Question 4

In [1]:
```
import numpy as np
x = np.array([[1, 1, 1, 1], [4, -2, 3, -5]]).T
x
```

Out[1]:
```
array([[ 1,  4],
       [ 1, -2],
       [ 1,  3],
       [ 1, -5]])
```

In [5]:
```
P = x@np.linalg.inv(x.T@x)@x.T
P
```

Out[5]:
```
array([[ 0.5462963 ,  0.10185185,  0.47222222, -0.12037037],
       [ 0.10185185,  0.32407407,  0.13888889,  0.43518519],
       [ 0.47222222,  0.13888889,  0.41666667, -0.02777778],
       [-0.12037037,  0.43518519, -0.02777778,  0.71296296]])
```

In [8]:
```
M = np.identity(P.shape[0]) - P
M
```

Out[8]:
```
array([[ 0.4537037 , -0.10185185, -0.47222222,  0.12037037],
       [-0.10185185,  0.67592593, -0.13888889, -0.43518519],
       [-0.47222222, -0.13888889,  0.58333333,  0.02777778],
       [ 0.12037037, -0.43518519,  0.02777778,  0.28703704]])
```

In [9]:
```
M@P # Almost equal to zero
```

Out[9]:
```
array([[ 2.34199567e-17, -3.03636098e-18,  1.62639249e-17,
        -2.13401922e-17],
       [-3.03636098e-18,  4.98762744e-18, -4.01199421e-18,
         4.53645656e-17],
       [-1.14916507e-17, -4.01199421e-18,  1.24750176e-18,
        -1.60051442e-17],
       [-2.82790861e-17,  3.14867778e-17, -1.42704207e-17,
         3.60555969e-17]])
```

In [10]:
```
# (a)
np.isclose(M@P, 0)
```

Out[10]:
```
array([[ True,  True,  True,  True],
       [ True,  True,  True,  True],
       [ True,  True,  True,  True],
       [ True,  True,  True,  True]])
```

(a) MP = 0

In [11]:
```
Q = np.array([[1, 3], [2, 8]])
Q
```

Out[11]:
```
array([[1, 3],
       [2, 8]])
```

In [15]:
```
x2 = x@Q
```

In [16]:
```
P2 = x2@np.linalg.inv(x2.T@x2)@x2.T
P2
```

Out[16]:
```
array([[ 0.5462963 ,  0.10185185,  0.47222222, -0.12037037],
       [ 0.10185185,  0.32407407,  0.13888889,  0.43518519],
       [ 0.47222222,  0.13888889,  0.41666667, -0.02777778],
       [-0.12037037,  0.43518519, -0.02777778,  0.71296296]])
```

In [17]:
```
M2 = np.identity(P2.shape[0]) - P2
M2
```

Out[17]:
```
array([[ 0.4537037 , -0.10185185, -0.47222222,  0.12037037],
       [-0.10185185,  0.67592593, -0.13888889, -0.43518519],
       [-0.47222222, -0.13888889,  0.58333333,  0.02777778],
       [ 0.12037037, -0.43518519,  0.02777778,  0.28703704]])
```

(b) P and M are unchanged after the transformation (multiplication by Q)