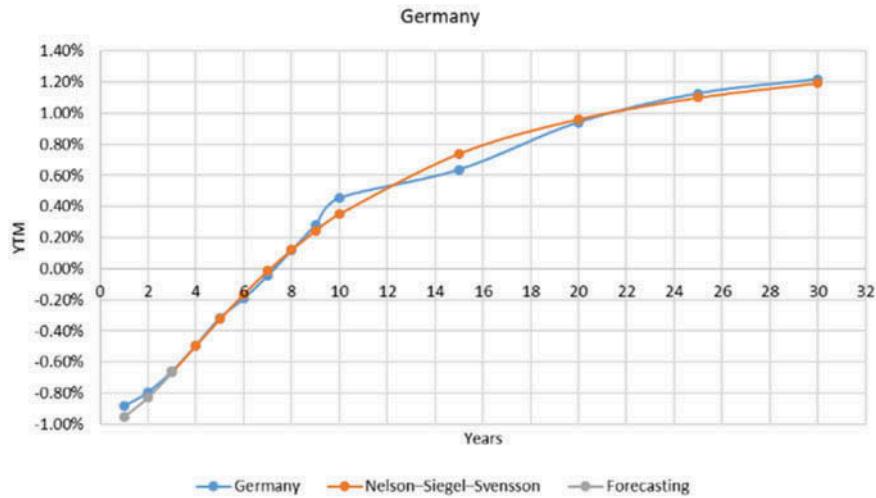


Figure A29. Ireland market and NSS yield curve (5 May 2017)—STF.

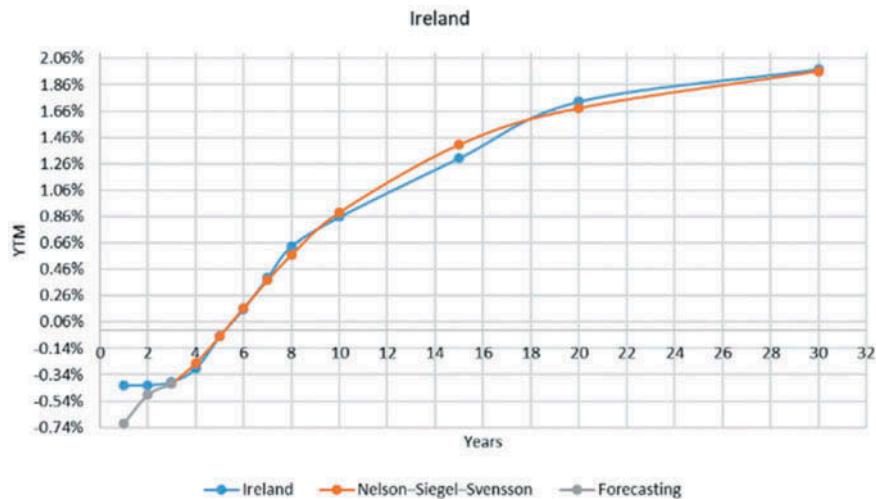


Figure A30. Italy market and NSS yield curve (5 May 2017)—STF.

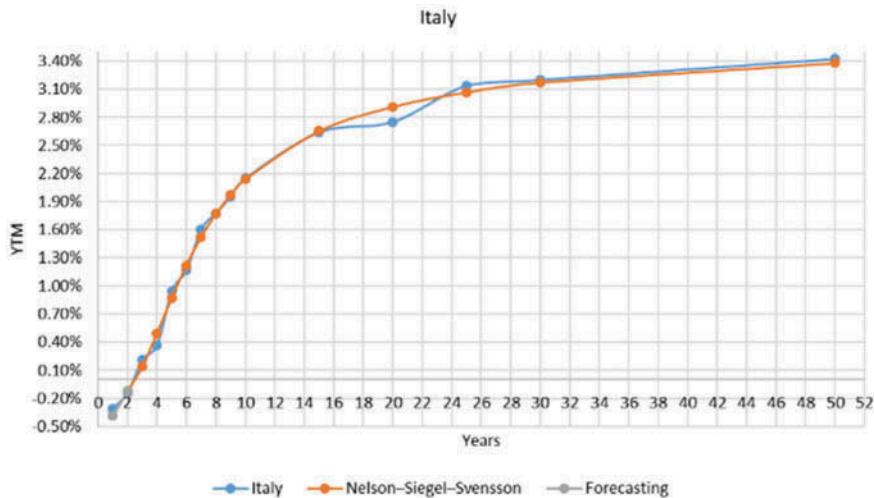


Figure A31. Japan market and NSS yield curve (16 March 2017)—STF.

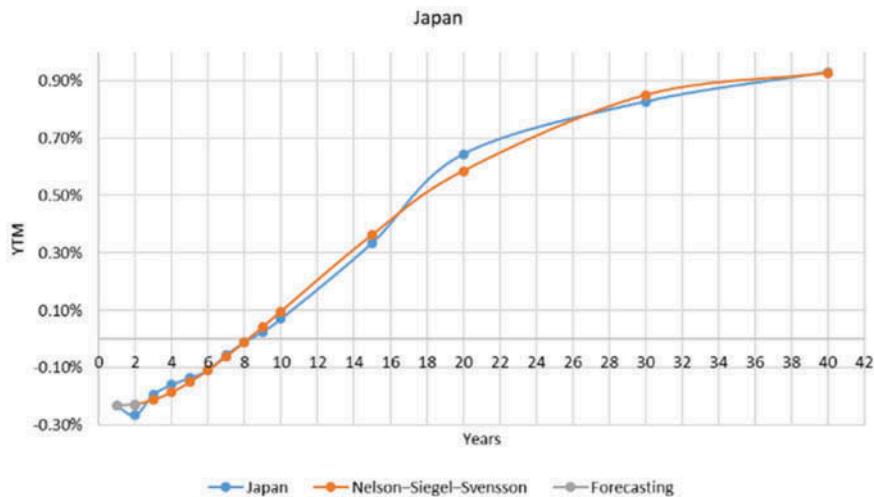


Figure A32. Lithuania market and NSS yield curve (5 May 2017)—STF.

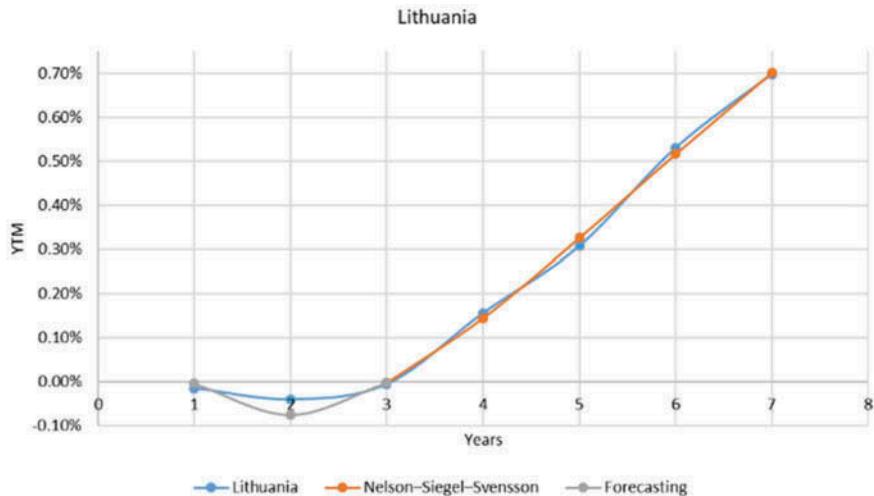


Figure A33. Luxembourg market and NSS yield curve (5 May 2017)—STF.

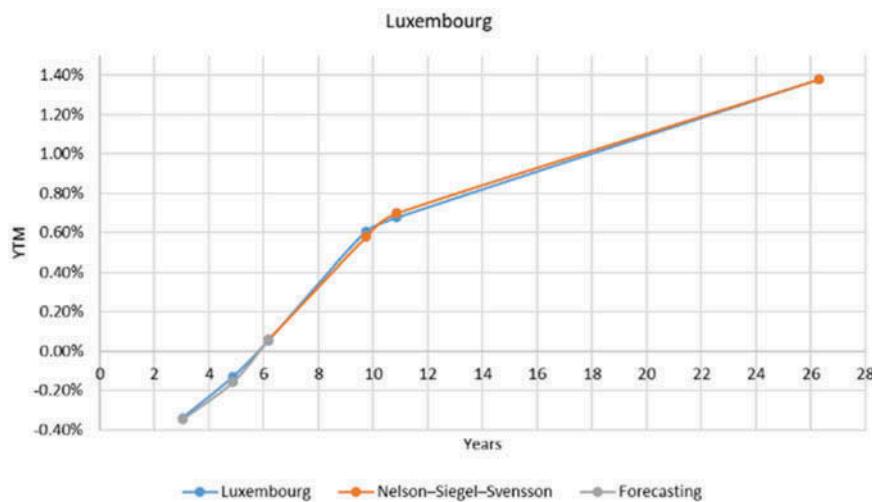


Figure A34. The Netherlands market and NSS yield curve (15 March 2017)—STF.

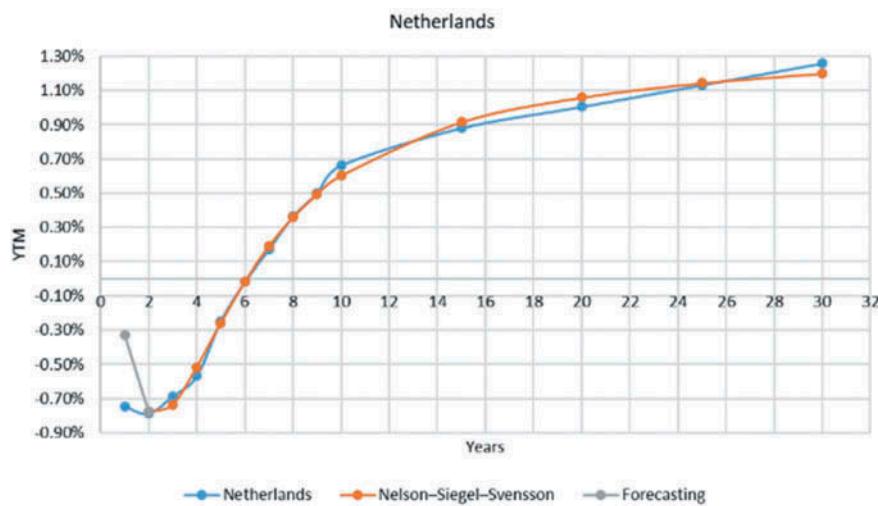


Figure A35. Portugal market and NSS yield curve (5 May 2017)—STF.

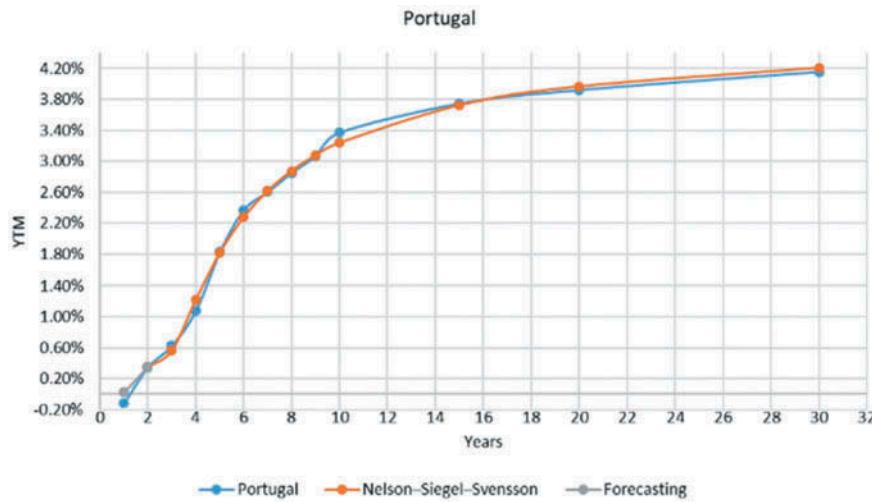


Figure A36. Slovakia market and NSS yield curve (5 May 2017)—STF.

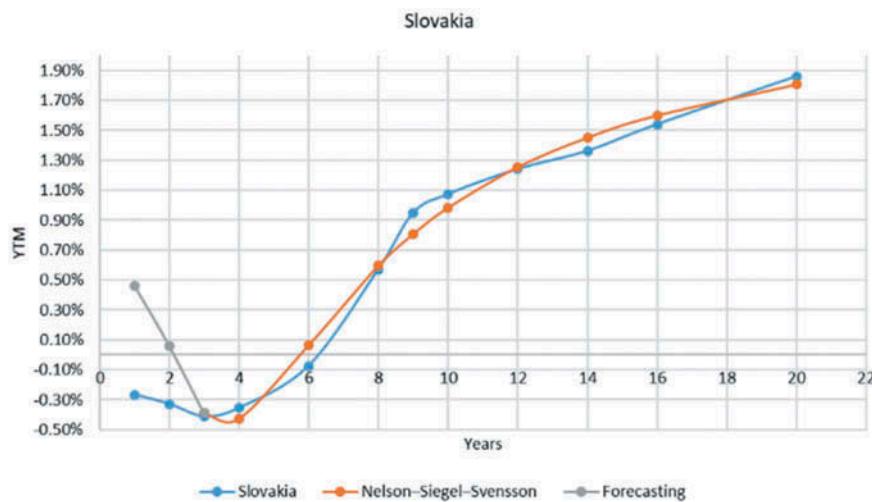


Figure A37. Slovenia market and NSS yield curve (5 May 2017)—STF.

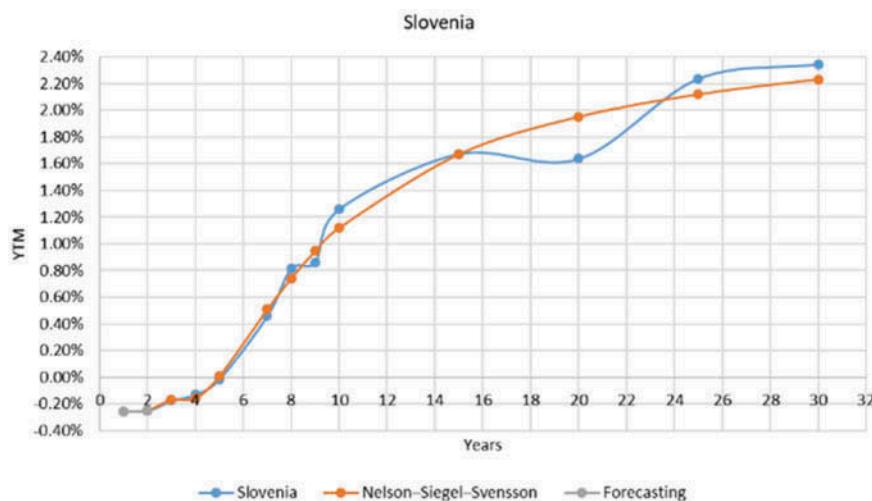


Figure A38. Spain market and NSS yield curve (5 May 2017)—STF.

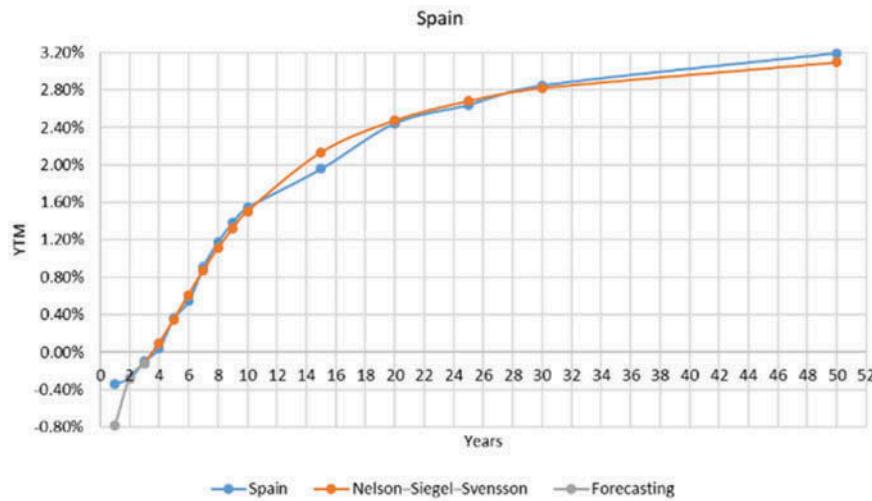


Figure A39. Sweden market and NSS yield curve (15 March 2017)—STF.

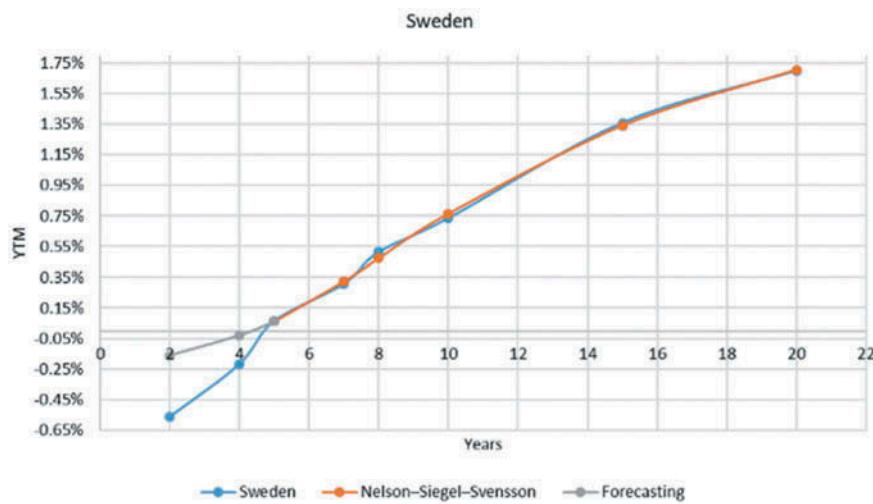


Figure A40. Switzerland market and NSS yield curve (15 March 2017)—STF.

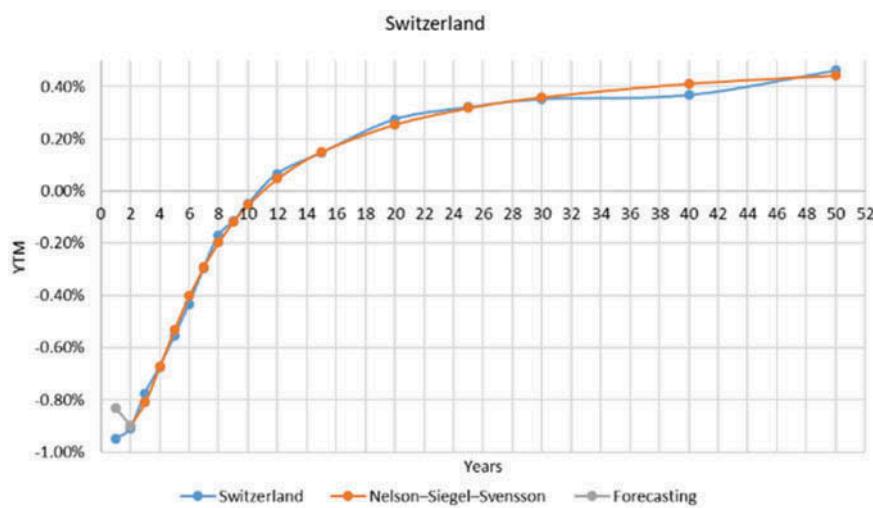


Table A3. NSS model $\theta_{1,2,3,4}$ and $\gamma_{1,2}$ factors (intermediate-term maturities forecast)

	θ_1	θ_2	θ_3	θ_4	γ_1	γ_2
Austria	0.020162	-0.019143	-0.116890	-0.077807	0.086461	1.729619
Belgium	0.023496	-0.029780	-0.884799	-0.078639	0.017087	2.023738
Bulgaria	0.029918	-0.018936	0.006211	-0.077114	1.476223	1.531002
Czech Republic	0.030088	-0.027950	-0.058276	-0.087537	0.316866	3.134442
Denmark	0.016939	-0.002638	-0.000051	-0.079249	0.009988	1.345077
Finland	0.022371	-0.029255	-0.039402	-0.011852	2.291256	19.348220
France	0.025399	-0.030448	0.002341	-0.040293	2.503659	2.320827
Germany	0.016482	-0.026273	0.00792	-0.025125	2.714309	2.723765
Ireland	0.025917	-0.034485	-0.047909	-0.085281	0.179730	1.968295
Italy	0.036582	-0.036717	-0.059474	2.155394	1.633825	77.820987544
Japan	0.000658	-0.003019	0.291665	-0.269408	11.1597784	10.501835
Lithuania	0.029082	-0.027602	0.046827	-0.065106	5.709136	3.564919
Luxembourg	0.018629	-0.019933	-0.308299	-0.069999	0.009942	1.799840
Netherlands	0.015123	-0.014851	-0.061748	-0.067745	0.082956	1.248862
Portugal	0.046554	-0.243638	1.245474	-1.042822	0.545787	0.662713
Slovakia	0.027724	-0.038131	0.742350	-0.789988	1.292437	1.393741
Slovenia	0.027570	-0.034833	-0.207497	-0.083209	0.057211	1.768952
Spain	0.035623	-0.035633	-0.245482	-0.103250	0.070087	1.878328
Sweden	0.027821	-0.030039	-1.216174	-0.074959	0.024818	2.475677
Switzerland	0.005543	-0.013433	-0.024206	-0.000303	1.620451	0.002583

Figure A41. Austria market and NSS yield curve (15 March 2017)—ITF.

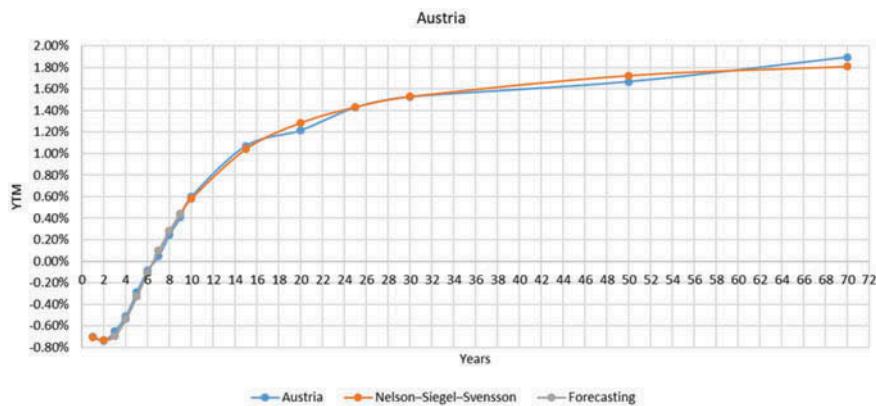


Figure A42. Belgium market and NSS yield curve (5 May 2017)—ITF.

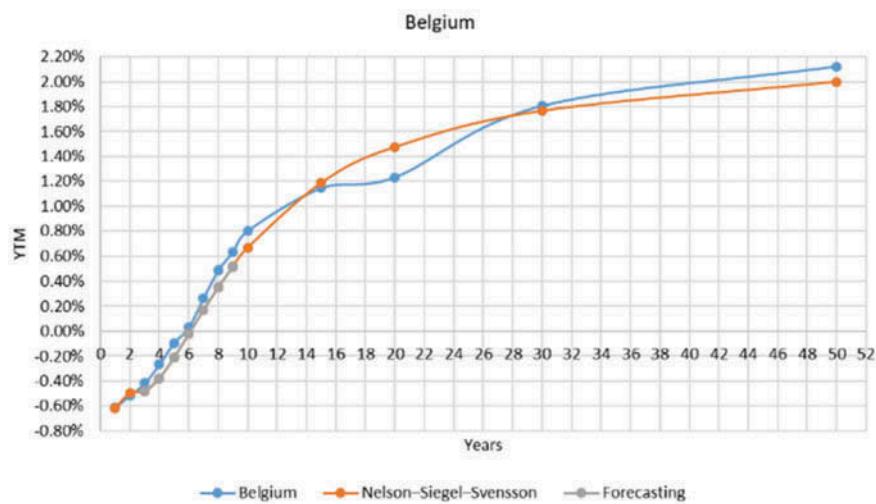


Figure A43. Bulgaria market and NSS yield curve (5 May 2017)—ITF.

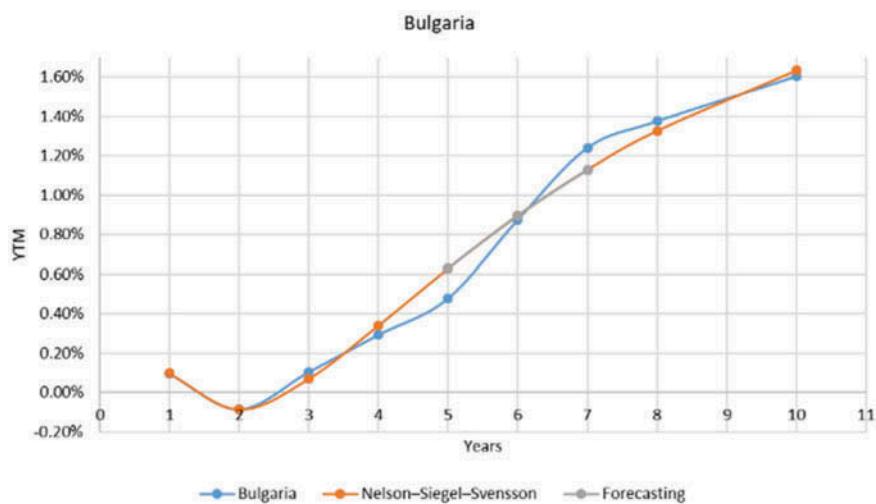


Figure A44. The Czech Republic market and NSS yield curve (5 May 2017)—ITF.

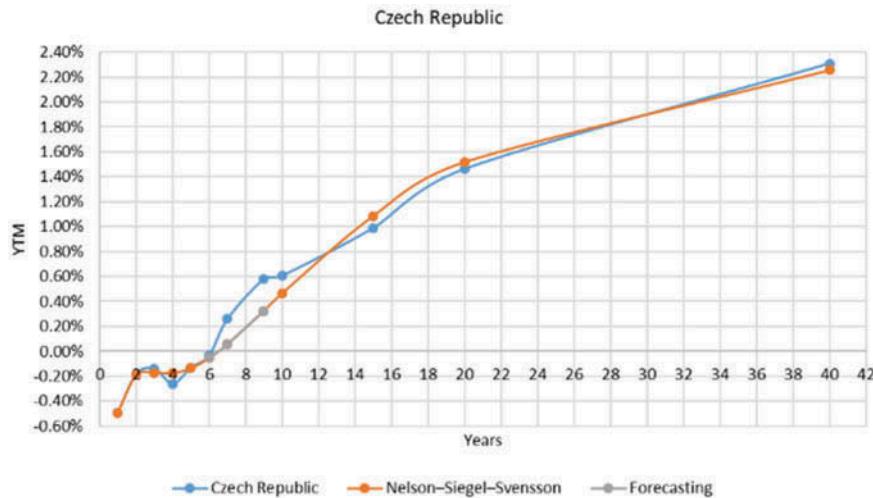


Figure A45. Denmark market and NSS yield curve (15 March 2017)—ITF.

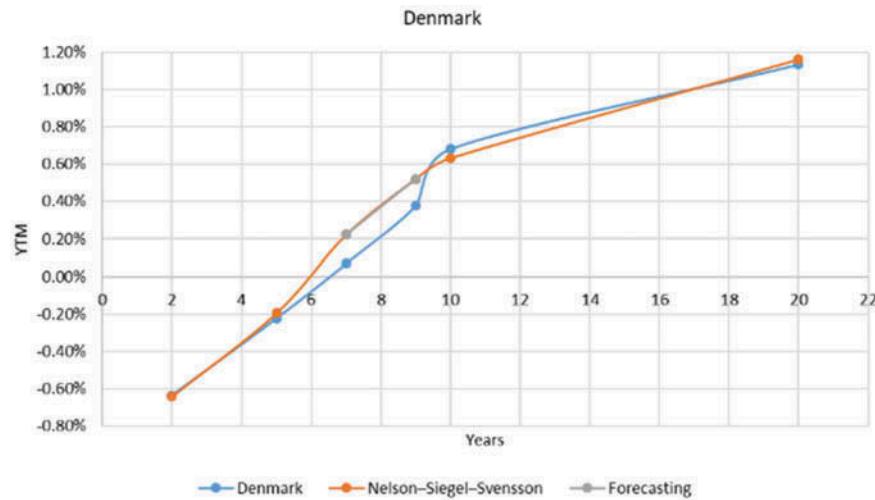


Figure A46. Finland market and NSS yield curve (15 March 2017)—ITF.

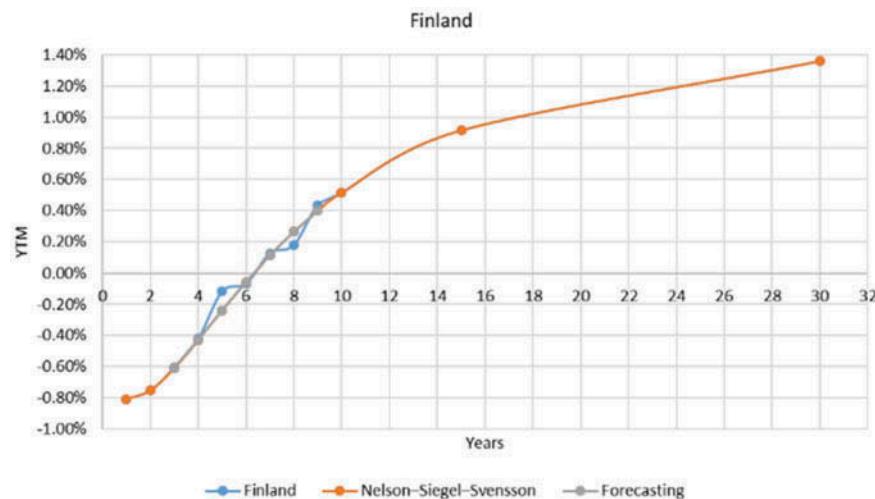


Figure A47. France market and NSS yield curve (15 March 2017)—ITF.

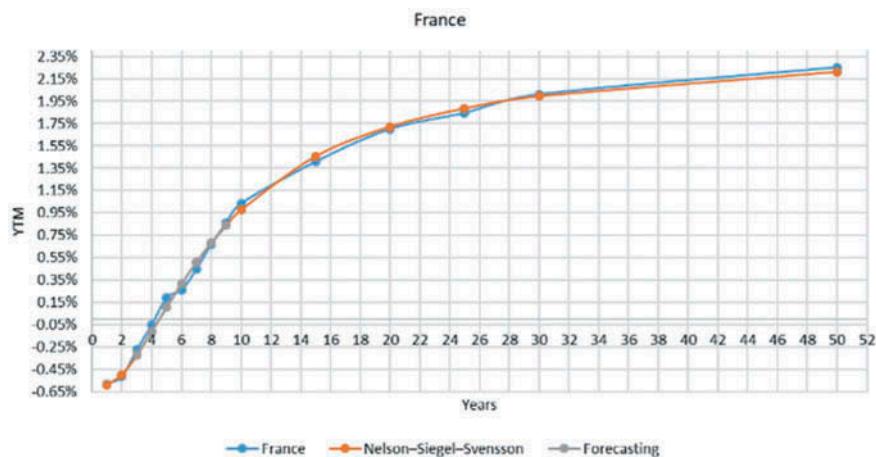


Figure A48. Germany market and NSS yield curve (16 March 2017)—ITF.

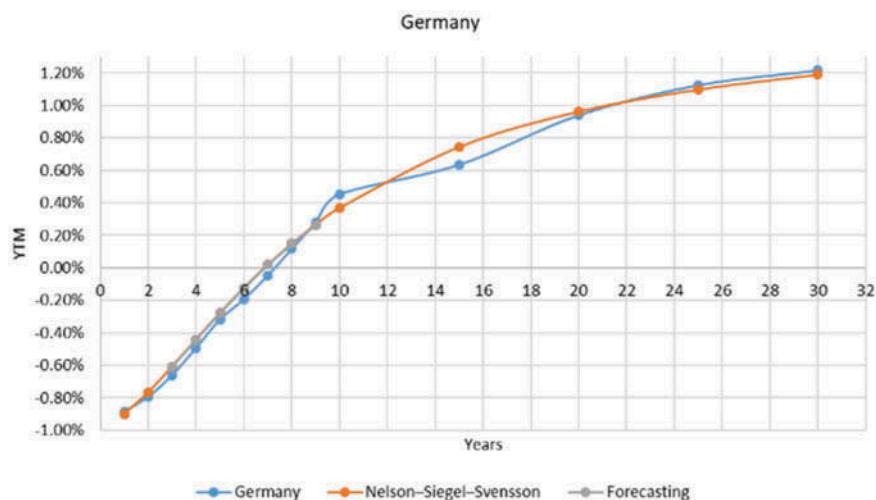


Figure A49. Ireland market and NSS yield curve (5 May 2017)—ITF.

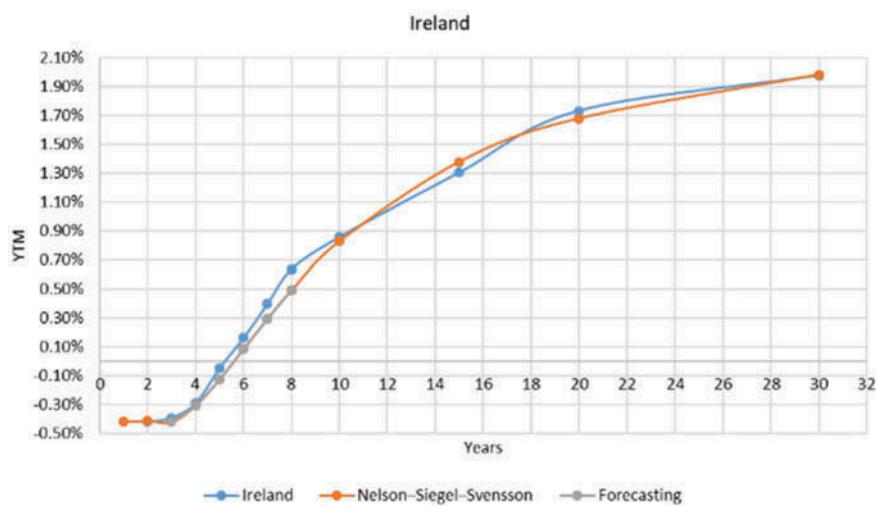


Figure A50. Italy market and NSS yield curve (5 May 2017)—ITF.

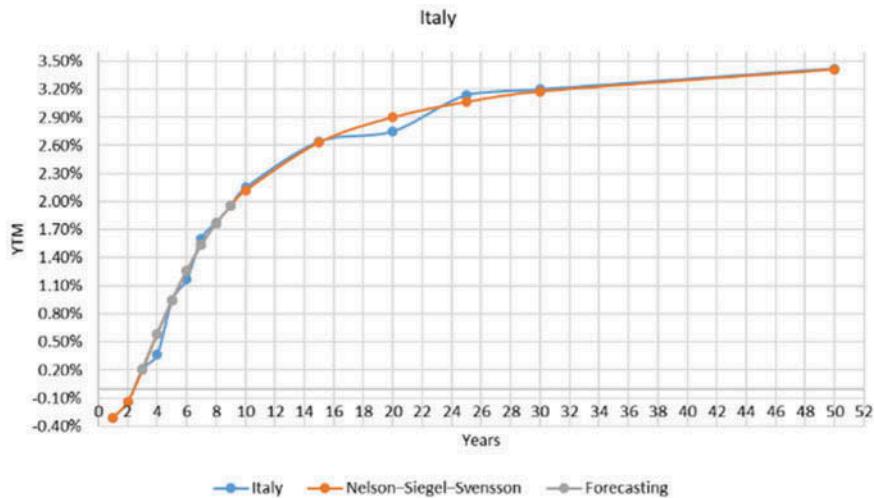


Figure A51. Japan market and NSS yield curve (16 March 2017)—ITF.

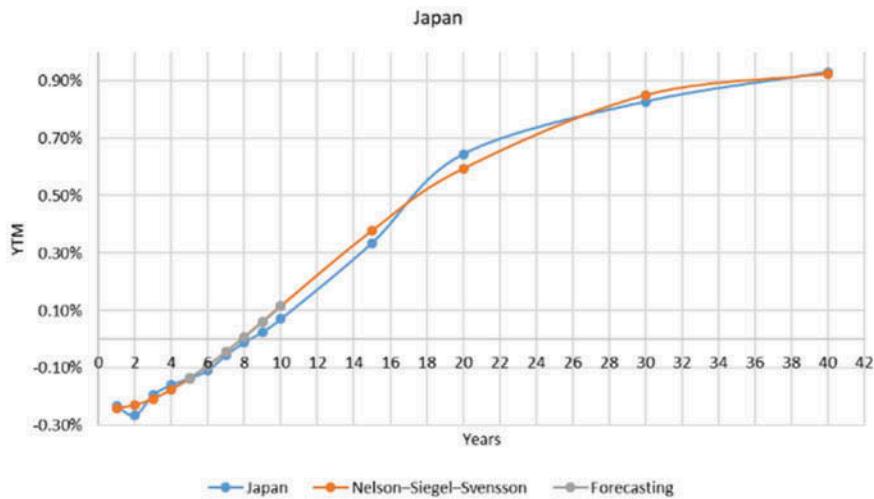


Figure A52. Lithuania market and NSS yield curve (5 May 2017)—ITF.

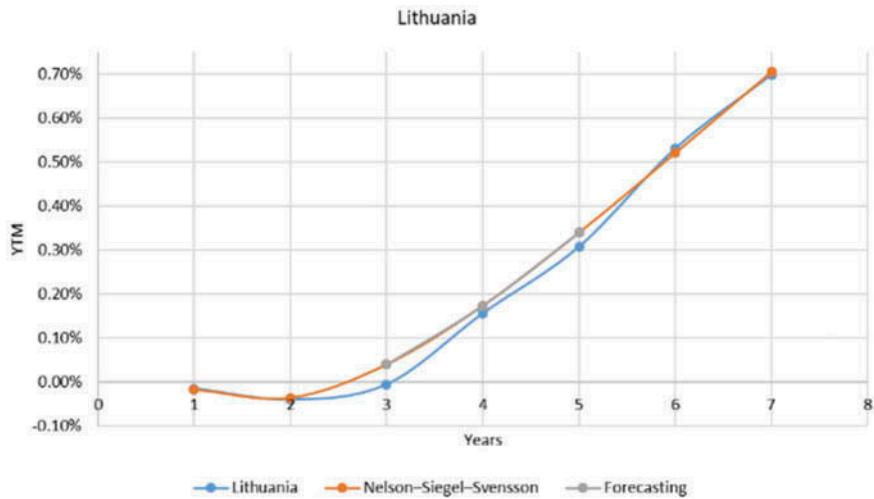


Figure A53. Luxembourg market and NSS yield curve (5 May 2017)—ITF.

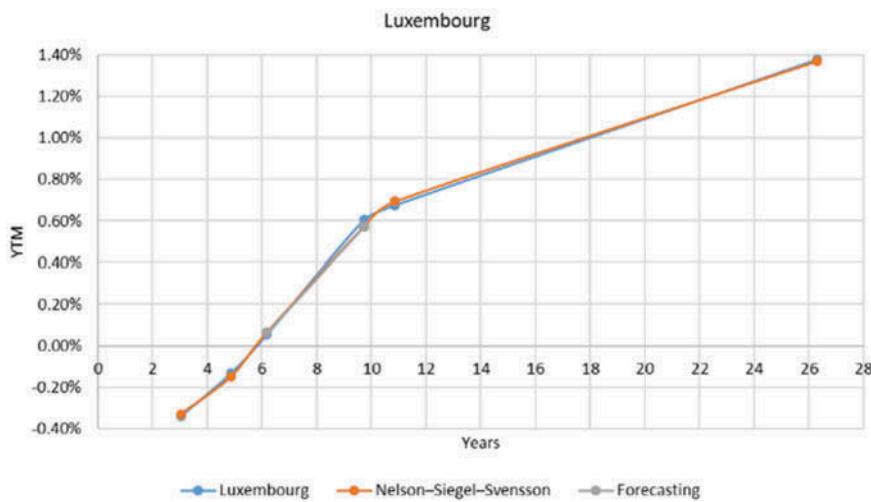


Figure A54. The Netherlands market and NSS yield curve (15 March 2017)—ITF.

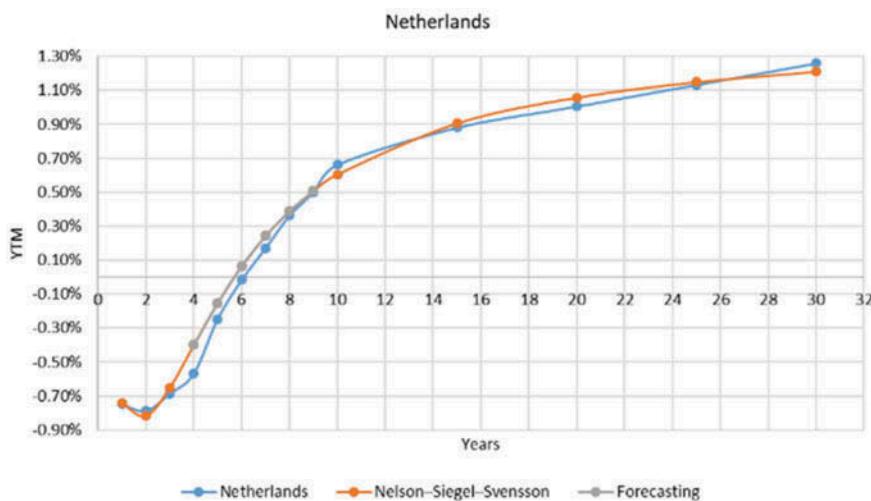


Figure A55. Portugal market and NSS yield curve (5 May 2017)—ITF.

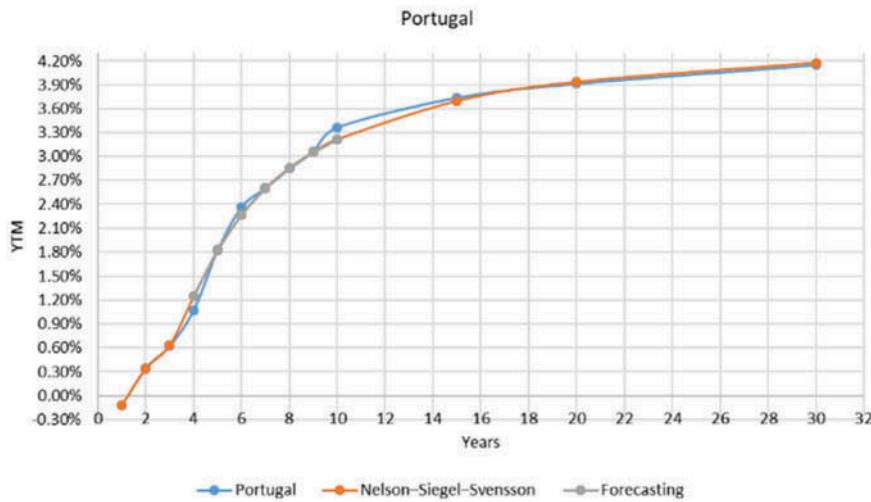


Figure A56. Slovakia market and NSS yield curve (5 May 2017)—ITF.

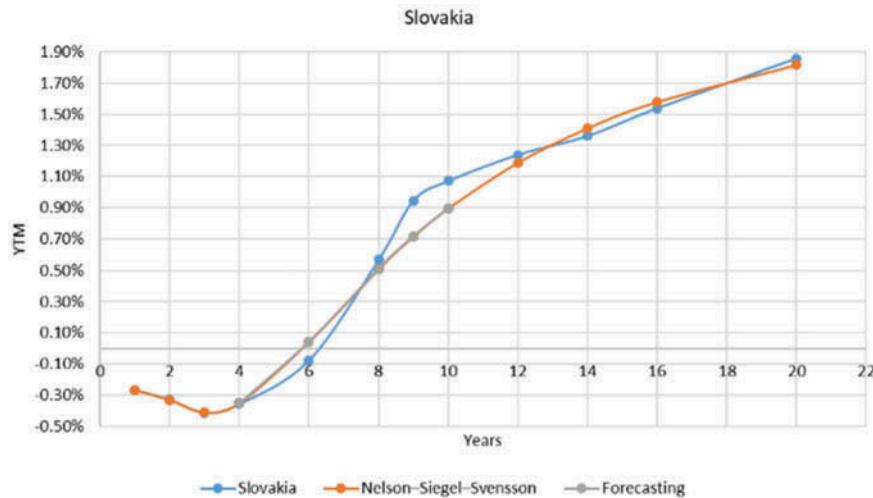


Figure A57. Slovenia market and NSS yield curve (5 May 2017)—ITF.

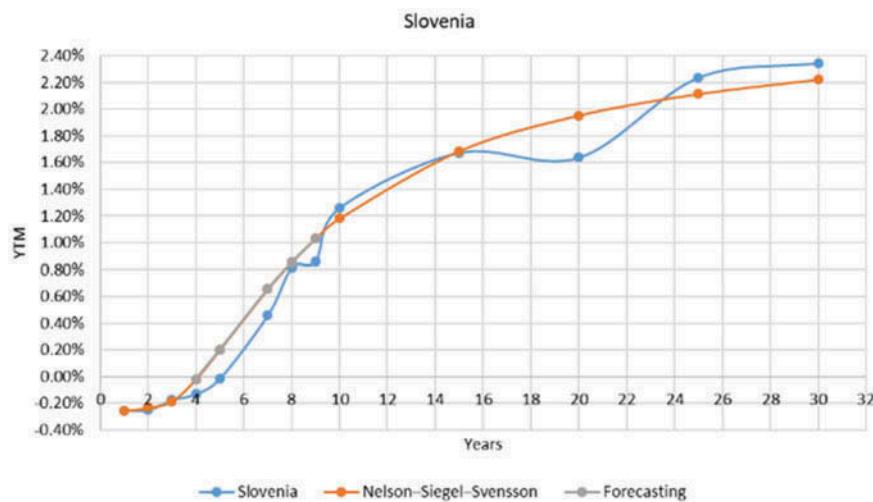


Figure A58. Spain market and NSS yield curve (5 May 2017)—ITF.

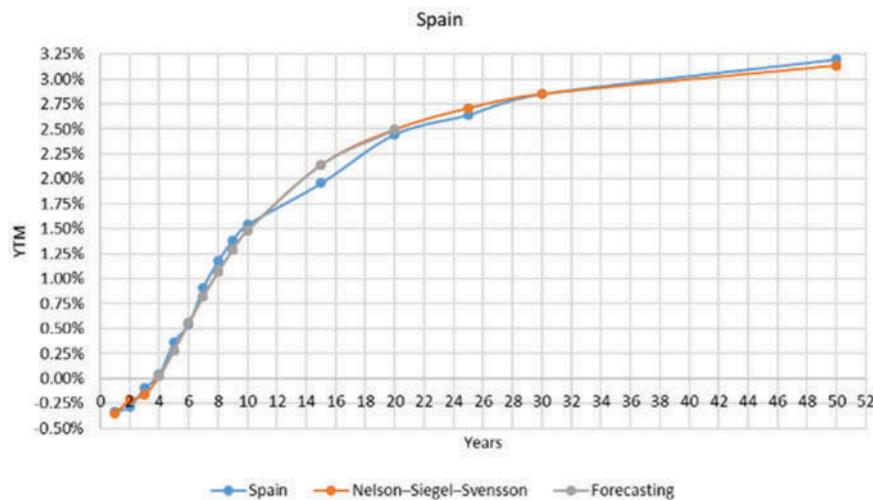


Figure A59. Sweden market and NSS yield curve (15 March 2017)—ITF.

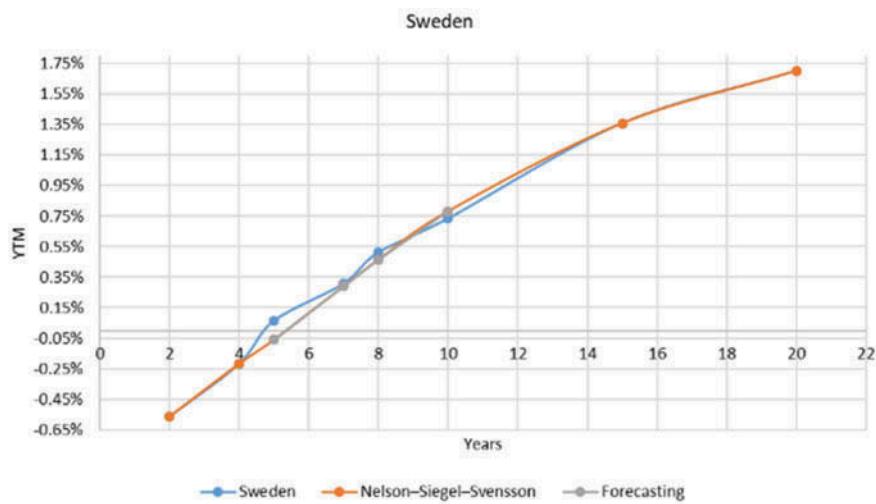


Figure A60. Switzerland market and NSS yield curve (15 March 2017)—ITF.

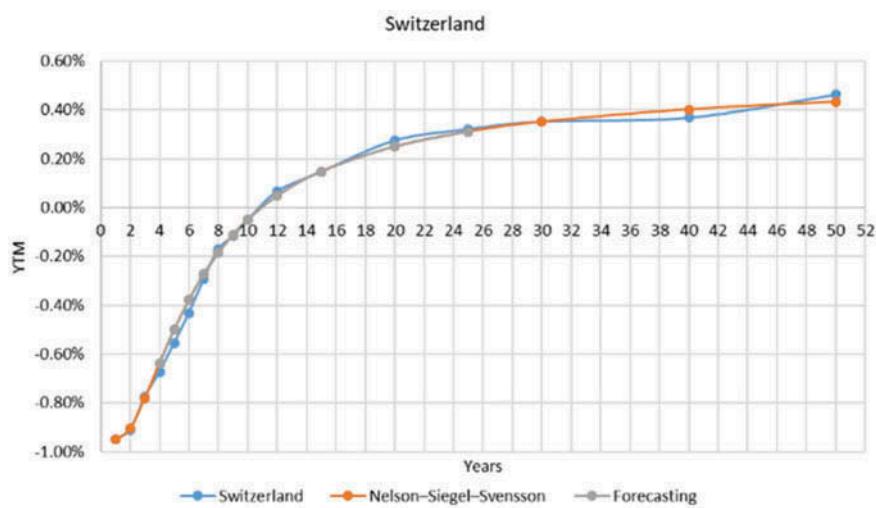


Table A4. NSS model $\theta_{1,2,3,4}$ and $\gamma_{1,2}$ factors (long term maturities forecast)

	θ_1	θ_2	θ_3	θ_4	γ_1	γ_2
Austria	0.019247	-0.019139	-0.116741	-0.073730	0.086327	1.733630
Belgium	0.022702	-0.029756	-0.864602	-0.074378	0.016690	1.885299
Bulgaria	0.042973	-0.036530	0.006278	-0.102254	1.309432	2.074813
Czech Republic	0.030279	-0.228187	0.009047	-0.090860	0.099065	2.629441
Denmark	0.017726	-0.002688	-0.000051	-0.082415	0.009988	1.456279
Finland	0.022365	-0.029326	-0.037821	-0.015600	2.100928	12.573914
France	0.025001	-0.030761	0.002349	-0.038693	2.402382	2.414755
Germany	0.013610	-0.029384	0.004170	-0.054150	0.495952	1.652236
Ireland	0.024499	-0.033156	-0.045477	-0.081230	0.165971	1.796850
Italy	0.037199	-0.035035	-0.067227	-0.003852	1.576561	194.113267
Japan	0.000680	-0.002844	0.291656	-0.270609	11.352144	10.239932
Lithuania	0.029534	-0.027475	0.046842	-0.066333	5.816622	3.420271
Luxembourg	0.019445	-0.019994	-0.308483	-0.072912	0.009948	1.800200
Netherlands	0.016811	-0.015129	-0.065832	-0.071701	0.096109	1.416197
Portugal	0.048890	-0.247341	1.246504	-1.044196	0.565397	0.693006
Slovakia	0.030081	-0.051019	0.746092	-0.781254	1.150972	1.281889
Slovenia	0.030972	-0.035226	-0.223980	-0.094916	0.063076	1.996553
Spain	0.035312	-0.035484	-0.239406	-0.102905	0.068051	1.781474
Sweden	0.027667	-0.030075	-1.273167	-0.070311	0.026011	2.613614
Switzerland	0.006376	-0.014388	-0.024799	-0.000303	1.7977835	0.002583

Figure A61. Austria market and NSS yield curve (15 March 2017)—LTF.

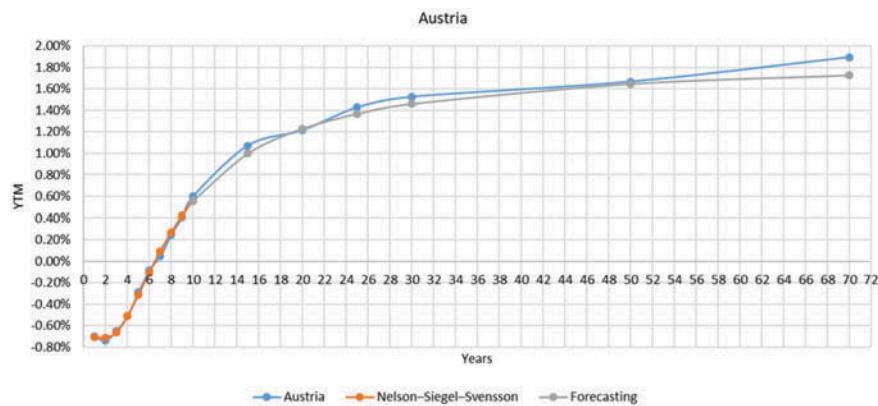


Figure A62. Belgium market and NSS yield curve (5 May 2017)—LTF.

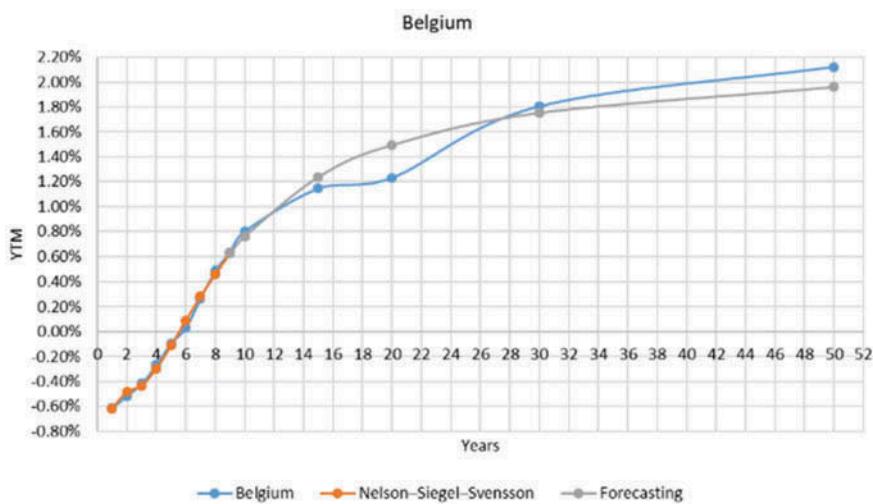


Figure A63. Bulgaria market and NSS yield curve (5 May 2017)—LTF.

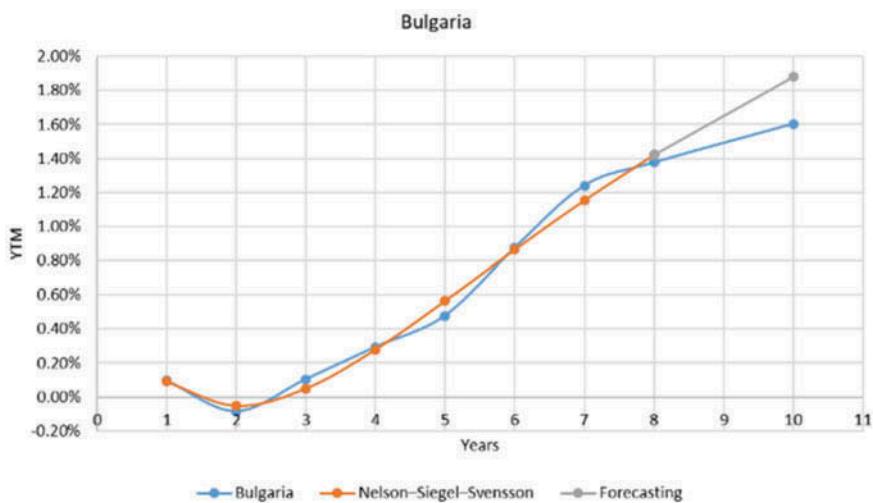


Figure A64. The Czech Republic market and NSS yield curve (5 May 2017)—LTF.

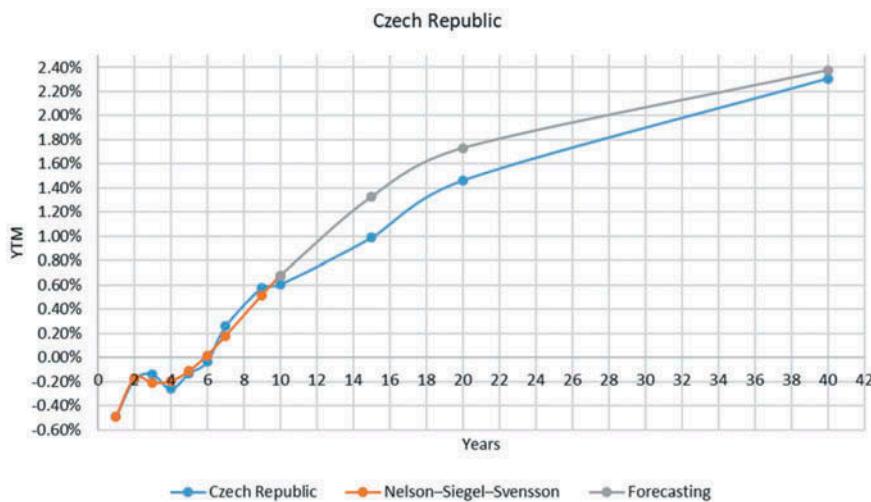


Figure A65. Denmark market and NSS yield curve (15 March 2017)—LTF.

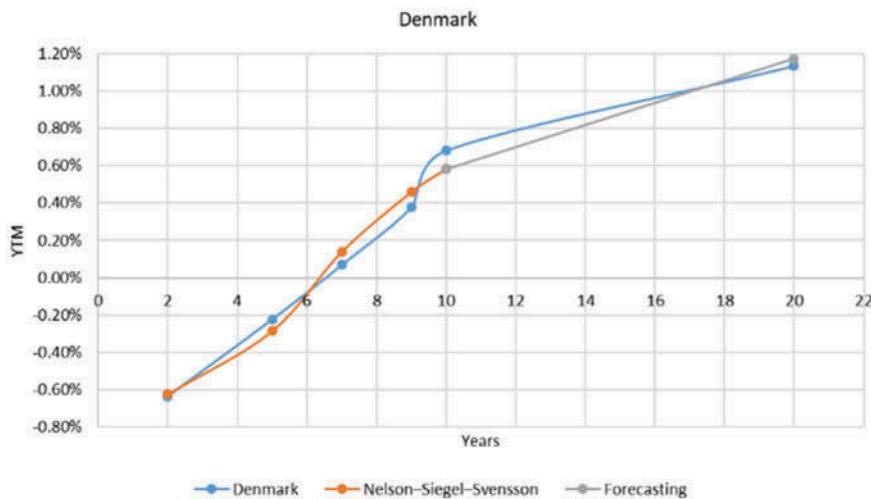


Figure A66. Finland market and NSS yield curve (15 March 2017)—LTF.

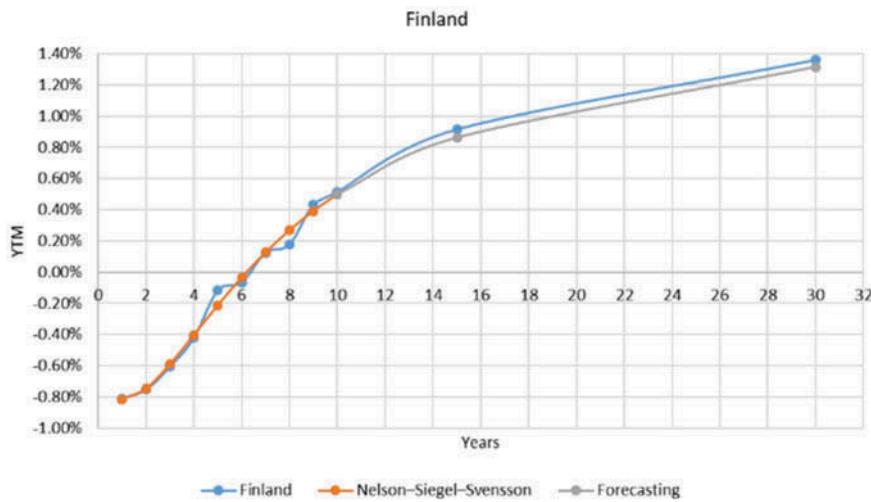


Figure A67. France market and NSS yield curve (15 March 2017)—LTF.

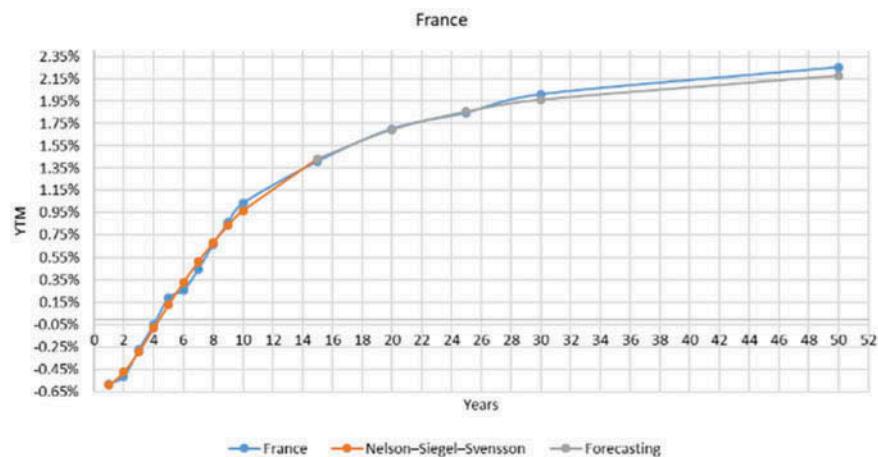


Figure A68. Germany market and NSS yield curve (16 March 2017)—LTF.

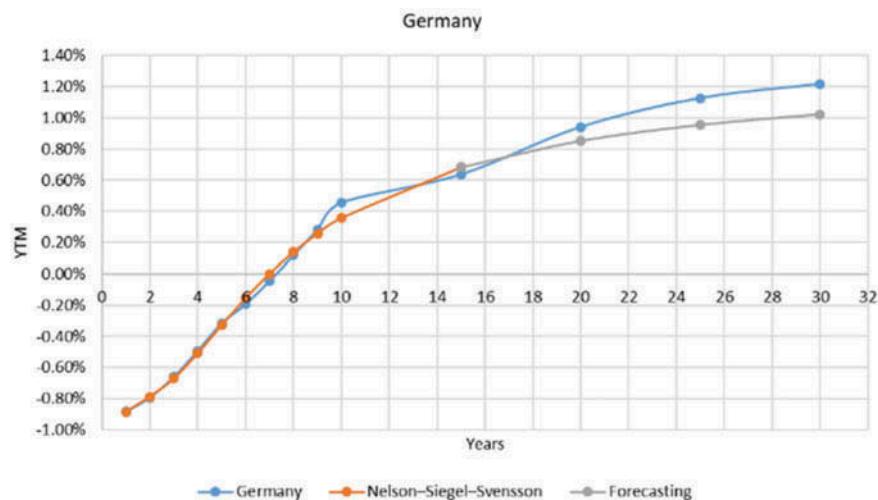


Figure A69. Ireland market and NSS yield curve (5 May 2017)—LTF.

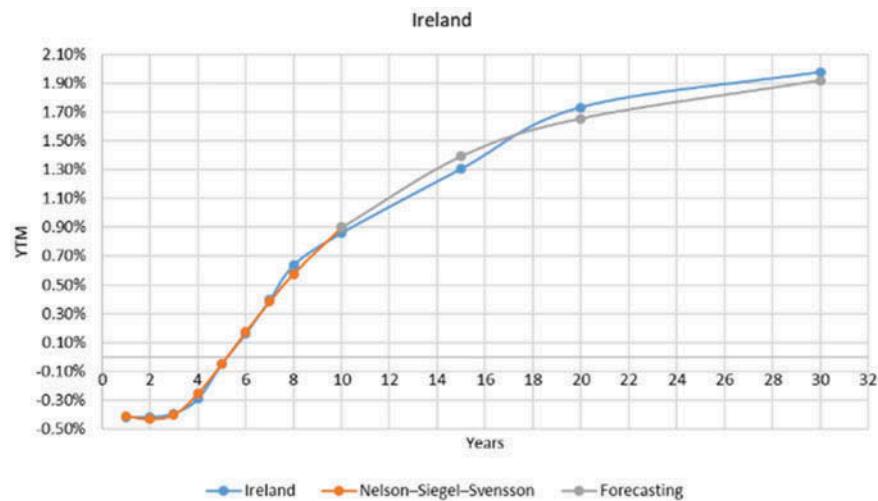


Figure A70. Italy market and NSS yield curve (5 May 2017)—LTF.

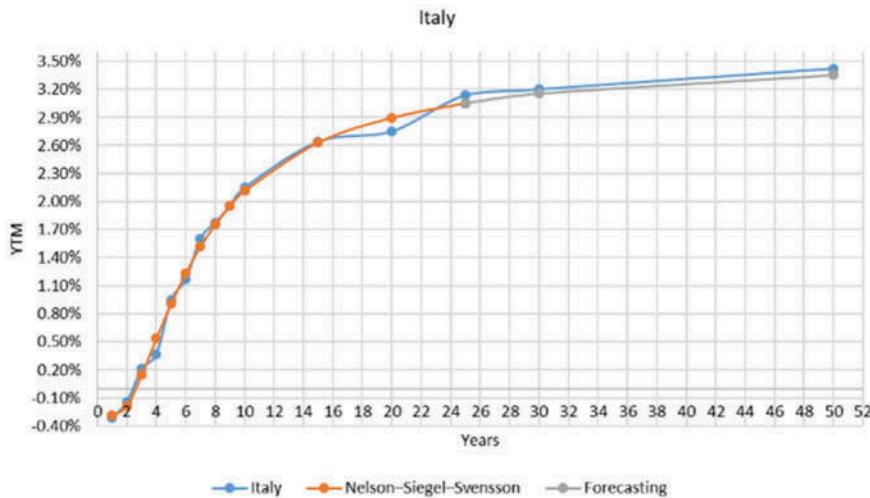


Figure A71. Japan market and NSS yield curve (16 March 2017)—LTF.

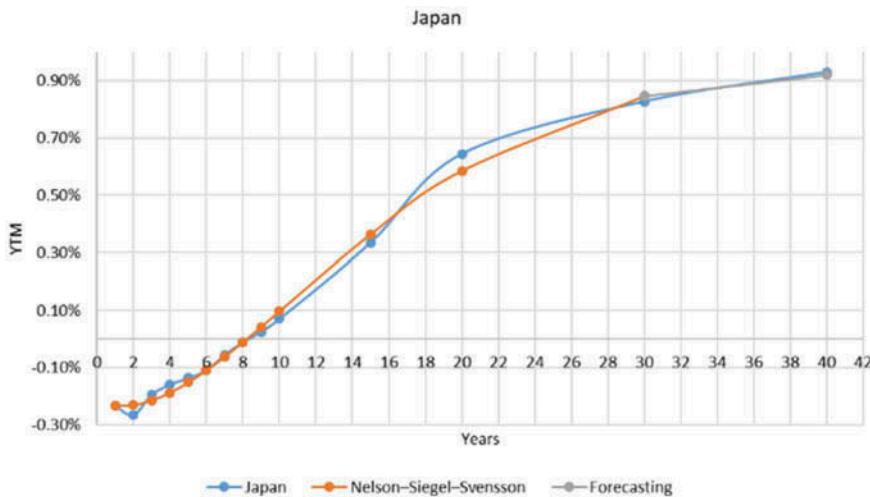


Figure A72. Lithuania market and NSS yield curve (5 May 2017)—LTF.

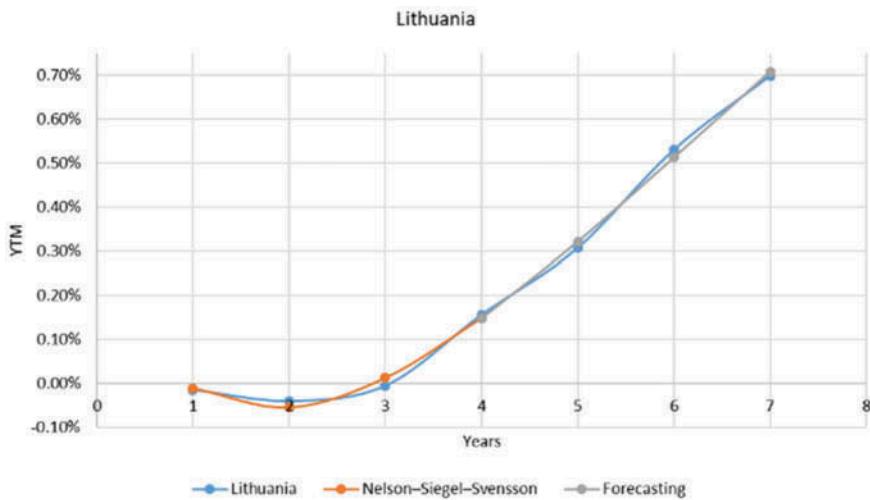


Figure A73. Luxembourg market and NSS yield curve (5 May 2017)—LTF.



Figure A74. The Netherlands market and NSS yield curve (15 March 2017)—LTF.

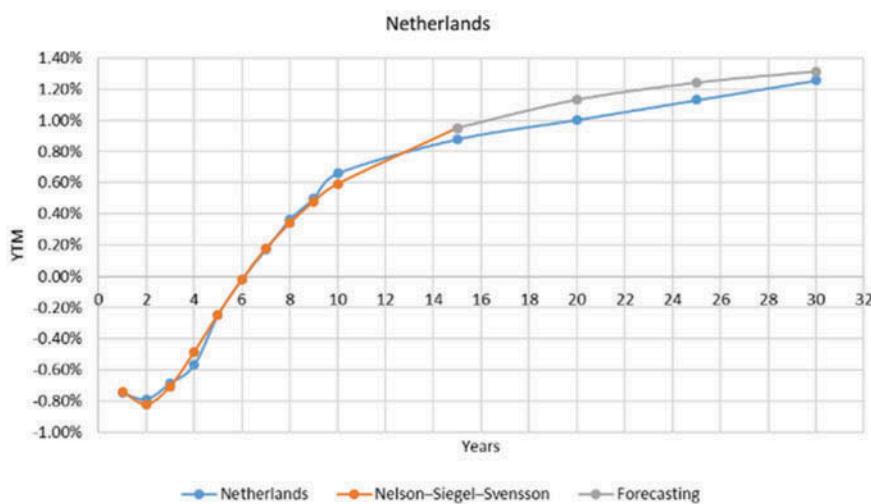


Figure A75. Portugal market and NSS yield curve (5 May 2017)—LTF.

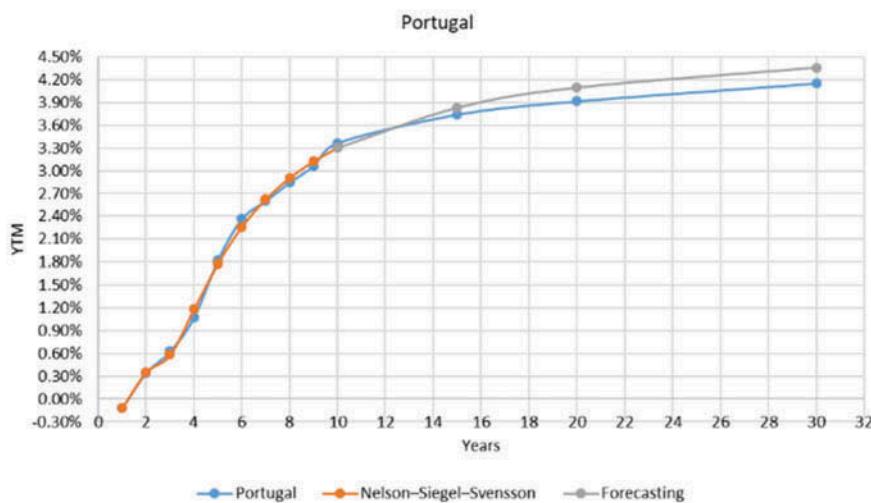


Figure A76. Slovakia market and NSS yield curve (5 May 2017)—LTF.

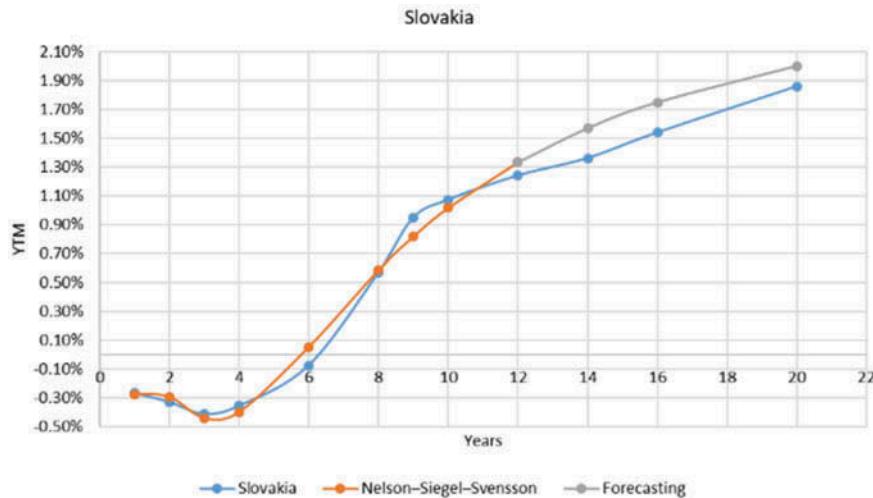


Figure A77. Slovenia market and NSS yield curve (5 May 2017)—LTF.

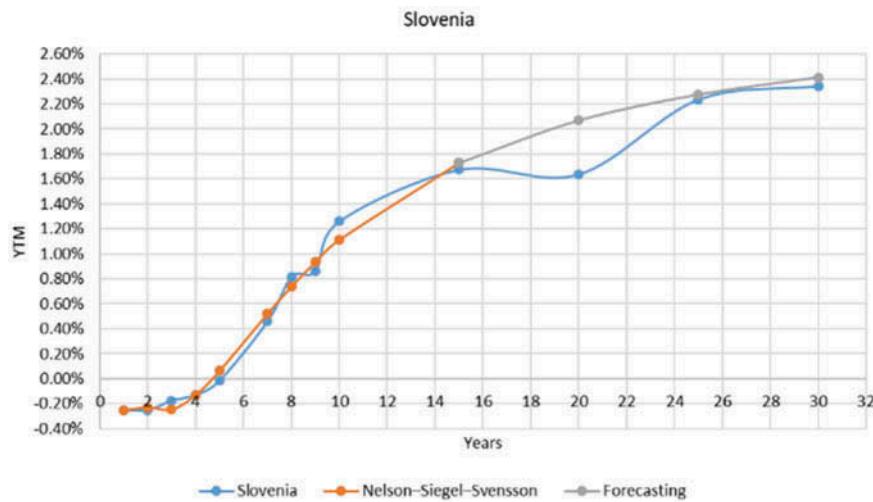


Figure A78. Spain market and NSS yield curve (5 May 2017)—LTF.

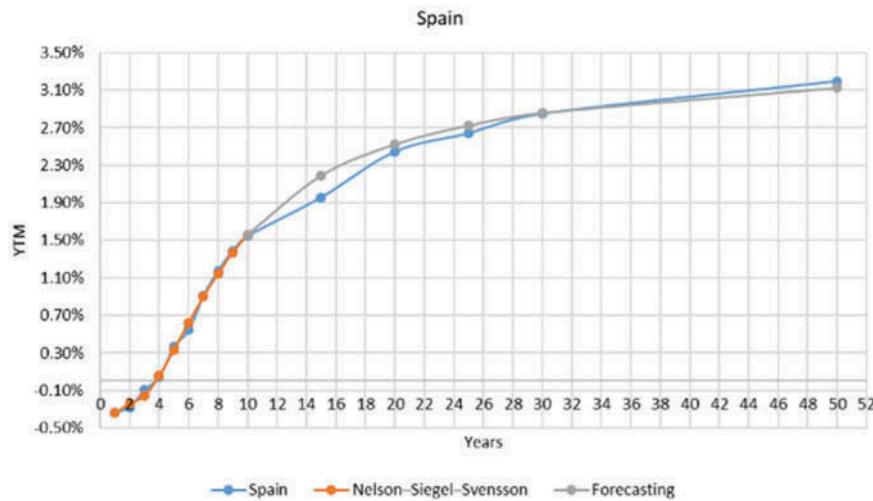


Figure A79. Sweden market and NSS yield curve (15 March 2017)—LTF.

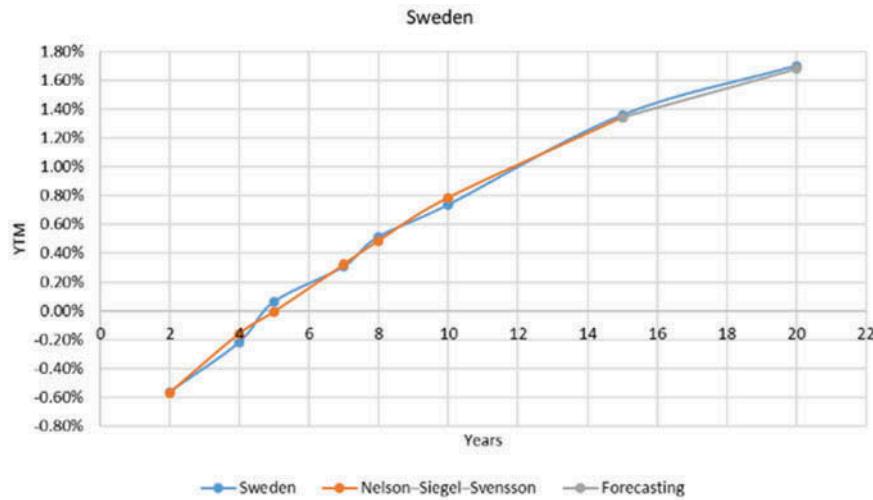
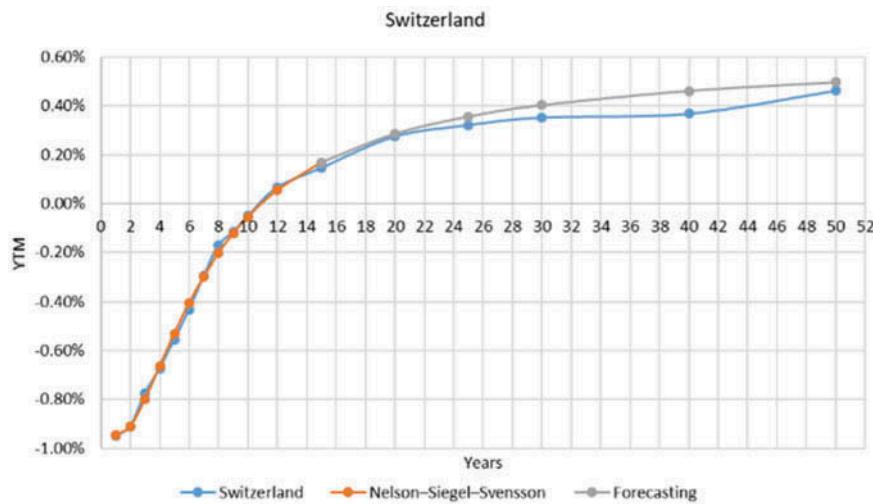


Figure A80. Switzerland market and NSS yield curve (15 March 2017)—LTF.





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