UCLA College | Physical Sciences Statistics

STATS 10: Introduction to Statistical Reasoning
Chapter 8

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Statistical Inference for Proportion

Research question: What is the population proportion?

1. **Point estimation:** suggests a single number $\hat{p}$.
2. **Interval estimation:** suggests a range of values with some confidence.
3. **Hypothesis testing**

Answer a more specific research question

e.g., “Is the population proportion 0.5?”

The result of the testing suggests a statistical ‘conclusion’ to the question.

The Essential Ingredients of Hypothesis Testing

A Criminal Trial as a Hypothesis Test

Criminal Trial Process
Claim 1: the defendant is innocent **Claim 2**: the defendant is guilty
Find clue/evidence.
Assess the evidence: how unlikely to have the evidence if the person is really innocent?
Make decision base on if the evidence is very unlikely when the defendant is innocent.

Hypothesis Testing

A **statistical hypothesis** is an assumption or claim about a population parameter.

Statistical hypothesis testing:

Assessing evidence provided by the data in favor of or against some claim about the population

1. **Hypothesize**
   - State a hypothesis (claim) that will be tested against a neutral “skeptical” claim.
2. **Collect and summarize data**
   - Choose a sample, collect relevant data and summarize them.
3. **Assess evidence**
   - Figure out how likely it is to observe data like the data we got, had the neutral “skeptical” claim been true.
4. **Conclude and Interpret**
   - Make conclusions based on the results.
   - Do you believe the claim, or do you find that the claim doesn’t have enough evidence to back it up?

HT Step 1-- Hypothesize

- **H_0 -- The Null Hypothesis (the benefit of the doubt)**

The neutral, status quo, skeptical statement about a population parameter.

  - "The coin is fair"
  - "The proportion of defective products produced by the machine is 0.20."

- **H_a -- The Alternative Hypothesis**

The research hypothesis; the statement about a population parameter that we intend to demonstrate is true.

  - "The coin is not fair"
  - "The proportion of defective products produced by the machine is less than 0.20"

One-sided (left)	One-sided (right)	Two-sided
$H_0: p = p_0$	$H_0: p = p_0$	$H_0: p = p_0$
$H_a: p < p_0$	$H_a: p > p_0$	$H_a: p \neq p_0$

p_0 -- null value, the value of the population proportion that the null hypothesis claims to be true.

Note: Hypotheses are always about **population parameters**; they are never about sample statistics.

Exercises

State the null and alternative hypotheses.

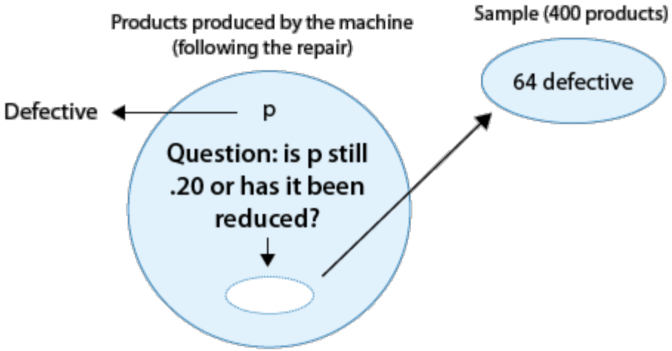
1. A 2014 Pew Poll found that 61% of Americans believed in global warming. A researcher believes this rate has declined.

2. The proportion of movie-goers who watch 3D movies used to be about 20%. Since the release of Avatar, we think the proportion has increased.

3. A government official claims that the dropout rate for local schools is 25%. Last year, 190 out of 603 students dropped out. Is there enough evidence to support the government official's claim.

Example (1/4)

A factory machine is known to produce 20% defective products, and is therefore sent for repair. After the machine is repaired, 400 products produced by the machine are chosen at random and 64 of them are found to be defective.
Do the data provide enough evidence that the proportion of defective products produced by the machine has been **reduced** as a result of the repair?



1. Stating the hypotheses

Has the proportion of defective products been reduced as a result of the repair?

$$\mathrm{H_0}: p = 0.20$$

A. $\mathrm{H_a}: p \neq 0.20$
B. $\mathrm{H_a}: p < 0.20$
C. $\mathrm{H_a}: p > 0.20$

HT Step 2 – Collect and summarize data

- **Sample statistic**
  e.g., sample proportion($\hat{p}$), sample mean($\bar{x}$)

- **Test statistic** -- a value (a statistic) that compares our observed outcome with the outcome we would get if null hypothesis is true.

The z-test statistic:

$$z = \frac{\text{observed value } - \text{null value}}{SE}$$

The z-test statistic for one population proportion:

$$z = \frac{\hat{p} - p_0}{SE}$$

- $\hat{p}$ is the sample proportion (from data)
- p_0 is the proportion from the null hypothesis (H_0)
- $SE = \sqrt{\frac{p_0(1-p_0)}{n}}$

One-Proportion Z-Test Statistic

Under the null hypothesis $H_0: p = p_0$, the **sampling distribution** for the z-test statistic is approximately the **standard normal** distribution $N(0, 1)$ if the following conditions are met:

1. Random and Independent
   - The sample is randomly selected from the population, and observations are independent of each other.
2. Large sample size

$$np_0 \geq 10 \text{ and } n(1 - p_0) \geq 10$$

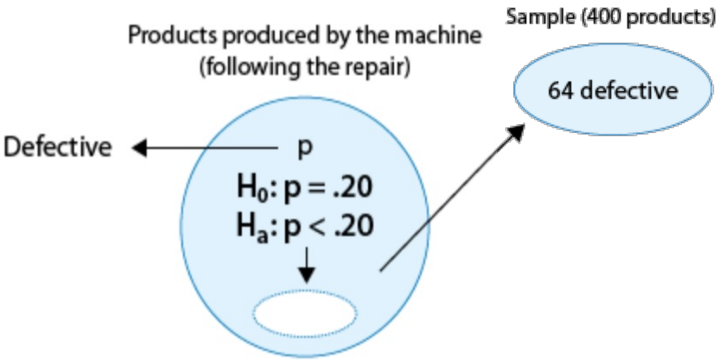
3. Large population

$$N \geq 10n$$

- The further the test statistic is from 0, the more we doubt the null hypothesis.
- If the test statistic is 0 (or close to 0), there is little to no evidence to doubt the null hypothesis.

Example (2/4)

A factory machine is known to produce 20% defective products, and is therefore sent for repair. After the machine is repaired, 400 products produced by the machine are chosen at random and 64 of them are found to be defective.
Do the data provide enough evidence that the proportion of defective products produced by the machine has been reduced as a result of the repair?



2. **Collect data, check the conditions, and summarize data with test statistic if the conditions are met.**

HT Step 3 – Assessing of Evidence

Measuring Surprise: The p-Value

It reports the probability of obtaining data like those observed assuming that the null hypothesis is true.

- Small *p*-values (close to 0) mean we are really surprised. If the null hypothesis is true, what we observed rarely happens.
- Large *p*-values (close to 1) mean we are not surprised at all. If the null hypothesis is true, what we observed happens quite often.

$$P(\text{obtaining a test statistic at least as extreme as the observed} \mid \mathrm{H}_0 \text{ true})$$

By "extreme" we mean extreme in the direction of the alternative hypothesis.

One-Tailed *p*-Value

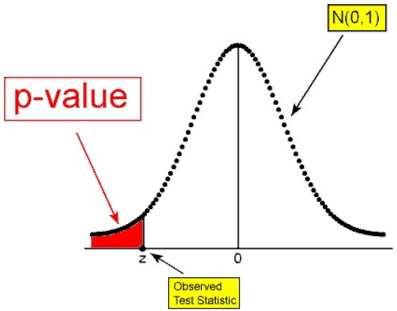
If $\mathrm{H_a}$ contains ">" or "<", we have a **one-tailed *p*-value**.

$\mathrm{H_a}: p < p_0$ -- left-tailed test.

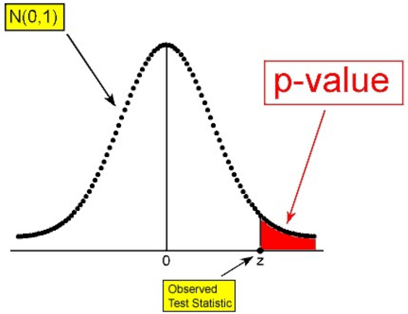
"Left-tailed" *p*-value: $P(Z \le z)$

$\mathrm{H_a}: p > p_0$ -- right-tailed test

"Right-tailed" *p*-value: $P(Z \ge z)$



The *p*-value is the area to the left of the test statistic



The *p*-value is the area to the right of the test statistic.

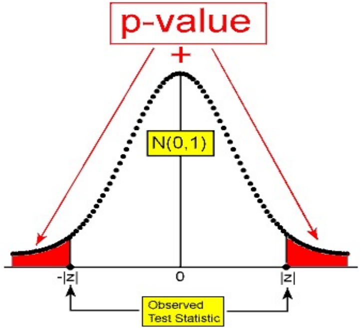
Two-Tailed *p*-Value

If the alternative hypothesis $\mathrm{H_a}$ contains "≠", we have a **two-tailed *p*-value**.

$$\mathrm{H_a}: p \neq p_0$$

"Two-sided" or "Two-tailed" *p*-value:

$$P(Z \le -|z|) + P(Z \ge |z|) = 2P(Z \ge |z|) = 2P(Z \le -|z|)$$

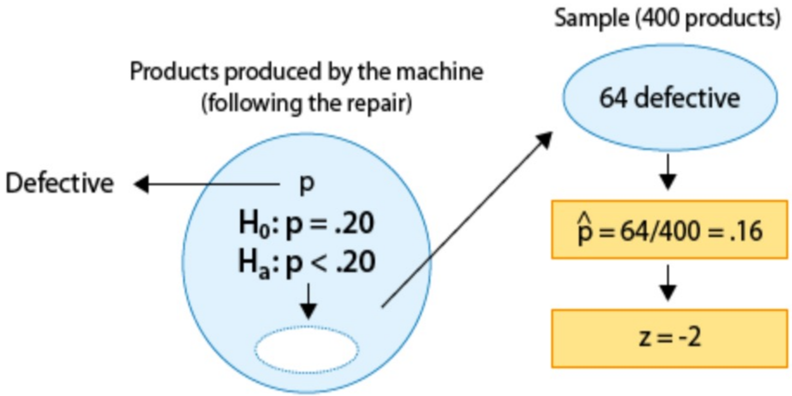


The *p*-value would be the total of the shaded region

In general, it is harder to reject H_o against a two-sided H_a because the p-value is twice as large.

Example (3/4)

A factory machine is known to produce 20% defective products, and is therefore sent for repair. After the machine is repaired, 400 products produced by the machine are chosen at random and 64 of them are found to be defective.
Do the data provide enough evidence that the proportion of defective products produced by the machine has been reduced as a result of the repair?



3. Assessing of evidence

Calculate the *p*-value

HT Step 4 – Conclude and Interpret

Determine whether we have enough evidence to reject $\mathbf{H_0}$ or not

The significance level α -- a cutoff point below which the p-value is considered small enough to reject $\mathrm{H_0}$ in favor of $\mathrm{H_a}$.

The significance level sets a standard for how extreme the data must be before we can reject the null hypothesis.

- If the p-value $\leq \alpha$, then the data we got is considered to be "rare or surprising enough" under $\mathrm{H_0}$, so we **reject** $\mathrm{H_0}$.

  -- The results are statistically significant

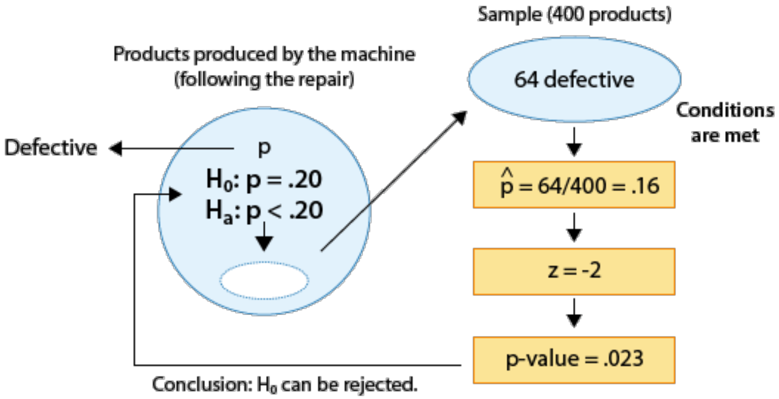
- If the p-value $> \alpha$, then the data is **not** considered to be "surprising enough" under $\mathrm{H_0}$, so we **fail to reject** $\mathrm{H_0}$.

  -- The results are not statistically significant

Commonly used significance level is $\alpha = 0.05, 0.01, 0.10$

Example (4/4)

A factory machine is known to produce 20% defective products, and is therefore sent for repair. After the machine is repaired, 400 products produced by the machine are chosen at random and 64 of them are found to be defective.
Do the data provide enough evidence that the proportion of defective products produced by the machine has been reduced as a result of the repair?



4. Conclude and Interpret

Using a significance level of 0.05.

Example: Increased Library Funding

Suppose a poll of 100 randomly selected voters in a city found that 56 of them favored increased funding for public libraries. Based on this poll, could we conclude that a majority of voters in this city (more than 50%) favor increased funding for public libraries?
Use a significance level of 0.05.

I. **Hypothesize**:

II. **Check conditions and Summarize**

III. **Assess evidence**

IV. **Conclude and Interpret**

Example: Increased Library Funding

I. Hypothesize:

$$\mathrm{H_0}: p = 0.50 \text{ versus } \mathrm{H_a}: p > 0.50$$

II. Check conditions and Summarize

Random sample of 100 voters, $\hat{p} = \frac{56}{100} = 0.56$, $SE = \sqrt{\frac{0.50\times(1-0.50)}{100}} = 0.05$

- Conditions are met (check this)
- Compute the z-test statistic: z = 1.2

III. Assess evidence

$$p\text{–value} = P(Z \geq 1.2) = 0.1151$$

IV. Conclude and Interpret

The p–value 0.1151 is greater than the significance level 0.05, we do **NOT** reject $\mathrm{H_0}$.

We cannot conclude that the majority of voters in this city (more than 50%) support increased public library funding