IITJAM 2025 Result: Fitting a Logistics Growth Model to the Data

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Abstract

The result for IITJAM 2025 Examination is out. We try to fit a logistics growth model [1] to the marks scored by candidates. For this purpose, data has been taken from various sources (result score-cards) available online.

1   Introduction

The logistics growth model is defined by the differential equation

$$\frac{\mathrm{d}R(m)}{\mathrm{d}m} = r\left(1 - \frac{R(m)}{K}\right)$$

with $R(0) = R_0$, where K is the capacity of $R(m)$, and r is the rate at which $R(m)$ changes with respect to m. We are denoting the rank at marks m by $R(m)$. Then, the value of K will be 1 and r is to be estimated. When $R(m) > 1$, the rate of change will be negative. As $R(m) \to 1$, $R'(m) \to 0$. We solve this system along with initial data $R(30.33) = 1212$ (rank-marks pair corresponding to one of the candidates) using the Mathematica code

```
DSolve[{R'[m] == r (1 - R[m]), R[33.33] == 1212}, R[m], m]
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to get

$$R(m) = e^{-mr}\left(e^{mr} + 1211e^{33.33r}\right). \tag{1}$$

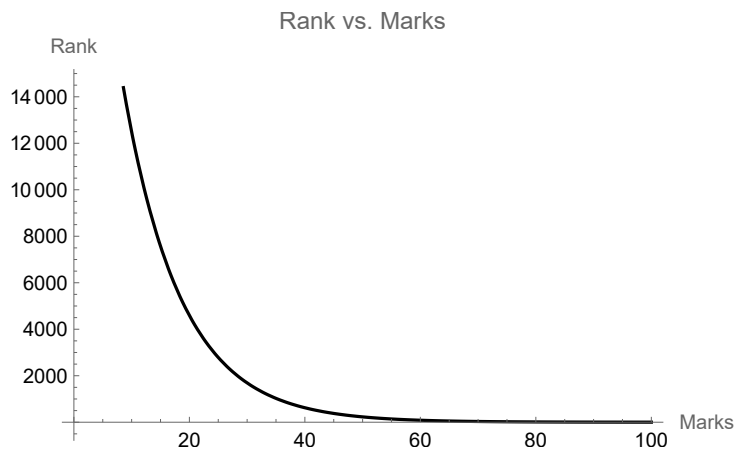The overall dynamics then are shown in figure 1, with imaginary value of r:

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Figure 1: Behaviour of $R(m)$ at $r = 0.1$.

2 Simulation

We use the data extracted from 20 actual result score-cards found online (table 1) to estimate the value of r in (1). Using **NonlinearModelFit**

Marks	Rank	Marks	Rank
30.33	1521	48.67	336
33.33	1212	54.33	187
79	4	54	194
58	140	62.67	86
95.33	1	49.33	319
94	2	30.33	1521
45.44	467	39.67	734
48	364	45.33	467
38	841	49.33	317
66	44	64.67	57

Table 1: Scores data.

command of Mathematica, we receive $r = 0.0828338$. Using this value, we plot the correct graph in figure 2. The model can be used to predict rank of varying marks, as shown in table 2. Of course, these are only approximate results, yet fairly accurate.
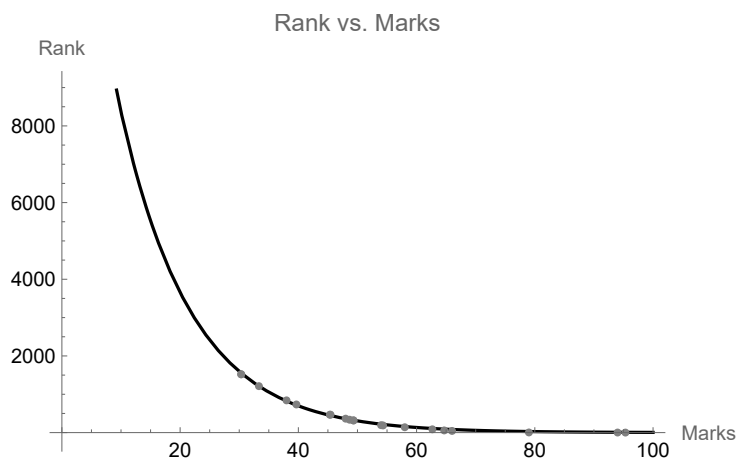
Figure 2: Comparison of the model (black) at $r = 0.0828338$ with actual data (gray).

Marks	10	20	30	40	50	60	70	80	90	100
Rank	8365	3654	1596	697	305	133	59	26	12	5

Table 2: Model result.

References

[1.] Barnes, Belinda, and Glenn R. Fulford. *Mathematical Modelling with Case Studies: A Differential Equations Approach Using Maple and MATLAB.* 2nd ed., CRC Press, 2009.