Quantum Computing Induction Problem Statement & Solutions

Section A: Quantum Mechanics

Question 1: Game of apparatus?

Let`s say you and your friend are performing an experiment. The experiment is to observe electrons. Both of you use different apparatus to measure position and momentum of electrons. Now say, your apparatus has a scale of picometers and shutter speed (time interval for measuring change in state) of femtoseconds. while, your friend's apparatus has scale of femtometres and shutter speed of attoseconds. You both calculate the uncertainties in the position and momentum using your respective apparatus and then try to verify the Heisenberg uncertainty principle (HUP).

1) Now, who do you think has more precise values for the position and momentum? your friend? if yes, then that would mean that he/she has lesser values of uncertainty for both position and momentum (because it's a more precise measurement). (3 marks)

   Now consider a third person who has even more precise measuring apparatus, so that the product of uncertainties in position and momentum measured, comes out to be of the order of plank's constant.

2) So, is there a possibility that this person can disprove HUP? if yes, then why? and if no? then what's wrong with our arguments. Give detailed explanation of your answer with relevant assumptions. Try to come with different scenarios*, to justify it. (15 marks)
   (Hint - try to think about the origin of the principle and its dependence / independence on measuring apparatus. No need to prove/disprove HUP using complex mathematics.) (* means scenarios to explain HUP intuitively) (We don't just want statements about whether it is a property of quantum states or not, but also expect you to present different scenarios and diagrams. Marks will be awarded based on the UNIQUENESS of the idea, so it is recommended to make your own scenarios)
   (Can refer to- Ch-1, lectures on physics by Richard Feynman)

3) Briefly explain the case with large scale objects? Is the principle still valid? (2 marks)

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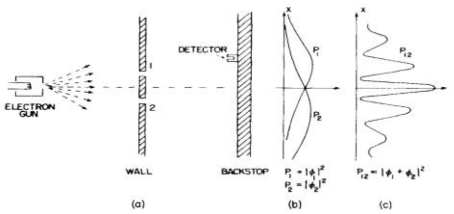
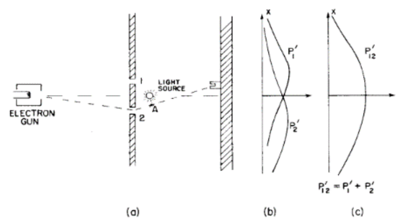
Solution: -

1. (A) Yes, my friend will have has more precise values for the position and momentum since his apparatus are much more accurate than mine.
   (B) Again yes, since his/her values are more accurate that would mean that he/she has lesser values of uncertainty for both position and momentum than mine.

2. No, even though the third person has highly precise values of position and momentum he can never disprove the HUP. These are the following reasons

   - Nature of HUP: The HUP is not a limitation of our measuring devices, but a fundamental property of quantum systems. It arises from the wave-like nature of matter at the quantum scale.
   - Wave-particle duality: When electrons are fired one at a time in a double experiment, they show the property of interference just like a wave. But when we observe from which slit each electron is coming out in the experiment, we see that the interference patterns are now gone Even when we dim the light and try to observe the electron, the observed electron doesn`t follow the interference pattern.



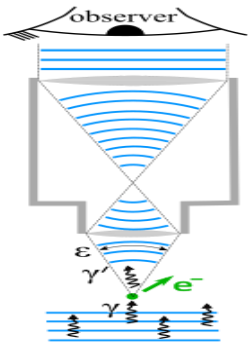


(Source: Feynman`s Lectures on Quantum Physics)

Conclusion:

This scenario shows that the act of measurement itself affects the outcome. When we try to pinpoint the position of the electron (by determining which slit it passes through), we lose information about its momentum (as evidenced by the loss of the interference pattern). If we try to use light of larger wavelength to observe the electron, then the interference pattern slowly reappears but then we are not able to observe from which slit the electron is emitted.

- <u>Thought experiment - The Heisenberg Microscope:</u>
  Even this experiment gives similar conclusion like the previous one.



(Source: Wikipedia)

This diagram illustrates the Heisenberg microscope thought experiment. To observe an electron, we must bounce a photon off it and detect the scattered photon with our microscope. However, this interaction changes the electron's momentum. The more precisely we try to measure the electron's position (by using shorter wavelength photons), the more we disturb its momentum.

This thought experiment shows that the uncertainty in position (Δx) and the uncertainty in momentum (Δp) are inversely related. As we decrease one, the other increases, always keeping their product above a certain minimum value $h/4\pi$.

Reference: <u>The Uncertainty Principle (Stanford Encyclopaedia of Philosophy)</u>

- Quantum superposition: Another way to think about the HUP is through the concept of quantum superposition. Before measuring position, a quantum system exists in a state of superposition of all possible states let`s call it as Ψ. The act of measurement "collapses" this superposition into a definite state let's call this state as $\Psi 1$. Now when we measure the momentum of this system, we are actually measuring the momentum of electron in $\Psi 1$ state and not Ψ. So, HUP cannot be applied when Δx and Δp are measured by two different states.

- I have another interesting analogy to explain this. Consider a musical note, the more precisely we define it`s frequency, the longer the sound must play, making its position in time less certain. That is a very short sound, like a click or a pop, has precise location in time. However, its frequency content is spread out over a wide range. Whereas, a long, sustained note(like from flute) has a very precise frequency, but it's harder to pinpoint exactly where it starts or stops. Thus, just as with sound, where the trade-off between frequency and duration exists, quantum particles have a trade-off between position and momentum, as described by the Heisenberg Uncertainty Principle.

In all these scenarios, we see that the HUP is not a limitation of measurement but a fundamental property of quantum systems, arising from the wave-particle duality. Even with an "ideal" or "perfect" measuring device, we cannot simultaneously know both position and momentum with arbitrary precision.

3. HUP is still valid for large scale objects but it`s effects are negligible and are not observable in practical cases. The reasons for this are;

- The uncertainty given by HUP is in the order of 10-34 which is very very small for macroscopic objects compared their size and momentum.
- For example: If we consider a ball of 100g with an uncertainty in position of 1 mm then the uncertainty in velocity would be 5.27 x 10^-31 m/s which is negligible.
- Decoherence: Large objects constantly interact with their environment, causing rapid decoherence of quantum states. This makes quantum effects, including those related to the HUP, extremely difficult to observe in macroscopic systems.

In conclusion, while the HUP is universally valid, its effects are only noticeable and relevant at the quantum scale. For everyday objects, classical physics provides an excellent approximation of reality without needing to consider quantum uncertainties.

<u>Question 2: MY SCI-FI THEORY!</u>

So, internet defines entanglement as - "Quantum entanglement refers to the phenomenon in quantum physics where two or more particles become connected in such a way that the state of one particle cannot be described independently of the state of the other particles, regardless of the distance between them" Basically that means that if you have any two entangled particles in space, and if you change the state of one particle, then the state of other particle also changes, regardless of the distance between them. And this change is instantaneous. fascinating, right?

On internet there are a lot of complex theories explaining this, but none of them are fully accepted. That means you can give your own theory and argue that to be correct!!! Give possible simple arguments/theories* about how this phenomenon is possible? (10 marks)

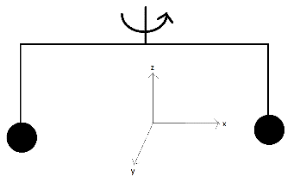
(* means any theory or argument that you give should be explained using conceptual physics) (it could be anything, yeah, even marvel or Star Wars, as long as it is SUPPORTED by physics arguments. We don't want it to be very fancy, just think how you will do it)

(Okay, so as an example, let's consider a theory - if the information about change in state has to travel billions of light years to reach the other entangled particle and that too instantly, means that it has to travel at a speed, which is much much greater than c (speed of light in vacuum). But by relativity theory, which applies to space-time, it cannot move at speed greater than c. This means that there is no space-time that could be defined, in which the information travels. So, we have a rupture in space-time itself (which theoretically is a wormhole). That means that this theory says that there is a wormhole between the 2 entangled particles. But do wormhole even exist, and if they do.........)

<u>Solution:</u>
Quantum Memory Link Theory

The quantum particles are not discrete particles instead they are all connected to each other through a connection in some higher dimension which is beyond the scope of classical physics. Let us call this connection as **link**. Through this link, quantum entanglement is possible.

Let`s understand it by an example:



In the above case the two spheres are revolving around vertical axis connected by light weighted rods. If we assume that we are in a 2D dimension and we try to rotate one of the sphere in 2D space then automatically the second sphere would rotate instantaneously. Now if we assume that the horizontal rod is infinitely long even then when we try to rotate one sphere around vertical axis the other sphere would instantaneously revolve. Now for a creature living in 2d space (x-y plane) in some other universe who is unaware of higher dimensions like 3d and beyond (and are not able to see and feel the concept of height) will assume that sphere we tried to rotate will send information to other sphere that even it has to rotate. For this to be instantaneous, information has to be transferred at speed more than that of light. But for 3d living beings like us we know the exact reason behind it.

Similarly, we humans living in 3D space have no idea of what higher dimensions look like. So, there is a possibility that all particles are connected to each other through a connection of some higher state which we are unable understand.

Hence there’s no need to break the speed of light because no information is physically transmitted. Instead, both particles are simply reflections of the same fundamental entity, just projected differently across space-time.

We can also understand it by taking the example of sharing cloud documents. If I and my colleague is working on the same document stored in cloud, this document exists in cloud (some higher, shared space). So, if my colleague does some changes on the document from his side then it would automatically reflect on mine document too, no matter how far I am.

In this analogy, the cloud represents the link(connection) between the entangled particles and the computer represents the entangled particles. The instant update reflects how the state of one particle immediately affects the other, just like changes in the document show up on both screens without any delay.

In conclusion, this theory presents entanglement not as a mysterious long-distance interaction, but as a natural consequence of particles sharing a unified quantum state. It reframes our understanding of separation in the quantum realm, suggesting that entangled particles maintain their connection through a higher-dimensional "quantum memory link" that is beyond our classical notions of space and time.

Section B: Quantum Computing

Question 1: Python

a) Elaborate what is pennylane and qiskit. write the similarities and differences between them. which one would you prefer and why? (5 marks)

b) Write a simple code to generate a Bloch Sphere for a qubit with state vector pointing 45 degrees from 1 , 135 degrees from 0 and phi=0 degrees (thus it remains in the circle joining 1 and 0 only). U can use Matplotlib or any library. Make sure the output comes out and you attach that graph of bloch sphere too with code! (15 marks)

c) Demonstrate Dirac notation with some examples in quantum computing. (5 marks)

Solution:

a) PennyLane and Qiskit are both open-source frameworks for quantum computing, but they have different focuses and features:

(i) PennyLane:

- It was developed by Xanadu.
- It focuses on ML and optimisation.
- It provides a device-independent approach
- It is used with classical machine learning libraries like PyTorch and TensorFlow
- It works great for projects that mix regular computers and quantum

(ii) Qiskit:

- It was developed by IBM
- It mainly focuses on general quantum computing tasks
- It allows us to control the quantum circuits in much better way
- Includes tools for quantum circuit visualization and quantum hardware access
- It works really well on actual quantum computers.

(iii) Similarities:

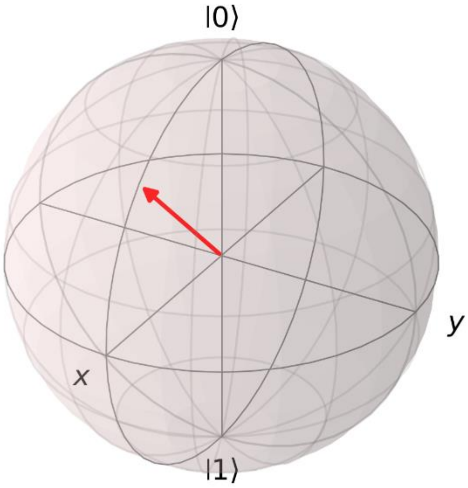
- Both support circuit-based quantum computing
- Both offer simulation capabilities
- Both have active communities and documentation

(iv) Differences:

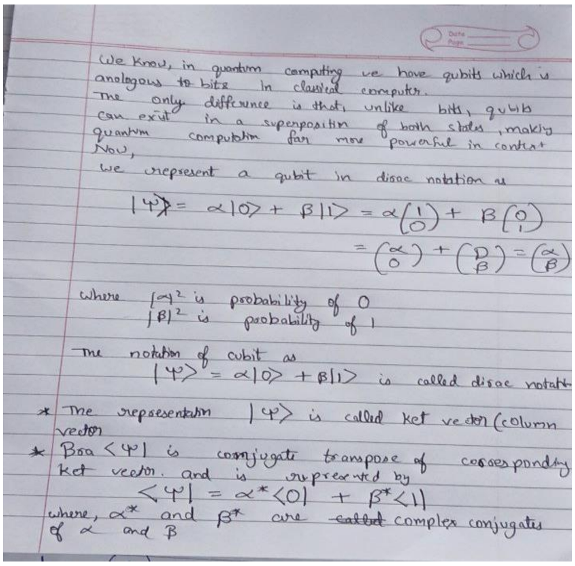
- PennyLane is more focused on quantum machine learning, while Qiskit is more general-purpose
- Qiskit has better integration with IBM's quantum hardware
- PennyLane has better integration with classical machine learning frameworks

Preference: For project that need a mix of regular machine learning and quantum computing, Pennylane is easier to use. But if we need to perform all kinds of quantum computing tasks, especially if we want to run it in an actual quantum computer then Qiskit will be a better choice.

b) Code: [QUANTUM-COMPUTING/Bloch_sphere.py at main · namanshetty25/QUANTUM-COMPUTING (github.com)]



c) Dirac notation provides a convenient framework for representing quantum states, operations, and calculations, especially in quantum computing. As the number of qubits increases it is convenient to represent them in dirac notation instead of matrices.



We Know, in quantum computing we have qubits which is anologous to bits in classical computer.
The only difference is that, unlike bits, qubits can exist in a superposition of both states, making quantum computation far more powerful in context
Now,
we represent a qubit in dirac notation as

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix}$$
$$= \begin{pmatrix}\alpha\\0\end{pmatrix} + \begin{pmatrix}0\\\beta\end{pmatrix} = \begin{pmatrix}\alpha\\\beta\end{pmatrix}$$

where $|\alpha|^2$ is probability of 0
$|\beta|^2$ is probability of 1

The notation of cubit as
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ is called dirac notation

* The representation $|\psi\rangle$ is called ket vector (column vector
* Bra $\langle\psi|$ is conjugate transpose of corresponding ket vector. and is represented by
$$\langle\psi| = \alpha^*\langle 0| + \beta^*\langle 1|$$
where, α^* and β^* are ~~called~~ complex conjugates of α and β

Question 2: Algorithms

a) How are Quantum Gates different from normal Logic Gates? Explain how they can be used for Ramsey experiment on qubit. (5 marks)

b) Write a code to demonstrate qubit manipulation through a simple quantum circuit. Also attach the circuit with the code. (15 marks)

c) What is Grover Algorithm? What can be some other uses of it than the usual purpose. Demonstrate Grover Algorithm with a code and include necessary circuit diagrams too. (15 mark)

Solution:

a) Quantum gates are different from the normal logic gates in the following ways

- Superposition: Quantum gates can manipulate qubits in superposition states, whereas classical gates operate on definite binary states.
- Reversibility: Most quantum gates are reversible, while many classical gates are not.
- Entanglement: Quantum gates can work with entanglement of qubits whereas logical gates cannot.

Ramsey experiment:

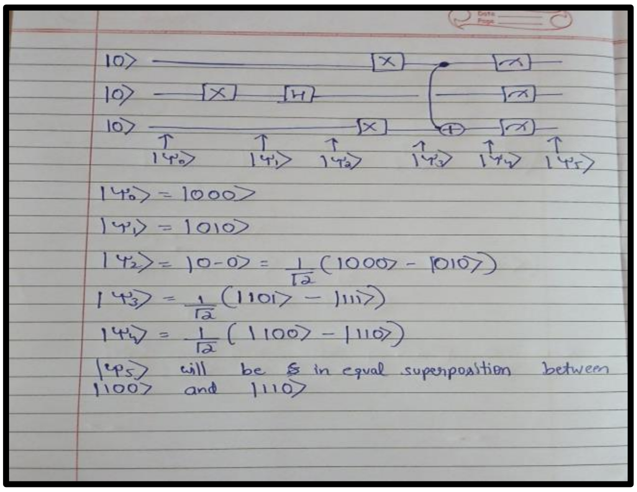
This experiment is performed for checking how long a qubit can maintain its quantum state before it decoheres (loses its quantum information to the environment).

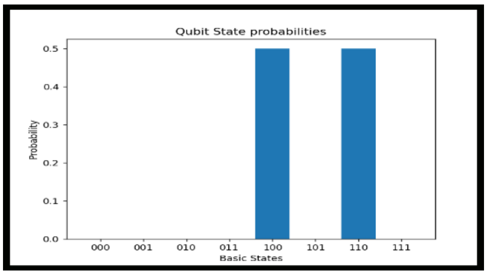
In this experiment we first take a qubit in its ground state $|0\rangle$. Then we apply Hadamard gate to it so the qubit will be in equal superposition. Now we let the qubit evolve freely for a time τ. During this time, the qubit's state rotates around the z-axis of the Bloch sphere due to its energy difference between $|0\rangle$ and $|1\rangle$ states. After this, we measure the qubit's state. The probability of measuring $|0\rangle$ or $|1\rangle$ depends on the rotation during the free evolution period. Again, we apply the Hadamard gate to with a different τ. As we plot the probability of measuring $|0\rangle$ versus τ, we see oscillations. These are called Ramsey fringes.

The frequency of oscillation describes the energy difference between $|0\rangle$ and $|1\rangle$ states and the decay of oscillation amplitude gives an idea of how long can a qubit maintain its quantum information.

The longer we can maintain these oscillations, the better our qubit is at preserving quantum information, which is very important for building larger and more complex quantum computers.

b) Code: [QUANTUM-COMPUTING/Simple_quantum_circuit.py at main · namanshetty25/QUANTUM-COMPUTING (github.com)]()





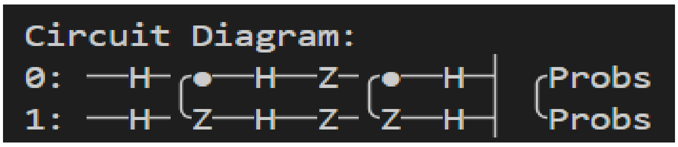
c) Grover's Algorithm is a quantum algorithm typically used to find a specific item in an unsorted database, reducing the time complexity from O(N) [in classical algorithm] to O(√N).

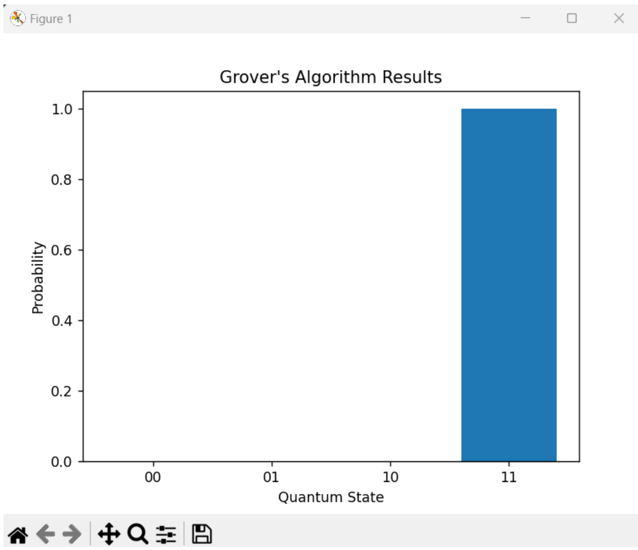
Other potential uses of Grover's Algorithm:

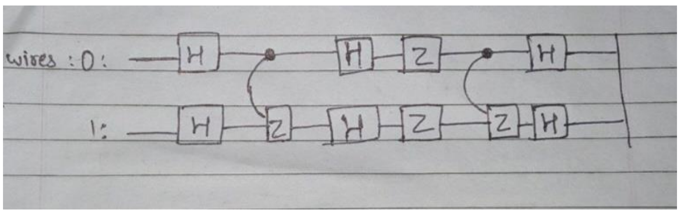
1. Solving systems of linear equations
2. Estimating the mean and median of a set of numbers
3. Collision finding in cryptography
4. Minimum finding in an unsorted array
5. Speeding up solutions to NP-complete problems
6. It can be used to find hash collisions, which has implications for cryptographic hash functions.

Code: [QUANTUM-COMPUTING/GroverAlgorithm.py at main · namanshetty25/QUANTUM-COMPUTING (github.com)]()

"The above code is designed to search for the target state |11⟩ from a set of possible states |00⟩, |01⟩, |10⟩ and |11⟩. By amplifying the probability of the target state, the algorithm increases the likelihood of measuring |11⟩ compared to the other states."







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