UCLA College | Physical Sciences Statistics

STATS 10: Introduction to Statistical Reasoning
Chapter 5

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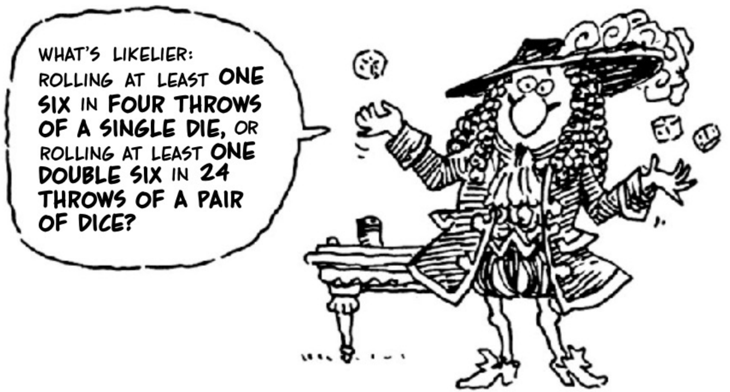
A Little History on Probability Theory

Antoine Gombaud, Chevalier de Méré (1607-1684) – two problems

Solved by Blaise Pascal (1623-1662) and Pierre de Fermat (1601-1665)

1. **The Dice Problem**



(By Larry Gonick & Woollcott Smith)

2. **The Problem of Points**

Two players play a game of chance with the agreement that each player puts up equal stakes and that the first player who wins a certain number of rounds (or points) will collect the entire stakes. Suppose that the game is interrupted before either player has won. How do the players divide the stakes fairly?

Randomness and Probabilities

Randomness

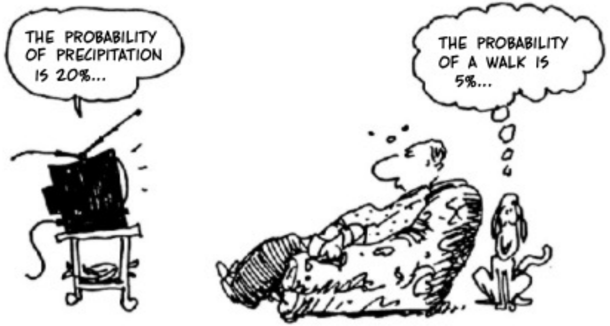
No predictable pattern occurs, and no outcome is more likely to appear than any other

Randomness can be achieved with help from computer or some other randomizing device.

- Pseudo-random numbers are generated by algorithm that we do not know.
  - The algorithm starts with an initial number (seed). In R: set.seed()
  - Specifying the same seed will give us the same set of pseudo-random numbers.
- For most practical purposes, these pseudo-random numbers are as random as we need.

Probability

- What is the chance that it will rain tomorrow?
- What is the chance that I will live longer than 90 years?
- What is the probability that I will win the lottery?
- What are the chances that at least two of the students in class share the same birthday?



Probability is a measure of how likely an event is to occur.

Notation:

- **P(rain)**: the probability that it will rain tomorrow
- **P(A)**: the probability that event A will occur
- **P(B)**: the probability that event B will occur

Theoretical Probabilities

Long-run relative frequencies

The relative frequency at which an event occurs after infinitely many repetitions

Example:
Suppose we have a coin with two sides and the coin is fair
(i.e., either side has equally chance of landing).

The theoretical probability is
P(heads) = P(tails) = 0.5 or 50%

Empirical Probabilities

Relative frequencies based on experiment or observations of a real-life process

Example:

- In one experiment, we flip a fair coin 20 times and get 8 heads.

The empirical probability of getting heads is $\frac{8}{20} = 0.4 = 40\%$.

- In another experiment, we flip a fair coin 20 times and get 11 heads.

The empirical probability of getting heads is $\frac{11}{20} = 0.55 = 55\%$.

Theoretical vs. Empirical

Difference

- Theoretical probabilities are always the same value.
- Empirical probabilities could change from experiment to experiment.

Connection

We can use empirical probabilities to estimate and test theoretical probabilities.

- **Estimate**
  - Theoretical probabilities may be too difficult to compute.
  - Empirical probabilities can help us estimate the theoretical probabilities base on what we observe.
- **Test**
  - The assumptions we make to compute theoretical probabilities could be incorrect.
  - The Empirical probabilities can help us verify the correctness of a theoretical value