More about Hypothesis Testing

The Effect of Sample Size

Example: Increased Library Funding

1. Suppose a poll of 100 randomly selected voters in a city found that 56 of them favored increased funding for public libraries. Based on this poll, could we conclude that a majority of voters in this city (more than 50%) favor increased funding for public libraries?

2. Suppose a poll of 250 randomly selected voters in a city found that 140 of them favored increased funding for public libraries. Based on this poll, could we conclude that a majority of voters in this city (more than 50%) favor increased funding for public libraries?

Use a significance level of 0.05.

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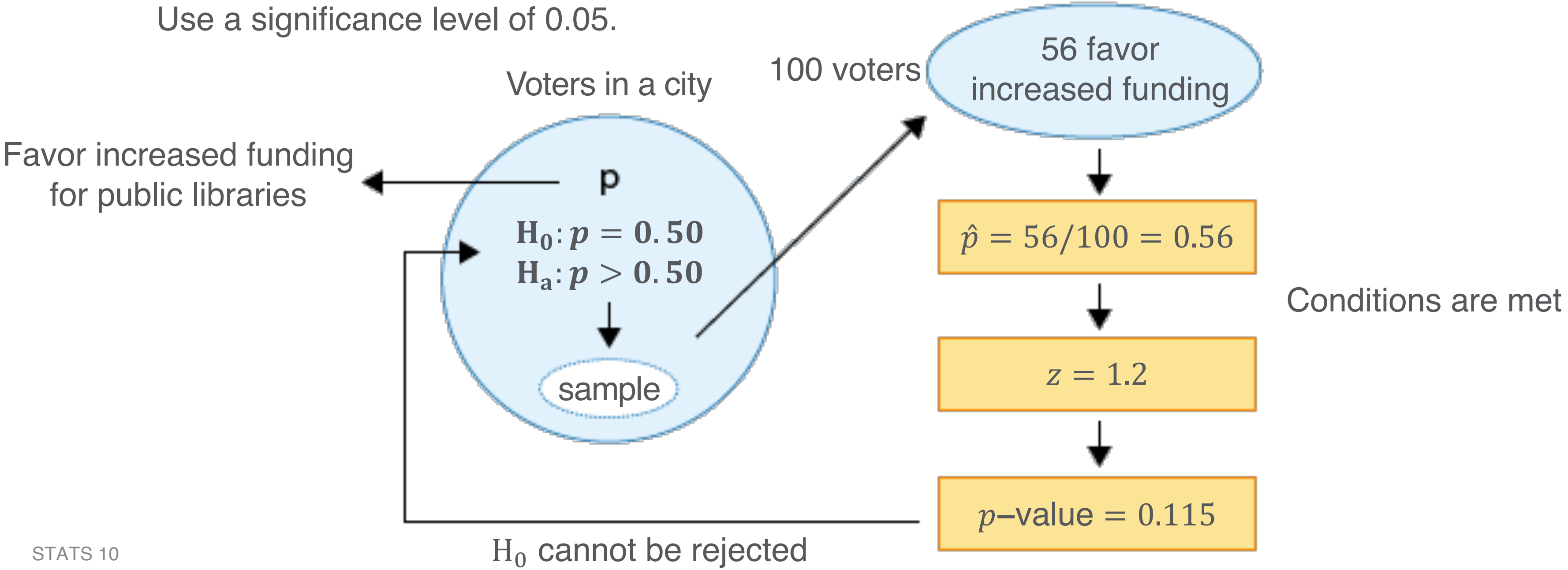

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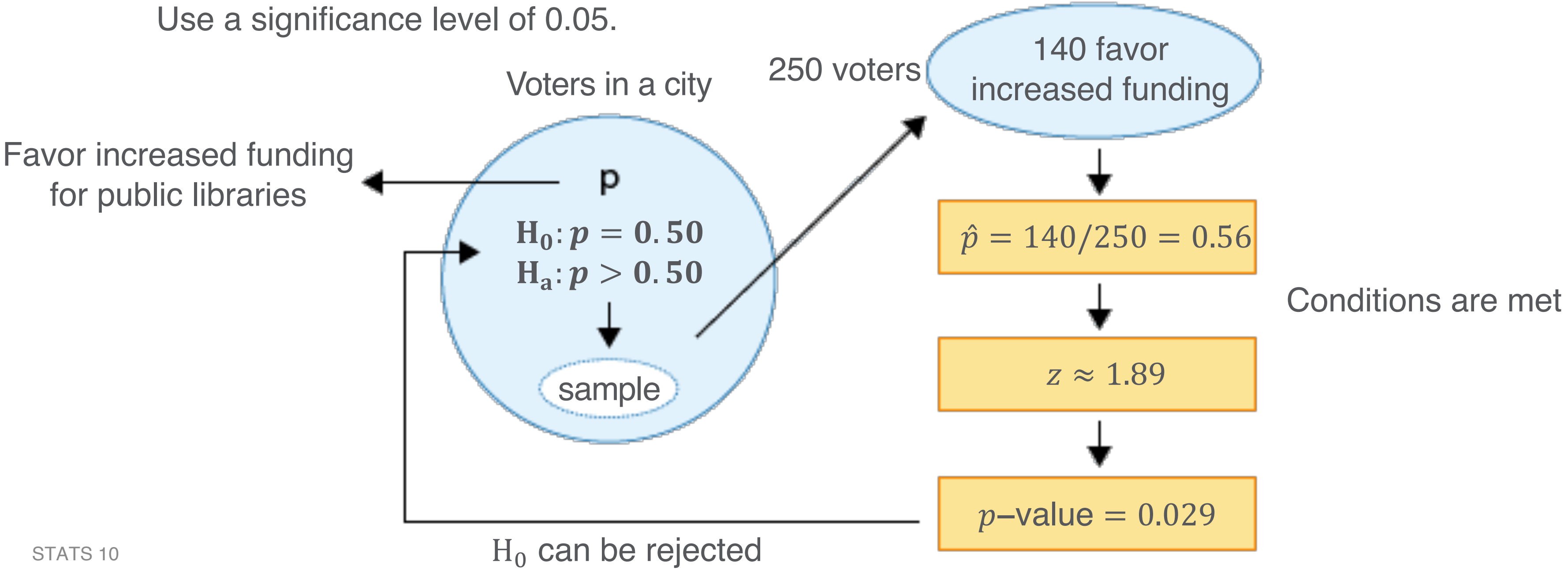


Two Types of Errors

There are two types of potential mistakes that can occur in hypothesis testing:

- Type I error (false positive): the rejection of a true null hypothesis H_0.
- Type II error (false negative): fail to reject a false null hypothesis H_0.

	H_0 is true	H_0 is false
Reject H_0 (p–value $\leq \alpha$)	Type I error (false positive)	Correct inference (true positive)
Do not reject H_0 (p–value $> \alpha$)	Correct inference (true negative)	Type II error (false negative)

The probability of Type I error is α – the significance level.

The probability of Type II error is denoted by β. The power of a test is $1 - \beta$, which represents the ability of the test to correctly reject the null hypothesis.

Example: New Drug

Before a new drug is released to the market its safety must be ensured. Scientists testing the safety of a new drug use the following hypotheses for their hypothesis testing:

$$H_0\text{: drug is not safe}$$
$$H_a\text{: drug is safe}$$

Describe two types of errors scientists might make in conducting this test.

- Type I error: concluding that the drug is safe when it is not safe.
- Type II error: concluding that the drug is not safe when it actually is safe.

As a consumer, which one should you be more worried about, scientists making a Type I error or a Type II error?

A. Type I B. Type II

Comparing Proportions from Two Populations

Hypothesis Test for Two Proportions

We now want to compare two population proportion:

- p_1 represents the proportion of the first population
- p_2 represents the proportion of the second population

We are interested in the relationship between p_1 and p_2

Hypothesis	Symbols	The Alternative in Words
Two-sided	$\mathrm{H_0}: p_1 = p_2$ $\mathrm{H_a}: p_1 \neq p_2$	The proportions are different in the two populations.
One-sided (Left)	$\mathrm{H_0}: p_1 = p_2$ $\mathrm{H_a}: p_1 < p_2$	The proportion in population 1 is less than the proportion in population 2.
One-sided (Right)	$\mathrm{H_0}: p_1 = p_2$ $\mathrm{H_a}: p_1 > p_2$	The proportion in population 1 is greater than the proportion in population 2.

Two Proportions Test Statistic

Under H_0, there is no difference in proportions, the two population proportions are the same: $p_1 = p_2$. So $p_1 - p_2 = 0$. The null value is 0.

Thus, we have

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{SE} = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{SE}$$

$$SE = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$$

- n_1, n_2 are the sample sizes in sample 1 and sample 2 respectively
- x_1, x_2 are the number of successes in sample 1 and sample 2 respectively
- $\hat{p}$ -- **pooled sample proportion,** an estimator for the common population proportion if H_0 is true.

Checking Conditions

The sampling distribution for the two-proportion test statistic is approximately the standard normal distribution N(0, 1) if the following conditions are met and H_0 is true.

1. **Random samples.** The samples are randomly selected from the appropriate populations
2. **Independent samples.** The samples are independent of each other.
3. **Independent within samples.** The observations within each sample are independent of one another.
4. **Large samples.** Both sample sizes must be large enough. Under H_0, $p_1 = p_2$, so we use the **pooled sample proportion $\hat{p}$** to estimate the common null proportion.

   Check that

   $$n_1\hat{p} \geq 10, \qquad n_1(1-\hat{p}) \geq 10, \qquad n_2\hat{p} \geq 10, \qquad n_2(1-\hat{p}) \geq 10$$
5. **Big Populations.** Each population is at least 10 times as big as its sample.

Example: Medication for Hives

Two types of medication for hives are being tested to determine if there is a difference in the proportions of adult patient reactions.

20 out of a random sample of 200 adults given medication A still had hives 30 minutes after taking the medication. 12 out of another random sample of 200 adults given medication B still had hives 30 minutes after taking the medication.

	Medication A	Medication B	Total
Still had hives after 30 mins	20	12	32
Hives disappeared after 30 mins	180	188	368
Total	200	200	400

Example – Confidence Interval

Sample data

$x_1 = 20, n_1 = 200$
$\hat{p}_1 = 0.10$

$x_2 = 12, n_2 = 200$
$\hat{p}_2 = 0.06$

Conditions met

Point estimate

$\hat{p}_1 - \hat{p}_2 = 0.04$

Standard error

$\mathrm{SE} = 0.027$

Confidence multiplier

95% C.I:
$z^* = 1.96$

95% C.I.

$0.04 \pm 1.96 * 0.027$
$= (-0.013, 0.093)$

Example – Hypothesis Test

Hypothesize

$H_0: p_1 = p_2$

$H_a: p_1 \neq p_2$

Collect and summarize data

$x_1 = 20, n_1 = 200$
$\hat{p}_1 = 0.10$

$x_2 = 12, n_2 = 200$
$\hat{p}_2 = 0.06$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = 0.08$$

Conditions met

$$z = 1.47$$

Assessing evidence

p–value $= p(z < -1.47) + p(z > 1.47)$
$= 0.142$

Fail to reject H_0

Confidence Interval
and
Hypothesis Testing

Confidence Intervals and Hypothesis Testing

Confidence intervals and hypothesis tests are closely related but often used to answer slightly different questions.

- **Confidence intervals** – estimate the value of a parameter
- **Hypothesis test** – decide between two different claims about a parameter value

Even though they are designed to answer different questions, a confidence interval with $(1 - \alpha)$ confidence level can lead us to the same type of conclusion as a two-sided hypothesis test with significance level α.

Reject the null hypothesis —— the null parameter value is not captured by the interval.

Confidence Level $(1 - \alpha)$	Alternative Hypothesis	Significance Level (α)
90%	Two-Sided ≠	10%
95%	Two-Sided ≠	5%
99%	Two-Sided ≠	1%

Example: Death Penalty

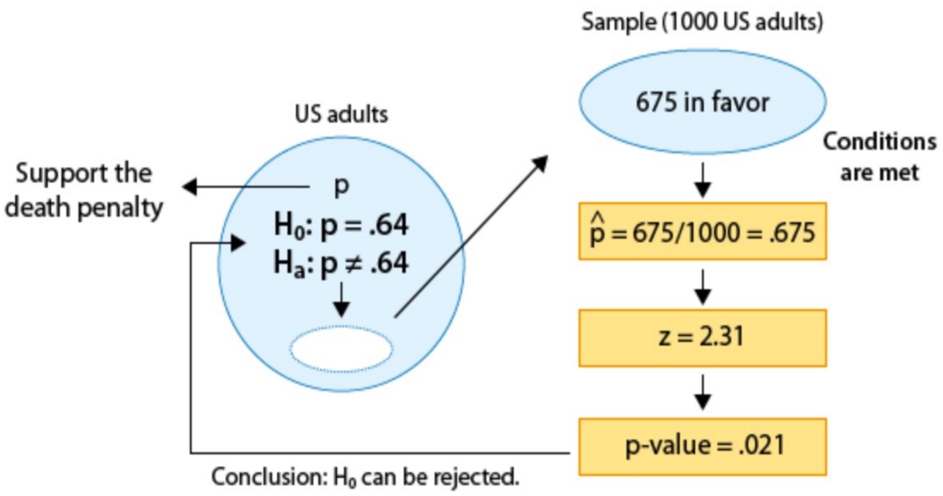
In 2003 a poll estimated that 64% of U.S. adults support the death penalty for a person convicted of murder. In a more recent poll, 675 out of 1,000 U.S. adults chosen at random were in favor of the death penalty for convicted murderers.

Do the results of this poll provide evidence that the proportion of U.S. adults who support the death penalty for convicted murderers (p) changed between 2003 and the later poll? (Use a significance level 0.05)

1. Conduct a hypothesis test.
2. Construct a 95% confidence interval for the proportion of Americans who support the death penalty.
3. Explain how your confidence interval supports the conclusion of your hypothesis test.

Example: Death Penalty

Hypothesis Test



95% Confidence Interval

$$0.675 \pm 1.96 \times \sqrt{\frac{0.675(1-0.675)}{1000}}$$
$$= 0.675 \pm 0.029$$
$$= (0.646, 0.704)$$