Practical Dirty Paper Coding Schemes Using One Error Correction Code With Syndrome

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***Abstract*— Dirty paper coding (DPC) offers an information-theoretic result for pre-cancellation of known interference at the transmitter. In this letter, we propose practical DPC schemes that use only one error correction code. Our designs focus on practical use from the viewpoint of complexity. For fair comparison with previous schemes, we compute the complexity of proposed schemes by the number of operations used. Simulation results show that compared to previous DPC schemes, the proposed schemes require lower transmission power to maintain the bit error rate to be within 10^{-5}.**

***Index Terms*— Dirty paper coding, error correction code, interference cancellation.**

I. Introduction

DIRTY paper coding (DPC) is an interference pre-cancellation scheme for an additive white Gaussian noise (AWGN) channel in the presence of interference. Costa proved that if a transmitter non-causally knows a Gaussian distributed interference, the capacity of the AWGN channel could be achieved [1]. Also, the generalized result for an arbitrarily distributed interference channel was the subject of [2]. Ever since Willems [3] first suggested practical DPC for causally known interference in 1988, there have been several researches into a practical DPC scheme.

Controlling transmission power regardless of interference power requires the use of shaping codes, such as lattice codes. Lattice codes are the Euclidean space analogue of linear codes, and can achieve the capacity of the AWGN channel [4], [5]. From the expectation of practical use, low-density lattice codes [6] were proposed for efficient decoding. The general structure of a DPC scheme consists of channel codes and shaping codes. DPC schemes can be divided into two groups according to the relationship between the channel codes and the shaping codes: nested, and superposition. The nested approach [7]– [9] has a design constraint in that the codeword set of shaping codes is a subset of that of channel codes. Since the constraint of shaping codes makes it hard to design, there were several challenges in solving this problem. Erez and ten Brink [8] tried to design a nested structure using coset dilution. Sun *et al.* [9] addressed a code design based on nested turbo codes.

The superposition approach is much simpler than the nested approach. This approach independently selects the channel codes and the shaping codes. In [10], they used low-density parity-check (LDPC) codes as channel codes, and convolutional codes as shaping codes. To get better performance, density evolution can be used to design LDPC codes, which is concatenated with shaping codes [11]. For low transmission rate, a scheme [12] using irregular repeat accumulate (IRA) codes as channel codes, and trellis coded quantization as shaping codes, showed near capacity performance. On the other hand, there were schemes that targeted high transmission rates. Chung [13] proposed a DPC scheme that was a special case of a superposition scheme, and showed capacity-approaching performance at a transmission rate of 3.0 bit/symbol. Similarly, there was a scheme for high transmission rate using LDPC codes with convolutional codes [14].

Most previous researches used the joint iterative decoding process between channel decoder and shaping decoder to achieve capacity-approaching performance. However, high decoding complexity makes it hard to use in practice, due to the importance of decoding complexity, and the latency problem in communication systems. A recent work [15] introduced simple DPC schemes using one error correction code (ECC). These schemes put additional bits into a message bit sequence for shaping operation. Although demonstrating lower complexity than previous schemes using two ECCs, the error performance was not sufficient to use.

In this paper, we propose practical DPC schemes that focus on low complexity. The proposed schemes consist of only one ECC, like the schemes in [15]. Although the error performance is degraded compared to two-ECC schemes, the performance gap from the capacity is significantly reduced compared to previous one-ECC schemes.

The rest of this paper is organized as follows. Section II introduces the encoder and decoder structure of the proposed schemes. Section III provides the complexity and performance analysis, while Section IV concludes the paper.

II. Proposed Schemes

In order to perform the shaping operation efficiently, the proposed schemes use recursive systematic convolutional (RSC) codes that are decodable by Viterbi algorithm. First, we address a simple DPC scheme that uses one RSC code (DPC-RSC). The RSC code performs not only as channel code, but also as shaping code by using the syndrome former of short length linear block codes. For performance improvement, turbo-like codes can also be used. At the end of this section, we present the encoder and decoder structure of a DPC scheme based on turbo-like codes (DPC-TLC).

Note that the proposed schemes do not use a minimum mean square error (MMSE) factor, which is used for performance

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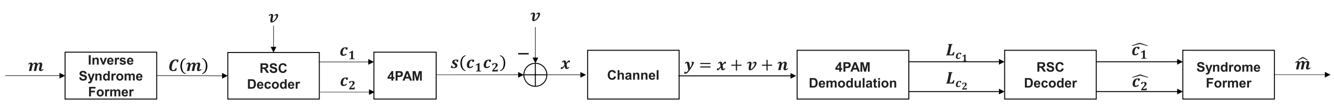

Fig. 1. Dirty paper coding based on recursive systematic convolutional codes.

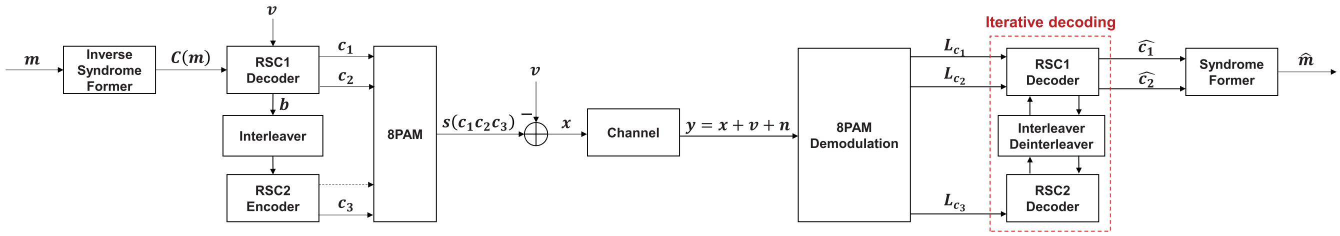

Fig. 2. Dirty paper coding based on turbo-like codes.

improvement in the previous two-ECC schemes. The MMSE factor converges to 1 at the high signal-to-noise ratio (SNR) region, which is the target SNR region of the proposed schemes. Therefore, the MMSE factor provides no significant performance gain.

A. Dirty Paper Coding Based on RSC Codes

Figure 1 represents an example of the proposed DPC-RSC. Let $C(\alpha)$ denote a subset of a codeword set whose syndromes are α. The length of codewords is in reciprocal relation to the code rate of RSC codes, R_{RSC}. In other words, the scheme uses the syndrome former of $(1/R_{RSC}, 1)$-linear block codes. The encoder finds the corresponding codeword set $C(m)$, where m is a message bit.

The RSC decoder finds an RSC codeword that not only is the closest to the interference v, but also satisfies the condition that the output of each step in Viterbi algorithm must be a codeword in $C(m)$. To satisfy the conditions, branch metrics for the RSC decoder are calculated as follows:

$$BM(c) = \begin{cases} |v - s(c)|, & \text{if } c \in C(m) \\ \infty, & \text{if } c \notin C(m) \end{cases} \tag{1}$$

where c is a linear block code codeword $c_1 c_2 \ldots c_{(1/R_{RSC})}$, and $s(\cdot)$ represents a symbol mapping function. The modulation is pulse amplitude modulation (PAM) with Gray code mapping. When the interference is uniformly distributed and its power is P_v, the range of interference value is $[-\sqrt{3P_v}, \sqrt{3P_v}]$. For the proper shaping operation, the constellation points have to be uniformly placed in that range. Therefore, the distance between neighboring symbols Δ is

$$\Delta = 2\sqrt{3P_v}/M, \tag{2}$$

where M is the number of constellation points. After subtracting the interference from the modulated signal, $x = s(c) - v$, the encoder transmits the signal.

The decoder receives signal $y = x+v+n = s(c)+n$, where n represents channel noise. Since DPC-RSC does not require iterative decoding, the Viterbi algorithm is used rather than BCJR algorithm, from the viewpoint of complexity. Then to find original message bits, we apply the syndrome former of linear block code, which is used at the encoder, to the decoded RSC codeword, $\hat{c}$.

B. Dirty Paper Coding Based on Turbo-Like Codes

In order to improve the error performance, we concatenate additional RSC codes to DPC-RSC as turbo codes. Figure 2 represents the encoder and decoder structure of the proposed DPC-TLC. The RSC1 decoder, which is in the encoder side, operates the Viterbi algorithm for the shaping operation to achieve a codeword. In addition, an input bit sequence b, which is unused in previous DPC schemes, can also be obtained. In the proposed DPC-TLC, we use b to improve the error performance. As conventional turbo encoding procedure, the RSC2 encoder encodes interleaved b.

In DPC-TLC, the bit-to-symbol mapping significantly affects the shaping gain, since the RSC2 codeword is not related to the interference. To minimize inevitable shaping loss due to the RSC2 codeword, we place the RSC2 code bits at the least significant bit in the modulation. The distance between two neighboring constellation points is the function of interference power as (2). After subtracting the interference from the modulated signal, $x = s(c) - v$, the encoder transmits the signal.

As for a conventional turbo decoder, iterative decoding between RSC1 decoder and RSC2 decoder is used. The syndrome former of linear block code is then applied to the decoded RSC1 codewords.

III. Simulation Results

In this section, we show the required transmission power reduction of the proposed schemes compared to previous DPC schemes [15] based on convolutional codes (DPC-CC) and LDPC codes (DPC-LDPC). First, we analyze the complexity of each scheme to compare their error performance when they have similar complexity. Since iterative decoding algorithms achieve powerful error correction performance at the cost of complexity, we separate the schemes into two groups for comparison, on the basis of whether the iterative decoding algorithm is used, or not.

A. Complexity Analysis

The entire operations of both DPC-RSC and DPC-CC are based on the Viterbi algorithm. Therefore, when an equal constraint length convolutional code is used, DPC-RSC and DPC-CC have similar complexity. On the other hand, the algorithms of DPC-TLC and DPC-LDPC are quite different. Therefore, we have to precisely check their complexity. Although the complexities of both encoder and decoder should be considered, we analyze only the complexity of the decoding operation in detail, since in most communication systems, the decoding complexity is a more important issue than the encoding complexity. In DPC-TLC and DPC-LDPC, different decoding algorithms are used to decode messages: the BCJR algorithm, and the belief propagation (BP) algorithm. To compare two different algorithms, we first check the number of used $\min^*(= -\ln(e^x + e^y))$ operations and addition operations, which are commonly used in both. Then, we decide the weight of each operation, since the computation complexity of the min* operation and addition operation are different.

Assume that there is a k-bit message. When the constraint length of RSC codes is L, the complexity of the min* operation C_{min^*} and addition operation C_{add} per message bit are

$$C_{min^*} = W_{min^*} \cdot (4 \cdot 2^L - 2) \cdot I_{max}, \tag{3}$$
$$C_{add} = W_{add} \cdot 8 \cdot 2^L \cdot I_{max}, \tag{4}$$

where I_{max} is the maximum number of iterations. The parameters W_{min^*} and W_{add} represent the weight of each operation. The complexity of DPC-TLC is the summation of C_{min^*} and C_{add} of RSC1 and RSC2. In the same manner, the complexity of BP algorithm can be represented as

$$C_{min^*} = W_{min^*} \cdot \frac{\sum_i^{d_{c,max}} (i-2) \cdot i \cdot c_i}{k} \cdot I_{max}, \tag{5}$$
$$C_{add} = W_{add} \cdot \frac{\sum_i^{d_{v,max}} 2i \cdot v_i}{k} \cdot I_{max}, \tag{6}$$

where c_i and v_i represent the number of degree-i check nodes and variable nodes, respectively. The parameter $d_{c,max}$ is the maximum check node degree, and $d_{v,max}$ is the maximum variable node degree. The min* operation consists of two exponential, one addition, and one logarithm operation. From the simulation, we obtain that the execution times for the exponential operation and logarithm operation are 7 times longer than that of the addition operation. Therefore, the weight of the min* operation W_{min^*} is 22 when that of the addition operation W_{add} is 1.

The RSC1 code is primary code in the proposed structure, and the RSC2 code supports the performance of the RSC1 code. Therefore, we select RSC2 codes that have a relatively smaller constraint length compared to the RSC1 code. Figure 3 shows the complexities when the constraint length of RSC2 codes varies from 4 to 6, while the constraint length of RSC1 codes is 6. The LDPC code used in DPC-LDPC is specified in DVB-S2X standard [16] of length $N = 64,800$, and its code rate is 1/2. The parameter I_{max} for DPC-TLC and DPC-LDPC is 8 and 50, respectively. As a result, when we select the constraint length of RSC2 codes as 5, the complexities of both schemes are the most similar.

Before analyzing the error performance, we simply compare the encoding complexity of DPC-TLC and DPC-LDPC. DPC-LDPC flips the auxiliary bits, and computes the distance between the resulting symbol sequence and the interference

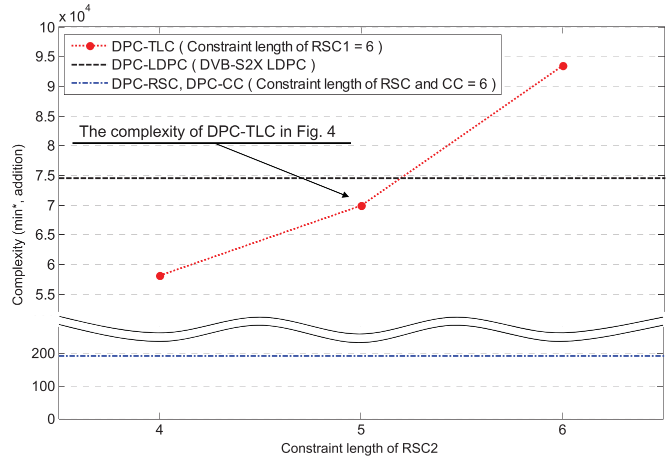

Fig. 3. Complexity comparison of DPC schemes.

each time, until proper auxiliary bits are found that minimize the transmission power. Because of the large number of LDPC encoding processes, it has high encoding complexity. On the other hand, the proposed DPC-TLC requires only one Viterbi decoding, and one convolutional encoding process.

B. Error Performance Analysis

The octal generation polynomial of RSC codes used in DPC-RSC is $(101, 147)_8$ [17]. DPC-TLC uses RSC1 codes with generator polynomial $(101, 147)_8$, and RSC2 codes with generator polynomial $(45, 67)_8$. The syndrome former of linear block code is equal to $H = [1 \quad 0]$. The information transmission rate of whole simulations is 1.0 bit/symbol. The modulation of DPC-RSC and DPC-CC is 4-PAM, and that of DPC-TLC and DPC-LDPC is 8-PAM.

Figure 4 compares the error performance of the proposed schemes with those of DPC-CC and DPC-LDPC when the interference power is 7.5 dB. In comparison to DPC-CC, the proposed DPC-RSC requires 2.1 dB less transmission power to achieve BER of 10^{-5}. When iterative decoding is used, DPC-TLC requires 3.5 dB less transmission power than DPC-LDPC. Moreover, the proposed DPC-RSC shows better performance than DPC-LDPC, even though LDPC codes have much better error correcting performance than RSC codes.

The gap in error performance can be explained by the shaping gain. In DPC-CC and DPC-LDPC, the auxiliary bits provide a number of code vectors for a message sequence. The number of code vectors to transmit a k-bit message is equal to $2^{\beta k}$, where β is an auxiliary rate. In this simulation, the parameter β is equal to 0.5. On the other hand, the number of code vectors in the proposed schemes is equal to 2^k. Since the parameter β is smaller than 1, the proposed schemes have more code vectors to transmit a message sequence. The larger auxiliary rate leads to a reduction of transmission power. However, the scheme could be forced to increase the transmission power in order to attain the desired performance, since a higher order constellation is necessary to maintain the transmission rate.

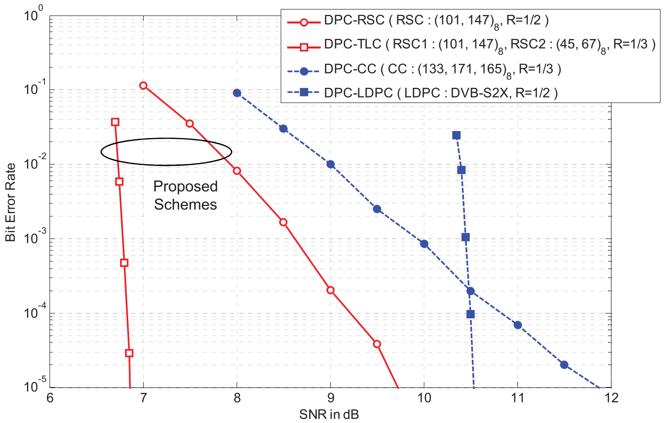

Fig. 4. BER performance of DPC-RSC, DPC-TLC, and the schemes in [15].

TABLE I
SHAPING GAIN FOR DPC SCHEMES WHEN THE TRANSMISSION RATE IS 1.0 BIT/SYMBOL

Scheme	Base Algorithm	Modulation	Shaping Gain
DPC-RSC(Proposed)	Viterbi	4-PAM	7.38 dB
DPC-CC[15]	Viterbi	4-PAM	4.37 dB
DPC-TLC(Proposed)	Viterbi	8-PAM	6.90 dB
DPC-LDPC[15]	Random Flip	8-PAM	1.02 dB
Hypercube Shaping	-	BPSK	6.99 dB

The influence of the number of code vectors on the shaping gain is shown in Table I. The shaping gain is obtained by calculating the transmission power reduction compared to the transmission power, when the interference and encoded symbol are independent. The gap in shaping gain shows a similar tendency to the error performance represented in Fig. 4. While DPC-RSC shows the most shaping gain among the schemes in Table I, DPC-LDPC shows a great gap from DPC-RSC. Since the coding gain of DPC-LDPC is not sufficient to overcome this gap, the DPC-RSC shows better performance with significantly lower complexity. Similarly, the gap of shaping gain between DPC-TLC and DPC-LDPC provides evidence for the difference of error performance while using ECCs that have similar error correction capability. We also check the hypercube shaping method, which is one of the lattice shaping methods. The represented shaping gain is obtained when the lattice size is equal to 4. Although more shaping gain can be achieved by reducing the lattice size, error performance degradation occurs due to the reduced distance between neighboring constellation points. Since a higher order constellation increases degradation, the hypercube shaping method is not suitable for recent communication systems.

Aside from the error performance, the proposed schemes also show an advantage in flexibility. Flexibility is one of the evaluation parameters in communication systems. Although recent LDPC codes, such as quasi-cyclic LDPC codes, provide frame length flexibility, turbo codes still show benefits in this evaluation.

IV. CONCLUSION

In this paper, two practical DPC schemes based on RSC codes and turbo-like codes were proposed. The proposed schemes use Viterbi algorithm with a syndrome former for efficient shaping operation. Since the structure of the proposed decoders is analogous to the existing ECC decoder, it can be easily implemented. Furthermore, our schemes do not require auxiliary bits, which cause the decrease of efficiency, but are essential for the shaping operation of DPC-CC and DPC-LDPC. Our simulation results show that compared with existing schemes, the proposed schemes provide excellent error performance. The shaping gain of each scheme provides evidence of that performance.

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