Selection of Error-Correcting Codes Parameters for Data Channels

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***Abstract*— Models of error-correcting coding (Reed-Solomon codes, LT-codes) are realized. Interference affecting the quality of the signal transmission through the data channel is also realized. The results of the various characteristics of code with different encoding methods, when changing the redundancy of Reed-Solomon code, when changing the encoding parameters of code LT, the transmission results obtained with various types of errors are obtained.**

Keywords*— *error-correcting coding, transmission channel, the probability of decoding.

I. INTRODUCTION

Nowadays, areas that related to data processing and transfer - local wired networks, mobile communications, wireless networks and data storage devices have proliferated and continue to develop rapidly. An important task is to increase the efficiency of existing delivery methods, including algorithm design that allows to enhance reliability of the transmitted information. Also parallel problem of the influence of communication channels was always relevant, because they may be distorted information. To ensure reliable transmission can either improve the reliability of the channel, or effect some operations with the transmitted information.

The error-correcting coding procedure allows the required error probability, however, requires the use of encryption devices an encoder, a decoder, an interleaver and hence additional processing costs. In circumstances where you want to save high-speed transmission while ensuring given immunity, you must have the codes to deal effectively with errors occurring, and have fast encoding/decoding[1].

Reed-Solomon codes and LT codes are powerful error correction technique, which outperforms many well-known coding schemes. Due to their error-correcting capability codes have become part of some modern transmission standards. However, decoders of such codes currently have many limitations, and their design is a difficult task.

Work purpose is researching complex questions regarding the design and analysis of the characteristics of error-correcting encoder in order to save hardware resources and devices to provide a given level of quality. We use probability theory methods and random processes, the algebraic theory of error-correcting codes and information theory.

II. FEATURES OF CHECK MODELS

A. *Reed-Solomon code*

Reed-Solomon code is a block code in which characters are composed of k bits. If these characters are viewed as message packets, the code may be used to deliver messages to the erasure channel. The main feature of the code is the following: for the delivery of K information symbols it is enough to take any K symbols of N. Otherwise for the proper reception of the message from the K symbols in a block of N packets of any M=N–K symbols may be peeling off.

Optimality of the code in the above sense is achieved its hard algebraic structure. As a result, there is the problem of adding "on the fly" a small number of check symbols. In the transition from M=N–K parity to a larger number M'=N'–K all is required to recalculate the parity. The algebraic structure of the code and prevents unlimited increase in the number of check symbols in the code. Code exists only when N<q =2k. The rigid structure of the code leads to a significant computational cost for encoding and decoding. Each verification code symbol includes all K source symbols (blocks) of the message. Therefore, when coding requires K(N-K) symbols over operations (additions and multiplications) [1].

The chance of error in the decoded symbol, *Pe* can be written in terms of the probability of error in the channel symbol, *p*, is given by

$$Pe = \frac{1}{n} \times \sum_{j=t+1}^{n} j \times \left(\frac{n!}{j!(n-j)!}\right) \times p^{j} \times (1-p)^{n-j} \quad (1)$$

where: m – a positive integer greater than unity; p – the probability of error in the channel symbol; n – the number of code symbols in the encoded symbol; t – number of erroneous bits per symbol, which can fix the code; n – the number of pilot symbols. [2] Graph depending on a number of *Pe* is shown in Figure 1

Analysing Figure 1, we can conclude that by increasing the number of erroneous bits in the symbol, which can correct the code, the probability of correct symbol increases (at low values of channel errors).

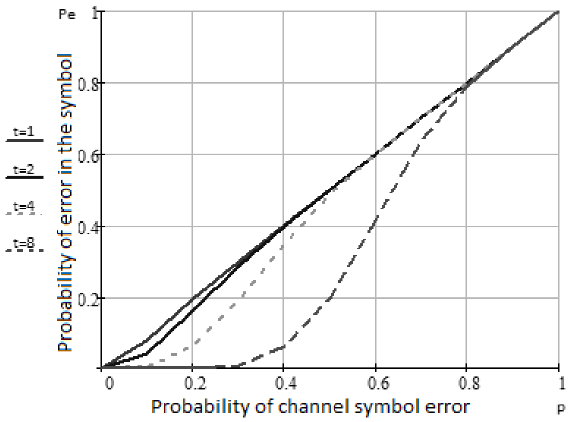

Fig. 1. Probability of error Pe in the symbol of the probability of channel errors in the symbol p, for Reed-Solomon code with the characteristics m=4, n=15. [3]

B. LT-code

The code was created by M. Luby (Michael Luby) in 1998 Its name comes from «Luby transform». Encoding algorithm can be formulated with the help of the generating graph in Figure 2 shows an example of such a graph.

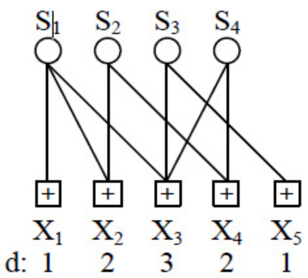

Fig. 2. Generating graph of LT codes

The basis of a finding of *ρ (d)* and the decoding algorithm is simple probability problem. There is a report of *K* source symbols of this set with probability *1/K* produced *K′* random samples characters. The result is a set of *K′* code symbols. $K' \sim = K\ln(K/\delta)$ for sufficiently large *K*, this is done to ensure that, with probability $1-\delta$ each of all the *K* source symbols appeared at least once among the *K′* code symbols. With such a number of code symbols it is possible to reconstruct the original message with a probability of $1-\delta$. The reconstruction algorithm is extremely fast and uses only the information about the numbering symbol.

A key aspect of the LT-codes design provides a probability density function *f(d)*. Ideal distribution is called the distribution, the probability density function is given by the following formula:

$$\rho(d) = \begin{cases} \dfrac{1}{K}, d = 1, \\ \dfrac{1}{d(d-1)}, d = 2,3..., K \end{cases} . \quad (2)$$

This distribution is ideal in terms of coding cost, which is minimal and is equal to *lnK* [4]. Robust density distribution μ(d) is given by:

$$\mu(d) = \frac{\rho(d) + \tau(d)}{\sum_d \rho(d) + \tau(d)}, \quad (3)$$

where *ρ(d)* is defined according to (2), and *τ(d)* of the (4):

$$\tau(d) = \begin{cases} \dfrac{S}{Kd}, d = 1,2,..., \dfrac{K}{S} - 1, \\ \dfrac{S}{K}\ln(\dfrac{S}{\delta}), d = \dfrac{K}{S}, \\ 0, d > \dfrac{K}{S}. \end{cases} \quad (4)$$

In the formula (4) *S* is given by:

$$S(K, \delta) = c\ln(\frac{K}{\delta})\sqrt{K}, \quad (5)$$

where c – parameter of the distribution; is defined as a positive number having a value less than unity. The cost of decoding for the code is the order of *ln(K/δ)* XOR operations[5].

III. Analyzed model description and results

For the analysis of the error-correcting codes and data channels has been developed programming model, allowing to investigate the behavior of error-correcting codes LT and RS on the background of the major errors in the communication channels.

Let us briefly describe the model developed by the error-correcting codes. Encoding and decoding is performed by such codes as:

- Reed-Solomon code, the code for the ability to change the redundant information added to the end of the encoded message;
- LT code, the code for the ability to change the probability of errors in the transmission channel.

Created a program which allows simulate communication channel with several variable parameters for a specific file (txt or bmp). As the channel models 3 mathematical models are used:

- channel with additive Gaussian noise;
- channel with multiplicative noises;
- erasure channel.

It is possible to change the error variance for the additive noise and make the change in the probability of error for the multiplicative noise and channel erasure.

Investigations were carried out using the implemented models. In the beginin using models of error-correcting coding we encode the file (txt or bmp) with one of the codes, and then using a model that simulates the data channels, we make mistakes, corresponding to one of the realized channels, encoded file, then decode the file. The simulation results are listed in Table 1.

TABLE I. SIMULATION RESULTS

Error-correcting code	**Redundancy, %**	**The data channel, the error variance of the signal amplitude**		
		with AWGN	***with mult. noises***	***with erasures***
RS	7	0,185	0,170	0,1
	14	0,0028	0,0015	0,001
	30	0,0067	0,0055	0,003
LT	7	0,260	0,245	0,22
	14	0,1	0,0985	0,078
	30	0,168	0,145	0,121

From these results, we can conclude that the decoding has better properties with LT-code. It allows you to correct the transmitted message with the error variance of 0.26 on the amplitude of the signal in an AWGN channel, with an error probability of 0.245 in the channel with multiplicative error and probability of error of 0.22 in the channel erasure. It was also determined that the time spent on coding and decoding of messages from the LT-code is much less than Reed-Solomon code.

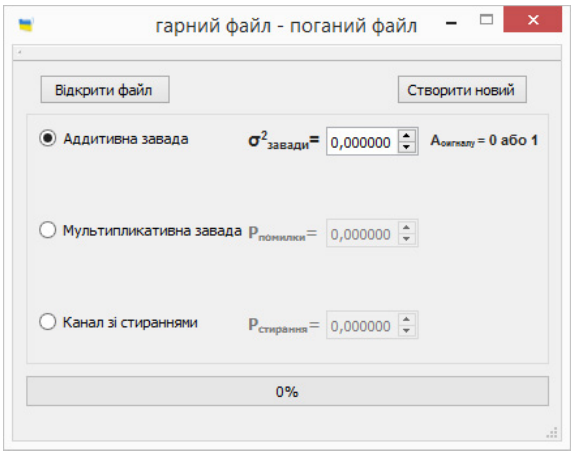

Fig. 3. Appearance of the program, which simulates the data channels

Fig. 4. Example publication AWGN with a variance of the error signal amplitude 0.5

IV. Conclusion

Development of communication channels, which entails reducing the number of errors, also the ever-increasing volume of transmitted information open up broad prospects for further implementation and using of error-correcting codes. Work of error-correcting codes such as Reed-Solomon code and the LT-code was simulated. The simulation results showed that with the introduction of random error correcting codes for noise-proof it.

Scientific novelty of the research is to build recommendations for optimizing the parameters of error-correcting codes for data channels, as well as and finding the decoding method, which is optimal criterion for the time decoding / probability of erroneous decoding.

According to the results obtained using these models can be concluded that better characteristics provides LT code, it corrects more errors, but it is necessary to gather of K'=1,1K symbols or packets.

The practical significance of the results is to improve the quality of data transmission using error-correcting coding for a given signal/noise in the transmission channel. The results of the dispersion error and the error probability for different data channels, which are able to fix the noise-resistant codes RS and LT.

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