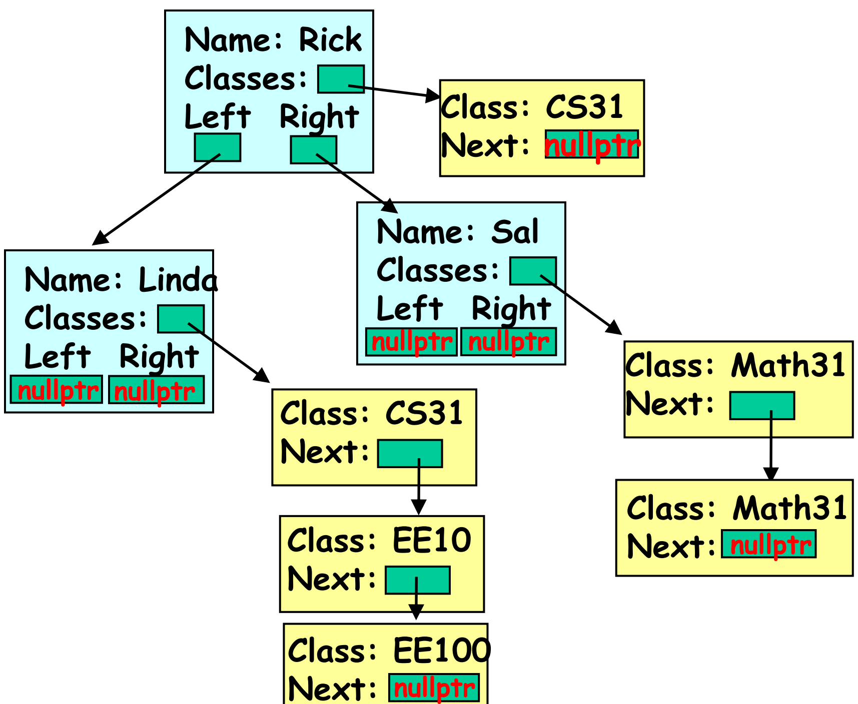
Lecture #14

- Hash Tables
  - The Modulus Operator
  - Closed hash tables
  - Open hash tables
  - Hash table efficiency and "load factor"
  - Hashing non-numeric values
  - unordered_map: A hash-based STL map class
- (Database) Tables

Big-OH Craziness

Consider a *binary search tree* that holds N student records, all indexed by their name.

Each student record contains a linked-list of the L classes that they have taken while at UCLA.

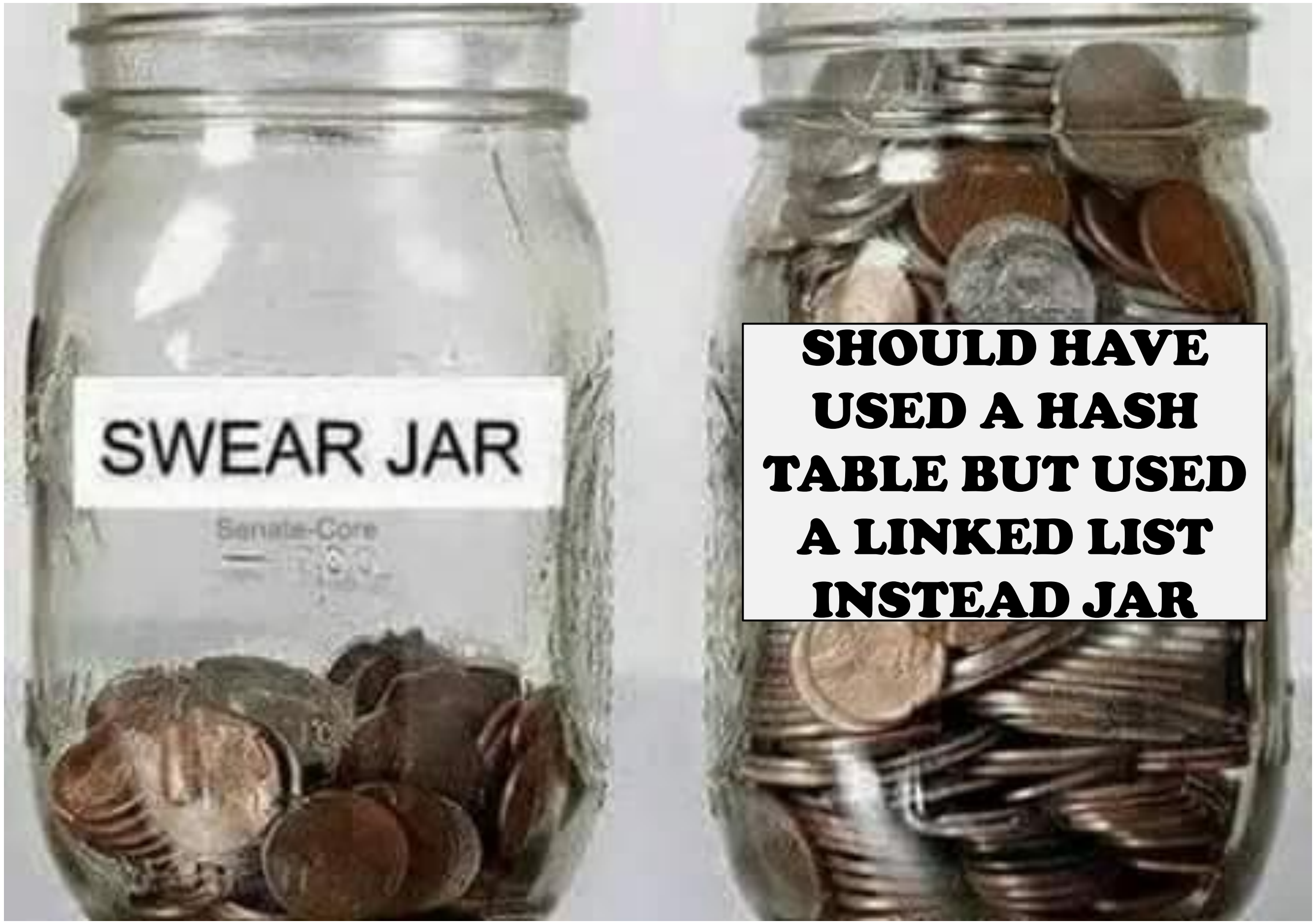


What is the big-oh to determine if a student has taken a class?

```
bool HasTakenClass(
        BTree &b,
        string &name,
        string &class
        )
```

Answer: O(log N + L). Why? You can find any student in log N time in a balanced binary search tree with N nodes. Once we find the student's node, we then need to search through that student's list of classes, which is O(L). Notice we didn't say O(log N * L) – that would imply that for ALL of the students we search through from the root of the tree, we need to go through their L classes. Instead, we only go through the L classes of the student we're interested in. THIS IS A CRITICAL POINT TO LEARN. ASK CAREY OR YOUR TA IF YOU DON'T GET THIS!

Hash Tables


SWEAR JAR
SHOULD HAVE USED A HASH TABLE BUT USED A LINKED LIST INSTEAD JAR

Hash Tables
Why should you care?

Hash tables are often THE most efficient way to search for data!

You can search a hash table with billions of items in just microseconds!

They're used in search engines, antivirus scanners, navigation systems, social network sites, etc.

And because you'll be asked about them in job interviews and on exams.

So pay attention!

The Modulus Operator

In C++, the % operator is used to divide two numbers and obtain the remainder.

For example, if we compute:

`int x = 1234 % 100;`

the value of x will be 34.

```
       12 R 34
     _________
100 ) 1234
      100
      ____
       234
       200
       ____
        34
```

Now, as it turns out, the modulo operator has an interesting property!

Let's see if you can figure out what it is…

The Modulus Operator

Let's modulus-divide a bunch of numbers by 5 and see what the results are!

0 % 5 = 0
1 % 5 = 1
2 % 5 = 2
3 % 5 = 3
4 % 5 = 4
5 % 5 = 0
6 % 5 = 1
7 % 5 = 2
8 % 5 = 3
9 % 5 = 4
10 % 5 = 0
11 % 5 = 1

What do you notice?

When we divide numbers by 5, all of the remainders are less than 5 (between 0-4)!

Let's try again with 3 for fun!

When we divide numbers by 3, all of the remainders are less than 3 (between 0-2)!

And as you'd guess, if you divided a bunch of numbers by 100,000, the remainders would all be less than 100,000 (between 0-99,999)!

Rule: When you divide by a given value N, all of your remainders are guaranteed to be between 0 and N-1!

The "Hash Table"

OK… So far, what's the most efficient ADT we know of to insert and search for data?

Right! The (balanced) Binary Search Tree – it gives us $O(\log_2 N)$ performance!

Can we do any better? If so, how much better?

Challenge:

Build an ADT that holds a bunch of 9-digit student ID#s such that the user can add new ID#s or determine if the ADT holds an existing ID# in just 1 step – not $O(N)$ or $O(\log_2 N)$ but $O(1)$.

The (Almost) Hash Table

How can we create an ADT where we can insert the 9-digit student ID#s for all 50,000 UCLA students…

and then find if our ADT holds a given ID# in just one algorithmic step?!?!?

That can't be done… can it?

It can, and let's see how!

Let's use a really, really large array to hold our #s.

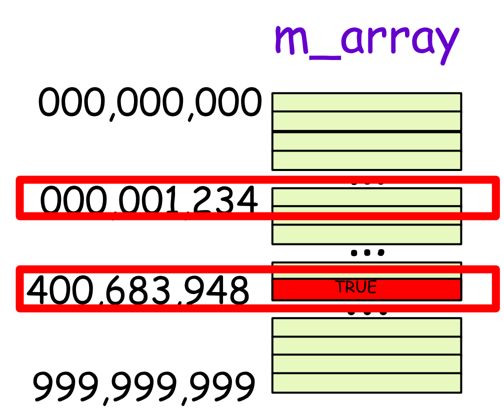
The (Almost) Hash Table

```
class AlmostHashTable
{
public:
   void addItem(int n)
   {
      m_array[n] = true;
   }
   bool holdsItem(int q)
   {
      return m_array[q] == true;
   }
private:
   bool m_array[100000000]; // big!
};

int main()
{
   AlmostHashTable x;

   x.addItem(400683948);
   if (x.holdsItem(1234) != true)
      cout<< "Couldn't find it!";
}
```

Idea:
Let's create an array with 1 billion slots - one slot for each valid ID#.

To add a new ID# with a value of N, we'll simply set array[N] to true.

To determine if our array holds a previously-added value Q, simply check if array[Q] is true.



The (Almost) Hash Table

OK – so now we know how to build an O(1) search!
But what's the problem with our ADT?

It's really, really inefficient:
Our array has 1 billion slots
yet there are only 50,000 UCLA student IDs
we could possibly add to it,
so we're wasting 999,950,000 of the slots…

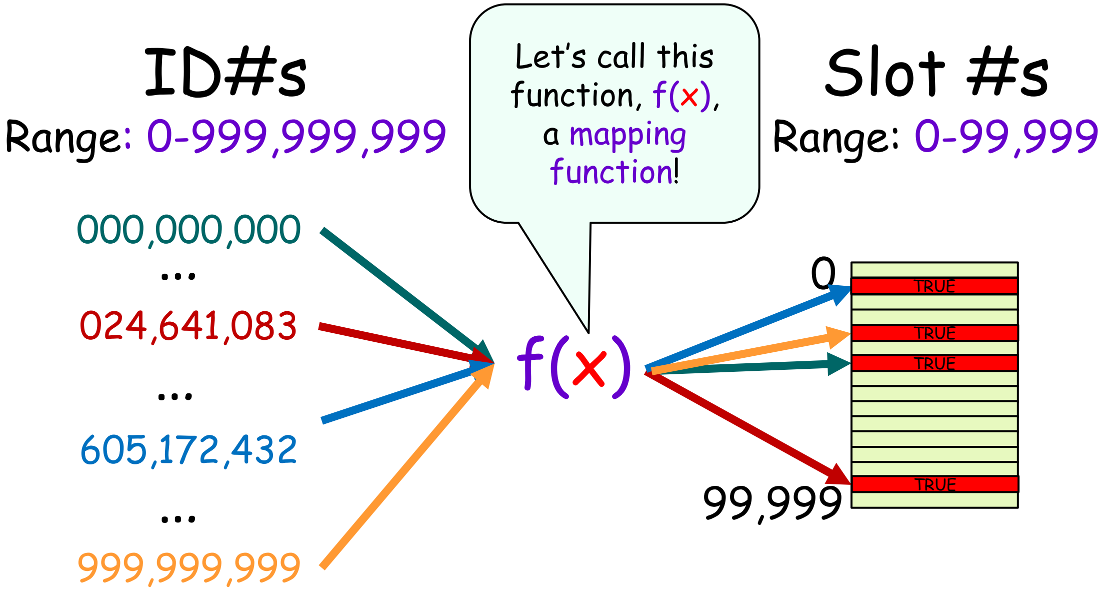
It would be great if we could use the same algorithm
but with a smaller array, say one with 100,000 slots
instead of 1 billion!

The (Almost) Hash Table

Lets say we want to keep track of our 50,000 ID#s in an array with just 100,000 slots.

If we just try to use our 9-digit number to index the array, there won't be room!

What we need is some cool mathematical function that takes in a 9-digit ID# and somehow converts it to a unique slot number between 0 and 99,999 in the array!



The (Almost) Hash Table

```
class AlmostHashTable2
{
public:
   void addItem(int n)
   {
      int slot = mapFunc(n);
      m_array[slot] = true;
   }
   bool containsItem(int q)
   {
      int slot = mapFunc(q);
      return m_array[slot] == true;
   }
private:
   int mapFunc(int idNum)
     { /* ??? */ }

   bool m_array[100000]; // not so big!
};
```

This is the "almost" hash table, v2. It's still NOT a valid hash table, but it's getting closer!

Assuming we can come up with such a mapping function (mapFunc), we can use a (small) 100,000 element array to hold our data…

For a given student ID x, we compute slot=mapFunc(n) to get its slot number in the array.

We then set the slot to TRUE to indicate that a student with that ID is in our table.

By the way, the official CS lingo for a "slot" in the array is a "bucket." So that's what we'll call our slots from now on! ☺

Ok, so what does mapFunc() look like? ☺

The Mapping Function

How can we write a mapFunc that converts our large ID# into a bucket # that falls within our 100,000 element array?

```
int mapFunc(int idNum)
{
   const int ARRAY_SIZE = 100000;

   int bucket = idNum % ARRAY_SIZE;
   return bucket;
}
```

This line takes an input value idNum and returns an output value between 0 and ARRAY_SIZE - 1. (0 to 99,999)

RIGHT! The C++ % operator (aka the modulus division operator) does exactly what we want!!!

So now for each input ID# we can compute a corresponding value between 0-99,999!

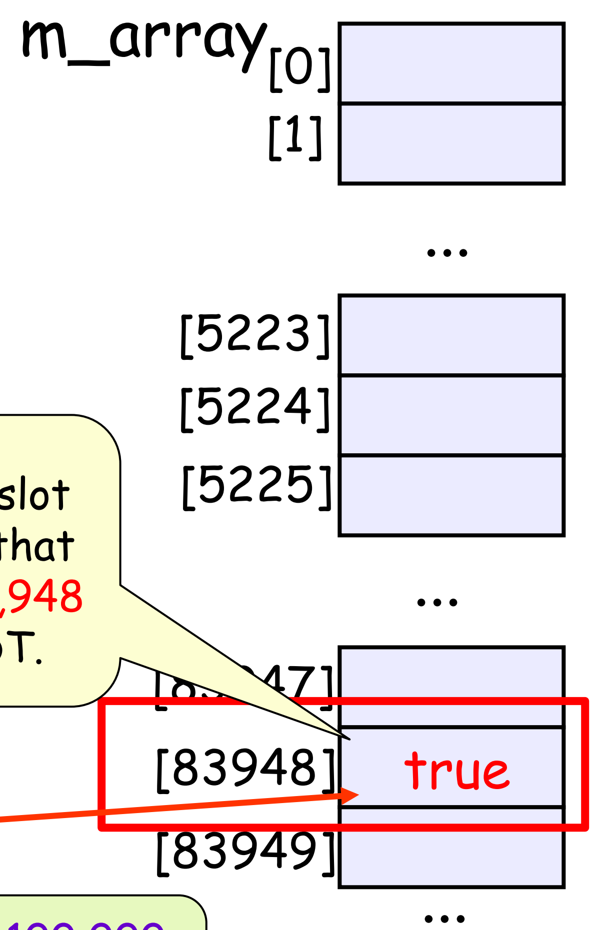
And this corresponding value can be used to pick a bucket in our 100,000 element array!

The (Almost) Hash Table

Let's see how it works.

```
class AlmostHashTable2
{
public:
   void addItem(int n)
   {
      int bucket = mapFunc(n);
      m_array[bucket] = true;
   }

private:
   int mapFunc(int idNum)
   {
      return idNum % 100000;
   }

   bool m_array[100000]; // not
};
```

```
int main()
{
   AlmostHashTable2 x;
   x.addItem(400683948);
   x.addItem(111105224);
   x.addItem(222205224);
}
```



The true value in slot 83,948 indicates that the value 400,683,948 is held in our ADT.

400,683,948 % 100,000
= 83,948

The (Almost) Hash Table

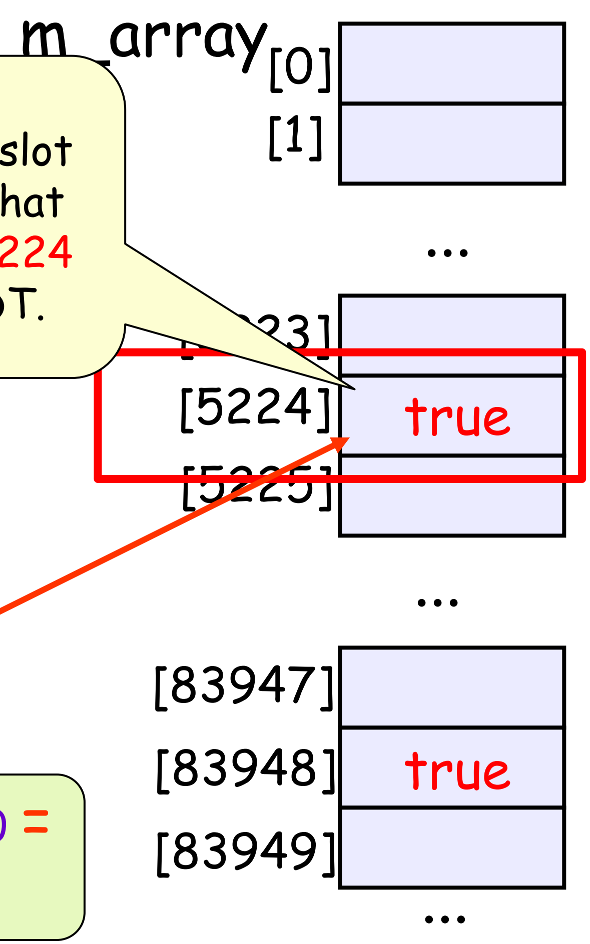
```
class AlmostHashTable2
{
public:
   void addItem(int n)
   {
      int bucket = mapFunc(n);
      m_array[bucket] = true;
   }

private:
   int mapFunc(int idNum)
   {
      return idNum % 100000;
   }

   bool m_array[100000]; // not so big!
};
```

```
int main()
{
   AlmostHashTable2 x;
   x.addItem(400683948);
   x.addItem(111105224);
   x.addItem(222205224);
}
```

Let's see how it works.

The true value in slot 5,224 indicates that the value 111,105,224 is held in our ADT.

111,105,224 % 100,000 =
5,224



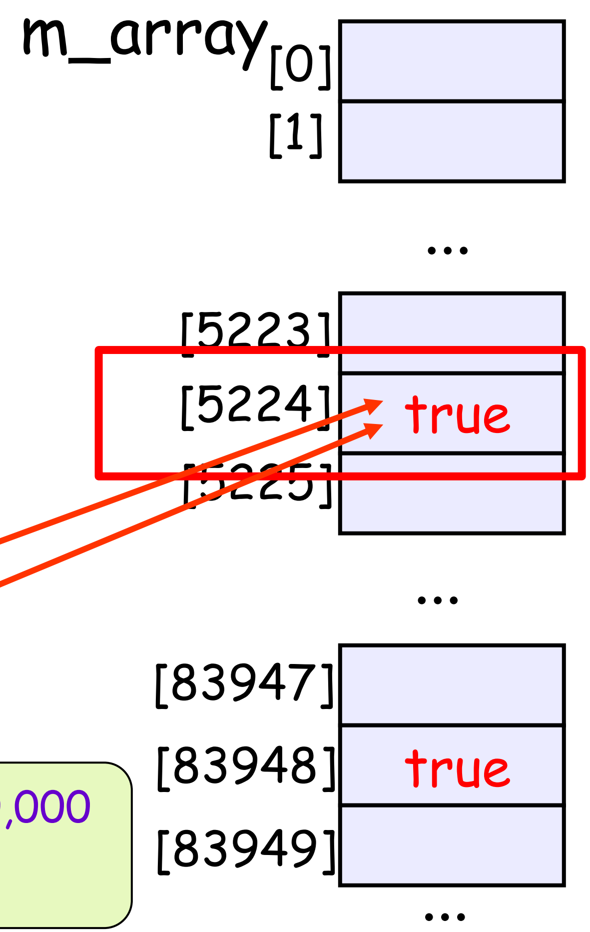
The (Almost) Hash Table

```
class AlmostHashTable2
{
public:
   void addItem(int n)
   {
      int bucket = mapFunc(n);
      m_array[bucket] = true;
   }

private:
   int mapFunc(int idNum)
   {
      return idNum % 100000;
   }

   bool m_array[100000]; // no so big!
};
```

```
int main()
{
   AlmostHashTable2 x;
   x.addItem(400683948);
   x.addItem(111105224);
   x.addItem(222205224);
}
```

222,205,224 % 100,000
= 5,224

Ok, let's try to add the last ID# (222205224) to our almost hash table…
Like 111,105,224, mapFunc() computes a slot for it of 5,224 (222,205,224 % 100,000 == 5,224)!
But wait! We already stored a true value in bucket 5,224 to represent 111,105,224!
Nnow things are ambiguous! How can we tell if my hash table holds 222,205,224 or 111,105,224?
This is called a "collision" and it's a real problem!

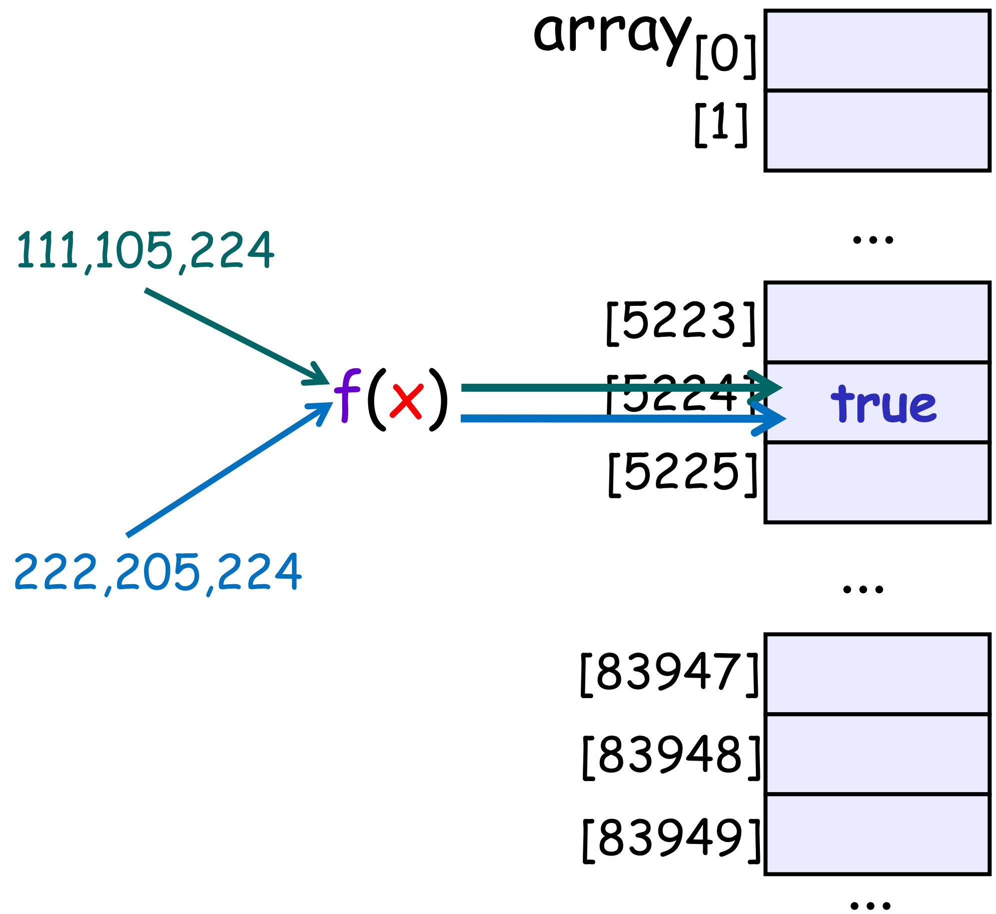


The (Almost) Hash Table: A problem!

A collision is a condition where two or more values both map to the same bucket in the array.

This causes ambiguity, and we can't tell what value was actually stored in the array!

Let's see how to fix this problem!



REAL Hash Tables

There are many schemes for dealing with collisions, and today we'll learn two of the most popular…

The Closed Hash Table with "Linear Probing"

The "Open Hash Table"

Closed Hash Table with Linear Probing: Insertion

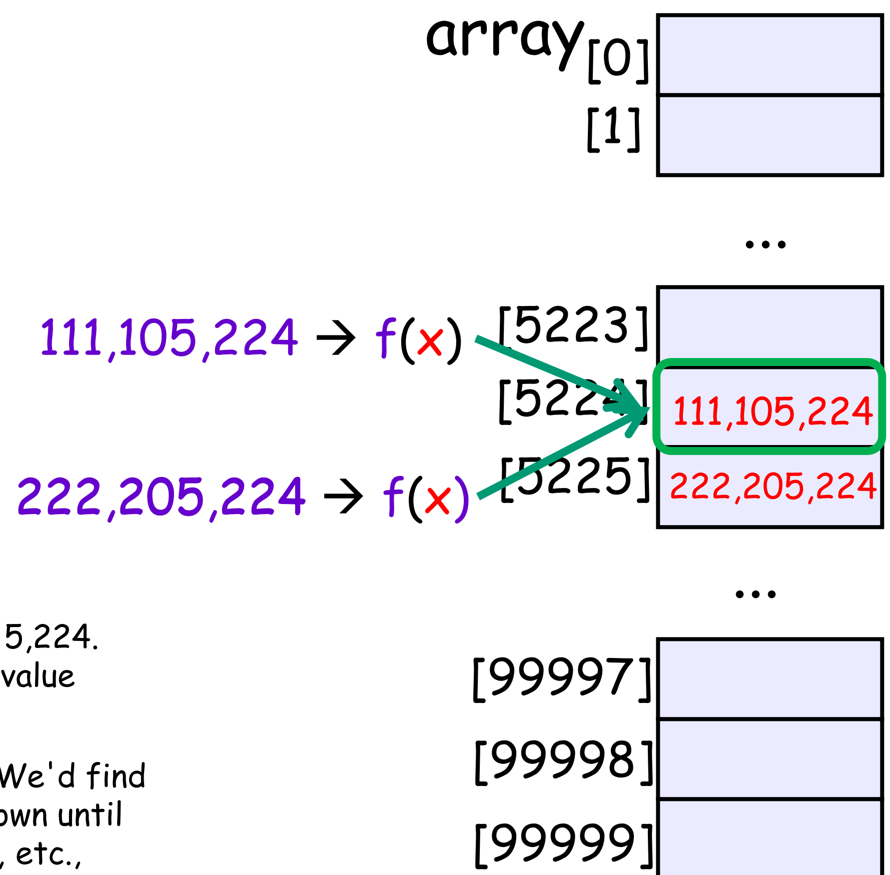
Linear Probing Insertion:

As before, we use our mapping function to locate the right bucket in our array.

If the target bucket is empty, we can store our value there.

However, instead of storing true in the bucket, we store our full original value – this prevents ambiguity!

If the bucket is occupied, we scan down from that bucket until we hit the first empty bucket. We put the new value there.



So first we'd add 111,105,224 by computing its slot number, 5,224. We'd find that this slot is empty, and then we'd stick the value 111,105,224 in that slot.

Then we'd compute the slot for 222,205,224 and get 5,224. We'd find that slot 5,224 is already occupied. So we start "probing" down until we find an empty slot (e.g., look in slot 5,225, then 5,226, etc., wrapping around at slot 99,999 back to zero). The moment we find an empty slot, we place our value (222,205,224) in that slot.

In this case, slot 5,225 is unoccupied so we place our value 222,205,224 in that slot.

Closed Hash Table with Linear Probing: Insertion

Linear Probing Insertion:

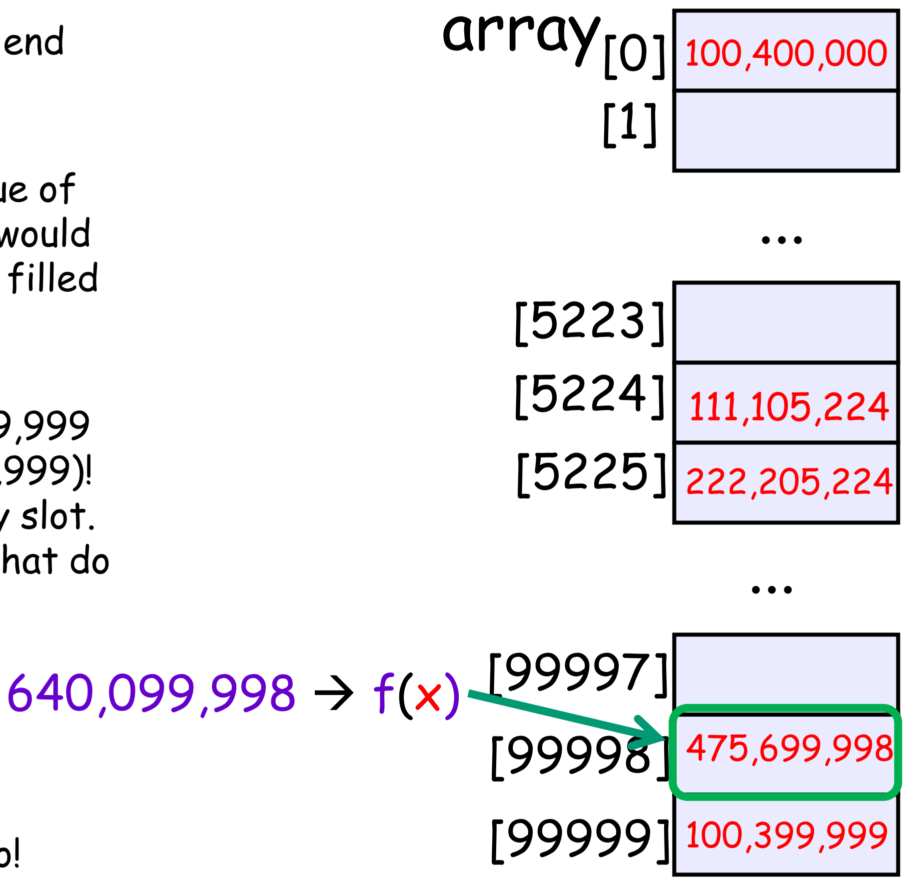
Sometimes, you'll need to insert an item near the end of the table…

For instance, let's say we want to insert a new value of 640,099,998 into our hash table. Notice that this would normally go into slot 9,998. But that slot is already filled with 475,699,998.

So we start probing downward. We look at slot 99,999 and see that it's already filled too (with 100,399,999)! So we want to keep probing down to find an empty slot. But if we do, we'll go past the end of the array. What do we do?

Well, if you run into a collision on the last bucket, and go past the end…

You simply wrap back around the top to slot zero!



Closed Hash Table with Linear Probing: Searching

Linear Probing Searching:

To search our hash table, we use a similar approach.

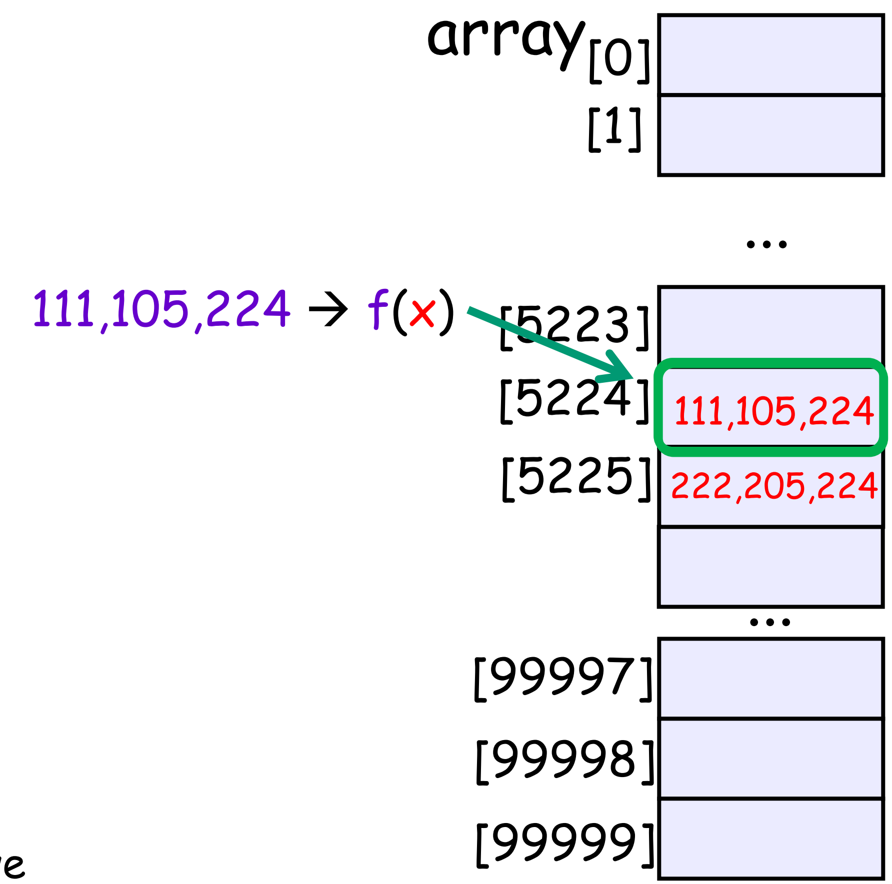
We compute a target bucket number with our mapping function.

We then look in that bucket for our value. If we find it, great!

If we don't find our value, we probe linearly down the array until we either find our value or hit an empty bucket.

If while probing, you run into an empty bucket, it means: your value isn't in the array.

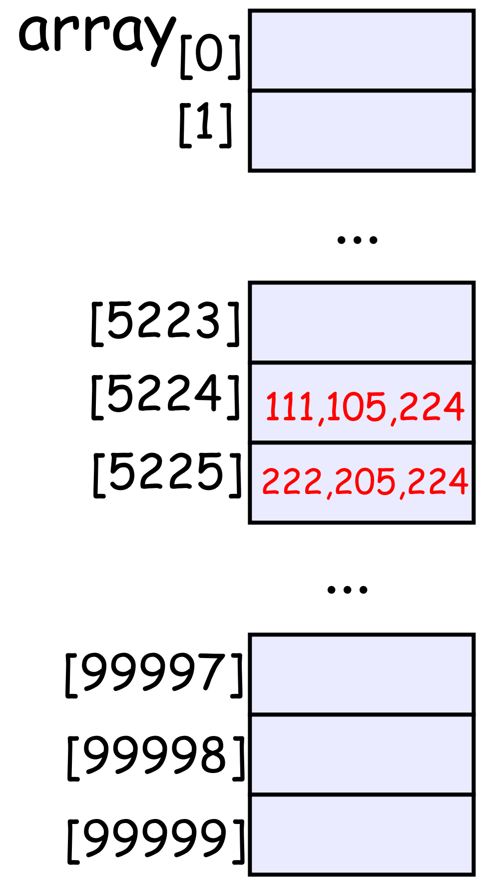
As before, if we go past the end of the array, we just wrap around back to slot zero and continue searching.



Closed Hash Table with Linear Probing

This approach addresses collisions by putting each value as close as possible to its intended bucket.

Since we store every original value (e.g., 111,105,224) in the array, there is no chance of ambiguity.



Closed Hash Table with Linear Probing

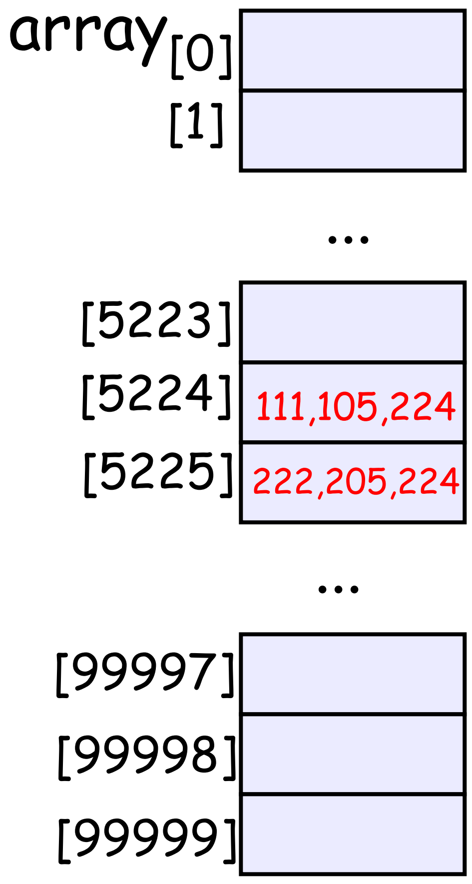
So why do we call this a
"Closed" hash table???

Since our data is stored in a
fixed-size array, there are a
fixed (closed) number of buckets
for us to put values.

Once we run out of empty buckets,
we can't add new values...

Linked lists and binary search trees
don't have this problem!

Ok, let's see the C++ code now!



Linear Probing Hash Table: The Details

In a Linear Probing Hash Table, each bucket in the array is just a C++ struct.

Each bucket holds two items:

1. A variable to hold your value (e.g., an int for an ID#)
2. A "used" field that indicates if this bucket in the hash table has been filled or not.

```
struct BUCKET
{
    // a bucket stores a value (e.g. an ID#)
    int idNum;

    bool            used;  // is bucket in-use?
};
```

If this field is false, it means that this Bucket in the array is empty. If the field is true, then it means this Bucket is already filled with valid data.

```
const in NUM_BUCK = 10;

class HashTable
{
public:
   void insert(int idNum)
    {
      int bucket = mapFunc(idNum);

      for (int tries=0;tries<NUM_BUCK;tries++)
       {
         if (m_buckets[bucket].used == false)
          {
            m_buckets[bucket].idNum = idNum;
            m_buckets[bucket].used = true;
            return;
          }
         bucket = (bucket + 1) % NUM_BUCK;
       }
      // no room left in hash table!!!
    }

private:
   int mapFunc(int idNum) const
       {  return idNum % NUM_BUCK;   }

   BUCKET m_buckets[NUM_BUCK];
};
```

First we compute the starting bucket number.

Since our array has 10 slots, we will loop up to 10 times looking for an empty space. If we don't find an empty space after 10 tries, our table is full!

We'll store our new item in the first unused bucket that we find, starting with the bucket selected by our mapping function.

If the current bucket is already occupied by an item, advance to the next bucket (wrapping around from slot 9 back to slot 0 when we hit the end).

Here's our mapping function.
As before, we compute our bucket number by dividing the ID number by the total # of buckets and then taking the remainder (%).

Our hash table has 10 slots, aka "buckets."

Linear Probing: Inserting

```
const in NUM_BUCK = 10;

class HashTable
{
public:                          29
   void insert(int idNum)            bucket [ 9 ]
   {
      int bucket = mapFunc(idNum);

      for (int tries=0;tries<NUM_BUCK;tries++)
      {
         if (m_buckets[bucket].used == false)
         {
            m_buckets[bucket].idNum = idNum;
            m_buckets[bucket].used = true;
            return;
         }
         bucket = (bucket + 1) % NUM_BUCK;
      }
      // no room left in hash table!!!
   }

private:
   int mapFunc(int idNum) const
       {  return idNum % NUM_BUCK;  }

   BUCKET m_buckets[NUM_BUCK];
};
```

	idNum:	used:
0		f
1		f
2		f
3		f
4		f
5		f
6		f
7		f
8		f
9	29	f

```
main()
{
   HashTable ht;

   ht.insert(29);
   ht.insert(65);
   ht.insert(79);

}
```

Linear Probing: Inserting

```
const in NUM_BUCK = 10;

class HashTable
{
public:                          65
   void insert(int idNum)             bucket [ 5 ]
   {
      int bucket = mapFunc(idNum);

      for (int tries=0;tries<NUM_BUCK;tries++)
      {
         if (m_buckets[bucket].used == false)
         {
            m_buckets[bucket].idNum = idNum;
            m_buckets[bucket].used = true;
            return;
         }
         bucket = (bucket + 1) % NUM_BUCK;
      }
      // no room left in hash table!!!
   }

private:
   int mapFunc(int idNum) const
       {  return idNum % NUM_BUCK;   }

   BUCKET m_buckets[NUM_BUCK];
};
```

	idNum:	used:
0		f
1		f
2		f
3		f
4		f
5	65	f
6		f
7		f
8		f
9	29	T

```
main()
{
   HashTable ht;

   ht.insert(29);
   ht.insert(65);
   ht.insert(79);

}
```

Linear Probing: Inserting

```
const in NUM_BUCK = 10;

class HashTable
{
public:
   void insert(int idNum)
   {
      int bucket = mapFunc(idNum);

      for (int tries=0;tries<NUM_BUCK;tries++)
      {
         if (m_buckets[bucket].used == false)
         {
            m_buckets[bucket].idNum = idNum;
            m_buckets[bucket].used = true;
            return;
         }
         bucket = (bucket + 1) % NUM_BUCK;
      }
      // no room left in hash table!!!
   }

private:
   int mapFunc(int idNum) const
       {   return idNum % NUM_BUCK;   }

   BUCKET m_buckets[NUM_BUCK];
};
```

bucket = 79 % NUM_BUC

bucket = 79 % 10

bucket = 9

79

bucket 0

	idNum:	used:
0	79	f
1		f
2		f
3		f
4		f
5	65	T
6		f
7		f
8		f
9	29	T

```
main()
{
   HashTable ht;

   ht.insert(29);
   ht.insert(65);
   ht.insert(79);

}
```

```
const in NUM_BUCK = 10;

class HashTable
{
public:

  bool search(int idNum)
  {
    int bucket = mapFunc(idNum);
    for (int tries=0;tries<NUM_BUCK;tries++)
    {
      if (m_buckets[bucket].used == false)
        return false;
      if (m_buckets[bucket].idNum == idNum)
        return true;
      bucket = (bucket + 1) % NUM_BUCK;
    }
    return false;// not in the hash table
  }

private:
   int mapFunc(int idNum) const
       {  return idNum % NUM_BUCK;   }

   BUCKET m_buckets[NUM_BUCK];
};
```

Compute the starting bucket where we expect to find our item.

Since we may have collisions, in the worst case, we may need to check the entire table! (10 slots)

If we reach an empty bucket (and haven't yet found our item) then we know our item is not in the table!

Otherwise, the bucket is in-use. If it also holds our ID# then we've found our item and we're done.

If we didn't find our item, advance to the next bucket in search of it. Wrap around when we reach the end of the array.

If we went through every bucket and didn't find our item, then it's not in the hash table! Tell the user.

```
const in NUM_BUCK = 10;

class HashTable
{
public:

  bool search(int idNum)
  {
     int bucket = mapFunc(idNum);

     for (int tries=0;tries<NUM_BUCK;tries++)
     {
       if (m_buckets[bucket].used == false)
         return false;
       if (m_buckets[bucket].idNum == idNum)
         return true;

       bucket = (bucket + 1) % NUM_BUCK;
     }
     return false;// not in the hash table
  }

private:
   int mapFunc(int idNum) const
       {  return idNum % NUM_BUCK;   }

   BUCKET m_buckets[NUM_BUCK];
};
```

29

bucket 9

0	idNum:	79	used:	T
1	idNum:		used:	f
2	idNum:		used:	f
3	idNum:		used:	f
4	idNum:		used:	f
5	idNum:	65	used:	T
6	idNum:	15	used:	T
7	idNum:	175	used:	T
8	idNum:		used:	f
9	idNum:	29	used:	T

```
main()
{
  HashTable ht;
  …
  bool x;
  x = ht.search(29);
  x = ht.search(175);
  x = ht.search(20);
}
```

```
const int NUM_BUCK = 10;

class HashTable
{
public:

  bool search(int idNum)
  {
     int bucket = mapFunc(idNum);

     for (int tries=0;tries<NUM_BUCK;tries++)
     {
       if (m_buckets[bucket].used == false)
         return false;
       if (m_buckets[bucket].idNum == idNum)
         return true;

       bucket = (bucket + 1) % NUM_BUCK;
     }
     return false;// not in the hash table
  }
private:
   int mapFunc(int idNum) const
        {  return idNum % NUM_BUCK;  }

   BUCKET m_buckets[NUM_BUCK];
};
```

175

0	idNum:	79	used:	T
1	idNum:		used:	f
2	idNum:		used:	f
3	idNum:		used:	f
4	idNum:		used:	f
5	idNum:	65	used:	T
6	idNum:	15	used:	T
7	idNum:	175	used:	T
8	idNum:		used:	f
9	idNum:	29	used:	T

```
main()
{
  HashTable ht;
  …
  bool x;
  x = ht.search(29);
  x = ht.search(175);
  x = ht.search(20);
}
```

```
const int NUM_BUCK = 10;

class HashTable
{
public:
                                 20
  bool search(int idNum)
  {
     int bucket = mapFunc(idNum);

     for (int tries=0;tries<NUM_BUCK;tries++)
     {
       if (m_buckets[bucket].used == false)
         return false;
       if (m_buckets[bucket].idNum == idNum)
         return true;

       bucket = (bucket + 1) % NUM_BUCK;
     }
     return false;// not in the hash table
  }

private:
   int mapFunc(int idNum) const
       {  return idNum % NUM_BUCK;  }

   BUCKET m_buckets[NUM_BUCK];
};
```

0	idNum: 79	used: T
1	idNum:	used: T
2	idNum:	used: f
3	idNum:	used: f
4	idNum:	used: f
5	idNum: 65	used: T
6	idNum: 15	used: T
7	idNum: 175	used: T
8	idNum:	used: f
9	idNum: 29	used: T

```
main()
{
  HashTable ht;
  …
  bool x;
  x = ht.search(29);
  x = ht.search(175);
  x = ht.search(20);
}
```

What Can you Store in your Hash Table?

Oh, and if you like, you can include additional associated values (e.g., a name, GPA) in each bucket!

For instance, what if I want to also store the student's name and GPA in each bucket along with their ID#?

You can do that!

Now when you look up a student by their ID# you can ALSO get their name and GPA!

```
struct Bucket
{
    int         idNum;
    string      name;
    float       GPA;
    bool        used;
}
```

```
bool search(int id, string &name, float &GPA)
{
  int bucket = mapFunc(idNum);

  for (int tries=0;tries<NUM_BUCK;tries++)
  {
    if (m_buckets[bucket].used == false)
      return false;
    if (m_buckets[bucket].idNum == idNum)
    {
      name = m_buckets[bucket].name;
      GPA = m_buckets[bucket].GPA;
      return true;
    }
    bucket = (bucket + 1) % NUM_BUCK;
  }
  return false;// not in the hash table
}
```

Linear Probing: Deleting?

So far, we've seen how to insert items into our Linear Probe hash table.

What if we want to delete a value from our hash table?

Let's take a naïve approach and see what happens...

To delete a value, let's just zero out our value and set the used field to false... For instance, let's delete 65.

To delete the value of 65 in slot #5.

	idNum:	used:
0	79	T
1		f
2		f
3		f
4		f
5	65	T
6	15	T
7	175	T
8		f
9	29	T

	idNum:	used:
0	79	T
1		f
2		f
3		f
4		f
5	0	f
6	15	T
7	175	T
8		f
9	29	T

We zeroed out idNum and set used to false.

If we delete a value where a collision happened...

When we try to search again, we may prematurely abort our search, failing to find the sought-for value. For example, if we search for 15 (in the table on the right), our algorithm will go to slot #5. We'd find that slot #5 is empty, and we'd abort our search. Eek!

So, as you can see, if we simply delete an item from our hash table in a naïve, we have problems!

There are ways to solve this problem with a Linear Probing hash table, but they're not recommended!

So, in summary, only use Closed/Linear Probing hash tables when you don't intend to delete items from your hash table.

Like if you're building a hash table that holds words for a dictionary...You'll just add words, never delete any, right?

The "Open Hash Table"

We just saw how to use linear probing to deal with collisions in our closed hash table.

Our closed hash table + linear probing works just fine, but it still has a few problems:

It's difficult to delete items

It has a cap on the number of items it can hold... That's a bummer.

It'd be nice if we could find a way to avoid both of these problems, yet still have an O(1) table!

We can! And it's called the "Open Hash Table."
Let's see how it works!

The "Open" Hash Table

Idea: Instead of storing our values directly in the array, each array bucket points to a linked list of values.

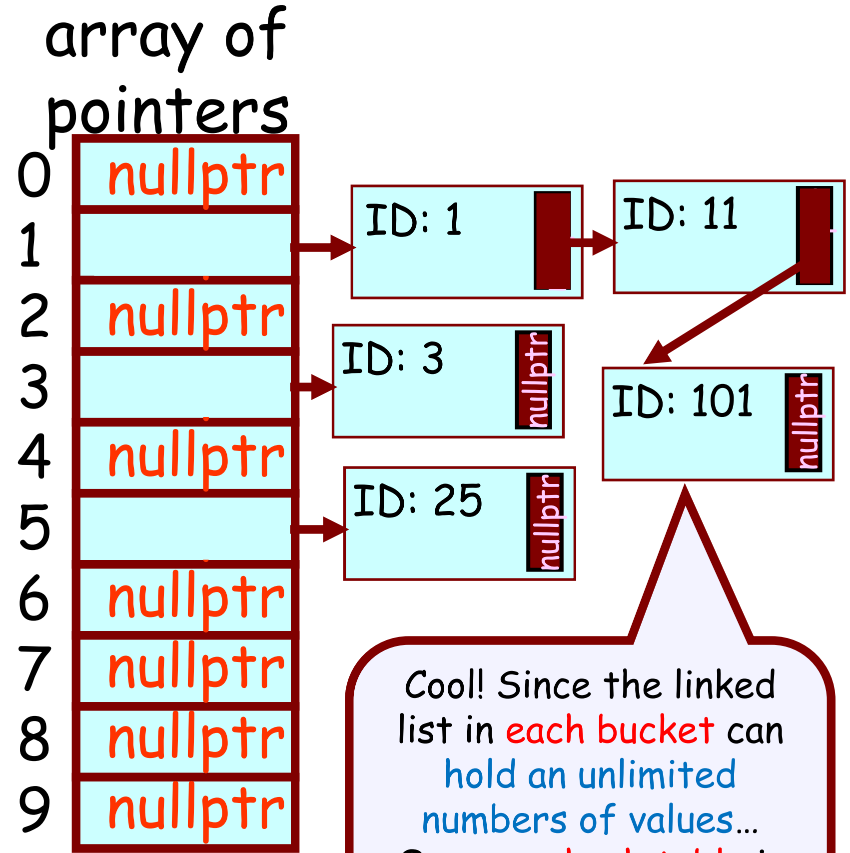
To search for an item:

1. As before, compute a bucket # with your mapping function:

```
bucket = mapFunc(idNum);
```

2. Search the linked list at array[bucket] for your item

3. If we reach the end of the list without finding our item, it's not in the table!



Cool! Since the linked list in each bucket can hold an unlimited numbers of values… Our open hash table is not size-limited like our closed one!

Insert the following values: 1, 3, 11, 25, 101

The "Open" Hash Table: Deletions

Question:
How do you delete an item from an open hash table?

Answer:
You just remove the value from the linked list.

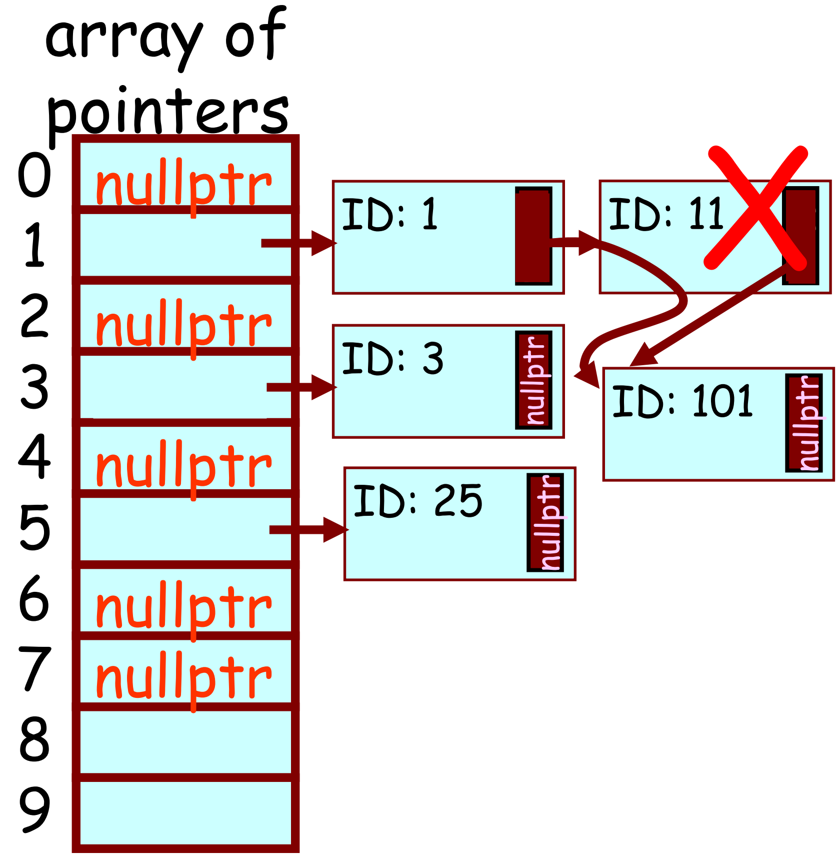
Let's delete the student with ID=11. We just relink ID=1 to ID=101 and use the C++ delete command to delete node 11.

Cool! Unlike a closed hash table, you can easily delete items from an open hash table!

If you plan to repeatedly insert and delete values into the hash table, then the Open table is your best bet!

Also, you can insert more than N items into your table (and still have great performance)!

Why? Because each bucket can hold more than one item!



Hash Table Efficiency

Question: How efficient is the hash table ADT?
How long does it take to locate an item?
How long does it take to insert an item?

Answer:

It depends upon:

(a) The type of hash table (e.g., closed vs. open),

(b) how full your hash table is, and

(c) how many collisions you have in the hash table.

Hash Table Efficiency

Let's assume we have a completely (or nearly) empty hash table…

What's the maximum number of steps required to insert a new value ?

Right! There's zero chance of collision, so we can add our new value in one step!

And finding an item in a nearly-empty hash table is just as fast!

We have no collisions so either we find an item right away or we know it's not in the hash table…

0	idNum: -1 Name:	GPA: etc…
1	idNum: -1 Name:	GPA: etc…
2	idNum: -1 Name:	GPA: etc…
3	idNum: -1 Name:	GPA: etc…
4	idNum: -1 Name:	GPA: etc…
5	idNum: -1 Name:	GPA: etc…
6	idNum: -1 Name:	GPA: etc…
7	idNum: -1 Name:	GPA: etc…
8	idNum: -1 Name:	GPA: etc…
9	idNum: -1 Name:	GPA: etc…

Hash Table Efficiency

Ok, but what if our hash table is nearly full?

What's the maximum number of steps required to insert a new value ?

Right! It could take up to N steps. Why? Well let's say I want to insert a new item in slot #6. Since the table is already full, we have to keep probing down until we hit the first open slot. That might be N slots away, in this case in slot 5.

And searching can take just as long in the worst case…

So technically, a hash table can be up to O(N) when it's nearly full!

So how big must we make our hash table so it runs quickly? To figure this out, we first need to learn about the "load" concept…

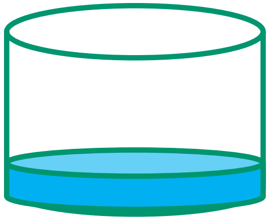
0	idNum: 89	GPA: 3.87	Name: Tad	etc…
1	idNum: 21	GPA: 4.0	Name: Abe	etc…
2	idNum: 12	GPA: 3.2	Name: Ben	etc…
3	idNum: 42	GPA: 3.9	Name: Liz	etc…
4	idNum: 34	GPA: 1.10	Name: Al	etc…
5	idNum:	GPA:	Name:	etc…
6	idNum: 06	GPA: 3.89	Name: Jill	etc…
7	idNum: 67	GPA: 3.4	Name: Hoa	etc…
8	idNum: 78	GPA: 1.7	Name: Bill	etc…
9	idNum: 29	GPA: 2.1	Name: Nat	etc…

Hash Table Efficiency: The Load Factor

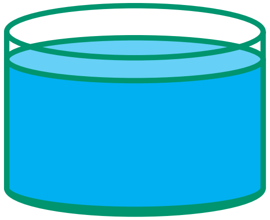
The "load" of a hash table is the maximum number of values you intend to add divided by the number of buckets in the array.

$$L = \frac{\text{Max \# of values to insert}}{\text{Total buckets in the array}}$$

Example: A load of L=.1 means your array has 10X more buckets than you need (you'll only fill 10% of the buckets).



Example: A load of L=.9 means your array has 10% more buckets than you need (you'll fill 90% of the buckets).



Closed Hash w/Linear Probing Efficiency

0	idNum: 89 Name: Tad	GPA: 3.87 etc…
1	idNum: -1 Name:	GPA: etc…
2	idNum: 42 Name: Liz	GPA: 3.9 etc…
3	idNum: 12 Name: Ben	GPA: 3.2 etc…
4	idNum: -1 Name:	GPA: etc…
5	idNum: -1 Name:	GPA: etc…
6	idNum: -1 Name:	GPA: etc…
7	idNum: -1 Name:	GPA: etc…
8	idNum: -1 Name:	GPA: etc…
9	idNum: 29 Name: Nat	GPA: 2.1 etc…

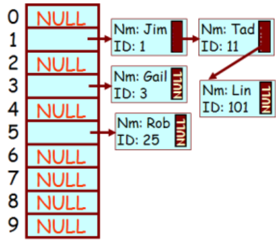
Given a particular load L for a Closed Hash Table w LP, it's easy to compute the average # of tries it'll take you to insert/find an item:

Average # of Tries = $\frac{1}{2}(1 + 1/(1-L))$ for $L < 1.0$

So, if your closed hash table has a

load factor of	your search will take
.10 (your array is 10x bigger than required)	~1.05 searches
.20 (your array is 5x bigger than required)	~1.12 searches
.30 (your array is 3x bigger than required)	~1.21 searches
…	
.70 (your array is 30% bigger than required)	~2.16 searches
.80 (your array is 20% bigger than required)	~3.00 searches
.90 (your array is 10% bigger than required)	~5.50 searches

Open Hash Table Efficiency



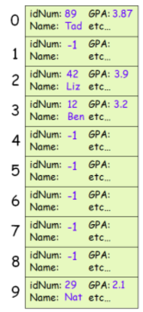
Given a particular load L for an Open Hash Table, it's also easy to compute the average # of tries to insert/find an item:

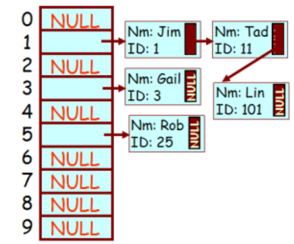
Average # of Checks = 1 + L/2

So, if your open hash table has a

load factor of	your search will take
.10 (your array is 10x bigger than required)	~1.05 searches
.20 (your array is 5x bigger than required)	~1.10 searches
.30 (your array is 3x bigger than required)	~1.15 searches
…	
.70 (your array is 30% bigger than required)	~1.35 searches
.80 (your array is 20% bigger than required)	~1.40 searches
.90 (your array is 10% bigger than required)	~1.45 searches

Closed vs. Open Hash Table





Closed Hash w/L.P.

Load	Avg Steps
.10	~1.05 searches
.20	~1.12 searches
.30	~1.21 searches
	...
.70	~2.16 searches
.80	~3.00 searches
.90	~5.50 searches

Open Hash

Load	Avg Steps
.10	~1.05 searches
.20	~1.10 searches
.30	~1.15 searches
	...
.70	~1.35 searches
.80	~1.40 searches
.90	~1.45 searches

Moral: Open hash tables are almost ALWAYS more efficient than Closed hash tables!

Sizing your Hash Table

Challenge:

If you want to store up to 1000 items in an Open Hash Table and be able to find any item in roughly 1.25 searches, how many buckets must your hash table have?

Remember: Expected # of Checks = 1 + L/2

Answer:

Part 1: Set the equation above equal to 1.25 and solve for L:

$$1.25 = 1 + L/2 \longrightarrow .25 = L/2 \longrightarrow .5 = L$$

Part 2: Use the load formula to solve for "Required size":

$$L = \frac{\text{\# of items to insert}}{\text{Required hash table size}} \rightarrow .5 = \frac{1000}{\text{Required hash table size}} \rightarrow \text{Required hash table size} = \frac{1000}{.5}$$

$$\boxed{= 2000 \text{ buckets}}$$

If our hash table has 2000 buckets and we're inserting a maximum of 1000 values, we are guaranteed to have an average of 1.25 steps per insert/search!

This result means:

"If you want to be able to find/insert items into your open hash table in an average of 1.25 steps, you need a load of .5, or roughly 2x more buckets than the maximum number of values you'll put into your table."

So basically it's a tradeoff!

You could always use a really big hash table with way-too-many buckets and ensure really fast searches...

But then you'll end up wasting lots of memory...

On the other hand, if you have a really small hash table (with just barely enough room), it'll be slower.

Finally, when choosing the exact size of your hash table (the number of buckets)...

Always try to choose a prime number of buckets...

Instead of 2000 buckets,
give your hash table 2017 buckets.

This causes more even distribution and fewer collisions!

What Happens If…

What happens if we want to allow the user to search by the student's name instead of their ID number?

Well, our original mapping function won't quite work:

```
int mapFunc(int ID)
{
   return(ID % 100000)
}
```

A hash function is a function that takes an arbitrary input (like a string)…

And produces an integer output, like a value between 0 and 2 billion.

```
int mapFunc(string &name)
{
   int h = hash(name);
   return h % 100000;
}
```

Well, we need a two-step process!

First, we need to compute a unique numeric value from our string using a "hash" function!

Second, we use our modulo as before to compute a bucket number that fits into our hash table.

A Hash Function for Strings

Here's one possibility for a hash function that can convert a string into an integer value.

```
int hash(string &name)
{
   int i, total=0;

   for (i=0;i<name.length(); i++)
         total = total + name[i];

   return(total);
}
```

But this hash function isn't so good. Why not?

Hint:

What happens if we hash "BAT"?

What happens if we hash "TAB"?

How can we fix it?

Answer: This hash function produces the same output value for many of the same inputs. For example, hash("BAT") == hash("TAB") == hash("SAC"). We ideally want a hash function that gives very different results for even similar inputs.

A Better Hash Function for Strings

Here's better version of our string hashing function – while not perfect, it disperses items more uniformly in the table. Notice that it takes the position of each character into account when computing its result.

```
int hash(string &name)
{
   int i, total=0;

   for (i=0;i<name.length(); i++)
         total = total + (i+1) * name[i];

   return(total);
}
```

Now "BAT" and "TAB" hash to different slots in our array since this version takes character position into account.

But this function still isn't great. Coming up with good hash functions is a PhD-level exercise!

A GREAT Hash Function for Strings

Rather than write your own hash function from scratch, why not use one written by the pros?

C++ provides a great string hashing function:

Make sure to
#include<functional>
to use C++'s hash function!

We'll define our own mapping function, but leverage C++'s hash algorithm under the hood.

```
#include <functional>

unsigned int yourMapFunction(const std::string &hashMe)
{
   std::hash<std::string> str_hash;                  // creates a string hasher!
   unsigned int hashValue = str_hash(hashMe);        // now hash our string!

   hen just add your own modulo
   unsigned int bucketNum = hashValue % NUM_BUCKETS;
   return bucketNum;
}
```

First you define a C++ string hashing object.

This returns a hash value between 0 and 4 billion.

Then you use the object to hash your input string.

Finally, you apply your own modulo function and return a bucket # that fits into your hash table's array.

Writing Your Own Hash Function

Great! But what if you need to write a hash function for some non-standard data type?

```
unsigned int yourMapFunction(const SomeCrazyTypeOfData &hashMe)
{
                                ????
}
```

Like hashing…

Geospatial coordinates
An array of N numbers
The contents of a data file

This is a non-trivial exercise!

You really need to understand the "nature" of the data you're hashing…

Then design your algorithm, analyze it, and iterate.

Choosing a Hash Function: Tips

1. The hash function must always give us the same output value for a given input value:

Today: hash(400683948) → 83,948

Tomorrow: hash(400683948) → *still* 83,948

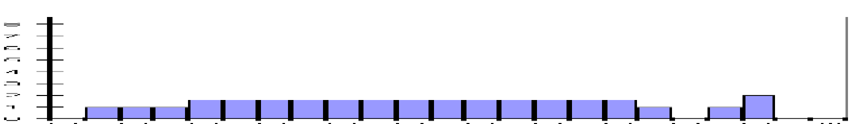
2. The hash function should disperse items throughout the hash array as randomly as possible.

hash("abc") = 294

hash("cba") = 294

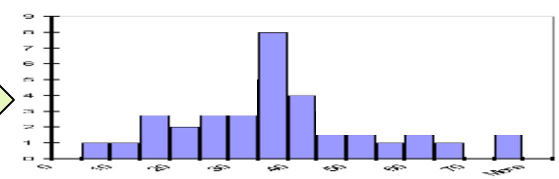
Not good!

3. When coming up with a new hash function, always measure how well it disperses items (do some experiments!)



Good!

Bad!



The unordered_map: A hash-based version of a map

```
#include <unordered_map>
#include <iostream>
#include <string>

using namespace std;

int main( )
{
   unordered_map <string,int> hm;                  // define a new U_M
   unordered_map <string,int>::iterator iter; // define an iterator for a U_M

   hm["Carey"] = 10;                              // insert a new item into the U_M
   hm["David"] = 20;

   iter = hm.find("Carey");                       // find Carey in the hash map

   if (iter == hm.end())                          // did we find Carey or not?
      cout << "Carey was not found!";             // couldn't find "Carey" in the hash map
   else
   {
      cout << "When we look up " << iter->first;      // "When we look up Carey"
      cout << " we find " << iter->second;            // "we find 10"
   }
}
```

Hash Tables vs. Binary Search Trees

	Hash Tables	Binary Search Trees
Speed	O(1) regardless of # of items	$O(\log_2 N)$
Simplicity	Easy to implement	More complex to implement
Max Size	Closed: Limited by array size Open: Not limited, but high load impacts performance	Unlimited size
Space Efficiency	Wastes a lot of space if you have a large hash table holding few items	Only uses as much memory as needed (one node per item inserted)
Ordering	No ordering (random)	Alphabetical ordering

Tables


EVERYTIME YOU CHANGE THE DATABASE TABLE
GOD KILLS A KITTEN
memegenerator.net

Tables

Why should you care?

Tables are the building block of databases (like Oracle & MySQL)

They're used to organize large amounts of data and make it quickly searchable.

Tables are used to:
Hold your $$ bank account data
Store your student transcripts
Hold your credit card transactions
Hold usernames/pws for most sites

So pay attention!

"Tables"

Let's say you want to want to write a program to keep track of all your BFFs…

Of course, you want to remember all the important dirt about each BFF:

And you want to quickly be able to search for a BFF in one or more ways…

" Find all the dirt on my BFF 'David Johansen' "

" Find all the dirt on the BFF whose number is 867-5309 "

Name: Carey Nash
Phone number: 867-5309
Birthday: July 28
iPhone or 'droid: iPhone
Social Security #: 111222333
Favorite food: …

"Tables"

In CS lingo, a group of related data is called a "record."

Each record has a bunch of "fields" like Name, Phone #, Birthday, etc. that can be filled in with values.

If we have a bunch of records, we call this a "table." Simple!

While you may have many records with the same Name field value (e.g., John Smith) or the same Birthday field value (e.g., Jan 1st)…

Some fields, like Social Security Number, will have unique values across all records - this type of field is useful for searching and finding a unique record!

A BFF Record

Name: Carey Nash
Phone number: 867-5309
Birthday: July 28
iPhone or 'droid: iPhone
Social Security #: 111222333
Favorite food: …

Table of BFF Records

Name: Carey Nash

Name: David Small
Phone number: 555-1212

Name: John Rohr
Phone number: 999-9191
Birthday: Jan 1
iPhone or 'droid: Droid
Social Security #: 47372727
Favorite food: …

A field (like the SSN) that has unique values across all records is called a "key field."

Implementing Tables

```
struct Student
{
   string name;
   int IDNum;
   float GPA;
   string phone;
   …
};
```

How could you create a record in C++?

Answer: Just use a struct or class to represent a record of data!

How can you create a table in C++?

Answer: You can simply create an array or vector of your struct!

```
vector<Student> table;
```

```
// algorithm to search by the name field
int SearchByName(vector<Student> &table, string &findName)
{
   for (int s = 0; s < table.size(); s++ )
      if (findName == table[ s ].name)
              return( s );// the student you're looking for is in slot s
   return( -1 );                    // didn't find that student in your table
}
```

```
// algorithm to search by the phone field
int SearchByPhone(vector<Student> &table, string &findPhone)
{
   for (int s = 0; s < table.size(); s++ )
      if (findPhone == table[ s ].phone)
              return( s );// the student you're looking for is in slot s
   return( -1 );                    // didn't find that student in your table
}
```

Implementing Tables

Heck, why not just create a whole C++ class for our table?

```
struct Student
{
   string name;
   int IDNum;
   float GPA;
   string phone;
   …
};
```

```
class TableOfStudents
{
public:
   TableOfStudents();        // construct a new table
   ~TableOfStudents();      // destruct our table
   void addStudent(Student &stud);  // add a new Student
   Student getStudent(int s);    // retrieve Students from slot s
   int searchByName(string &name);  // name is a searchable field
   int searchByPhone(int phone);  // phone is a searchable field
   …
private:

   vector<Student> m_students;
};
```

Tables

In the TableOfStudents class, we used a vector to hold our table and a linear search to find Students by their name or phone.

This is a perfectly valid table – but it's slow to find a student! How can we make it more efficient?

Well, we could alphabetically sort our vector of records by their names…

Then we could use a binary search to quickly locate a record based on a person's name.

But then every time we add a new record, we have to re-sort the whole table. Yuck!

And if we sort by name, we can't search efficiently by other fields like phone # or ID #!

Name: David
ID #: 111222333
GPA: 2.1
Phone: 310 825-1234

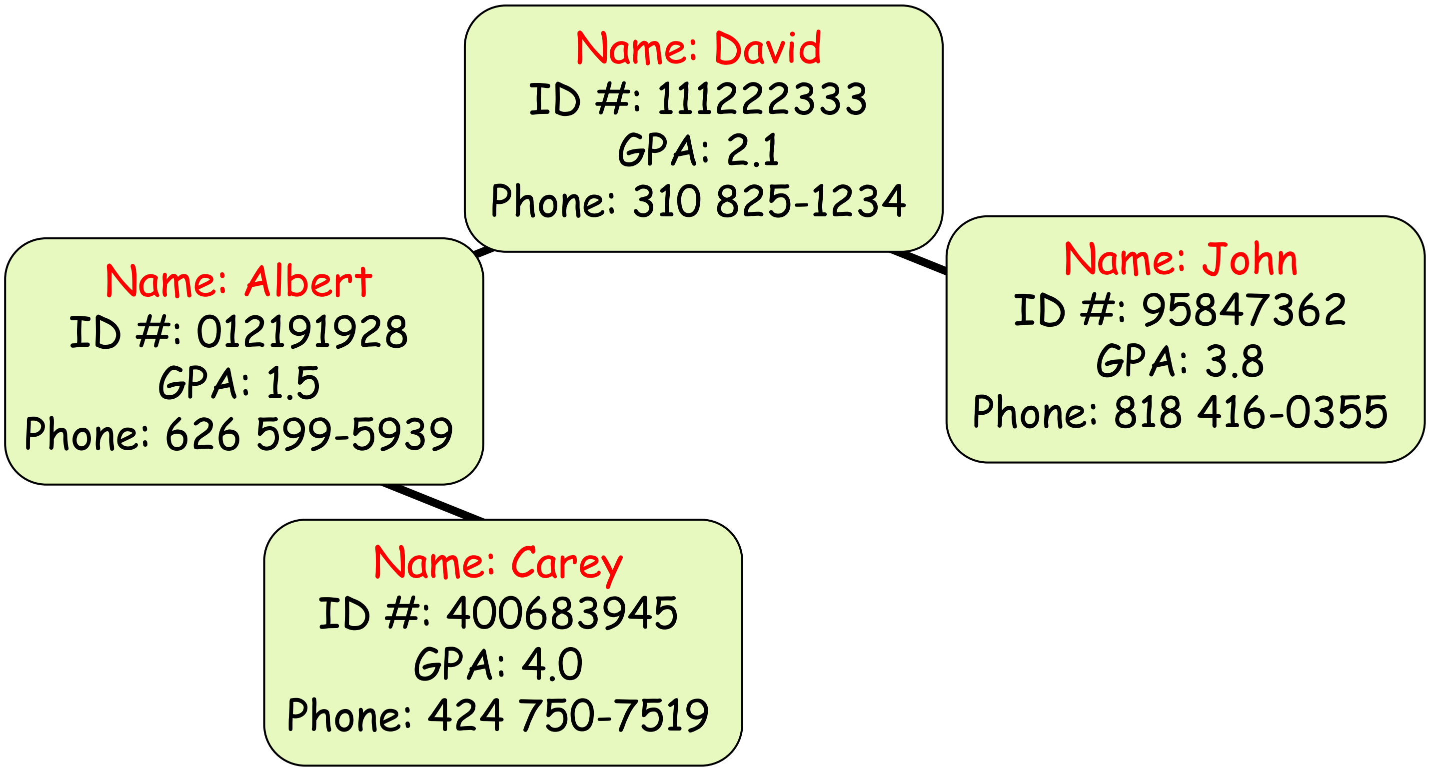
Name: John
ID #: 95847362
GPA: 3.8
Phone: 818 416-0355

Name: Carey
ID #: 400683945
GPA: 4.0
Phone: 424 750-7519

Name: Albert
ID #: 012191928
GPA: 1.5
Phone: 626 599-5939

Tables

Hmmm… What if we stored our records in a binary search tree (e.g., a map) organized by name? Would that fix things?

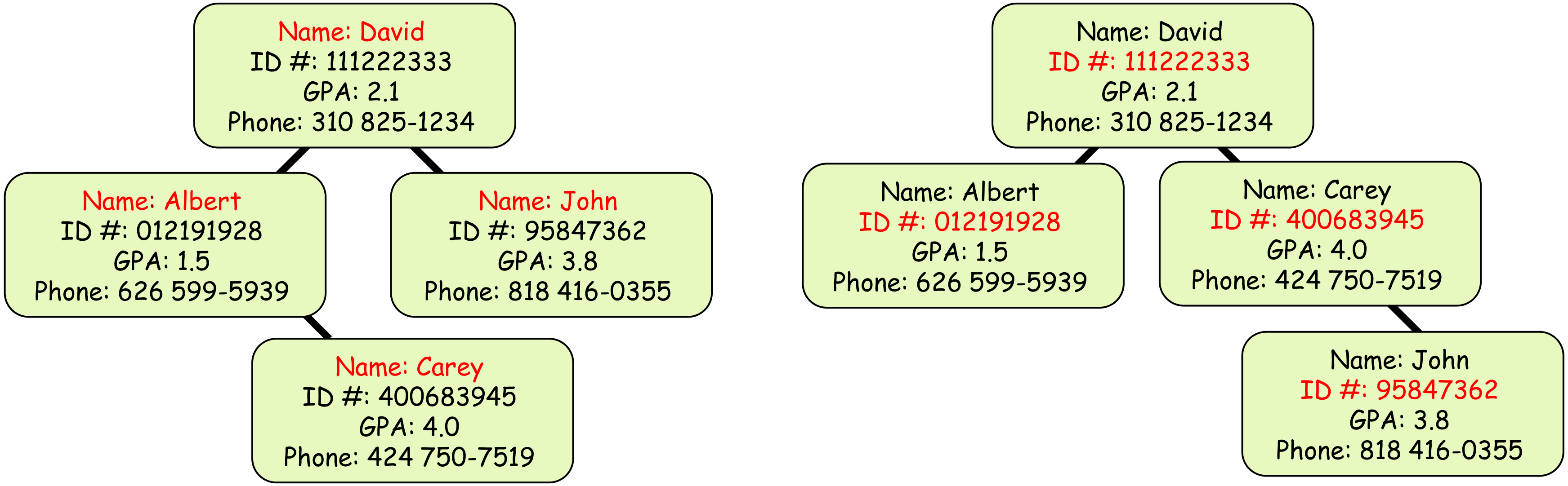


Well, now we can search the table efficiently by name…

But we still can't search efficiently by ID# or Phone #….

Tables

Hmmm… What if we create two tables, ordering the first by name and the second by ID#?



That works… Now I can quickly find people by name or ID#!

But now we have two copies of every record, one in each tree! If the records are big, that's a waste of space!

So what can we do? Let's see!

Making an Efficient Table

1. We'll still use a vector to store all of our records…
2. Let's also add a data structure that lets us associate each person's name with their slot # in the vector…
3. And we can add another data structure to associate each person's ID # with their slot # too!

m_students

0	name: Alex GPA: 2.05 ID: 7124 …
1	name: Linda GPA: 3.99 ID: 0003 …
2	name: Jason GPA: 1.55

```
class TableOfStudents
{
public:
    TableOfStudents();
    ~TableOfStudents();
    void addStudent(Student &stud);
    Student getStudent(int s);
    int searchByName(string &name);
    int searchByPhone(int phone);

private:
   vector<Student> m_students;
   map<string,int>   m_nameToSlot;
   map<int,int>      m_idToSlot;
   map<int,int>      m_phoneToSlot;
};
```

Our second data structure lets us quickly look up a name and find out which slot in the vector holds the related record.

Our third data structure lets us quickly look up an ID# and find out which slot in the vector holds the related record.

These secondary data structures are called "indexes."
Each index lets us efficiently find a record based on a particular field.
We may have as many indexes as we need for our application.

Making an Efficient Table

So what does our addStudent method look like now?

Well, we have to add our new student record to our vector just like before.

But now, every time we add a record, we've also got to add the name to slot # mapping to our first map!

Finally, every time we add a record, we've also got to add the ID# to slot # mapping to our second map!

```
class TableOfStudents
{
pu
    void addStudent(Student &stud)
    {
        m_students.push_back(stud);

        int slot = m_students.size()-1;   // get slot # of new record
        m_nameToSlot[stud.name] = slot;   // maps name to slot #
        m_idToSlot[stud.IDNum] = slot;    // maps ID# to slot #
    }
private:
    vector<Student> m_students;
    map<string,int>  m_nameToSlot;
    map<int,int>     m_idToSlot;
};
```

ID: 7124
...
GPA: 1.55
ID: 1054
...
name: Abe
GPA: 4.00
ID: 9876
...
name: Zelda
GPA: 3.43

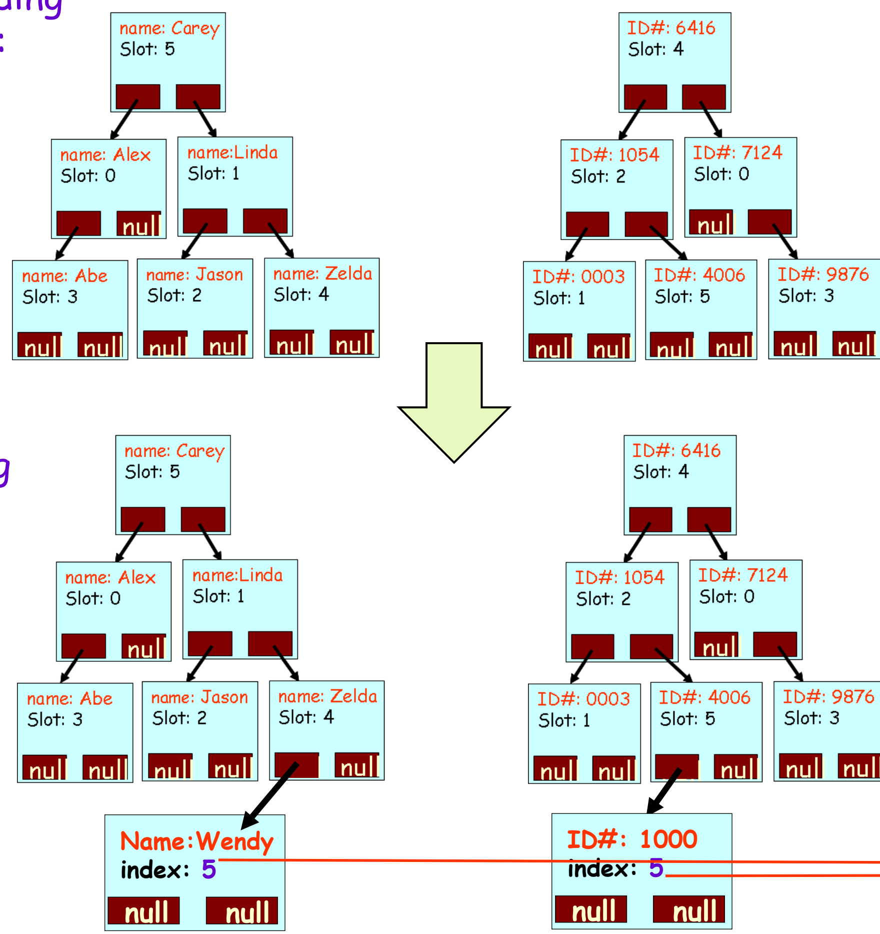
Complex Tables

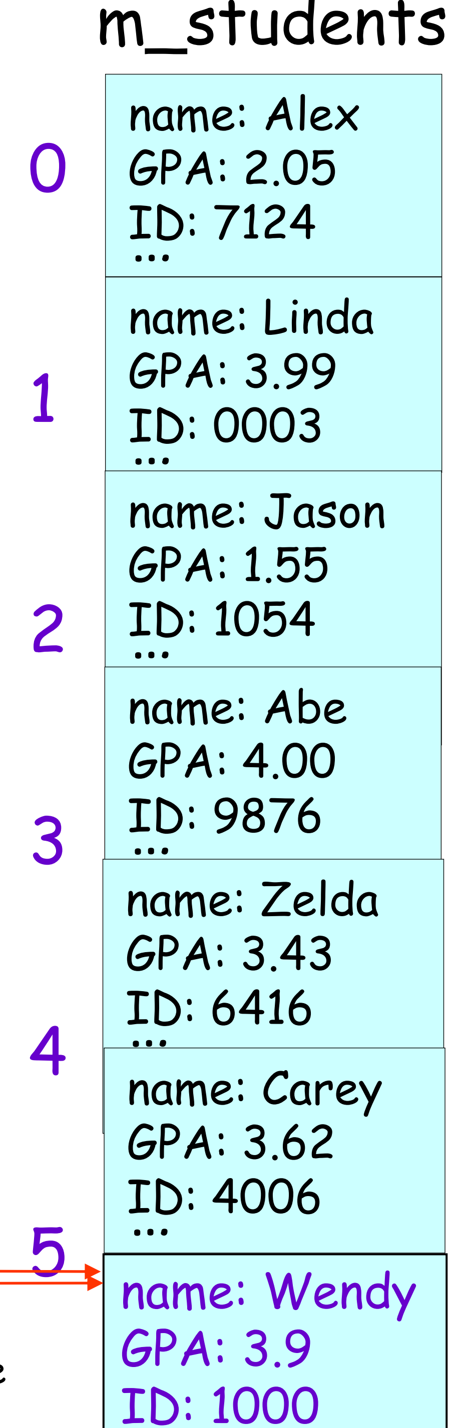
So to review, what do we have to do to insert a new record into our table?

Let's add: Wendy, ID=1000, GPA=3.9

Before adding Wendy:

After adding Wendy:





But wait!!!! - Any time you delete a record or update a record's searchable fields, you also have to update your indexes!

Tables

As it turns out, databases like "Oracle" use exactly this approach to store and index data!

(The only difference is they usually store their data on disk rather than in memory)

And by the way... While my example used binary search trees to index our table's fields...

You could use any efficient data structure you like!

For example, you could use a hash table!

Using Hashing to Speed Up Tables

0
name: Alex
GPA: 2.05
ID: 7124
...

1
name: Linda
GPA: 3.99
ID: 0003
...

2
name: Jason
GPA: 1.55
ID: 1054
...

3
name: Abe
GPA: 4.00
ID: 9876

Can we use hash tables to index our data instead of binary search trees?

Of course!

Now we can have O(1) searches by name! Cool! But in that case why not just always use hash tables to index all of our key fields?

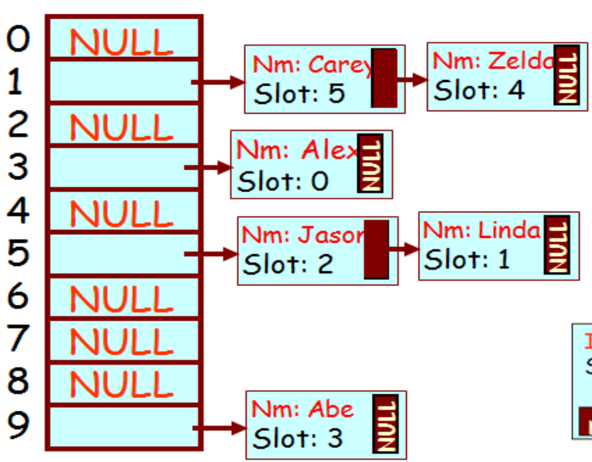
Answer: Because hash tables store the data in an essentially random order.

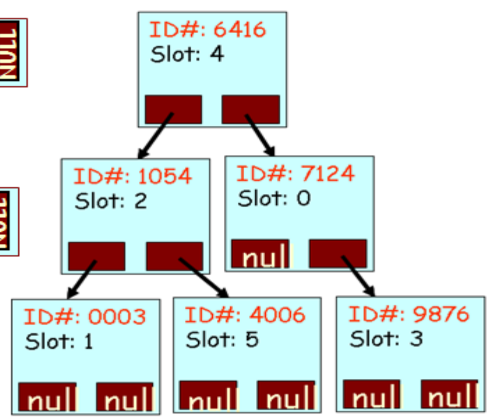
While a BST is slower, it does order the key fields in alphabetical order...

For instance, what if we want to be able to print out all students alphabetically by their name.

If our index data structure is a binary search tree, that's easy!

If we indexed with a hash table, we'd have to do a lot more work to do the same thing...





Moral: You need to understand how your table will be used to determine how to best index each field.

For example:

I'd use a BST for the name field so I can print people's names in alphabetical order. But I'd use a hash table for the phone field, cause I just need to search quickly but I don't need to order records by their phone #.

Challenges

Question: What is the big-oh of traversing all of the elements in a hash table?

Question: I have two hash tables: the first has 10 buckets, and the second has 20 buckets. If I insert each of the following IDs into each hash table, where will each ID number end up (which bucket #s)?

ID = 5
ID = 15
ID = 25
ID = 100

Question: How can you print out the items in a hash-table in alphabetical/numerical order.

Answers:

1. The big-O of traversing a simple hash table like we've covered is O(B+N) where B is the # of buckets in the table and N is the total # of items currently stored in the table. Why B+N? Well let's say you've just added 1 item to a hash table with B=1 billion buckets. To print out that one item, you'll have to loop through all 1 billion buckets. That said, most modern hash-table implementations add a bit of extra linking in the nodes to enable O(N) traversals. So if you get this question in an interview, say O(N).
2. first[5] = 5, first[6] = 15, first[7] = 25, first[0] = 100; second[5] = 5, second[15] = 15, second[6] = 25, second[0] = 100
3. Given that items are distributed essentially randomly through the hash table by a good hash function, you have to first copy all of the items from the hash table into a vector or array (which is O(N)), then sort them using a good sort function like quicksort (O(N*logN), then print the sorted items out. Or you could insert all of the items into a binary search tree and then traverse the tree in-order. Soon we'll also learn how to do this with a "heap" ADT!