Naman Singhal | Statistics for MFE : PS1

Q1

$$P(A \cup B \mid D)$$

$$= \frac{P((A \cup B) \cap D)}{P(D)}$$ (Using Conditional Probability)

$$= \frac{P((A \cap D) \cup (B \cap D))}{P(D)}$$ (using set distribution)

$$= \frac{P(A \cap D) + P(B \cap D) - P(A \cap B \cap D)}{P(D)}$$ $\left(P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)\right)$

$$= \frac{P(A \cap D)}{P(D)} + \frac{P(B \cap D)}{P(D)} + \frac{P((A \cap B) \cap D)}{P(D)}$$

$$= P(A \mid D) + P(B \mid D) + P(A \cap B \mid D)$$

Proved

Q2 Normal Case can be expressed as :

$$P_N(D_i) = 0.01 \quad \forall \; i \geq 1$$

where P_N is the probability in normal case.

Memory case can be expressed as :

$$P_M(D_1) = 0.01$$

$$P_M(D_i \mid D_{i-1}) = 2/5$$
$$\forall \; i > 1$$
$$P_M(D_i \mid D_{i-1}^c) = 1/165$$

and D_i is only dependent on D_{i-1}; independent of $D_1 - D_{i-2}$

(a) $P_M(D_1) = 0.01$ (Given)

Let's assume $P_M(D_K) = 0.01$ for some $K \geq 1$

$$P_M(D_{K+1}) = P_M(D_{K+1} \mid D_K)\, P(D_K) + P_M(D_{K+1} \mid D_K^c)\, P(D_K^c)$$

$$= 2/5 \times 0.01 + (1/165) \times 0.99$$

$P_M(D_{K+1}) = 0.01$

$\therefore$ By induction $P_M(D_K) = 0.01 \; \forall \; K \geq 1$

(b) Let's first calculate the $P_r(E|B)$ & $P_r(E|B^c)$, later we can use this to compute the desired result.

\# Normal case

$$P_r(E|B) = P(D_1^c \cap D_2^c \cap D_3 \cap D_4 \cap D_5^c \cap D_6^c | B)$$

since in the normal case, all D_i are independent

$$= P_r(D_1^c|B) \times P_r(D_2^c|B) \times \cdots$$

$$P_r(E|B) = (0.01)^2 (0.99)^4 \longrightarrow 1$$

\# Memory case.

$$P_r(E|B^c) = P(D_1^c \cap D_2^c \cap D_3 \cap D_4 \cap D_5^c \cap D_6^c | B^c)$$

We can apply conditional probability & the fact that D_i is only dependent on D_{i-1} to reduce this to

$$= P(D_1^c|B^c) \times P(D_2^c | D_1^c \cap B^c) \times P(D_3 | D_2^c \cap B^c)$$
$$\times P(D_4 | D_3 \cap B^c) \times P(D_5^c | D_4 \cap B^c)$$
$$\times P(D_6^c | D_5^c \cap B^c)$$

$$= (0.99) \times \left(\frac{164}{165}\right)^2 \times \left(\frac{1}{165}\right) \times \left(\frac{2}{5}\right) \times \left(\frac{3}{5}\right) \longrightarrow 2$$

Using Bayes theorem

$$P_r(B|E) = \frac{P(E|B)\,P(B)}{P(E)}$$

$$= \frac{P(E|B)\,P(B)}{P(E|B)\,P(B) + P(E|B^c)\,P(B^c)}$$

Substituting values using 1 and 2 and solving gives

$$= 0.11898$$

Q3] (a)

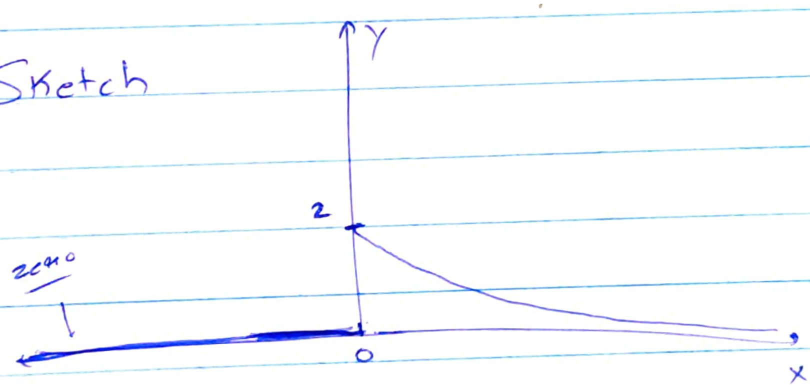
$$\int_{-\infty}^{\infty} f(x) = 1$$

$$\int_{0}^{\infty} f(x) + \underbrace{\cancel{\int_{-\infty}^{0} f(x)}}_{0} = 1$$

$$\frac{c\,e^{-2x}}{(-2)}\Bigg|_{0}^{\infty} = 1$$

$$\boxed{c = 2}$$

Sketch



(b) $P_r(1 < x < 2) = \int_{1}^{2} 2e^{-2x}$

$$= -e^{-2x}\Big|_{1}^{2}$$

$$\boxed{= \frac{e^{2}-1}{e^{4}}}$$

Given :

Q4 $\log(X) \sim N(\mu, \sigma^2)$

$$\log(1/X) = 1 - \log(X)$$

$$\sim 1 - N(\mu, \sigma^2)$$

Using $A \sim N(\mu, \sigma^2) \Rightarrow cA \sim N(c\mu, (c\sigma)^2)$

where c is a constant

and $c + A \sim N(c+\mu, \sigma^2)$

$$\log(1/x) \sim N(1-\mu, \sigma^2)$$

So $1/X$ is also log normally distributed,

but with a mean of $\underline{1-\mu}$